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MDCCCLXXIV.

ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public that their usual meetings were then continued, for the improvement of knowledge and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the

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thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

CONTENTS

OF VOL. 164.

PART I.

I. *On a Standard Voltaic Battery.* *By* Latimer Clark, *M.I.C.E. Communicated by Sir* William Thomson, *LL.D., F.R.S.* . . . . . page 1

II. *Researches in the Dynamics of a Rigid Body by the aid of the Theory of Screws. By* Robert Stawell Ball, *LL.D., Professor of Applied Mathematics and Mechanism in the Royal College of Science for Ireland. Communicated by Professor* Cayley 15

III. *On the Organization of the Fossil Plants of the Coal-measures.*—Part V. Asterophyllites. *By* W. C. Williamson, *F.R.S., Professor of Natural History in the Owens College, Manchester* 41

IV. *On the Action of Electricity on Gases.*—II. *On the Electric Decomposition of Carbonic-acid Gas. By Sir* B. C. Brodie, *Bart., D.C.L., F.R.S., late Waynflete Professor of Chemistry in the University of Oxford* 83

V. *On the Anatomy and Histology of the Land-Planarians of Ceylon, with some Account of their Habits, and a Description of two new Species, and with Notes on the Anatomy of some European Aquatic Species. By* H. N. Moseley, *M.A. Oxon. Communicated by* G. Rolleston, *M.D., Professor of Anatomy and Physiology in the University of Oxford* 105

VI. *On a newly discovered Extinct Ungulate Mammal from Patagonia,* Homalodontotherium Cunninghami. *By* William Henry Flower, *F.R.S.* 173

VII. *On the Atmosphere as a Vehicle of Sound. By* John Tyndall, *D.C.L., LL.D., F.R.S.* 183

VIII. *On the Fossil Mammals of Australia.*—Part VIII. *Family* MACROPODIDÆ: *Genera* Macropus, Osphranter, Phascolagus, Sthenurus, *and* Protemnodon. *By Professor* OWEN, *F.R.S. &c.* . . . . . page 245

IX. *On the Structure and Development of the Skull in the Pig* (Sus scrofa). *By* W. K. PARKER, *F.R.S.* 289

PART II.

X. *Contributions to the History of Explosive Agents.*—Second Memoir. *By* F. A. ABEL, *F.R.S., Treas. Chem. Soc.* 337

XI. *A Memoir on the Transformation of Elliptic Functions. By Professor* CAYLEY, *F.R.S.* 397

XII. *Studies on Biogenesis. By* WILLIAM ROBERTS, *M.D., Manchester. Communicated by* HENRY E. ROSCOE, *F.R.S.* 457

XIII. THE BAKERIAN LECTURE.—*Researches in Spectrum-Analysis in connexion with the Spectrum of the Sun.*—No. III. *By* J. NORMAN LOCKYER, *F.R.S.* 479

XIV. *On the Quantitative Analysis of certain Alloys by means of the Spectroscope. By* J. NORMAN LOCKYER, *F.R.S., and* W. CHANDLER ROBERTS, *Chemist of the Mint* 495

XV. *On Attraction and Repulsion resulting from Radiation. By* WILLIAM CROOKES, *F.R.S. &c.* 501

XVI. *On Electrotorsion. By* GEORGE GORE, *F.R.S.* 529

XVII. *The Winds of Northern India, in relation to the Temperature and Vapour-constituent of the Atmosphere. By* HENRY F. BLANFORD, *F.G.S., Meteorological Reporter to the Government of Bengal. Communicated by Major-General* STRACHEY, *R.E., C.S.I., F.R.S.* 563

XVIII. *On a Self-recording Method of Measuring the Intensity of the Chemical Action of Total Daylight. By* HENRY E. ROSCOE, *F.R.S.* 655

XIX. *On the Organization of the Fossil Plants of the Coal-measures.*—Part VI. Ferns. *By* W. C. WILLIAMSON, *F.R.S., Professor of Natural History in the Owens College, Manchester* 675

XX. *On Mr.* SPOTTISWOODE'S *Contact Problems. By* W. K. CLIFFORD, *M.A., Professor of Applied Mathematics and Mechanics in University College, London. Communicated by* W. SPOTTISWOODE, *M.A., Treas. & V.P.R.S.* 705

XXI. *On the Echinoidea of the 'Porcupine' Deep-sea Dredging-Expeditions. By Professor* WYVILLE THOMSON, *LL.D., D.Sc., F.R.S.* 719

XXII. *On the Structure and Development of* Peripatus capensis. *By* H. N. Moseley, *M.A., Naturalist to the 'Challenger' Expedition. Communicated by Professor* Wyville Thomson, *M.A., F.R.S., &c., Director of the Scientific Civilian Staff of the Expedition* . . . . . page 757

XXIII. *On the Fossil Mammals of Australia.*—Part IX. *Family* Macropodidæ: *Genera* Macropus, Pachysiagon, Leptosiagon, Procoptodon, *and* Palorchestes. *By Professor* Owen, *F.R.S. &c.* 783

XXIV. *Researches in Spectrum-Analysis in connexion with the Spectrum of the Sun.* —No. IV. *By* J. Norman Lockyer, *F.R.S.* 805

Index 815

LIST OF ILLUSTRATIONS.

Plates I. to IX.—Professor W. C. Williamson on the Organization of the Fossil Plants of the Coal-measures.

Plates X. to XV.—Mr. H. N. Moseley on the Anatomy and Histology of the Land-Planarians of Ceylon.

Plate XVI.—Professor Flower on a newly discovered Extinct Ungulate Mammal from Patagonia.

Plates XVII. to XIX.—Professor Tyndall on the Atmosphere as a Vehicle of Sound.

Plates XX. to XXVII.—Professor Owen on the Fossil Mammals of Australia.

Plates XXVIII. to XXXVII.—Mr. W. K. Parker on the Structure and Development of the Skull in the Pig.

Plates XXXVIII. to XL.—Mr. J. Norman Lockyer on Spectrum-Analysis in connexion with the Spectrum of the Sun.

Plate XLI.—Messrs. Lockyer and Roberts on the Quantitative Analysis of certain Alloys by means of the Spectroscope.

Plate XLII.—Mr. G. Gore on Electrotorsion.

Plates XLIII. to XLIX.—Mr. H. F. Blanford on the Winds of Northern India.

Plate L.—Mr. H. E. Roscoe on a Method of Measuring the Intensity of the Chemical Action of Total Daylight.

Plates LI. to LVIII.—Professor W. C. Williamson on the Organization of the Fossil Plants of the Coal-measures.

Plates LIX. to LXXI.—Professor Wyville Thomson on the Echinoidea of the 'Porcupine' Deep-sea Dredging-Expeditions.

Plates LXXII. to LXXV.—Mr. H. N. Moseley on the Structure and Development of *Peripatus capensis*.

Plates LXXVI. to LXXXIII.—Professor Owen on the Fossil Mammals of Australia.

Plates LXXXIV. to LXXXVI.—Mr. J. Norman Lockyer on Spectrum-Analysis in connexion with the Spectrum of the Sun.

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The Copley Medal to Professor Louis Pasteur, For. Memb. R.S., for his Researches on Fermentation and on Pebrine.

The Rumford Medal to Mr. Joseph Norman Lockyer, F.R.S., for his Spectroscopic Researches on the Sun and on the Chemical Elements.

A Royal Medal to Mr. Henry Clifton Sorby, F.R.S., for his Researches on Slaty Cleavage and on the minute Structure of Minerals and Rocks; for the construction of the Micro-Spectroscope, and for his Researches on Colouring-matters.

A Royal Medal to Professor William Crawford Williamson, F.R.S., for his Contributions to Zoology and Palæontology, and especially for his Investigations into the Structure of the Fossil Plants of the Coal-measures.

The Bakerian Lecture was delivered by Mr. J. Norman Lockyer, F.R.S.: it was entitled "Researches in Spectrum-Analysis in connexion with the Spectrum of the Sun.—No. III."

The Croonian Lecture was delivered by Professor David Ferrier, M.D.: it wa entitled "The Localization of Function in the Brain."

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PHILOSOPHICAL TRANSACTIONS.

I. *On a Standard Voltaic Battery.* By Latimer Clark, *M.I.C.E.* *Communicated by Sir* William Thomson, *LL.D., F.R.S.*

Received June 19,—Read June 19, 1873.

The object which the author had in view in pursuing the investigations alluded to in the following paper was to discover some form of voltaic battery which should have a perfectly constant electromotive force, and should maintain a uniform difference of electric potential between its poles. This want has been much felt by electricians; and the utility of such an investigation may be best shown by a brief reference to the recent history of electrical measurement.

In September 1861 a paper was read by the author before the British Association for the Advancement of Science advocating the adoption of a series of standard units of electrical measurement, and pointing out the mutual relations which should exist between such units. The subject was independently supported in Committee by Sir William Thomson, F.R.S., and the result was the appointment of a "Committee on Standards of Electrical Resistance," and a grant of money was set aside for the purposes of the Committee.

In 1862 the Committee presented their first Report; their numbers were then enlarged, further sums of money were voted for the continuation of their researches, and further Reports were presented in 1863, 1864, 1865, and 1867, after which the Committee was dissolved.

The Committee finally recommended the adoption of a system of natural electromagnetic units based on the metre and gramme*, in which the unit current flowing through a conductor of unit length exerts the unit force on the unit pole at the unit distance. As these units were unfitted in magnitude for practical use, certain multiples have been adopted in practice and have received names, and are now in almost universal use among electricians. These units are:—

1. Resistance.—The Ohm, equal to 10^7 absolute electromagnetic metre-gramme units.
2. Capacity.—The Farad, equal to 10^{-7} absolute electromagnetic units.
3. Potential.—The Volt, equal to 10^8 absolute electromagnetic units.

* They have since adopted the centimetre-gramme unit.

The measure of quantity is the same as that of electrostatic capacity, and in practice generally receives the same name, although it has been sometimes called the "Weber;" the weber or farad quantity is equal to 10^{-9} absolute units. Electrical currents are defined as currents of so many farads per second. In this system the volt electromotive force through the ohm resistance produces the unit current, or a current of one farad per second.

The Committee determined with great care the value of the ohm resistance and the farad capacity, and issued standards which have been very extensively copied and distributed. They would naturally have desired to issue a standard of electromotive force, or degree of potential, and thus complete the series; but in this they met with insuperable difficulties, and finally separated without accomplishing this part of their task.

This was a matter of regret, seeing that the electromotive forces of batteries and the strength of currents are among the measures most frequently required by the practical electrician.

The difference of potential between two bodies may be measured by measuring the force of attraction between two electrified planes of known dimensions at a known distance, or two coils conveying currents. It may also be determined by similar means to those employed by the Committee in their determination of the absolute unit of resistance—that is, by revolving a coil at a known speed in a field of known magnetic intensity. If the value of the earth's horizontal magnetic intensity (H) were uniform at different times and places, or easily obtained, and if the measurements were made at a distance from iron bodies, the tangent galvanometer would afford a means of absolutely measuring electromotive force.

All these methods, however, require complicated and expensive apparatus and great manipulative skill; and owing to these causes it may be safely asserted that not more than half a dozen absolute determinations of potential have ever yet been made. Practically electricians have been compelled to define electromotive forces by comparison with those of the Grove's or Daniell's cell, the copper and zinc cell, or other electromotive sources; and it is a curious circumstance that among the thousand galvanic combinations known to exist, not one has been hitherto found which could be relied upon to give a definite electromotive force: however pure the materials, and however skilful the manipulation, differences varying from four or five per cent. upwards constantly occur without any assignable cause; and different observers using different materials of course meet with still larger discrepancies.

The author, sustained by a conviction that this difficulty could not, in the nature of things, be insuperable, has carried on a course of experiments since 1867 with a view to discover and obviate the cause of these variations, and has devised a form of battery which he desires to lay before the Royal Society, and which appears to meet, in a very satisfactory manner, all necessary requirements.

The battery is formed by employing pure mercury as the negative element, the mercury being covered by a paste made by boiling mercurous sulphate in a thoroughly

saturated solution of zinc sulphate, the positive element consisting of pure distilled zinc resting on the paste.

The best method of forming this element is to dissolve pure zinc sulphate to saturation in boiling distilled water. When cool, the solution is poured off from the crystals and mixed to a thick paste with pure mercurous sulphate, which is again boiled to drive off any air; this paste is then poured on to the surface of the mercury, previously heated in a suitable glass cell; a piece of pure zinc is then suspended in the paste, and the vessel may be advantageously sealed up with melted paraffin-wax. Contact with the mercury may be made by means of a platinum wire passing down a glass tube, cemented to the inside of the cell, and dipping below the surface of the mercury, or more conveniently by a small external glass tube blown on to the cell, and opening into it close to the bottom. The mercurous sulphate (Hg_2SO_4) can be obtained commercially*; but it may be prepared by dissolving pure mercury in excess in hot sulphuric acid at a temperature below the boiling-point: the salt, which is a nearly insoluble white powder, should be well washed in distilled water, and care should be taken to obtain it free from the mercuric sulphate (persulphate), the presence of which may be known by the mixture turning yellowish on the addition of water. The careful washing of the salt is a matter of essential importance, as the presence of any free acid, or of persulphate, produces a considerable change in the electromotive force of the cell.

The electromotive force of the elements thus formed is remarkably uniform and constant, provided the elements be not connected up and allowed to become weakened by working. A long series of comparisons was made between various elements, some of which had been made many months, and it was found that the greatest variation among them all did not differ from the mean value more than one thousandth part of the whole electromotive force; such a difference as this was, however, unusual, and might have been due to slight differences of temperature.

The following Table gives some of the results obtained. Temperatures are not stated, as the elements were approximately at the same temperature as the standards with which they were compared at the time.

No. of element or letter.	Date of construction.	Date of comparison.	Value.
96.	March 23, 1871	March 25, 1871	1·0000
16.	February 16, 1871	March 24, 1871	·9997
89.	March 23, 1871	March 25, 1871	·9991
90.	"	"	·9993
91.	"	"	·9985
92.	"	"	·9988
93.	"	"	·9991
94.	"	"	·9988
95.	"	"	·9998
97.	"	"	·9996
98.	"	"	·9996
99.	"	"	·9995
100.	"	"	·9993

* The author has obtained it from Messrs. Hopkins and Williams, 5 New Cavendish Street.

TABLE (continued).

No. of element or letter.	Date of construction.	Date of comparison.	Value.
101.	March 24, 1871	March 25, 1871	1·0006
102.	"	"	1·0004
103.	"	"	1·0003
104.	"	"	1·0003
105.	"	"	1·0001
106.	"	"	·9995
115.	March 27, 1871	March 28, 1871	1·0008
116.	"	"	1·0005
117.	"	"	1·0002
118.	"	"	1·0005
119.	"	"	1·0002
120.	"	"	1·0001
A.	March 30, 1871	April 3, 1871	1·0003
C.	"	"	1·0003
E.	May 16, 1871	May 20, 1871	1·0005
F.	"	"	1·0005
D.	"	"	1·0003
L.	May 18, 1871	"	1·0004
155.	December 1, 1871	December 19, 1871	1·0001
156.	"	"	·9999
157.	"	"	1·0001
158.	"	"	1·0007
159.	"	"	1·0003
160.	February 17, 1872	February 26, 1872	1·0004
161.	"	"	1·0004
162.	"	"	1·0001
163.	February 24, 1872	"	1·0002
164.	"	"	·9999
165.	"	"	1·0001
166.	"	"	1·0001
167.	February 28, 1872	February 29, 1872	1·0007
W. 1.	September 11, 1872	October 9, 1872	·9999
2.	"	"	1·0001
3.	"	"	1·0003
4.	"	"	·9996
5.	"	"	·9996
6.	"	"	1·0001
7.	"	"	1·0003
8.	"	"	·9999
9.	"	"	·9997
10.	"	"	1·0006
11.	"	"	·9993
12.	"	"	1·0004
13.	"	"	·9993
14.	"	"	·9994
15.	"	"	1·0005
16.	"	"	·9996
17.	January 20, 1873	March 15, 1873	·9993
18.	"	"	1·0001
19.	"	"	1·0001
20.	"	"	·9993
21.	"	"	1·0004
22.	"	"	1·0001
23.	"	"	1·0005
24.	"	"	1·0000
25.	"	"	1·0005
26.	"	"	1·0005
		Mean value.....	·9999

Several experiments were made to determine the variation of the electromotive force at different temperatures; from the mean of these it appears that the force decreases with increase of temperature in the ratio of about ·06 per cent. for each degree Cent.; for example, an element gave relative values of ·9993 at 0° Cent. and of ·9412 at 100° Cent. The element varies much more at temperatures near 0° Cent. than at temperatures near 100° Cent. The variation for about 10 degrees above or below 15·5 is ·06 per degree Cent. When the temperature is lowered from 15°·5 to 0° the force increases at the rate of ·08 per cent. per degree; when raised from about 15°·5 to 100° Cent. it diminishes at the rate of ·055 per cent. per degree.

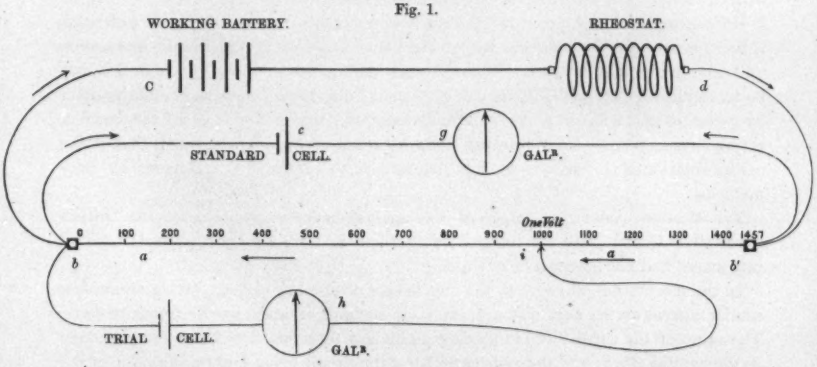
The element maintains a sensibly constant electromotive force for one or two years, and possibly longer if the salts be prevented from drying by an air-tight covering.

It is not intended that this element should supersede any of the existing combinations in practical use for the production of a current; for it, like the Marie Davy and many other batteries, falls rapidly in electromotive force when allowed to work through a circuit of small resistance, though it recovers its original electromotive force if allowed to remain inactive for a short time. It is intended to be used chiefly as a standard of electromotive force with which other elements or sources of potential can be compared by means of an electrometer or of instruments (similar to the one described below) which do not require any current.

It will, however, continue to supply a permanent current through a circuit of large resistance, say 10,000 ohms, without any sensible diminution of its force, and has been advantageously applied to the testing of submarine cables.

The instrument used in comparing the elements was one devised by the author in 1859 (see fig. 6, p. 14); the following diagram will explain its construction:—

Fig. 1.



a a represents a length of ten metres of platinum-iridium wire about ·5 millimetre

diameter wound on a cylinder of ebonite, the ends being connected to the axes $b\,b'$, which work in blocks of metal with mercury contacts: two batteries are also connected to the same blocks; the larger one, C, of several cells, sends a continuous current through the coil, the strength of which can be varied by means of the rheostat or resistance-coil, d; the smaller, c, is the standard element; it is connected with the terminal blocks, $b\,b'$, and it has a reflecting galvanometer, g, in circuit with it; as these two batteries are connected up in the same direction, they both tend to send a current through the coil $a\,a$. If the difference of potential maintained by the battery between the blocks $b\,b'$ be greater than that of the standard cell, the battery will of course overpower the cell and send a reversed current through it; if, on the other hand, the difference of potential be less, then both the battery and the cell will jointly send a current through $a\,a$. In practice, however, the resistance, d, is so adjusted that the difference of potentials at b and b' is exactly the same as the difference of potential between the poles of the standard cell—in other words, is equal to its electromotive force, in which case no current passes through the galvanometer, g, and the cell remains inactive.

In comparing a trial cell with the standard, one pole of the cell is connected with that end of the coil to which the similar pole of the standard is fixed; the other pole is connected through a second galvanometer, h, to a sliding piece, i. By means of this sliding piece contact can be made at any point of the coil, $a\,a$, which is calibrated into 10,000 equal divisions. The point along the wire is readily found at which the potential is the same as that of the trial cell, and consequently no current passes through the galvanometer, h; in this case the reading or number of divisions gives the value of the trial cell in ten-thousandth parts of the standard element. As it is necessary that the standard element should have a higher electromotive force than that which is compared with it, two or more cells may be employed as a standard.

Having thus obtained a constant and easily reproducible measure of electric potential, it became necessary to ascertain its precise value in terms of the British-Association units and in absolute measure. There are two well-known methods by which this may be accomplished; the one is by the use of Weber's electrodynamometer*, and the other by means of the sine or tangent galvanometer†, in which the force of the current, acting on a suspended needle through a known resistance, is compared with that of the earth's horizontal intensity. It was determined to measure the element by both methods.

The electrodynamometer employed was an instrument constructed for the British-Association Committee, and referred to in their Report for 1867, page 478. This instrument had not been previously used.

In the electromagnetic system, the unit length of the unit current, acting on another similar current at the unit distance, exercises the unit of attractive or repulsive force. The value of the current (C) in absolute units may therefore thus be determined from its mechanical effect; and the resistance (R) of the circuit being known, the value of the

* Taylor's Scientific Memoirs, vol. iii.

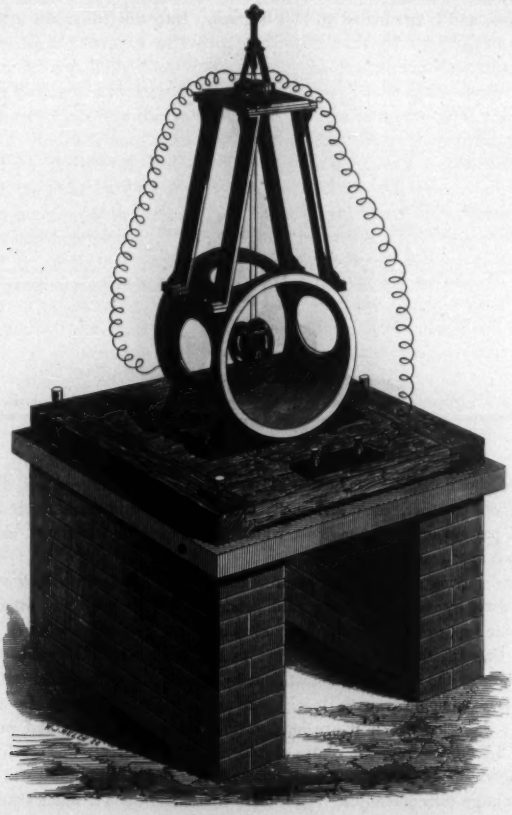
† British-Association Report, 1863, pp. 116, 141.

electromotive force (E) follows from OHM's formula,

$$C=\frac{E}{R} \text{ or } E=CR.$$

In the instrument in question (fig. 2) the large fixed coil is double, as in the arrange-

Fig. 2.

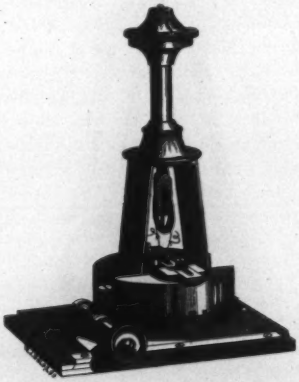


ment given by HELMHOLTZ and GAUGAIN to the tangent galvanometer, the two coils being in parallel vertical planes at a distance apart equal to their radius; the small coil is also

double, and is suspended bifilarly truly central to the fixed coils, the bifilar suspension-wires being used to convey the current between the fixed and movable coils.

The top of the instrument (fig. 3) is furnished with various contrivances for facilitating the central adjustment of the coils; these consist of two plates, forming a slide-rest movement fitted with verniers, by which horizontal motion can be given to the suspension in any direction. The upper plate carries a circular collar, which can be rotated by a tangent screw, and is graduated to 360 degrees. Into this collar fits a brass frame,

Fig. 3.



carrying two ebonite blocks, on which are two horizontal sliding pieces, diametrically opposite to one another, each furnished with a vernier, and terminating interiorly in small brass pulleys or rollers, against which the suspension-wires rest, and by which their distance apart can be regulated.

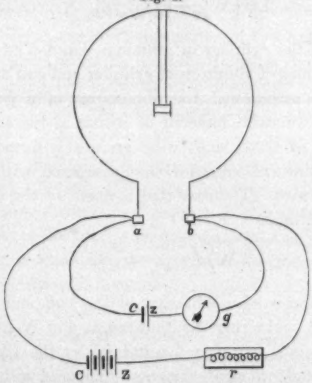
The frame carries a light pulley three centimetres diameter, which supports the suspension-wires by means of a silk cord passing over the pulley and attached to the wires near the top, so as to ensure an equality of tension on the two wires. The wires pass down through the collar and socket to the lower coil. The suspension-pulley admits of upward and downward adjustment by means of a milled head screw.

The electrodynamometer with its telescope and stand was supported on a solid brick foundation; the scale was a metre long, divided into millimetres, and was fixed at a distance of 2·7 metres from the centre of suspension. The scale was carefully adjusted at right angles to the axis of the telescope. A plate of silvered glass was fixed at the back of the large coils and adjusted parallel to them, and upon it was marked the centre of the coil accurately determined. When this centre mark, viewed through the telescope, was brought to coincide with the cross wires by moving the large coils, and when the

centre of the telescope reflected in the silvered glass also coincided with the cross wires, the telescope was of course directed normally to the centre of the plane of the coils, which were adjusted in the magnetic meridian; excessive care was taken with all the adjustments and readings, which were repeated with reversed currents and positions.

In order to maintain a current through the coils of the dynamometer and to ensure that the difference of potential between its poles should be precisely equal to that of the standard cell, an auxiliary battery was used in the manner before described. This consisted of five large DANIELL'S cells working through a circuit consisting of the dynamometer and a rheostat, r (fig. 4); a and b are the two terminals of the dynamometer, and the poles of the auxiliary battery, C, Z, are connected with them; the similar poles of the standard elements c, z, and the galvanometer, g, are also connected to the same terminals; and the rheostat r is adjusted so that no current passes through the galvanometer. In this case it is evident that the poles a and b are maintained at a difference of potential precisely equal to that of the standard elements. In this arrangement not the slightest difficulty is experienced in maintaining a perfectly steady and uniform current through the coils of the dynamometer. The poles of the dynamometer were so arranged that they could be connected immediately with a Wheatstone balance in order that its resistance could be measured promptly after each observation.

Fig. 4.



The winding of the large coils of the instrument was superintended by Professor CLERK MAXWELL, who kindly supplied me with the measurements as follows:—

	millimetres.
Mean circumference of first coil	1558·48
Mean circumference of second coil	1559·16
The depth of each coil is	12·90
The breadth of each coil is	15·00
The distance apart of the planes of the coils .	250

Each layer has 15 windings, and there are 15 layers, so that each coil has 225 windings.

The small coils were wound afresh by myself; the brass channels for the reception of the wire were of different sizes, and the same number of turns could not therefore be wound on it.

	millimetres.
Mean circumference of first coil	359·25
Mean circumference of second coil	357·45
Mean depth of the coils	6·67
Mean breadth of the coils	10·52
Mean distance apart	62·41
Number of windings on first coil	311
Number of windings on second coil	327

The moment of inertia of the suspended coil was determined from a great number of observations by different methods.

(1) The coil was vibrated on a fine steel wire, and the moment was then increased by a gun-metal cylinder passing through the centre of the coil; the increased time of vibration was then observed, and the moment of the coil calculated by the formula

$$I=\left(W\frac{l^2}{12}+\frac{d^2}{16}\right)\frac{t^2}{t'^2-t^2}, \quad \ldots\ldots\ldots\ldots \quad (1)$$

where W is the weight of the cylinder in grammes, l and d its length and diameter in metres, and t' and t the times of vibration of cylinder and coil and of coil.

(2) From the value so ascertained the dimensions of a gun-metal cylinder were calculated, having about the same moment of inertia as the coil when vibrating on a transverse diameter: two of these rings were accurately formed by Mr. Becker and carefully weighed; their times of vibration were compared with that of the coil when suspended from the same wire. The corrected moment of the coil was then calculated from these times by the formula

$$I=W\frac{t^2}{t'^2}\left(\frac{r^2+r'^2}{4}+\frac{a^2}{3}\right), \quad \ldots\ldots\ldots\ldots \quad (2)$$

where t and t' are the times of a vibration of the coil and the ring, r and r' the external and internal radii of the ring, $2a$ its breadth, and W its weight.

(3) The moment of inertia was also determined by the vibration about its longitudinal axis of a metal cylinder of small thickness compared with its radius, as suggested by Sir William Thomson, F.R.S. (Proc. Royal Society, vol. xiv. p. 294), the coil and cylinder being alternately vibrated on the same wire.

The following were the results of the observations:—

First system, mean of five observations	1·27641
Second system, mean of twenty observations	1·27680
Third system, mean of one observation	1·27795
Moment of inertia employed in calculation	1·27691 metre-gramme.

It is not necessary to give the mathematical formula used in calculating the values of E, but the following Table gives the results of the whole of the series of observations with the electrodynamometer.

Date.	Value of E in volts.	Remarks.
8 December 1871 ...	1·4585	3 cells.
9 " "	1·4651	3 cells.
14 " "	1·4616	3 cells.
15 " "	1·4561	3 cells.
15 " "	1·4579	2 cells.
16 " "	1·4586	3 cells.
" " "	1·4517	3 cells, coil turned 180°.
" " "	1·4552	2 cells, coil turned back 180°.
" " "	1·4565	3 cells.
" " "	1·4535	2 cells.
" " "	1·4564	3 cells.
18 " "	1·4649	3 cells.
19 " "	1·4562	3 cells, coil turned 180°.
" " "	1·4558	3 cells, coil turned back 180°.
20 " "	1·4615	3 cells.
" " "	1·4539	3 cells.
" " "	1·4551	2 cells.
21 " "	1·4549	3 cells.
Mean value of E	1·45735 volt.	Temperature 15°·5 Cent.

The cells were frequently changed during the course of the experiments. Values were also obtained when the suspended coil was moved two millims. in various directions about the centre, but they did not differ sensibly from the above.

As a verification of the results obtained with the electrodynamometer, the electromotive force of the new element was also determined by means of the sine galvanometer by a method which is well known, viz.

$$E=\left(\frac{K}{2\pi n} H \sin\theta\right)\times R, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where E is the electromotive force, K is the radius of the circle, n the number of turns, H the horizontal intensity of the earth's magnetism, θ the angle through which the coils must be turned in order to maintain the needle in the plane of the coils, and R the resistance of the circuit in absolute measure.

The instrument employed was specially constructed for these experiments, and presents some novelties. The needle was one centimetre in length, and was furnished with a mirror of parallel glass silvered by the chemical process, so that the reflection from either side could be observed in the telescope. The coil was 140 millimetres in diameter, and was furnished with a large mirror accurately parallel to its plane, silvered on the observing or front side, and having the centre of the coil marked upon it; by the aid of the telescope and these mirrors it was easy to adjust the needle accurately to the centre of the coil, and to ensure that the plane of the coil was truly vertical, and coincided with the magnetic meridian.

The telescope was carried on an arm one metre in length, which with the coil turned

on a pair of theodolite plates; and thus readings could be taken to half minutes. The experiments were performed within five miles of the Royal Observatory. The value of H, a knowledge of which is necessary for the determinations with this instrument, was kindly supplied to me by the Astronomer Royal for each day on which the observations were taken. No iron was near the instrument.

A difference of potential equal to that of one standard cell was maintained between the poles of the sine galvanometer, by the use of an auxiliary battery, rheostat, and galvanometer, in the manner described when treating of the dynamometer observations.

The following Table gives the results of these experiments.

Date.	Value of H.	Value of E.	Remarks.
9 Feb.	1·788	1·45605	
9 „	„	1·45457	
9 „	„	1·45400	Galvanometer wound with 8 turns German silver wire.
10 „	1·788	1·45809	
10 „	„	1·45669	
11 „	1·788	1·45799	
18 „	1·787	1·45566	Rewound with 28 turns German silver wire.
19 „	„	1·45671	
19 „	„	1·45680	
20 „	1·787	1·45752	Rewound with 27 turns German silver wire.
20 „	„	1·45645	
24 „	1·786	1·45522	
24 „	„	1·45492	
Mean value of E ...		1·4562 volt.	Temperature 15°·5 Cent.

The observations are corrected for the temperature of the element and of the coils; but the correction for the breadth and depth of the coil, according to Professor Clerk Maxwell's formula*, was so small as only to appear in the fifth place of decimals, and was therefore neglected. The instrument was rewound twice with various lengths of wire.

We have therefore the mean value of the electromotive force of the standard cell,—

	volt.
1. As determined by the electrodynamometer (18 observations)	1·45735
2. As determined by the sine galvanometer (13 observations) .	1·45621
Mean value of E	1·45678

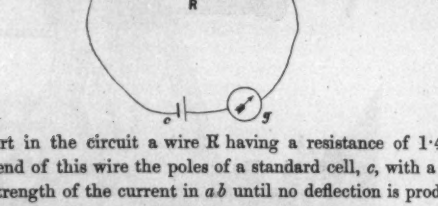
or, since no importance can be attached to the figures beyond the third place of decimals, 1·457 volt, equal to 145700 absolute electromagnetic units.

The uses of this standard element to practical electricians are sufficiently obvious. It may be used for determining the electromotive force of other elements by the use of an electrometer or by the discharge from a condenser. Or a condenser having a capacity of $\frac{1}{1\cdot457}$ farad charged by the standard cell would contain the B. A. unit quantity of electricity (one Weber), or $\frac{1}{100}$ of the absolute unit of quantity.

* British-Association Report, 1863, p. 170.

It is also of great value for maintaining a current of known strength in any circuit for the purposes of experimental research. Thus if it be desired to produce in any circuit ($a\ b$, fig. 5) a current equal to the B. A. unit of current ($\frac{1}{100}$ absolute units), it is only

Fig. 5.



necessary to insert in the circuit a wire R having a resistance of 1·457 ohm, and to connect to each end of this wire the poles of a standard cell, c, with a galvanometer, g, and to vary the strength of the current in $a\,b$ until no deflection is produced on the galvanometer; the current through $a\ b$ will then be equal to one B. A. unit of current, or 1 farad per second, whatever its length or resistance.

By varying the resistance of R, or by varying the number of elements c, any given current can be steadily maintained through $a\,b$ at pleasure; on the other hand, the value of any given current can be measured by so varying the resistance R that no deflection is produced on the galvanometer. The value of the passing current will then be

$$C=\frac{1\cdot457}{R}\text{ farad per second.}$$

It is also evident that, knowing the value of E, we may determine the horizontal intensity of the earth's magnetism, H, in any place quickly and simply by means of an ordinary sine or tangent galvanometer.

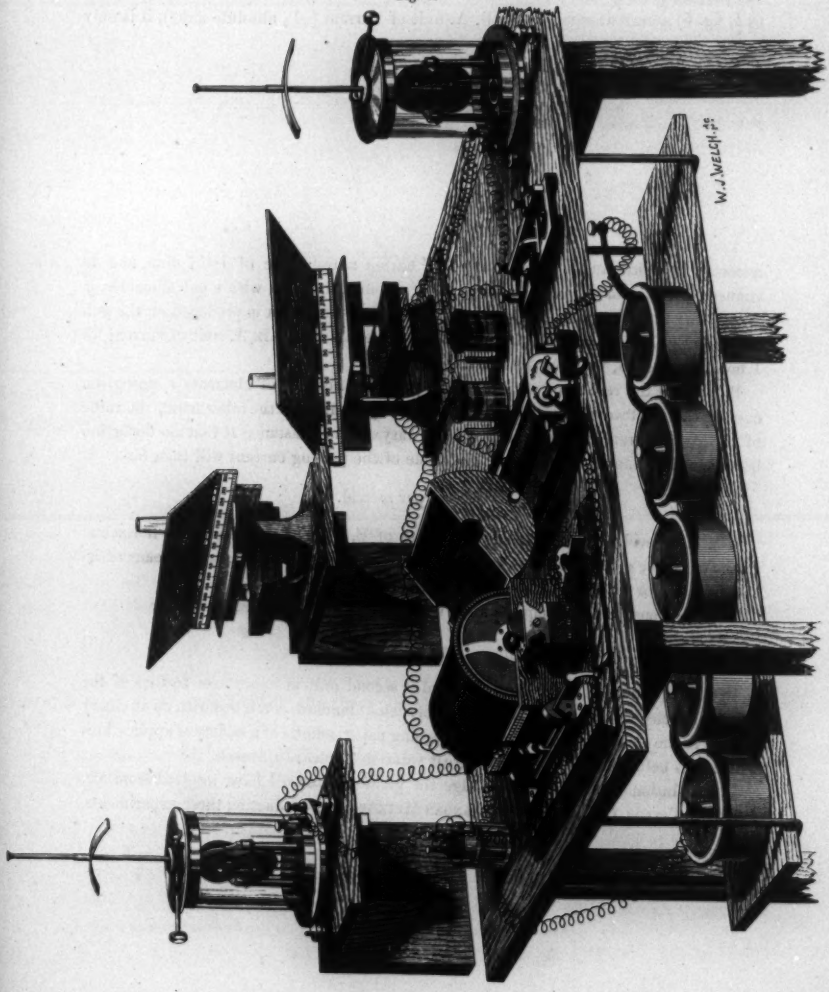
Thus by transposing the equation (3) we have for the tangent galvanometer

$$H=\frac{E\,2\pi n}{R\,K\tan\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In fact the standard of electric potential is second only in importance to that of the standard of electric resistance; and the use of such a standard, combined with an auxiliary battery in the manner described in the foregoing paper, admits of a variety of applications which it is believed will be found of great value in electrical research.

In conclusion I have to acknowledge the great assistance I have received from Mr. Herbert Taylor, C.E., and Dr. Alexander Muirhead, in conducting these experiments.

Fig. 6.



II. *Researches in the Dynamics of a Rigid Body by the aid of the Theory of Screws.*

By Robert Stawell Ball, *LL.D., Professor of Applied Mathematics and Mechanism in the Royal College of Science for Ireland. Communicated by Professor* Cayley.

Received May 29,—Read June 19, 1873.

Contents.

Introduction 15
I. On the Virtual Coefficient of a pair of Screws 16
II. Coreciprocal Screws 18
III. The Sexiant 21
IV. On Impulsive Screws and Instantaneous Screws 24
V. The principal Screws of Inertia 27
VI. Miscellaneous Propositions 32

INTRODUCTION.

In a paper communicated to the Royal Irish Academy ("The Theory of Screws—a geometrical study of the kinematics, equilibrium, and small oscillations of a rigid body," Transactions of the Royal Irish Academy, vol. xxv. p. 157) the chief features of what the writer has ventured to call the Theory of Screws were sketched*. It is the object of the present paper to give some further extensions and applications of that theory. The chief point which it is now proposed to illustrate is the *appropriateness* of the method to many problems in the dynamics of a rigid body. This will, to some extent, appear from the analogy subsisting between the conceptions of the theory and the familiar notions to which the conceptions degrade when the rigid body degrades to a particle. It should also be remarked that the complete generality of the method with reference to forces and constraints gives rise to many theorems of great interest, which could hardly be enunciated without the ideas which the theory embodies.

A *screw* is a straight line in space with which a definite linear magnitude termed the *pitch* is associated. The pitch may have any value from $-\infty$ to $+\infty$. A body is said to receive a *twist* about a screw when it is rotated about the screw, and is at the same time translated parallel to the screw through a distance equal to the product of the pitch and the angle of twist.

A *wrench* about a screw consists of a force and a couple: the force is along the screw, while the axis of the couple is parallel to the screw; and the moment of the couple is the product of the force and the pitch of the screw.

* References to the former paper are enclosed in square brackets thus—art. [12]. Reference to the articles of the present paper are enclosed in semicircular brackets thus—art. (12).

As a vector expresses the entire conception of the movement of a particle from one position to another, so a *twist* expresses all that is involved in the movement of a rigid body from one position to another (CHASLES).

As a force expresses the resultant of a number of forces applied to a particle, so a *wrench* expresses the resultant of a number of forces applied to a rigid body (POINSOT).

If a body receive a twist about the screw A, and then a twist about the screw B, the resulting position could have been produced by a twist about a third screw, C.

The three screws A, B, C lie upon the "cylindroid," a conoidal cubic surface of which the equation is

$$z(x^2+y^2)-2mxy=0.$$

The pitch of the generator which is inclined to the axis of x at the angle θ is

$$p+m\cos 2\theta,$$

where p is an arbitrary constant.

I. ON THE VIRTUAL COEFFICIENT OF A PAIR OF SCREWS.

1. *Definition of the virtual coefficient.*—If a body receive a twist about a screw A, of pitch a, through a small angle α, while acted upon by a wrench P about the screw B, of pitch b, the quantity of energy expended is [art. 18]

$$\alpha \,.\, \mathrm{P} \,.\, [(a+b)\cos\theta - d\sin\theta],$$

where d is the length of the common perpendicular to A and B, and θ is the angle between A and B.

Perhaps the simplest rule to distinguish between θ and its supplement is the following. Suppose the common perpendicular to be a screw, in the ordinary sense of the word, and that there is a nut on this screw to which A is attached. If, then, the nut be *turned so as to make* A *approach* B (that is, to make the length of the common perpendicular diminish), the angle through which A has turned when it has become parallel to B is the angle θ.

It is a remarkable consequence of the symmetry of this expression, that precisely the *same* quantity of energy is required to twist a body about B, through an angle α, against a wrench P about the screw A.

The quantity within the brackets may be called the *virtual coefficient* of the pair of screws. In the former paper considerable application was made of the case where the virtual coefficient vanished, and the screws were then said to be *reciprocal* [art. 18]. We now proceed to show some results which can be derived from the reciprocal character of the expression in cases where the virtual coefficient does not vanish.

2. *Analogy of the composition of rotations to the composition of forces.*—It is a matter of great interest that angular velocities are compounded like forces, and translations like couples. The source of the analogy in the general principles of virtual velocities has been traced by RODRIGUES (LIOUVILLE's Journal, t. 5, 1840, p. 436). In the former paper

[art. 16] this analogy was generalized into a theorem, which asserted that twists and wrenches are compounded by the same laws. We can now show that this theorem is a consequence of the reciprocal character of the virtual coefficient.

3. *Source of the identity of the rules for the composition of twists and the composition of wrenches.*—Let L, M, N be three screws, about which wrenches X, Y, Z equilibrate. Take any known screw S_m; let A_m, B_m, C_m be the virtual coefficients of S_m with L, M, N.

If a body receive a small twist ω about S_m the quantity of energy expended must be zero, since the three acting wrenches equilibrate; but the energy consumed is the algebraical sum of ωXA_m, ωYB_m, ωZC_m, whence we deduce

$$XA_m+YB_m+ZC_m=0;$$

six of the equations obtained by giving different values to m will determine the values of X, Y, Z, and also the conditions which must be fulfilled by the positions and pitches of the three screws L, M, N. These conditions we know [art. 16] are only fulfilled when L, M, N lie upon the same cylindroid. When six equations of this type are satisfied, then all similar equations must also be satisfied.

Let it now be proposed to examine the conditions under which three small twists α, β, γ about the same screws L, M, N can neutralize each other; in other words, that the last twist shall restore the body to the same position which it occupied before the first. The total quantity of energy expended in the three twists against a wrench F on any screw S_m must be zero; hence the algebraical sum of the three quantities, αFA_m, βFB_m, γFC_m, must be zero, or

$$\alpha A_m+\beta B_m+\gamma C_m=0.$$

We thus see that the quantities α, β, γ must be proportional to X, Y, Z, and that the conditions to be fulfilled by L, M, N are precisely the same in both cases.

4. *On the component wrenches about six screws of reference into which any wrench may be resolved.*—The properties of the virtual coefficient enable us to calculate with facility the magnitudes of seven wrenches about seven screws which equilibrate, or, in fact, to resolve a wrench into six wrenches about six given screws* [art. 46].

Let A_1 &c., A_7 be the given screws, and X_1 &c., X_7 the required wrenches. Let S be *any* screw and R_m be the virtual coefficient of S and A_m. If the body receive a twist ω about S, the quantity of energy expended must be zero, and therefore

$$R_1X_1+\&c.+R_7X_7=0;$$

six of these equations would determine the ratios of X_1 &c., X_7. By judicious selection of S the process is greatly simplified. If S be reciprocal to X_3 &c., X_7 (art. 5), then R_3 &c., R_7 vanish, and the equation

$$R_1X_1+R_2X_2$$

determines the ratio of X_1 and X_2.

* The problem of resolving a *force* along six given lines, where the lines form the edges of a tetrahedron, has been solved by Möbius (Crelle's Journal, t. xviii. p. 207).

It follows, from art. (3), that precisely the same investigation determines seven twist velocities about seven screws which neutralize each other.

II. CORECIPROCAL SCREWS.

5. *On a property of five twists analogous to a property of two vectors.*—That one vector γ can be determined, which is perpendicular to two given vectors α, β, is a proposition to which an analogue may be found in the Theory of Screws. The mechanical equivalent of the simple vector theorem just referred to expresses that a force directed along the vector γ is unable to disturb the equilibrium of a particle only free to be displaced parallel to α and β, or, in fact, to move in the plane of α and β. We may state the result thus: a particle which has only one degree short of absolute freedom can only be in equilibrium when the force acting on the particle occupies one position.

To the theorem expressed in this manner, we have an analogous proposition in the Theory of Screws: a rigid body which has only one degree short of absolute freedom can only be in equilibrium when the wrench acting on the body is about one determinate screw. This is demonstrated as follows. The body having only one degree short of absolute freedom must be capable of twisting about five screws [art. 94]. Any wrench which is unable to disturb the equilibrium of the body must be reciprocal to the five screws; and since a screw is determined by five elements, only a finite number of wrenches fulfilling this condition are possible. Now, as pointed out [art. 45], that number must be *one*; for if there were two, then every screw on the cylindroid determined by those two would fulfil the conditions [art. 21], and the number would be infinite.

6. *Calculation of the single screw reciprocal to five given screws.*—Let one of the five given screws be typified by

$$\frac{x-x_k}{\alpha_k}=\frac{y-y_k}{\beta_k}=\frac{z-z}{\gamma_k}\ (\text{pitch}=\varrho_k),$$

while the desired screw is defined by

$$\frac{x-x'}{\alpha}=\frac{y-y'}{\beta}=\frac{z-z'}{\gamma}\ (\text{pitch}=\varrho).$$

The condition of reciprocity (art. 1) produces five equations of the following type:—

$$\begin{aligned}&\alpha[(\varrho+\varrho_k)\alpha_k+\gamma_k y_k-\beta_k z_k]+\beta[(\varrho+\varrho_k)\beta_k+\alpha_k z_k-\gamma_k x_k]\\&+\gamma[(\varrho+\varrho_k)\gamma_k+\beta_k x_k-\alpha_k y_k]+\alpha_k(\gamma y'-\beta z')+\beta_k(\alpha z'-\gamma x')\\&+\gamma_k(\beta x'-\alpha y')=0.\end{aligned}$$

From these five equations the relative values of the six quantities

$$\alpha,\quad \beta,\quad \gamma,\quad \gamma y'-\beta z',\quad \alpha z'-\gamma x',\quad \beta x'-\alpha y'$$

can be determined by linear solution. Introducing these values into the identity

$$\alpha(\gamma y'-\beta z')+\beta(\alpha z'-\gamma x')+\gamma(\beta x'-\alpha y')=0,$$

gives the equation which determines ϱ.

To express this equation concisely we introduce two classes of subsidiary magnitudes. We write one magnitude of each class as a determinant.

$$\begin{vmatrix} \varrho_1\beta_1+z_1\alpha_1-x_1\gamma_1, & \varrho_1\gamma_1+x_1\beta_1-y_1\alpha_1, & \alpha_1, & \beta_1, & \gamma_1 \\ \varrho_2\beta_2+z_2\alpha_2-x_2\gamma_2, & \varrho_2\gamma_2+x_2\beta_2-y_2\alpha_2, & \alpha_2, & \beta_2, & \gamma_2 \\ \varrho_3\beta_3+z_3\alpha_3-x_3\gamma_3, & \varrho_3\gamma_3+x_3\beta_3-y_3\alpha_3, & \alpha_3, & \beta_3, & \gamma_3 \\ \varrho_4\beta_4+z_4\alpha_4-x_4\gamma_4, & \varrho_4\gamma_4+x_4\beta_4-y_4\alpha_4, & \alpha_4, & \beta_4, & \gamma_4 \\ \varrho_5\beta_5+z_5\alpha_5-x_5\gamma_5, & \varrho_5\gamma_5+x_5\beta_5-y_5\alpha_5, & \alpha_5, & \beta_5, & \gamma_5 \end{vmatrix} = P.$$

By cyclical interchange the two analogous functions Q and R are defined.

$$\begin{vmatrix} \varrho_1\beta_1+z_1\alpha_1-x_1\gamma_1, & \varrho_1\gamma_1+x_1\beta_1-y_1\alpha_1, & \varrho_1\alpha_1, & \beta_1, & \gamma_1 \\ \varrho_2\beta_2+z_2\alpha_2-x_2\gamma_2, & \varrho_2\gamma_2+x_2\beta_2-y_2\alpha_2, & \varrho_2\alpha_2, & \beta_2, & \gamma_2 \\ \varrho_3\beta_3+z_3\alpha_3-x_3\gamma_3, & \varrho_3\gamma_3+x_3\beta_3-y_3\alpha_3, & \varrho_3\alpha_3, & \beta_3, & \gamma_3 \\ \varrho_4\beta_4+z_4\alpha_4-x_4\gamma_4, & \varrho_4\gamma_4+x_4\beta_4-y_4\alpha_4, & \varrho_4\alpha_4, & \beta_4, & \gamma_4 \\ \varrho_5\beta_5+z_5\alpha_5-x_5\gamma_5, & \varrho_5\gamma_5+x_5\beta_5-y_5\alpha_5, & \varrho_5\alpha_5, & \beta_5, & \gamma_5 \end{vmatrix} = L.$$

By cyclical interchange the two analogous functions M and N are defined.

The equation for ϱ reduces to

$$(P^2+Q^2+R^2)\varrho+PL+QM+RN=0.$$

The reduction of this equation to the first degree is an independent proof of the important principle, that one screw and only one can be determined which is reciprocal to five given screws; ϱ being known, α, β, γ can be found, and also two linear equations between x', y', z', whence the reciprocal screw is completely determined.

7. *Coreciprocal screws.*—A set of six screws can be chosen, so that each screw is reciprocal to the remaining five. For take A_1 arbitrary; A_2 reciprocal to A_1; A_3 reciprocal to A_1, A_2; A_4 reciprocal to A_1, A_2, A_3; A_5 reciprocal to A_1, A_2, A_3, A_4; and A_6 reciprocal to A_1, A_2, A_3, A_4, A_5. A group constructed in this way is called a set of *coreciprocal screws.*

Thirty constants determine a group of six screws. If the group be coreciprocal, fifteen conditions must be fulfilled; we have therefore fifteen elements still disposable, so that we are always enabled to select a coreciprocal group with special appropriateness to the problem under consideration.

The facilities presented by rectangular axes for questions connected with the dynamics of a particle have perhaps their analogues in the conveniences which arise from referring the twist coordinates of a rigid body to a group of coreciprocal screws.

8. *Resolution of a wrench along six coreciprocal screws.*—The resolution of a wrench S (or of a twist velocity) into six wrenches (or six twist velocities) of magnitudes X_1 &c., X_6 along six reciprocal screws of reference A_1 &c., A_6 is thus effected geometrically. Draw the cylindroid (A_1, A_2), select on this cylindroid the screw P reciprocal to S [art. 44]; if a rigid body only free to twist about P be acted upon by wrenches about S, A_1, &c.,

A_6, the only operative wrenches are those about A_1, A_2, for all the others are reciprocal to P, and are destroyed by the reaction of the constraints. Hence the wrench X_1 must be such as to neutralize the effect of the wrench X_2 in its efforts to disturb the equilibrium of a body only free to twist about P. Therefore the wrenches X_1, X_2 must be such as compound into a wrench about the screw on the cylindroid reciprocal to P. Thus the ratio of X_1 to X_2 is completely determined.

9. *Expressions for the components about six coreciprocal screws into which any wrench may be decomposed.*—The use of the virtual coefficient will afford concise values of the components. Let A_1 &c., A_6 be the coreciprocal screws, and let X be the wrench about the screw S which is to be decomposed. Let p_m be the pitch of the screw A_m. Let R_m be the virtual coefficient of S and A_m. Let X_m be the component wrench about A_m.

The energy expended in giving a body a small twist ω around A_1 against the wrench X is

$$\omega X R_1;$$

this must equal the energy expended in giving the body the same twist in opposition to the component wrench X_1; for since A_2 &c., A_6 are reciprocal to A_1, the wrenches X_2 &c., X_6 cannot affect the quantity of energy required. The virtual coefficient of two coincident screws reduces to the sum of the pitches, and therefore

$$\omega X R_1 = 2\omega X_1 p_1,$$

whence

$$X_1 = X\frac{R_1}{2p_1}.$$

The analogy of this process to the resolution of a force or velocity along three rectangular axes may be noticed. The velocity of a point is resolved into three components parallel to the axes by multiplying the velocity into the direction cosines; so the twist velocity of the rigid body is resolved into six twist velocities about six coreciprocal screws by multiplying the original twist velocity into six functions analogous to the cosines.

10. *Relation between the square of a wrench and the squares of its components along a coreciprocal system.*—We can also detect a theorem analogous to the familiar truth that the sum of the squares of the three component velocities is equal to the square of the original velocity.

To twist a body through a small angle ω about the screw S, in opposition to the wrench X about the same screw, requires a quantity of energy equal to

$$2\omega . p . X;$$

but this must equal the quantity of energy necessary to overcome the components, namely,

$$\omega(X_1 R_1 + \&c. + X_6 R_6),$$

but (art. 9)

$$R_m = \frac{2p_m X_m}{X};$$

substituting, we find

$$pX^2 = p_1 X_1^2 + \&c. + p_6 X_6^2$$

III. THE SEXIANT.

11. *Definition of the Sexiant.*—When six screws, A_1 &c., A_6, are reciprocal to a single screw T, a certain relation must subsist between the pitches and the coefficients of A_1 &c., A_6. The function which, when equated to zero, gives the condition required, may be called the *sexiant* of the six screws.

The equations of the screw A_k are

$$\frac{x-x_k}{\alpha_k}=\frac{y-y_k}{\beta_k}=\frac{z-z_k}{\gamma_k}\ (\text{pitch}=\varrho_k).$$

The reciprocal screw T, which without loss of generality we may suppose to pass through the origin, is represented by

$$\frac{x}{\alpha}=\frac{y}{\beta}=\frac{z}{\gamma}\ (\text{pitch}=\varrho).$$

The condition that A_k and T be reciprocal is

$$(\varrho+\varrho_k)(\alpha\alpha_k+\beta\beta_k+\gamma\gamma_k)+x_k(\gamma\beta_k-\beta\gamma_k)+y_k(\alpha y_k-\gamma\alpha_k)+z_k(\beta\alpha_k-\alpha\beta_k)=0.$$

Writing the six equations of this type, found by giving k the values 1 to 6, and eliminating the six quantities

$$\varrho\alpha,\ \varrho\beta,\ \varrho\gamma,\ \alpha,\ \beta,\ \gamma,$$

we obtain the result:—

$$\begin{vmatrix}
\alpha_1\varrho_1+\gamma_1y_1-\beta_1z_1, & \beta_1\varrho_1+\alpha_1z_1-\gamma_1x_1, & \gamma_1\varrho_1+\beta_1x_1-\alpha_1y_1, & \alpha_1, & \beta_1, & \gamma_1 \\
\alpha_2\varrho_2+\gamma_2y_2-\beta_2z_2, & \beta_2\varrho_2+\alpha_2z_2-\gamma_2x_2, & \gamma_2\varrho_2+\beta_2x_2-\alpha_2y_2, & \alpha_2, & \beta_2, & \gamma_2 \\
\alpha_3\varrho_3+\gamma_3y_3-\beta_3z_3, & \beta_3\varrho_3+\alpha_3z_3-\gamma_3x_3, & \gamma_3\varrho_3+\beta_3x_3-\alpha_3y_3, & \alpha_3, & \beta_3, & \gamma_3 \\
\alpha_4\varrho_4+\gamma_4y_4-\beta_4z_4, & \beta_4\varrho_4+\alpha_4z_4-\gamma_4x_4, & \gamma_4\varrho_4+\beta_4x_4-\alpha_4y_4, & \alpha_4, & \beta_4, & \gamma_4 \\
\alpha_5\varrho_5+\gamma_5y_5-\beta_5z_5, & \beta_5\varrho_5+\alpha_5z_5-\gamma_5x_5, & \gamma_5\varrho_5+\beta_5x_5-\alpha_5y_5, & \alpha_5, & \beta_5, & \gamma_5 \\
\alpha_6\varrho_6+\gamma_6y_6-\beta_6z_6, & \beta_6\varrho_6+\alpha_6z_6-\gamma_6x_6, & \gamma_6\varrho_6+\beta_6x_6-\alpha_6y_6, & \alpha_6, & \beta_6, & \gamma_6
\end{vmatrix}=0.$$

By transformation to *any parallel axes* the value of this determinant is *unaltered*. The evanescence of the determinant is therefore a necessary condition *whenever* the six screws are reciprocal to a single screw.

The sexiant is only a function of the differences $\varrho_1-\varrho_2$ &c. of the pitches; and when the pitches are equal the evanescence of the sexiant expresses that the six lines which form the screws are in involution.

The property possessed by six screws when their sexiant vanishes may be enunciated in different ways, which are precisely equivalent.

(*a*) The six screws are all reciprocal to one screw.

(*b*) The six screws are members of a coordinate system of five degrees of freedom.

(*c*) Properly selected wrenches about the six screws equilibrate, when applied to a free rigid body.

(*d*) Properly selected twist velocities about the six screws neutralize, when applied to a rigid body.

(*e*) A body might receive six small twists about the six screws, so that after the last twist the body would occupy the same position which it had before the first.

(*f*) By extending the language of Professor SYLVESTER ('Comptes Rendus,' t. lii. p. 741), we might perhaps assert that six screws are in involution when their sexiant vanishes.

(*g*) *Any* wrench (or twist or twist velocity) can be resolved into six wrenches (or twists or twist velocities) about six screws when the sexiant of the six screws does *not* vanish.

12. *Application of the sexiant to the resolution of a wrench about six screws of reference.*—One of the most remarkable properties of the sexiant is enunciated in the following theorem:—

If seven wrenches (or twists) about seven screws equilibrate (or neutralize), the magnitude of each wrench (or twist) must be proportional to the sexiant of the remaining six screws.

We shall demonstrate this property for twists, the demonstration for wrenches being, of course, exactly similar.

Let one, A_k, of the seven screws be represented by

$$\frac{x-x_k}{\alpha_k}=\frac{y-y_k}{\beta_k}=\frac{z-z_k}{\gamma_k}\,(\text{pitch}=\varrho_k).$$

Suppose x_k, y_k, z_k be the coordinates of the foot of the perpendicular from the origin on A_k.

The body receives a small twist ω_k about A_k; the rotation element of the twist may be transferred to a rotation about a parallel axis through the origin by the introduction of translations, whose components are

$$\omega_k(\gamma_k y_k-\beta_k z_k),\quad \omega_k(\alpha_k z_k-\gamma_k x_k),\quad \omega_k(\beta_k x_k-\alpha_k y_k).$$

Each of the seven twists is thus decomposed into three translations parallel to the three axes, and three rotations about the axes. If the seven twists neutralize, we have the six equations:

$$\begin{aligned}
&\omega_1\alpha_1+\&c.+\omega_7\alpha_7=0,\\
&\omega_1\beta_1+\&c.+\omega_7\beta_7=0,\\
&\omega_1\gamma_1+\&c.+\omega_7\gamma_7=0,\\
&\omega_1(\varrho_1\alpha_1+y_1\gamma_1-z_1\beta_1)+\&c.+\omega_7(\varrho_7\alpha_7+y_7\gamma_7-z_7\beta_7)=0,\\
&\omega_1(\varrho_1\beta_1+z_1\alpha_1-x_1\gamma_1)+\&c.+\omega_7(\varrho_7\beta_7+z_7\alpha_7-x_7\gamma_7)=0,\\
&\omega_1(\varrho_1\gamma_1+x_1\beta_1-y_1\alpha_1)+\&c.+\omega_7(\varrho_7\gamma_7+x_7\beta_7-y_7\alpha_7)=0.
\end{aligned}$$

These equations will be satisfied if for each value of ω the sexiant of the remaining six screws be substituted. This will appear most satisfactorily by writing one of the equations a second time and eliminating ω_1 &c., ω_7 from the seven equations. We then get a result which is necessarily zero. But it will be found that this is precisely the same result as would have been obtained by the substitution already mentioned.

13. *Circumstances under which the sexiant vanishes identically.*—The sexiant vanishes if $k+1$ screws be members of a coordinate system of freedom k [art. 31]; for then a

screw reciprocal to k screws of the system will be reciprocal to the $k+1$ screws [art. 36], and therefore a screw can be chosen reciprocal to the six screws from which the sexiant is formed.

14. *Use of the sexiant in resolving a wrench into components about a group of screws with which the wrench is coordinate.*—Given k screws of a coordinate system of freedom of the degree $k-1$ [art. 31], determine expressions for the wrenches (or twists) ω_1 &c., ω_k about the k screws which will neutralize each other.

Let the given screws be A_1 &c., A_k.

Take $7-k$ screws X_1 &c., X_{7-k} from the group reciprocal to the given coordinate system [art. 37].

If wrenches ω_1 &c., ω_7 about the seven screws A_1 &c., A_k, X_1 &c., X_{7-k} equilibrate, we must have (art. 12)

$$\frac{\omega_1}{S(A_2 \&c., A_k, X_1 \&c., X_{7-k})} = \&c. = \frac{\omega_k}{S(A_1 \&c., A_{k-1}, X_1 \&c., X_{7-k})} = \frac{\omega_{k+1}}{S(A_1 \&c., A_k, X_2 \&c., X_{7-k})} = \&c.,$$

where the symbol S denotes the sexiant of the six screws inside the brackets.

The sexiants under ω_{k+1} &c., ω_7 vanish identically (art. 13); hence ω_{k+1} &c., ω_7 are each zero. The other equations determine the required quantities ω_1 &c., ω_k.

15. *On a function analogous to the sine of the angle between two vectors.*—We have pointed out (art. 9) in what respects the virtual coefficient of a pair of screws may be considered analogous to the *cosine* of the angle between a pair of vectors. We hope the following attempt to point out a function in the theory of screws in some respects analogous to the *sine* of the angle between a pair of vectors will not be considered to transcend the reasonable use of mathematical metaphor.

The determinant whose evanescence expresses that three vectors are coplanar has the sexiant for its analogue in the theory of screws. If the condition that three vectors, α, β, γ, be coplanar is satisfied for *every* vector γ, then the sine of the angle between α and β must vanish. It is remarkable that the vanishing of the sine really involves two conditions, for it can only occur when the direction cosines of α and β are identical. If now the sexiant of A_1 &c., A_6 vanish for every screw A_6, the remaining five screws must fulfil the two conditions known to be necessary, in order that they may constitute members of a coordinate system with four degrees of freedom. If, then, we can find *one* function, the evanescence of which will afford the two necessary conditions, such a function may be considered analogous to the sine of the angle between two vectors.

It can hardly be objected to this analogy that, while two vectors are concerned in the one case, five screws enter into the other; for it must be remembered that $2+1$ is the complete number of vectors of reference, while $5+1$ is the complete number of screws of reference.

The investigation of art. (11) indicates the function of which we are in search. If the five screws are really coordinate members of a lower degree of freedom, the value of ϱ must become indeterminate, and therefore

$$P^2+Q^2+R^2=0.$$

We deduce from this the equations

$$P=0, \quad Q=0, \quad R=0,$$

two of which, being independent, provide the two conditions required.

16. *On a relation between a sexiant and a virtual coefficient.*—We shall first state a simple vector problem. Given three vectors α, β, γ, determine the cosine of the angle between γ and the common perpendicular to α and β.

The required cosine is the quotient obtained by dividing the determinant whose evanescence shows α, β, γ to be coplanar by the sine of the angle between α and β.

The corresponding question in the theory of screws is as follows. Given six screws A_1 &c., A_6 of which S represents the sexiant. Let B_m be the screw reciprocal to the five screws A_1 &c., A_{m-1}, A_{m+1} &c., A_6, and let R_m be the virtual coefficient of A_m and B_m. Let K_m denote the function

$$\sqrt{P^2+Q^2+R^2},$$

computed for the five screws A_1 &c., A_{m-1}, A_{m+1} &c., A_6. Then R_m can only differ by a numerical factor from

$$\frac{S}{K_m}.$$

For R_m must vanish if S vanish, unless at the same time K_m vanish. S is of the third dimensions of linear magnitude and K_m of the second, so that the quotient is of the same dimensions as R_m.

IV. ON IMPULSIVE SCREWS AND INSTANTANEOUS SCREWS.

17. *The impulsive cylindroid and the instantaneous cylindroid.*—A rigid body M is at rest in a position P, from which it is either partially or entirely free to move. If M receive an impulsive wrench about a screw X_1, it will commence to twist about an instantaneous screw A_1. If, however, the impulsive wrench had been about X_2 or X_3 (M being in either case at rest in the position P), the instantaneous screw would have been A_2 or A_3. Then we have the following theorem:—

If X_1, X_2, X_3 lie upon a cylindroid S (which we may call the impulsive cylindroid), then A_1, A_2, A_3 lie on a cylindroid S′ (which we may call the instantaneous cylindroid)*.

For if the three wrenches are of suitable magnitude they may equilibrate, since they are cocylindroidal; when this is the case the three instantaneous twist velocities must of course neutralize, but this is only possible if the instantaneous screws be cocylindroidal.

18. *On an anharmonic property of the impulsive and instantaneous cylindroids.*—If we draw a pencil of four lines through a point parallel to four generators of a cylindroid, the lines forming the pencil will lie in a plane. We may define the *anharmonic ratio of four generators on a cylindroid* to be the anharmonic ratio of the parallel *pencil*. We shall now prove the following theorem:—

The anharmonic ratio of four screws on the impulsive cylindroid is equal to the anharmonic ratio of the four corresponding screws on the instantaneous cylindroid.

* When three screws are contained on a cylindroid, the screws may, for brevity, be said to be *cocylindroidal*.

Before commencing the proof we remark that,

If an impulsive wrench F acting on a rigid body about the screw X be capable of producing the unit of twist velocity about A, then a wrench $F\omega$ about X will produce a twist velocity ω about A.

Let X_1, X_2, X_3, X_4 be four screws on the impulsive cylindroid, the wrenches appropriate to which are $F_1\omega_1$, $F_2\omega_2$, $F_3\omega_3$, $F_4\omega_4$. Let the four corresponding instantaneous screws be A_1, A_2, A_3, A_4, and the twist velocities are ω_1, ω_2, ω_3, ω_4. Let φ_m be the angle on the impulsive cylindroid [art. 7] defining X_m, and let θ_m be the angle on the instantaneous cylindroid defining A_m.

If three impulsive wrenches equilibrate, the corresponding twist velocities neutralize: hence it must be possible for certain values of ω_1, ω_2, ω_3, ω_4 to satisfy the following equations [art. 10]:—

$$\frac{\omega_1}{\sin(\theta_2-\theta_3)}=\frac{\omega_2}{\sin(\theta_3-\theta_1)}=\frac{\omega_3}{\sin(\theta_1-\theta_2)},$$

$$\frac{F_1\omega_1}{\sin(\varphi_2-\varphi_3)}=\frac{F_2\omega_2}{\sin(\varphi_3-\varphi_1)}=\frac{F_3\omega_3}{\sin(\varphi_1-\varphi_2)},$$

$$\frac{\omega_2}{\sin(\theta_3-\theta_4)}=\frac{\omega_3}{\sin(\theta_4-\theta_2)}=\frac{\omega_4}{\sin(\theta_2-\theta_3)},$$

$$\frac{F_2\omega_2}{\sin(\varphi_3-\varphi_4)}=\frac{F_3\omega_3}{\sin(\varphi_4-\varphi_2)}=\frac{F_4\omega_4}{\sin(\varphi_2-\varphi_3)},$$

whence

$$\frac{\sin(\theta_1-\theta_2)\sin(\theta_3-\theta_4)}{\sin(\theta_3-\theta_1)\sin(\theta_4-\theta_2)}=\frac{\sin(\varphi_1-\varphi_2)\sin(\varphi_3-\varphi_4)}{\sin(\varphi_3-\varphi_1)\sin(\varphi_4-\varphi_2)},$$

which proves the theorem.

If, therefore, we are given three screws on the impulsive cylindroid and the corresponding three screws on the instantaneous cylindroid, the connexion between every other corresponding pair is geometrically determined.

19. *On the impulsive wrenches which arise from the reaction of constraints.*—Whatever the constraints may be, their reaction produces an impulsive wrench R_1 upon the body at the moment when the impulsive wrench X_1 acts. The two wrenches X_1 and R_1 compound into a third wrench Y_1. If the body were free, Y_1 is the impulsive wrench to which the instantaneous screw A_1 would correspond. Since X_1, X_2, X_3 are cocylindroidal, A_1, A_2, A_3 are cocylindroidal (art. 17), and therefore also are Y_1, Y_2, Y_3. The nine wrenches X_1, X_2, X_3, R_1, R_2, R_3, $-Y_1$, $-Y_2$, $-Y_3$ must equilibrate; but if X_1, X_2, X_3 equilibrate, then the twist velocities about A_1, A_2, A_3 must neutralize, and therefore the wrenches about Y_1, Y_2, Y_3 must equilibrate. Hence R_1, R_2, R_3 equilibrate, and are therefore cocylindroidal.

Following the same line of proof used in art. (18), we can show that

If impulsive wrenches act about any four cocylindroidal screws upon a partially free rigid body, the four corresponding initial reactions of the constraints also constitute wrenches about four cocylindroidal screws; and, further, the anharmonic ratios of the two groups of four screws are equal.

20. *On impulsive and instantaneous coordinate systems.*—Some of the preceding results may be generalized. If impulsive wrenches were to act upon a partially or entirely free rigid body about the different screws belonging to a coordinate system of freedom k, then the corresponding instantaneous screws, and also the wrenches produced by the initial reactions of the constraints, would each constitute a coordinate group of freedom k.

21. *On a special impulsive cylindroid.*—If the impulsive cylindroid be defined by a pair of screws along the same straight line, the surface becomes evanescent, and every screw on the straight line is contained on the cylindroid. We thus deduce the following theorem as a particular case of art. (17):—

If a rigid body, free or constrained in any way, were to receive impulsive wrenches about screws of every pitch on a straight line, then all the corresponding instantaneous screws lie on a cylindroid.

One of the screws of zero pitch on the instantaneous cylindroid, when the body is free, passes through the centre of inertia of the body, because this must be the instantaneous screw corresponding to infinite pitch along the impulsive screw.

22. *Calculation of the specific impulsive wrenches when a sufficient number of pairs of corresponding impulsive and instantaneous screws are known.*—By the words specific impulsive wrench we are to understand an impulsive wrench which, acting on a free quiescent rigid body, will communicate to the body the unit of twist velocity about an instantaneous screw.

Let A_1 &c., A_{k+1} be $k+1$ screws of a coordinate system of freedom k, and ω_1 &c., ω_{k+1} be a set of twist velocities about A_1 &c., A_{k+1} which neutralize. Let X_1 &c., X_{k+1} be corresponding impulsive screws for a free body; it is required to find the specific impulsive wrenches.

Let the impulsive wrenches be

$$F_1\omega_1 \text{ &c., } F_{k+1}\omega_{k+1}.$$

R_1 &c., R_{6-k} are screws reciprocal to the system A_1 &c., A_k.

Q_1 &c., Q_{6-k} are screws reciprocal to the system X_1 &c., X_k.

Adopting the notation for sexiants employed in art. (14),

$$S_1 = S(A_2 \text{ &c., } A_{k+1},\ R_1 \text{ &c., } R_{6-k}),$$

$$T_1 = S(X_2 \text{ &c., } X_{k+1},\ Q_1 \text{ &c., } Q_{6-k}),$$

we have the equations

$$\frac{\omega_1}{S_1} = \frac{\omega_2}{S_2} = \text{&c.} = \frac{\omega_{k+1}}{S_{k+1}}.$$

Since the body is free, the wrenches must equilibrate which could produce these velocities that neutralize; whence

$$\frac{F_1\omega_1}{T_1} = \text{&c.} = \frac{F_{k+1}\omega_{k+1}}{T_{k+1}}.$$

Combining the two sets of equations, we have for the specific impulsive forces F_1 &c., F_{k+1},

$$\frac{F_1}{T_1^1}=\frac{F_2}{T_2^2}=\&c.,\ \frac{F_{k+1}}{T_{k+1}^{k+1}}.$$

23. *Determination of the specific impulsive wrenches when the three impulsive screws lie on a cylindroid, and the corresponding instantaneous screws are known.*—In the case where $k=2$ the problem of the last article assumes a simple form.

Let θ_1, θ_2, θ_3 be the angles corresponding to the instantaneous screws on the instantaneous cylindroid. Let φ_1, φ_2, φ_3 be the angles corresponding to the impulsive screws on the impulsive cylindroid.

If the following equations be true,

$$\frac{\omega_1}{\sin(\theta_2-\theta_3)}=\frac{\omega_2}{\sin(\theta_3-\theta_1)}=\frac{\omega_3}{\sin(\theta_1-\theta_2)},$$

then we must also have

$$\frac{F_1\omega_1}{\sin(\varphi_2-\varphi_3)}=\frac{F_2\omega_2}{\sin(\varphi_3-\varphi_1)}=\frac{F_3\omega_3}{\sin(\varphi_1-\varphi_2)};$$

whence the relative values of F_1, F_2, F_3 are known.

24. *Relations between the impulsive screw and the instantaneous screw in certain special cases.*—When an impulsive *force* acts upon a free quiescent rigid body, the directions of the force and of the instantaneous screw are parallel to a pair of conjugate diameters in the momental ellipsoid.

When an impulsive wrench acting on a free rigid body produces an instantaneous *rotation*, the axis of the rotation must be perpendicular to the impulsive screw.

When an impulsive force acting on a free rigid body produces an instantaneous rotation, the direction of the force and the axis of the rotation are parallel to the principal axes of a section of the momental ellipsoid.

V. THE PRINCIPAL SCREWS OF INERTIA.

25. *On the locus of the impulsive screw corresponding to a given instantaneous screw.*—If an impulsive screw be given, the corresponding instantaneous screw is determinate, whether the body be free or constrained. If, however, the instantaneous screw be given, the impulsive screw is indeterminate, except in the case where the body is free. We have therefore the following problem:—To determine the locus (in a generalized sense) of an impulsive wrench which will cause a quiescent rigid body to commence to twist about a given screw.

Let A_1 &c., A_k be a group of screws defining the freedom of the body.

Let B_1 &c., B_{6-k} be screws reciprocal to the freedom.

S is the screw about which the body is to twist.

Let P be *any* impulsive wrench which would make the body commence to twist about S. Then any screw X coordinate with the $7-k$ screws,

$$P,\ B_1,\ \&c.,\ B_{6-k},$$

possesses the property required. This condition expresses the "locus" of X.

This is thus proved. An impulsive wrench about X can be decomposed into impulsive wrenches about the screws P, B_1, &c., B_{k-1}. All of these will be destroyed by the reactions of the constraints except the first; but by hypothesis an impulsive wrench about P will make the body twist about S.

For example, if the body had five degrees of freedom, then any impulsive screw on a certain cylindroid possesses the property required.

If the body had three degrees of freedom, then any screw reciprocal to a certain cylindroid would possess the required property.

26. *Definition of principal screws of inertia.*—A rigid body has k degrees of freedom; it will be shown that from the coordinate system expressing that freedom, k screws may be selected such that an impulsive wrench about any one of these screws will make the body commence to twist about that screw.

The k screws possessing this property may be called the *principal screws of inertia.*

We shall first demonstrate the existence of six principal screws of inertia in a perfectly free rigid body. Out of the thirty elements which in general determine a set of six screws, fifteen are still disposable in a set of six conjugate screws of kinetic energy. We are thus enabled to choose one set of conjugate screws of kinetic energy [art. 55] which are also coreciprocal (art. 7). The set thus chosen manifestly possesses the property that an impulsive wrench about any one of them makes the body commence to twist about that screw.

It can be shown that this set is in general unique. For let the group be represented by A_1 &c., A_6, and let S be another screw assumed to possess the same property. An impulsive wrench about S can be decomposed into impulsive wrenches about A_1 &c., A_6. These wrenches will produce twist velocities about A_1 &c., A_6. If these twist velocities could compound into a twist velocity about S, it would follow that equal twist velocities should be produced about each of the screws A_1 &c., A_6 by equal impulsive wrenches. If this were the case, then every impulsive screw would be its own instantaneous screw, which manifestly is not true. Hence there can be only one set of six principal screws of inertia.

27. *Construction of the six principal screws of inertia for a free rigid body.*—Let OA, OB, OC be the three principal axes through the centre of inertia of the rigid body, and let a, b, c be the corresponding radii of gyration. Let two screws, A′, A″, be formed along OA with pitches $+a$, $-a$, and let two other pairs of screws, B′, B″ and C′, C″, be similarly formed along the other principal axes, OB and OC. The six screws, A′, A″, B′, B″, C′, C″, are the set required.

28. *Principal screws of inertia of a constrained rigid body.*—Of the $k(k-1)$ arbitrary elements in the selection of any k screws from a coordinate system of freedom k, half the entire number still remain disposable in k conjugate screws of kinetic energy [art. 87]. The remaining disposable elements are just the number of conditions required that each of k screws should be reciprocal to all the rest. We have therefore just sufficient latitude for the determination of k conjugate screws of kinetic energy, which are also coreciprocal. We shall now show that the k screws thus defined possess the

property that an impulsive wrench about each of them will make the body commence to twist about the same screw.

We shall state the argument for three degrees of freedom, the general argument for any other degree of freedom being precisely similar.

Let A_1, A_2, A_3 be the three screws of the system which we have just determined.

B_1, B_2, B_3 are any corresponding impulsive screws.

R_1, R_2, R_3 are any three screws reciprocal to A_1, A_2, A_3 [art. 37].

Any impulsive wrench about a screw coordinate with B_1, R_1, R_2, R_3 will make the body twist about A_1 (art. 25). But the screws of the coordinate system containing B_1, R_1, R_2, R_3 are defined by being reciprocal to A_2 and A_3. Now A_1 is reciprocal to A_2, A_3, and therefore A_1 must be coordinate with B_1, R_1, R_2, R_3 [art. 89]. Therefore an impulsive wrench about A_1 will make the body twist about A_1.

It can be shown, by similar reasoning to that employed in art. (26), that the k principal screws of inertia are unique.

29. *On the pitch hyperbola in the principal plane of a cylindroid.*—We shall exemplify the preceding articles by determining the principal screws of a rigid body which has two degrees of freedom. The cylindroid expressing the freedom being drawn, we shall give the construction by which the two screws on that cylindroid may be found which possess the property in question.

To obtain, in the first place, a clear view of the distribution of the pitch upon a cylindroid, we introduce a conic which is to be known as the *pitch hyperbola.*

The cylindroid having as its equation

$$z(x^2+y^2)-2mxy=0 \text{ [art. 7]},$$

the generator in which the plane

$$y=x\tan\theta$$

cuts the surface has a pitch

$$p+m\cos 2\theta.$$

The pitch hyperbola has for its equation

$$\frac{x^2}{m-p}-\frac{y^2}{m+p}=k.$$

If ϱ be the radius vector of the pitch hyperbola in the direction θ,

$$p+m\cos 2\theta=\frac{k(m^2-p^2)}{\varrho^2};$$

hence we have the fundamental property of this conic, which is thus enunciated:—

The pitch of any generator of the cylindroid is proportional to the inverse square of the parallel diameter in the pitch hyperbola.

Given a screw A on a cylindroid, another screw B on the cylindroid reciprocal to A can always be determined [art. 44]. We shall now show that

A pair of reciprocal screws on a cylindroid are parallel to a pair of conjugate diameters in the pitch hyperbola.

The condition of reciprocity is (art. 1)

$$(a+b)\cos\theta - d\sin\theta = 0.$$

Substitute

$$a = p + m\cos 2\theta_1,$$
$$b = p + m\cos 2\theta_2,$$
$$d = m\sin 2\theta_1 - m\sin 2\theta_2,$$
$$\theta = \pi - (\theta_1 - \theta_2),$$

we readily deduce

$$\tan\theta_1 \tan\theta_2 = \frac{m+p}{m-p},$$

which proves the theorem (SALMON, 'Conics,' p. 152, 3rd edition):—

The asymptotes of the pitch hyperbola are parallel to the lines of zero pitch on the cylindroid. If $p > m$ the hyperbola changes into an ellipse. In this case there are no screws of zero pitch on the cylindroid.

30. *On the ellipse of equal kinetic energy for a body possessing two degrees of freedom.*—The pitch hyperbola is of merely kinematical significance; the ellipse now to be described involves the conceptions of kinetics.

Let θ be a screw upon the cylindroid and ω be the twist velocity about that screw; ω can be resolved into twist velocities ω_1 and ω_2 about any selected pair of screws θ_1, θ_2 upon the cylindroid. The velocity of any element of the rigid body can therefore be expressed as a linear function of ω_1 and ω_2. The kinetic energy of the rigid body must therefore have the form

$$A\omega_1^2 + 2B\omega_1\omega_2 + C\omega_2^2.$$

In the principal plane of the cylindroid draw through the centre two lines parallel to the generators θ_1 and θ_2, and with these two lines as axes construct the ellipse

$$Ax^2 + 2Bxy + Cy^2 = Mh^4.$$

If ϱ be the radius vector of the ellipse parallel to θ,

$$\frac{\omega}{\varrho} = \frac{\omega_1}{x} = \frac{\omega_2}{y};$$

whence the kinetic energy is

$$\frac{Mh^4\omega^2}{\varrho^2}.$$

We have thus proved that the kinetic energy due to a given twist velocity about any screw is proportional to the inverse square of the parallel diameter in the ellipse, which we may call the *ellipse of equal kinetic energy*. Or suppose a given quantity of energy is to be imparted by an impulsive wrench, the twist velocity that can be produced about any screw is proportional to the parallel radius vector.

If a pair of conjugate diameters of the ellipse of equal kinetic energy had been taken as axes, the expression for the kinetic energy becomes

$$A\omega_1^2 + C\omega_2^2.$$

By the properties of what we have called conjugate screws of kinetic energy [art. 56], we now see that every pair of conjugate diameters of the ellipse of equal kinetic energy are parallel to a pair of conjugate screws of kinetic energy on the cylindroid.

31. *Construction of the principal screws of inertia for a rigid body with two degrees of freedom.*—Draw the pitch hyperbola and the ellipse of equal kinetic energy; since these conics are concentric, a pair of common conjugate diameters can be drawn. The screws upon the cylindroid parallel to the common conjugate diameters are the principal screws of kinetic energy.

Let A_1, A_2 be the two screws thus determined, and let X_1, X_2 be the impulsive screws, wrenches about which, if the body were free, would make it commence to twist about A_1, A_2. Let R_1, R_2, R_3, R_4 be any four screws reciprocal to the cylindroid.

An impulsive wrench about any screw coordinate with X_1, R_1, R_2, R_3, R_4 will make the body twist about A_1. But since the screws are conjugate screws of kinetic energy, X_1 is reciprocal to A_2. Thus A_2 is reciprocal to the five screws X_1, R_1, R_2, R_3, R_4; and every other screw reciprocal to A_2 will therefore be coordinate with the five screws just written.

Since A_1 and A_2 are parallel to conjugate diameters of the pitch hyperbola, A_1 is reciprocal to A_2, and therefore coordinate with the five screws; an impulsive wrench about A_1 will therefore make the body commence to twist about A_1. In a similar manner it can be shown that A_2 is the other principal screw of inertia.

32. *Relation between a twist about a screw on a cylindroid and its components along a pair of reciprocal screws.*—Let ω be the twist, and ω_1, ω_2 the components. The pitch hyperbola referred to axes parallel to the screws corresponding to ω_1, ω_2 has for equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

If ρ be the radius vector parallel to the screw appropriate to ω,

$$\frac{\omega}{\rho}=\frac{\omega_1}{x}=\frac{\omega_2}{y};$$

whence

$$\frac{1}{a^2}\omega_1^2+\frac{1}{b^2}\omega_2^2=\frac{1}{\rho^2}\omega^2.$$

If p_1, p_2, p be the pitches of ω_1, ω_2, ω, we have

$$p_1\omega_1^2+p_2\omega_2^2=p\omega^2.$$

This result may be compared with art. (10).

We easily foresee, what a little calculation will verify, that, if the virtual coefficient of ω_1, ω_2, instead of vanishing, had the value R, the relation just written must be replaced by

$$p_1\omega_1^2+R\omega_1\omega_2+p_2\omega_2^2=p\omega^2.$$

Here, again, we are reminded of the analogy between the cosine and the virtual coefficient. This result may be generalized into a theorem which is of considerable interest.

Let A_1 &c., A_k be any k screws of pitches p_1 &c., p_k.

Let S be any other screw of pitch p coordinate with the system just written.

Let X be the magnitude of a wrench about S which is decomposed into wrenches X_1 &c., X_k about the k screws A_1 &c., A_k.

Giving the body a small twist ω about X_m, and denoting by $R_{m,n}$ the virtual coefficient of A_m, A_n, we have the following equations, art. (10):—

$$\begin{aligned} XR_{1,s} &= 2p_1X_1 + R_{1,2}X_2 + \&c. + R_{1,k}X_k, \\ XR_{2,s} &= 2R_{2,1}X_1 + 2p_2X_2 + \&c. \quad R_{2,k}X_k, \\ &\vdots \qquad\qquad \vdots \\ XR_{k,s} &= 2R_{k,1}X_1 + R_{k,2}X_2 + \&c. + 2p_kX_k. \end{aligned}$$

But giving the body a small twist ω about S, we have

$$2pX = R_{s,1}X_1 + R_{s,2}X_2 + \&c. + R_{s,6}X_6.$$

Eliminating $R_{s,1}$ &c., we have, finally,

$$pX^2 = p_1X_1^2 + \&c. + p_kX_k^2 + \Sigma R_{m,n}X_mX_n.$$

For six coreciprocal screws this result of course reduces to the relation of art. (10).

33. *Construction of the principal screws of inertia for a body with three degrees of freedom.*—We have demonstrated [art. 88] that all the screws parallel to a plane selected from the general system of three degrees of freedom lie on a cylindroid. A section of the pitch hyperboloid drawn through the kinematic centre gives the pitch hyperbola appropriate to the cylindroid. We hence infer the following theorem:—

Any set of three coreciprocal screws selected from the general system of three degrees of freedom must be parallel to three conjugate diameters of the pitch hyperboloid.

The principal screws of inertia must therefore be parallel to three conjugate diameters of the pitch hyperboloid; but they must also be parallel to three conjugate diameters of the ellipsoid of equal kinetic energy [art. 88], and hence the principal screws are completely determined.

VI. MISCELLANEOUS PROPOSITIONS.

34. *On the locus of the displacements of a point which can be produced by twists about the screws on a cylindroid.*—Let P be a point and A, B be any two screws on a cylindroid. If the body to which P is attached receive a small twist about A, the point P will be moved to P′. If the body received a small twist about B, P would be moved to P″. Then, whatever be the screw C on the cylindroid about which the body be twisted through a small angle, the point P will still be displaced in the plane PP′P″.

For the twist about C can be resolved into two twists about A and B, and therefore every displacement of P must be capable of being resolved along PP′ and PP″.

Thus, through every point in space a *locus plane* can be drawn to which the small movements of that point arising from twists about the screws on a cylindroid are confined.

The simplest construction for the locus plane is as follows:—Draw through the point P two planes, each containing one of the screws of zero pitch: the intersection of these planes is normal to the locus plane through P.

35. *Property of the screws of zero pitch on a cylindroid.*—The construction just given would fail if P lay upon one of the screws of zero pitch. The movements of P must then be limited, not to a plane, but to a line. The line is found by drawing a normal to the plane passing through P and through the other screw of zero pitch.

We thus have the following curious property of the lines of zero pitch, viz. that a point in the rigid body on the line of zero pitch will commence to move in the same direction whatever be the screw on the cylindroid about which the twist is imparted.

This easily appears otherwise. Appropriate twists about any two screws, A and B, can compound into a twist about the screw of zero pitch L, but the twist about L cannot disturb a point on L. Therefore a twist about B must be capable of moving a particle originally on L back to its position before it was disturbed by A. Therefore the twists about B and A must move the particle in the same direction.

36. *Equilibrium of a rigid body having two degrees of freedom and acted upon by gravity.*—The cylindroid is first to be drawn which expresses the freedom enjoyed by the body. It must be remembered that, whatever be the mechanical contrivances by which the body is constrained in its movements, so long as the position is completely defined by two coordinates all the displacements which it is competent for the body to accept could be communicated by twists about the screws of a cylindroid.

When a body with two degrees of freedom is in equilibrium, the wrench acting upon the body must be reciprocal to the cylindroid, but no other condition is required. In the case of gravity the wrench reduces to a force, or the wrench may be said to be about a screw of zero pitch passing through the centre of inertia of the rigid body. But a screw of zero pitch cannot be reciprocal to the cylindroid, unless it intersect both the screws of zero pitch on the cylindroid [art. 21].

The necessary and sufficient condition of equilibrium is, therefore, that the screws of zero pitch on the cylindroid should each intersect the vertical through the centre of inertia.

37. *Equilibrium of a rigid body having three degrees of freedom and acted upon by gravity.*—The vertical through the centre of inertia must be one of the screws of zero pitch belonging to the system reciprocal to the freedom. Draw the pitch hyperboloid appropriate to the freedom [art. 88]. Then all one system of generators are the coordinate screws with zero pitch, while all the other systems of generators are the reciprocal screws with zero pitch.

The necessary and sufficient condition of equilibrium is, therefore, that the vertical through the centre of inertia be one of the reciprocal system of generators on the pitch hyperboloid.

38. *Equilibrium of a rigid body having four degrees of freedom and acted upon by gravity.*—The rigid body can then be twisted about every screw reciprocal to a certain cylindroid. For the body to be in equilibrium, the wrench which acts upon it must be about a screw on this cylindroid. It is therefore necessary that the vertical through the centre of inertia of the rigid body coincide with one or other of the two screws of zero pitch on the cylindroid reciprocal to the freedom of the rigid body.

If, however, the two screws of zero pitch become imaginary on the cylindroid, as will be the case with certain dispositions of the constraints, it will not be possible for the body to remain in equilibrium, no matter how it may be placed.

If the body had five degrees of freedom, it can only remain in equilibrium when acted upon by a wrench about the single screw reciprocal to the freedom. If the restraints were such that the pitch of this screw were zero (which of course will not generally be the case), then when the vertical through the centre of inertia coincided with this line equilibrium would subsist. In general, however, it is impossible for a body with five degrees of freedom to be in equilibrium under the action of gravity.

On the other hand, if the body had only one degree of freedom, through every point in space a plane can be drawn such that every line in the plane passing through the point is a direction along which, if the vertical through the centre of inertia acted, equilibrium would subsist.

39. *Equilibrium of four forces applied to a rigid body.*—If the body be free, the four forces must be four wrenches about screws of zero pitch which are members of a coordinate system with three degrees of freedom. The forces must therefore be generators of an hyperboloid, and all belonging to the same system [art. 81]. The relative magnitudes of the four forces P, Q, R, S are easily determined when the positions are known. Draw the cylindroids (P, Q) and (R, S), then T, the common screw of these cylindroids, makes angles with P and Q, the sines of which angles are in the proportion of Q to P.

Three of the forces, P, Q, R, being given in position, S must then be a generator of the hyperboloid determined by P, Q, R. This proof of a well-known theorem is given to show the facility with which such results flow from the Theory of Screws.

Suppose, however, that the body have only five degrees of freedom, we shall find that somewhat more latitude exists with reference to the choice of S. Let X be the screw reciprocal to the freedom of the body. Then for equilibrium it will only be necessary that S be coordinate with the four screws

$$P,\ Q,\ R,\ X.$$

Now a cone of screws can be drawn through every point in space coordinate with the four screws just written, and on that cone one screw of zero pitch can always be found [art. 89]. Hence one line can be drawn through every point in space along which S might act.

If the body had only four degrees of freedom, the latitude in the choice of S is still greater. Let X_1, X_2 be two screws reciprocal to the freedom, then S is only restrained by the condition that it be coordinate with the five screws

$$P,\ Q,\ R,\ X_1,\ X_2.$$

Any line in space when it receives the proper pitch is a screw coordinate with the five screws just written. Through any point in space a plane can be drawn such that every

line in the plane passing through the point with zero pitch is a coordinate screw. This expresses the freedom enjoyed by S.

Finally, if the body had only three degrees of freedom, the four equilibrating forces P, Q, R, S may be situated anywhere.

The positions of the forces being given, their magnitudes are determined; for draw X_1, X_2, X_3 reciprocal to the freedom, and find the seven equilibrating wrenches about

$$P, Q, R, S, X_1, X_2, X_3.$$

The last three are neutralized by the reactions of the constraints, and the four former must therefore equilibrate.

40. *Equilibrium of five forces applied to a rigid body.*—The five forces must, if the body be free, form a coordinate system of four degrees of freedom. Draw the cylindroid reciprocal to the coordinate system of freedom. The five forces must therefore intersect both the screws of zero pitch on the cylindroid. We therefore have as a necessary condition that two straight lines can be drawn which intersect all the five forces. Four of the forces will determine the two lines, and therefore the fifth force may enjoy any liberty consistent with the requirement that it also intersects the two lines. This condition is also a sufficient one, so far as the positions of the forces are concerned.

If P, Q, R, S, T be the five forces, the ratio of P : Q is thus determined.

Let A, B be the two screws of zero pitch upon the cylindroid.

Let X, Y be two screws reciprocal to P, Q.

Let Z be a screw reciprocal to R, S, T.

Construct the screw I reciprocal to the five screws

$$X, Y, A, B, Z.$$

Now the four screws X, Y, A, B are reciprocal to the cylindroid (P, Q); therefore I, which is reciprocal to X, Y, A, B, must lie upon the cylindroid P, Q.

Since A, B, Z are all reciprocal to R, S, T, it follows that I, being reciprocal to A, B, Z, must be coordinate with R, S, T.

Hence I is coordinate with P, Q and also with R, S, T. If, therefore, forces along P, Q, R, S, T equilibrate, the forces along P, Q must compound into a wrench about I. This condition determines the forces along P, Q.

41. *Equilibrium of six forces applied to a rigid body.*—Professor SYLVESTER has shown ('Comptes Rendus,' tome lii. p. 816) that when the six lines P, Q, R, S, T, U are so situated that forces acting along these lines equilibrate when applied to a *free* rigid body, a certain determinant must vanish.

Using the ideas and language of the theory of screws, this determinant is simply the sexiant of the six lines, the pitches of course being zero.

If x_m, y_m, z_m be a point on one of the lines, the direction cosines of the same line being α_m, β_m, γ_m, the condition is therefore

$$\begin{vmatrix} \alpha_1, & \beta_1, & \gamma_1, & y_1\gamma_1 - z_1\beta_1, & z_1\alpha_1 - x_1\gamma_1, & x_1\beta_1 - y_1\alpha_1 \\ \alpha_2, & \beta_2, & \gamma_2, & y_2\gamma_2 - z_2\beta_2, & z_2\alpha_2 - x_2\gamma_2, & x_2\beta_2 - y_2\alpha_2 \\ \alpha_3, & \beta_3, & \gamma_3, & y_3\gamma_3 - z_3\beta_3, & z_3\alpha_3 - x_3\gamma_3, & x_3\beta_3 - y_3\alpha_3 \\ \alpha_4, & \beta_4, & \gamma_4, & y_4\gamma_4 - z_4\beta_4, & z_4\alpha_4 - x_4\gamma_4, & x_4\beta_4 - y_4\alpha_4 \\ \alpha_5, & \beta_5, & \gamma_5, & y_5\gamma_5 - z_5\beta_5, & z_5\alpha_5 - x_5\gamma_5, & x_5\beta_5 - y_5\alpha_5 \\ \alpha_6, & \beta_6, & \gamma_6, & y_6\gamma_6 - z_6\beta_6, & z_6\alpha_6 - x_6\gamma_6, & x_6\beta_6 - y_6\alpha_6 \end{vmatrix} = 0.$$

If $\alpha_1, \beta_1, \gamma_1$ be considered variable, all the other quantities remaining constant, we have the following theorem due to MÖBIUS:—

All the lines which can be drawn through a given point in involution with five given lines lie in a plane.

This is in reality only a particular case of the following theorem, which appears from equating the general expression for the sexiant to zero:—

All the screws of given pitch which can be drawn through a point so as to be coordinate with five given screws lie in a plane.

A single screw X must be capable of being found which is reciprocal to all the six screws P, Q, R, S, T, U. Suppose the rigid body were only free to twist about X, then the six forces would not only collectively be in equilibrium, but severally would be unable to stir the body only free to twist about X.

In general a body which was able to twist about six screws (of any pitch) would have perfect freedom; but the body capable of rotating about each of the six lines P, Q, R, S, T, U, which are in involution, is not perfectly free, since practically we have only five disposable coordinates.

If a rigid body were perfectly free, then a wrench about any screw could move the body; if the body be only free to rotate about the six lines in involution, then a wrench about every screw (except X) can move it.

The existence of the single screw X is the characteristic feature of six lines in involution which the theory of screws makes known to us.

The *conjugate axes* of Professor SYLVESTER (p. 743) are presented in the present system as follows:—Draw *any* cylindroid which contains the reciprocal screw X, then the two screws of zero pitch on this cylindroid are a pair of conjugate axes. For a force on any transversal intersecting this pair of screws is reciprocal to the cylindroid, and is therefore in involution with the original system.

Draw any two cylindroids, each containing the reciprocal screw, then all the screws of the cylindroids form a coordinate system with three degrees of freedom [art. 84]. Therefore the two pairs of conjugate axes, being four screws of zero pitch, must lie upon the same hyperboloid. This theorem is also due to Professor SYLVESTER.

The cylindroid also presents in a very clear manner the solution of the problem of finding two rotations which shall bring a body from one position to any other given position. Find the twist which would effect the desired change. Draw any cylindroid

through the corresponding screw, then the two screws of zero pitch on the cylindroid are a pair of axes that fulfil the required conditions. If one of these axes were given, the cylindroid would be defined and the other axis would be determinate.

42. *On a property of a body possessing four degrees of freedom.*—When a body has four degrees of freedom it is able to twist about every screw in space reciprocal to a certain cylindroid. Thus through any point a cone can be drawn the generators of which are perpendicular to the generators of a cylindroid; and by assigning the proper pitch to each of the cone generators we have a reciprocal screw [art. 89]. If, however, the point had been selected upon the double line of the cylindroid, the cone vanishes into a line and the pitch becomes indeterminate, thus giving the following general theorem in kinematics:—

When a body has four degrees of freedom, there is always one screw about which the body can be twisted whatever be the pitch assigned to that screw.

43. *A point* P *is always to be moved in a plane* A, *determine the "locus" of screws about which* P *may receive small twists.*—If the point P formed a portion of a rigid body M, the condition imposed on P would still leave M five degrees of freedom. There is therefore one screw, S, reciprocal to the freedom; S is to be found by the condition that a wrench about S shall be unable to disturb M. The only wrench which would not disturb M must be a force through P normal to A. The reciprocal screw S must therefore lie along this normal and have zero pitch. Any screw reciprocal to S will fulfil the required condition.

44. *A point* P *is always to move along a line* AB, *determine the "locus" of screws about which* P *may receive small twists.*—A rigid body attached to P would have four degrees of freedom. The reciprocal cylindroid in this case reduces to the plane through P normal to AB. All the screws in this plane pass through P and have zero pitch.

The "locus" required is evidently that of screws reciprocal to the cylindroid which has assumed this simple type.

45. *Generalization of a theorem due to* M. Chasles.—If a system of forces be resolved into two forces, the volume of the tetrahedron of which the two forces are opposite edges is constant. This may be generalized into the following:—If a system of forces be resolved into wrenches about two screws of equal pitch, the volume of the tetrahedron of which the wrenching forces are opposite edges is constant.

Let θ_1, θ_2, θ_3 be three cocylindroidal screws about which wrenches X, Y, Z equilibrate.

The volume of the tetrahedron formed by X, Y is

$$\tfrac{1}{3}m\,(\sin 2\theta_2 - \sin 2\theta_1)\sin(\theta_1-\theta_2)\,XY;$$

but

$$\frac{X}{\sin(\theta_2-\theta_3)}=\frac{Y}{\sin(\theta_3-\theta_1)}=\frac{Z}{\sin(\theta_1-\theta_2)};$$

thence the volume reduces to

$$\tfrac{2}{3}mZ^2\cos(\theta_1+\theta_2)\sin(\theta_2-\theta_3)\sin(\theta_3-\theta_1).$$

If the two screws θ_1, θ_2 have equal pitches (ϱ), we must have $\theta_1=-\theta_2$, and

$$\varrho=p+m\cos 2\vartheta_1,$$
$$\varrho_1=p+m\cos 2\vartheta_2;$$

whence the expression for the volume of the tetrahedron is finally

$$\tfrac{1}{3}(\varrho_1-\varrho)Z^2.$$

If $\varrho=0$ we have M. CHASLES's theorem. This generalization might have been deduced at once from the original theorem by the remark of [art. 82], that any coordinate system of screws is still a coordinate system when the pitches of all the screws have received a constant addition.

POSTSCRIPT.

Received January 27, 1874.

At the time the foregoing paper was read the writer was not aware of the close connexion between the Theory of Screws and the recent geometrical researches on the Linear Complex. His attention was kindly directed to this point by Dr. FELIX KLEIN, at the Bradford Meeting of the British Association.

PLÜCKER, in his 'Neue Geometrie des Raumes,' p. 24, thus introduces the word "Dyname:"—"Durch den Ausdruck 'Dyname' habe ich die Ursache einer beliebigen Bewegung eines starren Systems, oder, da sich die Natur dieser Ursache wie die Natur einer Kraft überhaupt, unserem Erkennungsvermögen entzieht, die Bewegung selbst: statt der Ursache die Wirkung, bezeichnet." Although it is not very easy to see the precise meaning of this passage, yet it appears that a "Dyname" may be either a twist or a wrench (to use the language of the present paper).

On page 25 (*loc. cit.*) we read:—"Dann entschwindet das specifisch Mechanische, und, um mich auf eine kurze Andeutung zu beschränken: es treten geometrische Gebilde auf, welche zu Dynamen in derselben Beziehung stehen, wie gerade Linien zu Kräften und Rotationen." There can be little doubt that the "geometrische Gebilde," to which PLÜCKER refers, are what we have called screws.

The surface used in the 'Theory of Screws' (page 161), and also in Phil. Mag. vol. xlii. p. 181, under the name of the cylindroid, had been already described by PLÜCKER, p. 97, *loc. cit.* PLÜCKER arrives at this surface by the following considerations:—Let $\Omega=0$ and $\Omega'=0$ represent two linear complexes of the first degree, then all the complexes formed by giving μ different values in the expression $\Omega+\mu\Omega'=0$ form a system of which the axes lie on the surface $z(x^2+y^2)-(k^2-k_0)xy=0$. The parameter of any complex of which the axis makes an angle ω with the axis of x is $k=k^2\cos^2\omega+k_0\sin^2\omega$. The writer was informed by Dr. KLEIN that PLÜCKER had also constructed a model of this surface (see note to p. 98).

PLÜCKER does not appear to have contemplated the mechanical and kinematical pro

perties of the cylindroid; but it is worthy of remark that the distribution of pitch which is presented by physical considerations is exactly the same as the distribution of parameter upon the generators of the surface, which was fully discussed by PLÜCKER in connexion with his theory of the linear complex.

On p. 130 (*loc. cit.*) PLÜCKER has arrived at the equation $k_1x^2+k_2y^2+k_3z^2+k_1k_2k_3=0$. This hyperboloid is the locus of lines common to three linear complexes of the first degree. The axes of the three complexes are directed along the coordinate axes, and the parameters of the complexes are k_1, k_2, k_3. On p. 132 we have the theorem that the parameter of any complex belonging to the "dreigliedrigen Gruppe" is proportional to the inverse square of the parallel diameter of the hyperboloid.

PLÜCKER had thus shown what is geometrically equivalent to some kinematical theorems proved in the 'Theory of Screws,' p. 203. When a body has three degrees of freedom, it may be rotated about all the generators of one system on the hyperboloid,

$$ax^2+by^2+cz^2+abc=0;$$

the body may also be twisted about three screws of pitches a, b, c along the axes, and the pitch of every other screw about which it can be twisted must be proportional to the inverse square of the parallel diameter in the pitch hyperboloid.

The conception on which the writer had founded the criterion of Reciprocal Screws ('Theory of Screws,' p. 166) had been previously employed by Dr. KLEIN (Math. Ann. Band iv. p. 413). "Es lässt sich nun in der That ein physikalischer Zusammenhang zwischen Kräftesystemen und unendlich kleinen Bewegungen angeben, welcher es erklärt, wie so die beiden Dinge mathematisch coordinirt auftreten. Diese Beziehung ist nicht von der Art, dass sie jedem Kräftesystem eine einzelne unendlich kleine Bewegung zuordnet, sondern sie ist von anderer Art, sie ist eine *dualistische*.

"Es sei ein Kräftesystem mit den Coordinaten Ξ, H, Z, Λ, M, N, und eine unendlich kleine Bewegung mit den Coordinaten Ξ', H', Z', Λ', M', N' gegeben, wobei man die Coordinaten in der im § 2 besprochenen Weise absolut bestimmt haben mag. *Dann repräsentirt*, wie hier nicht weiter nachgewiesen werden soll, *der Ausdruck*

$$\Lambda'\Xi+M'N+N'Z+\Xi'\Lambda+H'M+Z'N$$

das quantum von Arbeit, welche das gegebene Kräftesystem bei Eintritt der gegebenen unendlich kleinen Bewegung leistet. Ist insbesondere

$$\Lambda'\Xi+M'H+N'Z+\Xi'\Lambda+H'M+Z'N=0,$$

so leistet das gegebene Kräftesystem bei Eintritt der gegebenen unendlich kleinen Bewegung keine Arbeit. Diese Gleichung nun repräsentirt uns, indem wir einmal Ξ, H, Z, Λ, M, N, das andere Mal Ξ', H', Z', Λ', M', N' als veränderlich betrachten, den Zusammenhang zwischen Kräftesystemen und unendlich kleinen Bewegungen."

Dr. KLEIN has also shown (Math. Ann. Band ii. p. 368) that, if the principal axes of two complexes are at a perpendicular distance Δ, and are inclined at an angle φ, while the parameters of the complexes are k and k', the "simultaneous invariant" of the two

complexes is $\Delta \sin\varphi + (k+k')\cos\varphi$. If this invariant vanish, the two complexes are in "involution." The physical interpretation of this expression is found in what we have called the virtual coefficient of a pair of screws of which the pitches are k, k', the perpendicular distance Δ, and the angle $\pi-\varphi$. If the virtual coefficient vanish, the screws are reciprocal.

Dr. KLEIN (Math. Ann. Band ii. p. 204) has introduced the conception of six fundamental complexes, each pair of which are in involution. If the principal axes of the six complexes be regarded as screws of which the parameters of the corresponding complexes are the pitches, then these six screws will form what has been termed a coreciprocal group in the present paper.

In this postscript the writer's desire has been twofold. First to do justice to those authors from whose works he had found, after he had written these papers, what ought to have been ascertained before, namely, that in a few points he had merely rediscovered (though usually from a different point of view) what was already known. Second, to point out the intimate connexion which exists between the physical conceptions of the Theory of Screws and the modern geometrical speculations on the linear complex.

III. *On the Organization of the Fossil Plants of the Coal-measures.*—Part V. Asterophyllites. *By* W. C. Williamson, *F.R.S., Professor of Natural History in the Owens College, Manchester.*

Received May 17, 1873,—Read June 19, 1873.

In 1871 I published, in the 5th volume of the third series of the 'Memoirs of the Literary and Philosophical Society of Manchester,' a description of a new Cryptogamic fruit, to which I gave the provisional name of *Volkmannia Dawsoni*. One of the most remarkable features of this curious organism was seen in a transverse section of its central vascular axis, which appeared to be a triangular structure with truncated angles. When the above memoir was read (February 7, 1871) I had seen no stem having a similar structure; but after a careful study of the fruit, I said, "The verticillate arrangement of its bractigerous disks and bracts suggests the probability that we must seek for the parent plant amongst such as have their foliage arranged in corresponding verticils; and if this law of association be a sound one, we are apparently shut up to the three genera, *Asterophyllites*, *Annularia*, and *Sphenophyllum*"*. After reviewing the various features of these plants, I arrived at the conclusion that "it is the fruit either of *Asterophyllites* or of *Sphenophyllum*; and, judging from the general aspect of its bractigerous disks, I am more disposed to identify it with the former than with the latter"†. I was not at that time aware that Professor Renault, of Cluny, had found a stem at Autun having a similar triangular axis, and which he also had referred to *Sphenophyllum* ('Comptes Rendus,' 1870). My attention was at once directed to the discovery of true stems having a similar structure, and I soon found a few examples in the cabinets of Messrs. Butterworth and Whittaker of Oldham. But these gave me no such evidence as I required respecting the nature of the foliage with which these stems had been clothed, neither did any of them afford proof even that the plant had been a jointed one. I then turned to the coal-seam from which the fossil strobilus was obtained, and was at length rewarded by the discovery of a cluster of stems, each one of which was clothed with its peculiar bark, having the enlarged lenticular nodes exquisitely preserved, and the verticils of Asterophyllite-leaves radiating from the thin margin of each nodal disk in precisely the same way as the bracts had done from the corresponding portions of the fruit already described. What had previously been an inferred probability thus became an established fact; hence I have now no hesitation whatever in referring both the above fruit and the stems which I am about to describe to the genus *Asterophyllites*. Amongst the other specimens sent to me from Burntis-

* *Loc. cit.* p. 34. † *Ibidem*, p. 37.

land by G. GRIEVE, Esq., were some stems having a similar structure, but of larger size than any I had met with in Lancashire; but, though exquisitely preserved, the fragments were internodal ones, and not sufficiently long to exhibit the structure of the nodes.

I shall adopt in this memoir the plan which I followed in the third of this series of monographs, in which I treated of the Burntisland Lepidodendroid plants; I shall commence with the youngest twig of the Lancashire type which I have met with, and then trace the gradual development of the organism through the addition of successive exogenous growths by which the twig was thickened and finally converted into branch and stem.

The young Twig.—Plate I. fig. 1* represents a transverse section of a condition of which I have obtained several examples, and which evidently exhibits a twig or branch in its very young state; *c* is a central vascular bundle which has a maximum diameter of ·016† between the most distant tips of the slender divergent angles of its triangular section, the entire section, including the bark, being about ·06 in diameter. The bundle consists of a group of vessels, not arranged in any regular order; the maximum diameter of the largest of them is not more than ·0029, whilst that of the smallest is not more than ·0007. The triangle, as seen in this section, is very slender, its arms being long in proportion to their breadth. The larger vessels occupy its central portion, the smallest being found at the extremities of each of the arms.

A double inner bark (*g*) has disappeared from this specimen, though traces of it are seen in some similar but otherwise less perfect ones. It appears to be of the same nature as that which will be described in the more matured stems. This double inner layer has been surrounded by an outer bark (*k*), with a maximum thickness of about ·016. It is composed of compactly aggregated cells, with very thick walls, and which are of about the same diameters as the central vessels. A few of these cells have a diameter of ·0029; many others are yet smaller, whilst the great majority range between ·0015 and ·0007. A remarkable feature of this outer bark is its indentation, on each of its three sides, by a deep constriction (*k'*). These constrictions correspond with deep grooves which channel the surface of each of the internodes, but which disappear as they approach the nodes‡. The position of each groove is opposite each concave lateral surface of the central vascular bundle, when the latter has not been displaced from its normal

* I may observe that, to facilitate the comparison of these specimens, all the following figures have been drawn to the same scale, being enlarged 30 diameters, 1, 2, 3, 4, 5, 9, 10, 11, 12, 14, 15, 16, 18, 19, 20, 21, 27, 34, 45, 50, 51.

† These and all subsequent measurements are given in decimal parts of an inch.

‡ I find precisely similar grooves in a small plant of *Sphenophyllum Schlotheimi* (Plate V. fig. 26) in the museum of the Owens College. In this instance the grooves (Plate V. fig. 26, *k'*) are never vertical to any of the leaves of a verticil, but appear always to be intermediate between two of them. It also appears, from the same specimen, that the positions of these grooves do not change in alternate nodes, but that they form continuous lines down the stem, interrupted only at the nodes. Plate V. fig. 26, *k'*, is an enlarged view of part of the internode and lower node of *k*.

position. One of the grooves (k'') is, in this example, a double one, being divided by a central longitudinal ridge. This is a feature which often reappears in older branches.

Plate I. fig. 2 represents another section from the same coal-seam, which may possibly belong to a different species from the last one; at all events it differs somewhat both in size and form. The central vascular triangle (c) is much more robust, having a maximum diameter of ·05, the extreme diameter of the entire section, including the bark, being ·1. The largest vessels near the centre of the vascular triangle are about ·009 in diameter, those of the angles being smaller, whilst each of the extreme tips of the latter terminates in a cluster of very minute vessels, which do not exceed ·0006. External to the vessels of the central bundle, which, as in the last specimen, are not arranged in any linear order, is a single investing row of vessels (d) applied to each of the concave sides of the triangle, and arranged in a linear series; of these, the largest vessels occupy the centre of each concave line, the rest becoming smaller as they approach the apex of each angle. This is the first of a remarkable series of exogenous layers which successively invest the central bundle, and convert the triangular outline of this vascular axis into a circular one. The double inner bark (g) is again wanting. The outer one exhibits the same aspect as in the previous example, only the three lateral furrows (k') are larger and more concave than in Plate I. fig. 1, or indeed than in any other specimen that I have seen.

In Plate I. fig. 3 we have a transverse section of the vascular axis of another specimen, in which the exogenous zone (d) has received a second layer of vessels; and it will now be observed that whilst each of these concentric growths, composed of one lineal series of vessels, maintains its distinct individuality, nearly every vessel of the outer circle is placed in a direct line with one belonging to the circle on the inner side of it, the two combining as parts of a radiating series. This double concentric and radial arrangement is yet better seen in Plate I. fig. 4, where a third circumferential growth has been added. The central triangular axis is now larger than in the primary twig (Plate I. fig. 1). The enlargement is partly due to an expansion of each of its component vessels, and partly to an addition to their number. The extreme diameter of the axis between point and point is ·06, and that of its largest and more central vessels about ·005. Between these larger vessels and the exogenous layers we now find a thin series of very small vessels, forming the peripheral portion of the triangular axis, to which structure they certainly belong. These appear to me to have been developed (I know not how) subsequently to the appearance of the first exogenous layer. Of this latter portion we have three growths, of which the central one is, in this instance, the smallest. The maximum thickness of this united triple series is now ·0125; and, though at one point there is a slight irregularity in the arrangement of the vessels of the two outer rows, they maintain in general the radiating order already spoken of. Owing to the constant large size of the more central vessels of each row, the original triangle with concave sides is now being converted into a convex one, the greater portion of each of its lateral boundaries having become somewhat oblate. In this specimen we have the middle

bark (*h*) preserved. It consists of a very delicate parenchyma, the cells of which have a diameter of about ·0015. These are somewhat cubical, and are arranged in linear series, which radiate from the exogenous layers of the wood to the outer bark. This middle cortex does not appear to merge gradually with the outer one, but is separated from it by a somewhat sharply defined line. The outer bark (*k*) exhibits the same features that were seen in the previous specimens. The thick-walled cells are not arranged in linear series, as in the middle bark, but as in ordinary irregular parenchyma. In this example the three external internodal grooves (*k'*) are *all* double ones, the only instance in which I have found this to be the case.

Before proceeding to trace the further development of these stems, it will be convenient to examine the aspect which longitudinal sections of them present at this stage of their growth. This is shown in Plate I. fig. 5, which represents a section made through the centre of the triangular axis, and crossing one node and parts of two internodes. The vessels of the central triangle (*c*) are undistinguishable in this section, save by their central position, from those of the exogenous layers, *d d*. They are all of the reticulated type, seen more highly magnified in Plate I. fig. 6. I have satisfied myself that these reticulations are more distinct on the two sides of each vessel parallel with the direction of the medullary rays than on the sides at right angles to the latter. They are due to modifications of spiral deposits of lignine, and must not be confounded, as they have often been by WITHAM and some of his successors, with the disks of so-called glandular or discigerous fibres of the Conifera.

Though the delicate middle bark (*h*) is rarely well preserved, I have in my cabinet sufficient examples of it to show that it appears, in longitudinal sections, in the form of long narrow prismatic cells (Plate I. fig. 7), with nearly, if not absolutely, square ends. They vary considerably both in length and in diameter. They are usually from ·012 to ·006 long, the former much more frequently than the latter, the shorter and broader ones occurring near the nodes. One of my sections shows traces of the thin layer of parenchyma (*g*) within this prismatic layer. The outer bark (fig. 5, *k*) is now seen to consist of long, narrow, thick-walled prosenchyma. I have found it difficult to ascertain the greatest length of the component cells, but I have traced them as far as from ·03 to ·035: as is the case with the cells of the middle bark, they become shorter in the neighbourhood of each node (*k'*). At this latter point the bark swells out rapidly into a broad lenticular disk, from the thin margins of which verticils of long narrow leaflets (*l*) extend in very regular order. At these points the cells of the outer bark assume a very peculiar arrangement. Those which ascend from the internode below the nodal disk (Plate I. fig. 5, *k''*) proceed outwards in a very definite series, forming the lower portion of the disk, whilst those which descend in slightly curving lines (*k'*) from the internode above terminate rather abruptly against the curving lines of those of the lower series. These cellular tissues are prolonged uninterruptedly into the leaves, which I shall describe more fully by-and-by. In several of my specimens I have noticed that, a little below each node, a portion of the cells of the middle bark curve

upwards and outwards, as if to accompany some vascular bundle on its way to the leaves. This is seen in Plate I. fig. 5, *h'*. But in no one instance have I found the slightest trace of an actual vessel accompanying them. Though my sections, made in various directions, are nearly a hundred in number, I cannot trace a solitary example of a divergent vessel. I will not venture to affirm that the leaves of this plant, like those of the living *Psilotum*, were non-vascular; but it is certain that I cannot detect a trace of a vessel, either in the cellular leaves and disks or diverging from the vascular axis in the direction of those organs. In all the transverse sections the vessels retain the undisturbed arrangement shown in the figures, whether such sections are made at the nodes or at the internodes*. It is obvious that these stems have not been jointed or articulated in the same way as the *Calamites*. In the latter plants, the existence of each node alters the entire arrangement of all the tissues, vascular and cellular, to the very centre of the stem. On the other hand, the appearance of articulations seen in the common examples of *Asterophyllites* found in the shales, is an altogether different thing. It is wholly due to modifications of the cortical tissues, which are expanded into disks by a centrifugal development, which does not visibly affect the true vascular axis of the stem. This is an important physiological distinction, to be borne in mind when endeavouring to determine the systematic relations of these plants. The appearance of the cells of the outer bark, as seen in longitudinal sections of the internodes, is represented in Plate II. fig. 8. Each internode is rather more than half an inch in length.

Returning to the circumferential growth of the stem, I find that layers were successively added to the preexisting ones in precisely the same regular manner as seen in the examples already figured. Plate II. fig. 9 represents one of the finest sections I have met with, and which Mr. Brinkwater, late of Salford, kindly permitted me to make from a specimen which he had obtained from the Oldham deposits. In this example the central triangle has now increased to a maximum diameter of ·08, the result apparently of a general increase in the dimensions of the smaller vessels, because the largest ones still retain the maximum diameter of ·005 already observed in Plate I. fig. 4. The exogenous zones are now seven in number on two sides of the stem, and eight on the third. With the exception of this intercalation of a zone, the vessels of these exogenous layers maintain the same remarkable regularity of arrangement, both radially and circumferentially, that I have already described. The bark (*h*) remaining in this specimen is but a portion of the middle layer. Plate II. fig. 10 represents a slightly crushed example, in which we find eight exogenous layers on one side, nine or ten on the second, and eleven on the third, indicating that, as the stems increased in diameter,

* Since the above description was written, Professor Renault's important memoir, entitled "Recherches sur l'Organisation des *Sphenophyllum* et des *Annularia*," presented to the Académie des Sciences of Paris in May 1870 and 1873, has reached me. It proves very clearly that vascular bundles are given off from the three angles of the central triangular axis, and that these pass outwards, through the exogeneous zones and bark, to the leaves. Hence there is little doubt that the arrangement represented in my fig. 5, *h'*, indicates the condition suggested in the text.—February 5, 1874.

the irregularity of these growths, owing to some of the layers being crescentic ones, instead of forming perfect circles, became greater. The bark (*h*) in this example also consists of the middle portion. Both the last figures show very clearly the radiating arrangement which the long prismatic cells of this tissue exhibit when divided transversely.

In Plate II. fig. 11 we not only advance yet further, but we now discover that a change has been introduced into the method of growth. The specimen, for which I am indebted to Mr. WHITTAKER, of Oldham, has been somewhat crushed, but this in no way interferes with the easy interpretation of it. Its central portion has developed in the manner described in the case of the preceding examples, the triple arrangement of large vessels alternating with corresponding radiating groups of smaller ones. But when, by successive growths, the vascular axis attained the condition of a perfect circle, an altogether new mode of procedure has manifested itself. The preexisting axis has become the starting-point for a new series of layers, in which all the vessels are abruptly reduced to a very small and nearly equal diameter. Hence the older and the newer growths are separated from one another by a very distinct circular line of demarcation. From the new starting-point the vessels have been disposed as before, but they have increased slowly in size as each succeeding layer invested its predecessors. The entire stem in this instance has only the same diameter as that of Plate II. fig. 9; but I have no reason for supposing, on this ground, that these two belong to distinct plants. I have elsewhere obtained clear evidence that stems having the same diameter may nevertheless have attained to very different stages of growth. Plate II. fig. 12 is a second section seen in the same slide as the last, and is evidently part of another stem or branch of the same plant. The remarkable difference in the sizes of the larger vessels of the central growths seen in the three radiating clusters of them, as compared with the very small vessels composing the later and more external series, is here very striking. At first sight it is difficult to avoid the conviction that the difference is due to some increase in the size of the former, as compared with the corresponding ones of the previous examples; but such is not the case. The largest of those in Plate II. fig. 12 is not more than ·005, which we have already found to be a usual size. The difference occasioning the striking contrast has resulted from the very small dimensions of those composing what may be designated the secondary growths. These last two examples exhibit the highest degree of development which I have met with in any stem found in the Lancashire district *.

I succeeded in obtaining a very accurate tangential section of the exogenous portion of Plate II. fig. 9, with the object of ascertaining the exact distribution of the medullary rays separating the several radiating laminæ of vascular tissue. Plate II. fig. 13 represents a portion of this section. The medullary rays are numerous, but of

* Since the above two figures were drawn, I have met with a very perfect example of the same stage of growth, in which the tissues are wholly undisturbed by external pressure. It fully confirms my conclusion that the conditions just described are not due to any such disturbance, excepting so far as the accidental displacement of some of the tissues is concerned, but are characteristic of this stage of the growth of the stem.

the simplest possible type. They rarely consist of more than two, and often of but one, linear series of cells; when the former, the two or more cells are rarely exactly vertical to each other; but, as shown in the drawing, the two are somewhat inclined, often in opposite directions, to the vertical line. I have not been able to detect any of these cells in the central triangle; they appear to be confined to the exogenous layers. Of course I am aware of the old argument likely to be renewed against my nomenclature, and to hear the objection that there can be no medullary rays where there is no medulla; but the obvious reply is that a medulla is not necessary. The rays in the thick, ligneous, pithless roots of an exogenous forest tree are the homologues of the similar structures in its stem, notwithstanding the entire absence of a pith from the former organ, just as they continue to be rays in the stem itself when some aged oak has become hollowed into a huge cylinder.

The Leaves.—Having had the good fortune to discover a number of stems invested by a perfect outer bark, with well-preserved nodes at intervals of about half an inch, I at once commenced a vigilant search for leaves, and I was soon rewarded beyond my most sanguine expectations. Having made a vertical section through one stem which exhibited a perfect node, from each side of the thin lenticular margin of which there projected a slender leaf-like organ, as seen in Plate I. fig. 5 and Plate III. fig. 15, I proceeded cautiously to grind away the matrix external to the bark of the same specimen, in hope of discovering evidences of a verticillate arrangement. I soon came upon sections of four leaf-like organs, arranged in regular verticillate order. These sections were of the same type as those represented in Plate III. figs. 14 & 17. But proof was needed that these objects actually belonged to the node seen on the opposite side of the thin slab of stone. To obtain this, I continued to grind until I reached the margin of the disk, which presented itself in the form represented by Plate III. fig. 14, *l l*. The actual relation of the leaf-like organs to the disk was now thoroughly established.

The sections of the individual leaves, as they appeared in the regularly disposed verticil before I ground away that portion of the section, were similar to those still seen at Plate III. fig. 14, *m* & *m'*, but in an inverted position. These were gradually traced backwards to the corresponding portions (*l l l*) of the same figure, in which we have a section of part of the margin of the foliar disk from which the leaves sprang, and the under surface of which has already become grooved at two points, defining the bases of the three leaves *l l l*. At *m''* we have a longitudinal section of one leaf from the side of the disk, whilst at *m' m'* we have the inverted tips of two of the leaf bases *l l*, which had been bent inwards by the pressure of the earthy matrix, and consequently were divided a second time by the same section. At *m'''* is a section of another leaf, which, having been intersected obliquely, is longer than would be the case with an exactly transverse section. Plate III. fig. 15 is a longitudinal and slightly tangential section of the outer bark (*k*) of a stem passing through a node. Here we have a considerable portion of a leaf (*m*) intersected longitudinally. Its upper portion has been bent back upon the stem by external pressure, in the manner already referred to in my description of Plate III.

fig. 14. The somewhat clavate appearance of the free extremity of the leaf (*l*) is due to a slight accidental obliquity in the section at that point due to another flexure of the leaf carrying it out of the plane of the section. The thickness of this leaf ranges between ·0043 and ·0086.

Besides the evidence afforded by the examples just described, I have obtained several transverse sections made more or less in the plane of the foliar disk, and which display the verticillate arrangement of the divergent leaves in the most definite manner. One of these is represented in Plate III. fig. 16; it will be observed that, in the expanded portion of the disk (*k'*), the cells of the tissue evince somewhat more of a tendency to assume a radial linear arrangement than is usual in the outer bark. The bases of six linear leaves (*m m*) are seen radiating from the lower and left-hand sides of the section with considerable regularity. The breadth of each leaf is about ·021. I discover no trace of vascular tissues, either in the leaves themselves or amongst the cellular tissues of the foliar disk from which they arise; and it is a further curious fact that no vacant spaces exist, either in the outer (*k*) or middle (*h*) bark, such as are common amongst the *Lepidodendra*, where they were originally occupied by vessels that have disappeared. The transverse section of the central vascular axis, at this important point of the stem, exhibits its usual appearance; and it is obvious that, if any of the very large vessels composing it had ever been deflected into the leaves, their large size must have made them conspicuous objects, whilst their divergence must have left some considerable gap in the regularly concentric and radial lines of vessels; but no traces of such conditions have been discovered. The leaves appear as if they consisted wholly of cells, which, as we have already seen from Plate I. fig. 5, are mainly, if not entirely, derived from the outer bark*. Unfortunately these foliar tissues are much disorganized and rendered indistinct through carbonization, consequently the exact arrangement of their component elements has yet to be made out; at the same time I have clear evidence that the central part of the leaf, at least, was chiefly composed of long, narrow, parallel-sided cells, and that the remainder was occupied by a somewhat coarse parenchyma.

I have stated that the diameter of the bases of the leaves in Plate III. fig. 16 is ·05. This is exactly the breadth of the bases of the leaves in a very distinct verticil of *Asterophyllites* which I possess in an ironstone nodule from the Lower Coal-measures of the Oldham district, and which I think belongs to the same species as the sections described. The leaves in the specimen referred to are more than three quarters of an inch in length, and with twenty-seven or twenty-eight in the verticil. In the example represented in Plate III. fig. 16, I calculate, from their angle of divergence, that there

* Since the above description was written, Professor Renault has informed me that he can demonstrate, in one of his stems of *Sphenophyllum*, eighteen vascular bundles given off from the central triangular axis, and passing outwards at a node to be distributed to six leaves, each of the latter receiving three primary bundles which pass outwards to three corresponding teeth at the extremity of the leaf. Of course this observation renders it exceedingly probable that the apparent absence of small vascular bundles from my specimens is due to mineralization and is not a primary feature of the plant. See the note on p. 45.

have been about twenty-four in each verticil. This close correspondence between the leaves in my sections and the size, number, and arrangement of those of ordinary examples of *Asterophyllites* is further maintained if we examine individual sections belonging to the former series. Plate III. fig. 17 represents a group of such sections, brought together from various specimens in my cabinet. It will at once be seen that these are generally identical with similar ones (*m*, *m'*) associated with Plate III. fig. 14. The diameters of these sections vary from ·05 to ·025, variations which would, of course, exist according to whether the sections were made obliquely near the broader part of the leaf or near its narrow and almost linear tip. In every instance the leaf consists of a single, very thick midrib, with two somewhat succulent and slightly incurved lateral margins. This is exactly the structure of the leaves preserved in ironstone nodules now in my cabinet, the Asterophyllitean character of which no one would venture to dispute.

Amongst the numerous interesting specimens from Burntisland in Fifeshire for which I have been indebted to G. Grieve, Esq., was one which contained two stems, similar in many respects to those which I have just described. Careful search in other portions of the Burntisland rock revealed other specimens, showing that this plant had passed through stages of growth similar to those of our Lancashire type. But the distinctness of the two is demonstrated by the circumstance that, in the Oldham specimens, the vessels of the central axis are always *reticulated*, whilst those of the Burntisland species are as invariably *barred*.

Plate III. fig. 18 is a transverse section of a young twig, obviously corresponding to Plate I. fig. 1. Its greatest diameter is about ·017, and that of the largest of its vessels about ·0029, dimensions which closely correspond with those of the Oldham plant. The bark (*k*) exists in this specimen, and I can readily make out its cellular structure; but its outline is not so clearly defined as I could have wished, owing to the density of the vegetable mass in which it is imbedded. I see traces of the lateral grooves, especially at *k*, but these are not very definite. Some other specimens, however, in the same stage of growth as fig. 18, exhibit these grooves very definitely, almost as much so as Plate I. fig. 1. Plate III. fig. 19 represents one of these, in which the triangle is somewhat more robust than in the previous example. The grooves (*k'*) are here clearly shown on two sides of the specimen, the third having been accidentally broken off in making the section. The maximum diameter of this twig, including the bark, is ·054, or rather more than Plate I. fig. 1. It will be noticed that, in this specimen, the angles of the central triangle are distinctly truncate, reminding us of the same condition in *Volkmannia Dawsoni*. The next stage of growth in which I find this plant is represented in Plate III. fig. 20. The central triangle has now enlarged to ·054, and the diameter of its largest vessels to ·005. The exogenous layers are three or four in number. In their general arrangement these layers correspond with those of the Lancashire type, but with the apparently specific distinction that the vessels of the former are much more uniform in size and less regular in their arrangement than those of the latter.

These peculiarities, which recur in every one of the Burntisland specimens, result in an absence of that regular parallelism of the successive concentric layers which renders the young states of the Oldham plant so remarkable for their symmetric beauty. The bark now distinctly exhibits its separation into an inner (*g*), a middle (*h*), and an outer layer (*k*). The tissues of the two former are almost entirely destroyed, but the latter obviously consists of the thick-walled prosenchyma characterizing the Lancashire examples. A series of intermediate conditions conducts us to the magnificent stem of which a section is represented in Plate IV. fig. 21. This stem, inclusive of the bark, has a maximum diameter of ·34, that of the ligneous zone is ·25, that of the primary, or inner, circular zone is ·091, and that of the primary triangular axis is ·063. All the large central vessels of the triangle have a maximum diameter of ·0053, or nearly double the size of those constituting the corresponding portion of the young twig, Plate III. fig. 18. The most conspicuous features to be noticed in this noble section are the extreme distinctness of the change in the growth of the exogenous layers, to which I have already directed attention when describing the Oldham example, Plate II. fig. 11, and a distinct lacuna near the extremity of each arm of the central triangle, indicative of a longitudinal canal at each of those points. We here see that the primary exogenous additions were made in the way already illustrated by Plate III. fig. 20. But when the ligneous axis of this stem attained an almost perfectly cylindrical form and a diameter of ·091, it experienced, through some unknown agency, an interruption to the regular continuity of its exogenous growths. Whatever the cause of this check to vegetative action was, it passed away, and circumferential growth was resumed, but under some altered conditions. The new laminæ continued to be developed along radiating curves, which followed the same directions as before; those which were continuous with the three sets of small vessels that diverged from the corresponding angles of the central triangle were little, if at all, altered; but the case is otherwise with the three intervening portions. Here the *large* vessels of the primary exogenous layers terminated suddenly, giving place to very small ones; so that the innermost portions of the newer growths consisted of concentric vascular zones composed of vessels of nearly equal size throughout the greater part of each ring. As further successive additions were made, these new wedges became slowly broader, until, at their peripheral portions, the vessels of such as radiated from the *concavities* of the central triangle opposite to *d d d* became almost as conspicuously larger than those proceeding from the *angles* of the triangle opposite to *d' d' d'*, as was the case in the more central primary portions of the woody axis.

The bark in this stem displays very clearly its separation into an indistinctly double inner layer (*g* and *h*) and an outer one (*k*). We also see very distinctly, especially in the former portion, the individual cells of which it is composed; but these have been flattened and more or less detached from each other by pressure or mineralization, so that their primary arrangement is indistinctly indicated. As in the younger branches, the inner and middle barks have suffered more from these causes than the outer one.

Tangential sections of the exogenous layers (Plate IV. fig. 22) show the intersected extremities of the medullary rays (*f*). These are arranged in precisely the same way as in the Oldham plant (Plate II. fig. 13), the longer axes of the individual cells being frequently inclined (*f'*) to the vertical line separating the two vessels enclosing the ray. The vertical length of the cells in this section varies from ·0017 to ·00087. Each ray frequently consists of a single cell, rarely of more than two or three: such cells are more densely crowded together at fig. 22, *f*, than is usual. Their more general aspect is that presented by those on the left of the same series, and by the second lower series in the figure. On endeavouring to trace these medullary rays in sections made parallel to their course (Plate IV. fig. 23), if we employ low magnifying-powers, we have no difficulty in seeing that such sections are crowded with them; but, owing partly to the small number of the cells disposed vertically in each ray, and partly to the exceeding delicacy of their cell-walls compared with the coarser tissues of the vessels, it is a more difficult thing to decipher their details where high powers are used. Plate IV. fig. 23, *f*, exhibits their appearance under such powers. In many instances we see that the cells in these latter sections appear to be almost cubical; but, owing to their limited number in each ray, and the consequent absence of mutual pressure, their sides are more turgid and their general aspect less mural than in other examples of Coal-measure plants. This difficulty in tracing the details of their form is further increased by the fact that their vertical divisions frequently coincide with those of the vessels, owing to the dimensions of the two being correspondent in the radial direction. Nevertheless there is no real difficulty in demonstrating that these organs exhibit substantially the ordinary structure and arrangement of exogenous medullary rays.

I have already stated that the vascular elements of these stems differ from those of the Oldham plant, in consisting wholly of barred vessels instead of reticulated ones. Their general aspect is seen in the highly magnified portion of one of them represented in Plate IV. fig. 24. But small portions of vessels frequently exhibit the aspect shown in Plate IV. fig. 25; and it is easy to see that the reticulations of Plate I. fig. 6 are but modifications similar to those of Plate IV. fig. 25, only carried to a much greater extent and occurring in every vessel. They are both modifications of fibro-vascular tissue, and have no relationship to the discigerous fibres of the Conifera.

Though my sections of the Burntisland plant are numerous, all the fragments have been short ones, and I have not been fortunate enough to meet with one exhibiting clearly a nodal foliiferous disk with associated leaves. One small and crushed twig, however, appears in one of my slides, which I do not doubt is of this character; but it is too distorted by pressure for me to make much use of it. It appears to me that this species has been characterized by very long internodes.

Attention has been directed to the fact that, amongst the Oldham specimens, I have in no one instance discovered any trace of a lateral deflection of the vessels of the vascular axis either into leaves or into branches. Amongst my Burntisland examples I

have obtained one such section, represented in Plate V. fig. 27. In it we find a branch (*x*) projecting through the bark from the exogenous vascular laminæ which radiate from one arm of the central triangle. It appears as if most of the small vessels of one of the three segments radiating from these several angles were more or less deflected into this divergent appendage*. I am not quite sure that any of the central vessels of the triangle take any part in the new structure; but I have little doubt that the smaller ones constituting the extremity of the angle do so, since I find them to be distinctly deflected in the direction of the new growth. It is obvious that the vascular axis of this branch consists of a mass of vessels, some of which may be derived from the extremity of one angle of the central triangle; but the greater number of which certainly consist of the small vessels of the exogenous layer, which here radiate in converging lines from that angle. We thus obtain our first glimpse of the special use of the three alternating groups of small vessels seen in transverse sections of these stems, viz. to afford the vascular supplies required by some of the lateral appendages†. If this is a correct interpretation, there arises for further inquiry the interesting question whether or not the verticillate lateral branches are grouped with angles of divergence of 120°; or, in other words, are they arranged in ternate verticils, as appears to be most probable? I have found no example like the one just described amongst my Oldham specimens. But this is doubtless accounted for by the fact that nearly every one of the latter, in connexion with which I have found the nodes, have been young terminal twigs, as indicated by the regularity with which the leaf-bases remain *in situ*: consequently all their lateral buds were as yet undeveloped.

The bark of Plate V. fig. 27 obviously exhibits the innermost layer (*g*) in a contracted or compressed condition, whilst the outer one (*k*) is of fully double the thickness of that seen in the large stem, Plate IV. fig. 21. This enlargement corresponds with what we have already seen in the transverse section of the nodal disk in Plate III. fig. 16, and leaves no doubt on my mind but that the former is also a nodal section. There is no doubt, therefore, that the Burntisland plants exhibited the same nodal expansion of the bark that we find in the Lancashire type. It will further be observed from Plate V. fig. 27, *g'*, that the inner and middle barks accompany the branch in its outward course, as might have been anticipated, supposing my interpretation of this structure to be correct.

After combining the facts which I have enumerated, I have no doubt that these

* It is interesting to find that M. Renault has discovered solitary branches in some of his Autun specimens occupying exactly this position. He says respecting them, "Les rameaux que j'ai eu occasion de rencontrer encore adhérents à la tige étaient *solitaires* sur leur articulation." "Ceux que j'ai observés étaient toujours orientés de façon à être contenus dans le plan de l'un des angles saillants de l'axe central de la tige" (*loc. cit* p. 8, plate i. fig. 3).—February 9, 1874.

† M. Renault's discovery of the position and origin of the true foliar bundles already referred to (pp. 45 & 48) leaves no room for doubting that these deflected laminæ have been connected with a lateral *branch*, and not merely with *leaves*, since we have seen that the exogenous laminæ do not contribute to the formation of the merely foliar bundles.—February 9, 1874.

Burntisland stems are examples either of the genus *Asterophyllites* or of *Sphenophyllum*. The absence of leaves in these Scotch specimens renders it impossible at present to determine with which of these two genera they ought to be associated. Before attempting to ascertain the bearing of these facts upon the question of the systematic position of the plants, we must examine some other organs which, I am now satisfied, belonged to similar plants.

The first inquiry which suggests itself is, What are the relations subsisting between the stems described in these pages and the numerous fruits that have been described and figured by various observers on previous occasions? In 1853 Dr. Hooker published, in the 'Quarterly Journal of the Geological Society of London,' vol. x. p. 199, a description and figure of a terminal strobilus from Carluke, to which he gave the name of *Volkmannia Morrissi*. Eichwald, in his 'Lethæa Rossica,' tab. xv. fig. 1, figures, under the name of *Annularia densifolia*, a stem with eight verticils of circular nodal scars, and with cones given off from each verticil, similar cones having obviously been detached from the scars referred to. Geinitz figures, in his 'Die Steinkohlen-Formation in Sachsen,' under Sternberg's name of *Asterophyllites rigidus*, a fruiting spike; and in tab. xx. figs. 7 & 7 *a*, a second cone-bearing plant under the name of *Sphenophyllum emarginatum*. Germar, in his 'Steinkohlen-Gebirges von Wettin und Löbejün,' tab. vi., figures, under the name of *Sphenophyllites Schlotheimi*, some narrow strobili, about two and a half inches long, and attached by short peduncles to the axils of leaves. At tab. viii. fig. 6, he figures another fruit-bearing axis, with both lateral and terminal cones, under the name of *Sphenophyllites angustifolius*. In 1867 Mr. Carruthers published, in the 'Journal of Botany' for December 1867, a paper "On the Structure of the Fruit of *Calamites*," describing and figuring under this name some fruits discovered by Mr. Binney, to which he gave the name of *Volkmannia Binneyi*. These same fruits were further illustrated by Mr. Binney himself, in his 'Observations on the Structure of Fossil Plants found in the Carboniferous Strata,' plates iv. and v.; whilst on plate vi. of the same publication he figured some other fruits of an allied character. Still later (1870) the same observer published, in the 'Transactions of the Literary and Philosophical Society of Manchester,' vol. iv., third series, a "Note on the Organs of Fructification and foliage of *Calamodendron commune* (?)." In 1870 I published, in the same volume of 'Transactions,' a description of "A new Form of Calamitean Strobilus from the Lancashire Coal-measures;" and the fifth volume of the same series (1871) contained another of my memoirs, "On the Organization of *Volkmannia Dawsoni*." In his 'Traité de Paléontologie Végétale,' Schimper has republished several of the figures to which I have referred. To the *Volkmannia Binneyi* of Carruthers he has given the name of *Calamostachys Binneyana*. Such, so far as I am aware, are the chief fruits hitherto described that claim to belong to the genus *Asterophyllites* or its allies.

In the great majority of the instances referred to, there is no doubt whatever that the fruits belong to the several genera (*Asterophyllites* and *Sphenophyllum*) to which they are ascribed by their describers, because the leaves of these respective plants are found

attached to the stems supporting the strobili. But, unfortunately, in no one of these cases is any thing known of the internal organization of the strobili. The only recorded examples in which such organization is preserved are Mr. BINNEY's *Calamostachys Binneyana*, my Calamitean strobilus, and the *Volkmannia Dawsoni*, also described by myself. In my memoir on Calamites, published in vol. clxi. of the 'Philosophical Transactions,' and forming the first part of the present series, I discussed briefly the probable affinities of these three fruits (p. 501); and in the subsequent memoir on *Volkmannia Dawsoni*, I again examined their respective claims. Since that memoir appeared, I have investigated a very considerable number of specimens, all of which have tended to confirm the conclusions at which I previously arrived, viz. *that the only British strobilus, of which the internal organization has hitherto been described, that has any claims to be regarded as the fruit of Calamites, is that which I figured in the fourth volume of the 'Transactions of the Manchester Literary and Philosophical Society.'* Such being my conviction, I may exclude this plant from further detailed consideration at the present time, and inquire what light my more recent investigations have thrown upon the two remaining ones.

With this aim I have given, in Plate V. fig. 28, a transverse section of the *Volkmannia Dawsoni* just referred to, but from a section not figured in my previous memoir. In order to comprehend this figure, it is necessary to understand that this fruit is a strobilus, giving off lenticular bractigerous disks from each node, whilst the intervening spaces, corresponding with the internodes, are occupied by successive layers of sporangia, each layer consisting of four concentric verticils and occupying a single internode. Since the bracts fringing the margin of each nodal disk curve upwards and outwards, the transverse section, which has crossed the fruit somewhat obliquely, has cut through the sporangia belonging to two such internodes: those marked *u* belong to two verticils of the inner and upper series, and those indicated by *u'* form a small portion of the peripheral part of the next inferior internodal series, ascending to the level of the central portions of the one above it, as it extended upwards and outwards; *c* indicates the central vascular axis, surrounded by a vacant space originally occupied by cellular tissue, of which the only traces now found are some of their innermost portions (*g*), which evidently constitute a thin inner series of cells closely investing the vascular bundle, but which have shrunk away from it through desiccation, and now adhere only to its angles. The part of the inner circle marked *k* is a transverse section of the outer bark at one of the internodes. On the upper side of the same circle this cylinder of bark appears to be much thickened, because the slightly oblique section has here passed through one of the nodal disks. Being so near the central axis, we here see no trace of the division of that disk into its peripheral verticil of bracts; but this is clearly seen at *t*, *t'*, which letters indicate the margin of the next inferior nodal disk which ascends as it is extended centrifugally. At *t'* we can trace the first division of this disk into separate bracts, the base of each of which here exhibits an almost square section, but with a slight central projection on the outside,

marking the position of a midrib. Following this circle round to *t t*, we now see that the bracts have become independent of each other, proving that the section has intersected the specimen at a higher point to the left of the figure than to the right hand. At *t''* we discover a third range of bracts, ascending from a yet lower node, but intersected at a point where this range is still separated from those of the intermediate node by the outermost circle of the sporangia (*u'*), which occupy the intermediate internodal space; at the upper part of the figure (*t t*) these two series of bracts (*t*, *t'* & *t''*) come into contact, because at this point they both project beyond the outermost verticils of sporangia. Such are the general features of this fruit; but, for the purpose of exact comparison with the stems of *Asterophyllites*, each point must be examined somewhat more in detail. The central bundle of vessels which appears to represent the central triangle in the axis of *Asterophyllites* consist wholly of vessels. Their state of conservation does not show clearly whether they were barred or reticulate, but several indications suggest that the latter was the case. The greatest diameter of this vascular axis is ·065, that of the largest of its component vessels being about ·0037. In my original memoir I stated that "its transverse section may be described as a triangle with concave sides and concavely truncated angles" (*loc. cit.* p. 30), which description, it must be remembered, was published before I had seen any stems of *Asterophyllites* suggesting the triangular idea. Plate V. fig. 29 is a still more enlarged representation of the transverse section of this axis. We see from it that the arrangement of the vessels in the bundle corresponds exactly with that of the central triangles of the stems. They are grouped without any definite order, except that the extremities of the three primary angles are occupied by much smaller ones than the more central portions. It is very obvious that the two angles, *c' c'*, are merely enlarged and subdivided representatives of the undivided angle *c*. I wish to call attention to this tendency towards an hexagonal division, from its bearing upon the affinities of *Calamostachys Binneyana*. I discover no evidence whatever of any vessels being given off from this vascular axis to supply the bractigerous disks; but the state of carbonization of the specimen gives this negative appearance but a limited value. That the circle immediately surrounding this vascular axis was composed of delicate cortical parenchyma, may be inferred from the almost entire disappearance of the tissue. The layers, *g*, alone remain to represent its innermost portion. The outer bark (*k*), on the other hand, is exquisitely preserved, and, like that of *Asterophyllites*, consists of thick-walled parenchyma, corresponding in every respect with the structures seen in the outer bark of the stems. At each node this outer bark is extended centrifugally into a thick lenticular disk, breaking up at its margin into a peripheral series of bracts, *t'*. These nodal disks were arranged in successive verticils about ·9 apart. I find it difficult to ascertain the exact number of the bracts in each verticil; but, judging from the size of the primary subdivisions of the disk *t*, compared with the arc of the circle, there appear to have been about thirty-two in each verticil. Each bract was thick at its base, becoming thinner and more foliaceous towards its extremity. It curved upwards and outwards in a some-

what rigid manner, projecting considerably beyond the verticils of sporangia. The sporangia, as seen in a fractured fragment not yet cut up into sections, are obtusely angular, especially at their inner sides, the result of the mutual compression of concentric circles of rounded spheres. They were attached to the bractigerous disks by numerous long, slender, cylindrical sporangiophores, *v*. Since writing my original memoir, I have ascertained that these sprang from each margin of the base of each bract so as to constitute two parallel series. This relation of the sporangiophores to the bracts from which they spring is well seen at *t'*, where the bases of the pairs of sporangiophores appear to project inwards (though they are really proceeding upwards and outwards) from each subdivision of the nodal disk. I have already pointed out that each bract obviously had one thick midrib especially conspicuous near its base. In my memoir, speaking of the spores, I stated that "at the first glance we should be tempted to infer that their exteriors had been spinous; but I have not been able to satisfy myself that such has been the case" (*loc. cit.* p. 34). But since the above sentence was penned I have obtained yet finer examples of these spores, some of which are represented in Plate V. fig. 30, affording proof that long spines projected from their outer surfaces, the bases of these spines being connected by a coarse network of thickened ridges, reminding us of the aspect of the macrospores of *Selaginella inæquifolia* and other Lycopodiaceous fruits. The size of my newly discovered spores, including the length of the spines, is about ·0044. In my memoir I stated that it was ·0037; but the small discrepancy is accounted for by the fact that in the latter the spines were imperfectly preserved, hence their remarkable length was not known, and consequently not included in the measurement*.

I think that the preceding description must satisfy the most incredulous that this fruit belongs to the same plants as the stems described in the earlier part of the present memoir†. The two agree exactly in the form and composition of the central, non-exogenous, vascular axis, in possessing a bark of which the inner portion was composed of delicate cells whilst the outer one consisted of coarse parenchyma, and in the expansion of the latter *at each node* into a prosenchymatous leaf-bearing disk. The identity of the triangular axis forming the centre around which the exogenous growths were developed in the stems of *Asterophyllites* with the entire axis in the fruit is precisely what we should expect. I have pointed out, in my previous memoir on the Burntisland *Lepidodendra*, that the structure of the vascular bundle or bundles of a strobilus, which is usually a deciduous *annual* growth, is, as might have been anticipated, almost identical with that of the corresponding twigs developed upon the branches within the same limited period.

* These spores appear to me to bear a resemblance to the objects from the Coal-measures described by Mr. Carruthers as radiolarian rhizopods, and to which he has given the generic name of *Traquairia*. See the Report of the British Association for the Advancement of Science for 1872 (Trans. of Sect. p. 126).—February 9, 1874.

† I conclude that the fruit belongs to *Asterophyllites* rather than to *Sphenophyllum*, from the circumstance that its nodal disks subdivide into numerous narrow bracts instead of into a small number of broader ones.

Such a fruit is but one of these twigs whose development has been arrested. Now we have seen from figs. 1, 2, and 18 that, in such twigs of *Asterophyllites*, the vascular bundle has essentially the same triangular form as in the fruit. Then, in addition to these important positive evidences, we have equally important negative ones. A vascular bundle of a triangular form, and having truncated angles, is a rare phenomenon, whether in a strobilus or in a true branch—a phenomenon of which no trace exists in the Oldham beds, save in *Asterophyllites* and, as Professor RENAULT appears to have discovered, in the allied genus *Sphenophyllum*. When, therefore, we find, in a stratum scarcely exceeding a foot in thickness, a fruit possessing such an axis associated with a verticillate arrangement of foliar appendages, and we also find, in the same bed, numerous stems of *Asterophyllites* having a similar primary axis, also associated with a verticillate foliage, we have every possible reason for concluding that the two objects are parts of the same plant; and when, after prolonged and careful search, we fail to find, either in this or in any other part of the Coal-measures, any other plant possessing such a triangular axis, we have as near an approximation to certainty as we can ever hope to attain in these fossils, short of finding the two growing upon the same stem. This has not yet been done in the case of specimens of which the minute organization is preserved; but we do find specimens so associated which exhibit every outward appearance of identity with that of which I have described the internal structure.

In two of his memoirs already referred to, Mr. BINNEY published figures of fruits of *Asterophyllites* from Brooksbottom in Lancashire, where a rich deposit of these curious organisms was discovered in a thin bed of shale by my friend Captain JOHN AITKEN, of Bacup. Captain AITKEN has kindly favoured me with an abundant supply of these interesting specimens, thus enabling me to subject these fruits to a very careful investigation. In my memoir on *Volkmannia Dawsoni*, as also in that on the organization of *Calamites*, I expressed my belief that Mr. BINNEY's figures of these specimens represented Calamitean fruits; but, after studying the specimens themselves, I am satisfied that such is not the case. I believe them to be of the same type as *Volkmannia Dawsoni*. In the Calamitean strobilus which I described the free ends of the bracts are closely pressed against the sporangia, whilst in the *Volkmannia Dawsoni*, on the other hand, they project considerably beyond the sporangia. A similar projection appears in Captain AITKEN's specimens, as shown in Plate V. fig. 31, which represents one of the finest of these fruits enlarged to twice its natural size. They are found attached to stems and branches which are undoubtedly Asterophyllitean; and, as Mr. BINNEY has correctly pointed out, some of them were terminal, whilst others were arranged in successive verticils springing laterally from the axils of leaves, where they appear to have been sessile. There is no doubt that in this fruit every node sustained its layer of sporangia, instead of their being developed on alternate nodes, as in the *Calamostachys Binneyana*.

Plate V. fig. 32 represents a fine specimen of the fruit of another species of *Asterophyllites*. I know that the specimen was obtained from one of the shales of the Coal-measures near Manchester, though I am not certain as to the exact locality. The

drawing represents the specimen of the size of nature, but I am convinced that it was originally much longer than now. My cabinet contains two other fragments, which I believe belonged to it; but whether they did or not, the specimen is obviously broken off abruptly at each end. As it is, we have seven verticils of pedunculated strobili, two or three of which strobili only appear on the surface of the shale at each node. That this fructification has belonged to a different species of *Asterophyllites* to Plate V. fig. 31, appears from the fact that in the former all the strobili were pedunculate, whilst in the latter most of the lateral ones are sessile. In this respect the latter plant seems to resemble the *Annularia densifolia* figured by Eichwald*, which also agrees with the Brooksbottom type in the presence of verticils of very distinct scars, marking the points from whence strobili have been detached. Fig. 32 appears to be very similar to the pedunculated fruit originally figured by Sternberg under the name of *Volkmannia polystachia*. The *Sphenophyllites Schlotheimi* of Germar† is also of the same type. In my specimen each slender peduncle is about ·37 in length. It is slightly thickened where it springs from a node of the central axis, and still more so as it enters the strobilus. It obviously consists of a single joint or internode. The principal axis (*k*) consists of a series of long internodes, most of which range between 1·1 and 1·2 in length, and which are delicately but distinctly striated longitudinally; at each node this axis enlarges somewhat into a small lenticular disk, and attached to several of these we observe isolated leaves of *Asterophyllites*, constituting in each case the leaf from the axil of which the adjacent strobilus has sprung. Each strobilus consists of a linear series of verticils of bractigerous disks, disposed at intervals of about ·125. Each disk on leaving the axis first curves somewhat downwards, and then upwards and outwards. The marginal fringe of bracts continues in the latter course, so that their tips stand out free, as in *A. Dawsoni*, instead of coming into close contact with the next superior verticil. Each of these bractigerous disks sustains a stratum of sporangia upon its upper surface. The exact arrangement of the latter cannot be ascertained with accuracy; but I am quite satisfied that they are disposed, as in *A. Dawsoni*, in more than one concentric ring. Neither have I been able to determine with absolute certainty the exact number of bracts in each verticil; but it is obviously about fifteen or sixteen, which is a rather smaller number than in *A. Dawsoni*.

The preceding facts suffice to show that whilst various authors have figured different modifications of a common type of Asterophyllitean fruits, none of them obtained any clue to the internal organization of that type. *A. Dawsoni* has given us that clue. Secondary modifications have doubtless existed amongst the various species; but that the external forms and the inward organization of these fruits of *Asterophyllites* and *Sphenophyllum* are now substantially correlated is a fact about which I entertain no doubt.

An important question still seeks an answer, viz. What is *Calamostachys Binneyana*? Mr. Binney and Mr. Carruthers both reply, it is the fruit of a Calamite. But my

* Lethæa Rossica, tab. xv. fig. 8.

† Steinkohlen-Gebirges von Wettin und Löbejün, tab. vi.

doubts expressed long ago on this point remain not only unremoved, but strengthened by further researches. My cabinet contains nearly fifty sections of these fruits, and I have examined others in the cabinets of my friends. That the sporangia of these strobili adhere to the sporangiophores in a manner closely resembling those of the modern *Equisetaceæ*, as pointed out by Mr. CARRUTHERS, is undoubtedly true; and, assuming the *Calamites* to be *Equisetaceæ*, the above fact might be deemed a strong confirmation of the conclusion arrived at by the above observers. Mr. CARRUTHERS, in addition, thinks that he sees elaters attached to the spores, which conclusion, if true, would be an important additional fact pointing in the same direction. But, for reasons to be given immediately, I am unable to accept this interpretation of the structures in question. Notwithstanding what has been done by the two eminent observers referred to, sufficient remains to be discovered in this fruit to justify my re-examination of it. Its general plan of construction is, as Mr. CARRUTHERS has shown, very different from that of *Asterophyllites Dawsoni*. It has a jointed axis with enlarged nodes, each of which gives off appendages which are alternately bractigerous disks, somewhat resembling those of *Asterophyllites Dawsoni* deprived of their sporangia, and verticils of sporangiophores, which are almost identical in their organization, as well as in their mode of sustaining their sporangia, with those of the living Equisetums. Mr. CARRUTHERS has figured all these structures in his memoir referred to, but on a small scale; Mr. BINNEY has figured them on a much larger scale, but without including some features which appear to me important, especially in connexion with the transverse sections. The only sections of this latter class which that gentleman has given in his monograph on *Calamites* and *Calamodendron* are two on plate iv., both of which have been made in the plane of the verticillate sporangiophores, so that those of the bractigerous disks and of the internodes are not represented by him. I have not deemed it necessary to reproduce any longitudinal sections, because Mr. BINNEY'S excellent figures on his plate v., combined with the smaller ones of Mr. CARRUTHERS, leave little to be desired in this direction.

Plate VI. fig. 33 is a transcript of a specimen in my cabinet, with about nine foliar disks; whilst at v and v', owing to the removal of the surface of the specimen and the consequent disappearance of the central series of leaves, tangential sections of some of the sporangiophores are brought into view. We here see, as Mr. CARRUTHERS has already pointed out, that the foliar disks (k'') reach nearly to the surface of the fruit, where they break up into verticils of leaves, the latter being abruptly bent upwards, almost at right angles to the plane of the disk. This illustration enables us to understand Plate VI. fig. 34, which represents a transverse section made in the plane of the foliar disk (fig. 33, k''). At c we have part of the central vascular axis; k is the cellular lenticular disk; at t we have the bases of the upturned bracts intersected transversely, whilst at t' we have sections of the ends of those of the next inferior verticil, which, as Mr. CARRUTHERS has described, always alternate with those of the two neighbouring foliar verticils. In Plate VI. fig. 35 we have part of the periphery of a

similar section to the last, but with portions of the leaves of three foliar disks. Thus at t we have the bases of those belonging to the principal disk; at t' we have the alternating series of the disk below; whilst at t'' we again have the tips of the leaves of the inferior disk but one opposite to t, showing that the leaves fringing the disks are arranged in an order equivalent to that of the decussate plan, in which each third verticil returns to the disposition exhibited by the first. In both the above figures the disks are seen to consist of two kinds of coarse, thick-walled, cellular tissue; broad lines leading from the central axis to the base of each leaf (t) consist of prosenchymatous cells elongated in the direction of the plane of the disk, the intermediate triangular spaces being occupied by a coarse parenchyma. In Plate VI. fig. 34 especially we observe that the sections of the leaves (t') exhibit transverse sections of the same elongated prosenchyma. It is of some importance to note that neither in these nor in any other similar sections of these disks have I succeeded in detecting either a barred vessel or a vacant space which such a vessel may originally have occupied; yet two longitudinal sections in my cabinet afford clear proof that Mr. CARRUTHERS is correct when he affirms that a very slender bundle of such vessels runs up the centre of each leaf, and, to do so, it must have traversed the disk. I have only succeeded in detecting their presence in two out of some fifty sections of the fruit under examination, yet they once existed in every fruit. At $u\,u$ in Plate VI. fig. 34, the section has passed out of the plane of the disk and intersected two sporangia. There have been fourteen leaves in each verticil of this specimen. Plate VI. fig. 36 is a transverse section made in the plane of a verticil of sporangiophores: of these there are usually six in each verticil; but in this verticil there are seven, being half the number of the bracts of the adjacent foliar disk (Plate VI. fig. 34); this numerical relation corresponds with one already observed by Mr. CARRUTHERS, though in the specimens examined by him the respective numbers appear to have been six and twelve. Mr. BINNEY'S figures also represent his transverse section as possessing six sporangiophores; but as the foliar appendages are not present in the latter sections, we have no evidence as to their number. On turning to the vascular axis (fig. 36, c), we discover a distinct triangular structure with concavely truncated angles, thus producing the hexagonal form which is ordinarily characteristic of the axis of these fruits. In the present specimen the axis has accommodated itself to the accidental modification which has produced *seven* sporangiophores instead of six, by the angle c' terminating in two arcs instead of one. At the first glance this axis would appear to be almost a repetition of that of *Asterophyllites Dawsoni* (Plate V. fig. 29); but, as will be seen shortly, this is not exactly the case. We find the vascular bundle under consideration separated into a central and a peripheral portion. This bundle is surrounded by the vacant space (g) representing the missing inner bark, external to which we have the outer prosenchymatous bark (k) arranged in a series of crescentic curves, portions of two of which unite to form the two sides of each of the sporangiophores (v). In the latter structures the prosenchymatous cells of the bark become long, narrow, and parallel-sided, being drawn out at great length

in the radial direction. At its outer extremity the two sides of each sporangiophore diverge in very regular curves, the peripheral intervening space (*v'*) being occupied by a mass of parenchyma which, especially at *v'*, *v''*, and *v'''*, is seen to constitute disks of considerable thickness. The same appearances recur in whatever direction the section is made; hence my specimens confirm Mr. CARRUTHERS's statement, that these sporangiophores correspond very closely with those of the living *Equiseta*, and do not sustain Mr. BINNEY's opinion, that the sporangia are enclosed in "six heart-shaped bags." These sporangiophores undoubtedly contain barred vessels, since, like Mr. CARRUTHERS, I have found them in the pedunculated portion of the organ; and they are still more copiously developed where its two surface-layers diverge in regular curves to embrace the peripheral extremities of the sporangia attached to its opposite sides. That the sporangia adhere by those outer extremities to the proximate concavities formed by the inner surface of the terminal disks of the sporangiophores is certain. Hence, when a section is accurately made in the central plane of a sporangium, detached from the sporangiophore, as at fig. 36, *u'*, the outer end of the sporangium is always the part ruptured. The transverse sections of the leaves of the next inferior foliar disk are represented in the above figure at *t t*, and the tips of those of the next but one at *t'*.

Plate VI. fig. 37 represents another transverse section of the fruit, made across the internode between a foliar disk and a verticil of sporangiophores, from the same specimen as Plate VI. fig. 36. We here find the vascular axis surrounded by a small double cortical ring, *k*. The sporangia are now reduced to twelve, showing that the verticil of sporangiophores to which they were attached were but six in number. Hence it appears that variations from six to seven sporangiophores may coexist in contiguous verticils of the same strobilus. The sporangia (*u*) are here arranged with great regularity; and seven of the twelve exhibit that rupture of their outer ends to which I have just referred. The thin margins of the sporangiophores are seen at *v*, and a circle of the sections of the leaf-tips at *t*.

The central vascular axis of this specimen exhibits an important feature, which has not been recorded in any of the specimens described either by Mr. BINNEY or by Mr. CARRUTHERS; and as the section fig. 37 shows the peculiar structure very clearly, I have further enlarged it in Plate VI. fig. 38. The centre of this section is composed of a cluster of barred vessels grouped in the usual way; its periphery consists of a ring (*d*) of similar vessels, but arranged in radiating laminæ, and which evidently represent an exogenous growth superadded to the central bundle. I have never met with this condition in any other specimen, but the presence of this circumferential growth is important. It will be noticed that many of the radiating laminæ follow a curvilinear direction, and that the inner extremities of those laminæ manifest a decided tendency to bend inwards towards the respective central points opposite to *d d*. What their condition may have been at *d'* I cannot tell, because some accidental pressure has disturbed their natural arrangement here. I shall call attention to this peculiar disposition after examining the root-structures of *Asterophyllites*.

Mr. CARRUTHERS has described the wall of the sporangium as "composed of irregularly elongated cells, with projections of a secondary deposit extending at right angles into the interior of the cells." This is strictly correct; but there is something beyond this to be noted. The longer axes of these cells are radial in relation to that of the fruit-axis, and parallel with the longer sides of the sporangium. When we see only the outer surface of the sporangium-wall, we observe that the latter appears as represented in Plate VI. fig. 39, and by Mr. CARRUTHERS in his fig. 5; but its inner surface frequently exhibits the aspect seen in Plate VII. fig. 42. When a transverse section, like Plate VII. fig. 40 (again represented by Mr. CARRUTHERS in his fig. 45), is made across the shorter diameters of these cells, we see that each cell has very thick walls on its inner and two lateral surfaces; whilst the outer wall, forming part of the outer surface of the entire sporangium, is very thin. But if the section is made through the long axes of these cells, so that their lateral walls are brought into view, we then obtain the appearance indicated by Plate VII. fig. 41, where the parallel bars are much more numerous and more perfectly parallel in their arrangement than in Plate VII. fig. 42. The bars in fig. 40 are obviously different from those of fig. 41. The former represent the cell-walls of a corresponding number of distinct cells; the latter represent the transverse bars thickening the walls of one individual cell. It is obvious that we have here a modification either of the spiral or of the annular cell; but instead of the lignine, forming the thickened bars of Plate VII. figs. 41 & 42, being continued entirely round the transverse circumference of the elongated cell, some unknown influence has prevented their deposition on the outer surface of every cell—a curious example of the effect of unknown forces in regulating the distribution of these secondary deposits in fibro-vascular structures.

Mr. CARRUTHERS has represented in his fig. 1 a cluster of spherical spores, from the outer wall of each of which there project delicate thread-like appendages. He describes these as follows:—"The spores are simple globular bodies, frequently exhibiting an outer and an inner wall. Sometimes, however, they appear to be composed of a single wall, and then the outer wall is represented by lines more or less separated from the spores. These I believe to be elaters, similar in structure to those of *Equisetum*." As to the first of these statements respecting the outer and inner walls, my observations agree with those of my indefatigable friend, but from his elater hypothesis I am compelled to dissent. The following appears to me to be the probable explanation of these appearances.

Plate VII. fig. 43 represents a number of these spores selected from two strobili in different stages of development. The larger ones exhibit a series of examples in which we have an inner cell enclosed within an outer one. Of these, Plate VII. fig. 43, *w* are the forms which approach nearest to the condition in which Mr. CARRUTHERS recognizes elaters. In these specimens the point of contact between the inner and outer cells represents, according to his view, that at which the elaters of an Equisetiform spore are united to the actual spore; but the other examples figured show that this contact is accidental and not essential. In most of them the spore lies free in the centre of the

outer cell. That the latter breaks up when matured, is undoubtedly true. In a ripe strobilus the difficulty is to find a spore of which this layer is not broken up. But in none of the fragments do we discover the four club-shaped organs known as elaters. I have in my cabinet thousands of these spores, but in no solitary instance have I found a true clavate elater. On the other hand, I think I have abundant evidence that what Mr. CARRUTHERS regards as such are but the shreds of the mother-cells which were derived primarily from the interior of the sporangium-wall. An examination of true Equisetiform spores during their development, and a perusal of the records of the same process as it is described by HOFFMEISTER and SACHS, tend to confirm my view; not that any exact parallellism can be traced between the fossil spores and those of the living *Equisetum*! far from it; but we do find in the one the broad outlines of what may have taken place in the other, as in many varied cryptogamic growths. Be this as it may, I affirm unhesitatingly that had the outer wall of the fossil spore ever resolved itself into true elaters, we should see their spiral lines, as they are accurately represented by SACHS*, crossing the circular disks of the spores in innumerable instances. The tissues of which these spiral bands consist are precisely such as would be likely to be preserved, and be conspicuous, in the fossil state; whereas, I repeat, no one has ever pretended to have seen them in this condition in any solitary instance.

The correctness of my explanation receives further support from the spores of a strobilus in my cabinet which has not attained to its full maturity. Four of these spores represented to the right of the group (Plate VII. fig. 43, w', w'') will be recognized by their smaller size and darker tint. Their inner spheres especially are little more than half the size of those constituting the rest of the group, and their outer cell-walls are as yet unruptured. We have here the precise stage of growth in which the transverse spiral bands should be conspicuously present; and from this stage up to the final disruption of the cell they ought to become increasingly obvious, but nothing of the kind is to be seen. In every instance we have a simple cell-wall of uniform density, instead of that alternation of thick and thin bands, the formation of which precedes the liberation of the former as true elaters. In addition to this negative evidence, some of these specimens exhibit testimony of a positive kind. It must be remembered that elaters are formed out of the outer investing layer of each individual spore; but at w' I have represented two examples in which each outer cell-wall encloses *two* distinct spores—a condition that would have been genetically impossible had these cells represented the outer layers of individual spores, as is necessary on the elater hypothesis; but on the supposition that they are mother-cells almost every difficulty disappears. SACHS correctly describes the mother-cells of *Equisetum* as "ceasing to be a continuous tissue, and not only isolating themselves, but floating freely in a liquid, filling the sac of the sporangium" (*loc. cit.* p. 355). The disputed structures appear to me to have originated in some similar manner; and what Mr. CARRUTHERS has described as elaters I believe to be merely fragments of their irregularly torn cell-walls, which naturally broke up when the contained spores became

* Lehrbuch, fig. 280, C.

matured and required liberation. Why a larger number of the mother-cells did not contain more than one spore is but one of the many anomalies seen in this very perplexing fruit. The further question now arises—To what plant does this strobilus belong?

Mr. Binney answers this question by affirming that it is the fruit of *Calamodendron*; and Mr. Carruthers, regarding it in the same light, says that it is the fruit of a Calamite. The latter observer arrives at this conclusion under the conviction that *Calamites* and *Asterophyllites* are identical plants. The spores throw little light upon this problem, because they exhibit too generalized a form to indicate any special relations. Mr. Carruthers has referred to the semispiral deposits in the cells of the sporangia as representing the true spiral fibres in the sporangial cells of *Equisetum*. But such deposits are not confined to *Equiseta*. We have them in the corresponding sporangial cells of *Marchantia conica*, in a condition much more closely resembling those of our fossil fruit than do the Equisetaceous ones. The sporangia of our fossil very closely resemble those of *Equisetum* both in their form, in the mode of their attachment to the sporangiophores, and in the general aspects of the latter organs; but they differ in the fact that only alternate verticils of the nodal expansions bear sporangia in the fossil, whereas they are attached to every verticil in the recent type. It appears to me that in the former the sporangia are ruptured by detachment from the sporangiophore, instead of by longitudinal ventral dehiscence as in the latter. When we come to the central axis, the difference between the two types becomes increasingly manifest; though even here we are not without some curious features of resemblance. If we make a transverse section through the node of a recent Equisetaceous strobilus, *in the plane of a verticil of sporangiophores*, we see that the bark from which these sporangiophores spring expands into a small lenticular disk, the structure of which is almost identical with that of the *sterile* disks of *Calamostachys*, as seen in Plate VI. fig. 34, and proportionately different from the corresponding section of the *fertile* sporangiophores represented by Plate VI. fig. 36. Thus we have agreement between the recent and fossil types in the structure of parts whose general aspects are so different, and differences of structure where the outward forms agree. The double layer of the bark of *Calamostachys* corresponds exactly with what we have in *Asterophyllites*, whilst this tissue consists of a single uniform layer in *Calamites**. This is a distinction of the greatest importance. It would be contrary to all experience to expect to find so great a change in a continuous *aërial* bark investing the stem and the fruit-axis of a Cryptogam as would be involved in the sudden separation of a single parenchymatous bark into two strongly marked layers of parenchyma and prosenchyma. Yet we have to admit the possibility of such an anomalous condition as is here suggested, if we are to recognize in *Calamostachys* the fruit of *Calamites*.

The vascular part of the axis differs from the corresponding portion of *Calamites* even yet more than the bark does, approximating in the same ratio to that of *Asterophyllites*.

* See my memoir on *Calamites*. Phil. Trans. 1871, Plate xxiii. fig. 9.

In even the smallest known branches of the former plants we find exactly the same type of organization as we do in the largest and most matured stems. We invariably have a central cellular medulla, which becomes fistular at a very early period, and which is surrounded by a ring of distinctly separated vascular bundles or wedges, each one of which has a well-marked longitudinal canal at its inner angle. The same conditions recur with the greatest exactness in the cone which I have described in the Transactions of the Literary and Philosophical Society of Manchester as being that of a Calamite; but in *Calamostachys* all these conditions are reversed. Here we have neither a cellular medulla nor a fistular cavity. In their place we invariably have a solid, central axis of barred vessels, such as we find only in the very young twigs of Lepidodendroid plants and in *Asterophyllites*. The only indication of the structure of the exogenous zone of the parent stem of *Calamostachys* which we have found is that represented in Plate VI. fig. 38. Here, at all events, we might have expected to discover some trace of Calamitean arrangements, had these been Calamitean strobili; but nothing of the kind appears. It is as unlike the exogenous zone of a Calamite as it is like the corresponding tissue found in the roots of *Asterophyllites*. The central axis does not display the very definitely triangular form seen in that of *Asterophyllites*, but there is a decided approach towards it; and we detect a further indication of a trifid arrangement in Plate VI. fig. 38, at the two points indicated by the letters *d d*. Each of these constitutes a centre, on either side of which the radiating laminæ of the exogenous zone converge towards the letter *d* as they proceed outwards. I presume that a similar point may have existed to the left of the lower part of the figure indicated by *d'*; but here, as already mentioned, the tissues have been accidentally displaced by some disturbing force masking their primary condition.

We shall find a similar tendency to that just described recurring in the roots of *Asterophyllites*.

It appears to me that the number and symmetrical grouping of the nodal appendages round the central axis of *Calamostachys* is not without significance. In its normal and almost invariable condition each verticil, whether of leaves or of sporangia, exhibits some multiple of 3. Thus we have six sporangiophores and twelve foliar divisions of the barren verticils. Whatever these facts indicate they are in harmony with the prevalence of the trimerous type of structure so characteristic of the stem of *Asterophyllites*.

After balancing these various facts and arguments, I am led to the conclusion that *Calamostachys Binneyana* has much closer affinities with *Asterophyllites* than with *Calamites*. With the latter it has no one structural feature in common. There is no solitary point in which the two plants resemble each other. The resemblance of the fertile sporangia of *Calamostachys* to those of *Equisetum* has been combined with the foregone conclusion that the *Calamites* were Equisetaceous plants, in leading to the belief that the two were parts of the same plant; but I cannot conceive of any conditions in which the stem of a *Calamites* could be prolonged into that of the *Cala-*

mostachys. I have carefully investigated the relations which the fertile stems of the *Equiseta* bear to axes of their terminal fruit-spikes, and I find that their respective structures are typically identical. The transition from the stem to the fruit-axis produces no structural changes save such as are of the most trivial kind; the general type remains unaltered and continuous. But to plant *Calamostachys Binneyana* upon the top of a Calamite would be as abnormal as to surmount the stem of an *Equisetum* with the strobilus of a Lycopod.

Whilst expressing my conviction that *Calamostachys* is allied to the Asterophyllitean plants, I cannot as yet correlate it with any particular stem. I have no evidence indicating that it belonged to any of those described in the preceding pages, and suspect that the plant of which it is the fruit has yet to be discovered. Diligent search must continue to be made in the narrow vertical zone of beds within which the specimens occur, until some absolutely demonstrative facts are obtained removing the obscurity which still invests this very remarkable and interesting fruit.

I have already referred briefly to one or two examples of fruiting axes of *Asterophyllites* and *Sphenophyllum* in which circular scars, arranged in verticils at the nodes, marked the points from whence strobili had been detached. Plate VII. fig. 44 represents a similar stem, which I found amongst Captain AITKEN's Brooksbottom specimens. The scars (*r*) appear as if they were located below the node; but this appearance has, I think, merely arisen from the circumstance that the strobili were detached before the plant was enclosed in the shale, thus allowing the bases of the bracts to be compressed against the stem; the carbonaceous matter having subsequently become detached revealed the cicatrices in question*. Each cicatrix is nearly oval, but slightly narrower at its lower than at its upper extremity, and with a distinct central depression marking the position of a vascular bundle. There appears to me to be a close resemblance between this specimen and the plant figured by LINDLEY and HUTTON under the name of *Calamites verticillatus*, but respecting which I have already expressed my conviction that it had no affinity whatever with the genus *Calamites*†. The stem last mentioned appears to be the same as that figured by GEINITZ‡ under the name of *Equisetites infundibuliformis*. It becomes an interesting question whether this plant will not prove to be an arborescent stem of an *Asterophyllites* or *Sphenophyllum*; if so, are the characteristic scars large because they bore large strobili, or have they become larger through the expansion of the bark subsequently to the shedding of the strobili, and consequent upon the exogenous enlargement of the vascular portion of the stem? Each of the large oval cicatrices of this plant has a central impression more or less regularly defined, indicating the former passage of a central vascular axis at this point, such as is seen in each of the scars of Plate VII. fig. 44. In some instances they probably indicate the position of branches.

Plate VII. fig. 45 represents a fine example of *Calamites verticillatus* in the Museum

* It is possible that the young buds may have burst through the slightly confluent bases of the leaves, as the rootlets and branches of *Equiseta* do through their nodal sheaths.

† Phil. Trans. 1871, p. 507.

‡ Die Steinkohlen-Formation in Sachsen, tab. x. figs. 4, 5.

of the Owens College, and found in one of the upper coal-shales of Lancashire. The figure is reduced to two thirds the size of the original. It is not without significance that the only vegetable remains seen in the same large slab are an abundance of leaf-bearing stems and twigs of *Asterophyllites* of various sizes, which appear to be in all respects *facsimiles* of those described in the preceding pages. This example of the stem consists of a number of well-marked internodes of various sizes. These are marked by numerous faintly defined parallel longitudinal grooves, with others of a much more sharply defined character at irregular intervals. The latter have obviously been cracks in the original bark, into which the matrix has entered, causing the latter to stand out in projecting ridges. In this specimen the large cicatrices are planted in a verticillate manner upon each third node; in another and otherwise similar specimen in the Owens College collection they are only planted upon every eighth node, thus exhibiting variations in this plant which may possibly have been specific ones. The latter specimen further indicates what is but obscurely shown in the example figured, viz. that the cicatrices appear as if planted *upon* the node, and not *above* or *below* it.

The general resemblance of these large stems to Plate VII. fig. 44 is very striking. The facts now stated, combined with the difficulty of collocating these examples with any other known plant, strongly incline me to the belief that they are the arborescent stems of the *Asterophyllites* with which they are associated in the specimen figured.

In August 1869 I published in the 'Monthly Microscopic Journal' the brief memoir referred to in No. IV.* of the present series, in which I separated two very distinct groups of plants which had hitherto been regarded as allied to each other, viz. those belonging to Endlicher's genus *Dadoxylon*, whose wood-cells were truly discigerous, and those in which this character was only apparent, not real, it being due to a mere modification of fibro-vascular tissue. In that memoir I said:—"It appears necessary, therefore, to establish a new genus for all the plants whose woody tissues consist of reticulated fibres, and the name of *Dictyoxylon* appears an appropriate one for it. I should propose, for the present, to include in this genus *all* the reticulated types. At some future time their further separation into two or more genera may be requisite"†. When giving a sketch of this new genus at the Edinburgh Meeting of the British Association, I included in it, in accordance with the above determination, a new plant from the Coal-measures, to which I assigned the name of *Dictyoxylon radicans*. But in working out the details of the genus for the fourth memoir of the present series, I found that the necessity for further generic subdivision which I had anticipated was already urgent. In a letter to Dr. Sharpey‡, I observed, respecting the *D. radicans*:—"I think it not improbable that this has been the subterranean axis of some other plant, since I have succeeded in tracing its ultimate subdivisions into rootlets. I propose for the present to recognize it by the generic name of *Amyelon*. My specimens of this plant are very

* Philosophical Transactions, 1873, p. 378.

† *Loc. cit.* p. 68.

‡ Proceedings of the Royal Society, vol. xx. p. 436.

numerous." "They may prove to be rhizomes and roots of the Asterophyllite described in my last letter to you."

I think I have now accumulated most satisfactory evidence that the above hypothesis is correct, and am able to prove that *Amyelon is* the root and rootlets of the stems described in the preceding pages. I would observe at the outset that the first examination of the former plant would scarcely lead a naturalist to the above conclusion, since it exhibits no appearance of joints or special nodes, much less of verticillate arrangements of any kind, such as are seen in the rhizomes of *Calamites*. On the other hand, it is branched like the roots of an ordinary exogenous tree. Its bark also differs from that of *Asterophyllites*; but I think that I shall have no difficulty in tracing the homologous relations of the two conditions. In describing these specimens I will commence with the most characteristic example that I have seen, one of a series of valuable sections which I obtained from a nodule for which I am indebted to Mr. NIELD of Oldham. Plate VII. fig. 46 is a transverse section of a young root surrounded by a cluster of branching rootlets, which have been chiefly, if not wholly, given off from two points in the section. The central axis (*d*) consists of a series of reticulated vessels of small size, the largest of them not exceeding ·0018. These are arranged, in the usual exogenous manner, in very regular radiating laminæ or wedges. The centre, whence these laminæ spring, exhibits no trace of a medulla, nor, *in the present section*, of any other special tissue. In other sections to be described important differences present themselves in this respect. The vascular axis in fig. 46 consists of a large central mass of vessels, partly enclosed in a second crescentic layer representing a secondary exogenous growth. The latter portion, it will be observed, does not entirely enclose the central one—a condition to which I have referred in previous memoirs as not being uncommon amongst the Cryptogamic plants of the Coal-measures.

This vascular axis is surrounded by a bark consisting of two very distinct layers. The peculiar aspects of this tissue are better seen in Plate VIII. fig. 47, which is enlarged eighty diameters. Its inner layer (*g*) is an ordinary parenchyma, consisting of cells of very unequal sizes. Many of these are very small, but others are ·0038, these larger ones being especially predominant towards the exterior of the layer. Many of these external cells (*g'*) are subdivided tangentially to the circumference of the stem by a number of secondary cell-divisions. This parenchyma passes rapidly, but not abruptly, into the outer layer (*h*), in which cells, compressed tangentially, are arranged in irregular columns perpendicular to the surface of the structure. The boundary lines between contiguous columns are strongly marked, usually more so than the transverse septa of their component cells. The individual columns sometimes extend from the inner bark to the periphery; at others they terminate at various points midway, where they become intercalated between other similar ones.

Plate VIII. fig. 48 represents a radial longitudinal section of part of the root, a portion of a similar one being still further enlarged in Plate VIII. fig. 49. In the former figure the vessels of the axis (*d*) are seen to be crossed by numerous small medullary rays (*f*),

whilst the vessels themselves exhibit, in this aspect, very distinctly marked reticulations on the inner surfaces of their walls. In Plate VIII. fig. 49 we find the bark of this section exhibiting very similar appearances to those of fig. 47. We have the same inner layer of parenchyma (*g*), some of the cells of which (*g'*) are again subdivided by secondary septa, whilst those of the outer bark (*h*) are arranged in centrifugal columns, similar to those of the transverse section. We thus learn that the outer bark consists of numerous horizontally disposed lines of compressed cells, whose parallel sides are lines more or less parallel with the periphery of the bark. The origin of this peculiar structure is sufficiently obvious; but were it otherwise, Plate IX. fig. 54 would make it plain, as I shall shortly demonstrate.

Plate VIII. fig. 50 is a tangential section of part of the vascular axis, magnified seventy-five diameters. The vessels do not exhibit their characteristic reticulations on the surfaces which are parallel to this section. In every one of the sections which I have made in this plane I have obtained the same result. The reticulated structure is conspicuous in the vascular walls parallel with the medullary rays; but instead of being continued round the entire tube, the reticulations appear to terminate abruptly, and with defined though irregular margins, as represented at *d'*, on reaching the tangential surfaces, which latter merely exhibit a thin, uniform, and apparently structureless layer of carbonaceous matter. These facts must be correlated with what I have previously mentioned, viz. that in the stems of *Asterophyllites* (see *anteà*, p. 44) the reticulations of the vessels were more definite on such of their walls as are parallel with the medullary rays than are those tangential to them. The rays themselves (*f*) are numerous, and, like those of the stem (Plate II. fig. 13), consist of from one to four cells (rarely five) arranged in a single vertical line. Plate VIII. fig. 51 represents the lateral aspect of one of these medullary rays, where the cells (*f*) are seen to have a regular mural arrangement. They have a vertical diameter of about ·0015.

Returning to Plate VII. fig. 46, we find that at two points, one to the left and the other at the upper margin of the bark, the outer layer of that tissue divides and allows of the centrifugal extension of the inner parenchyma. The former rapidly thins out as it accompanies the latter for a very short distance, when it soon disappears. The inner parenchyma soon spreads out into a cluster of diverging rootlets (*o*), which stream in every direction. Some of these are seen in longitudinal (*o''*), others in oblique (*o'*), and others again in transverse (*o*) sections. In the longitudinal section (Plate VIII. fig. 48) we again see one of these root-clusters, commencing as a large globular mass (*o'*), dividing into separate rootlets. Extending directly inwards from this root-cluster we see a column of the cells of the inner bark (*g'*) prolonged through the outer one. In the particular portion of the section here represented much of this inner layer has been accidentally destroyed, but in the remainder of the specimen the tissue appears in its normal state. At *n* we see a bundle of vessels passing horizontally outwards through the vascular zone to reach the root-cluster. On turning to Plate VIII. fig. 52, we have one of these root-bundles divided tranversely (*n*), and pushing aside the vascular laminæ as it

emerges to reach the bark. These bundles appear in all tangential sections of the roots. I have endeavoured to discover some regularity in their taxis, but have failed to do so. The clusters of rootlets were evidently given off alike from all parts of the circumference of the primary axis and its branches, but not in quincuncial or any other regular order. Sometimes two or three of the vascular bundles appear in close approximation in one tangential section; in others they are fewer in number and much further apart. It thus appears obvious that the clusters of rootlets are arranged irregularly on the periphery of the bark. On making a tangential section of the outer bark-layer, *h* (Plate IX. fig. 55), this latter tissue appears as a thick-walled parenchyma, composed of very uniform cells having a tolerably mean diameter of ·0019. Where the vascular root-bundles pass outwards through this tissue, they do so enclosed in a sharply defined, round cylinder composed of the cells of the inner bark, and which ultimately expands to form the principal parenchyma of the individual rootlets. This cylinder has usually a diameter of about ·038 to ·04, and is obviously the same structure as that seen in Plate VIII. fig. 48, *g*. Though the individual rootlets correspond in their general aspects with those of *Stigmaria*, they differ in several points of detail, but especially in their smaller size, and in the circumstance that all their component tissues are almost invariably preserved, whilst the middle parenchyma is rarely seen in those of *Stigmaria*. Plate IX. fig. 53 represents a longitudinal section of part of a rootlet. The root-bundle (*n*) consists of a few slender vessels, which are sometimes barred and at others reticulated. This bundle is surrounded by a sheath (*o''*), which is often very clearly defined, especially in the transverse sections (Plate VII. fig. 46, *o*). It consists of long, narrow, thin-walled, and parallel-sided cells. This is invested by the principal parenchyma (*o'*), the cells of which are somewhat lengthened in the longitudinal direction; and the whole is enclosed in a well-marked and often very sharply defined parenchymatous cortical investment (*o*), which only differs from that upon which it rests in the cells being yet more elongated and compressed. These rootlets further differ from those of *Stigmaria* in being of a darker colour, and more suggestive of a robust and less delicate texture.

I have called attention to the peculiar aspect of the outer bark as seen in all vertical and transverse sections of it, and also to the fact that the largest and outermost cells of its inner parenchyma are frequently intersected by secondary cell-walls, which are parallel to each other as well as to the periphery of the axis. It appears obvious to me that we have here a meristem structure analogous to some peculiar bark-tissues described in two of my previous memoirs*. It also appears clear that the outer parenchymatous cells of the inner bark were the source from whence additions were constantly being made to the inner surface of the outer bark, by the elongation of those cells in a radial direction, and the contemporaneous development within each cell of the numerous parallel tangential cell-walls already referred to. It obviously follows that each long column of parallel cells, seen in Plate VIII. figs. 47, *k*, & 49, *k*, with their common, strongly marked, radially arranged boundary lines, represent what had originally

* Phil. Trans. 1872, Part I. Plate xxxi. figs. 54 & 57. Ibidem, Part II. p. 313.

been one single cell. It is only by some such explanation as this that I can reconcile the appearances of the two sections just referred to with those of a tangential section made through the same structures. The latter section, as I have already stated, merely exhibits a very ordinary thick-walled parenchyma (Plate IX. fig. 55), affording no indication of the peculiarities seen in the sections represented in Plate VIII. figs. 47 & 49. The robust strength of the cell-walls of fig. 55 is altogether different from the more delicate transverse divisions of fig. 47, *h*, making it probable that the genesis of the two had been different. It was difficult to believe that these radial and tangential sections belonged to the same structures. By making an obliquely radial section through the tissue, I obtained the result seen in Plate IX. fig. 54, which I think demonstrates the accuracy of my interpretation. In this section I obtained, practically, a foreshortened view of the oblong cellular columns of Plate VIII. figs. 47 & 49, which reduced each column to the condition which one of the cells of Plate IX. fig. 55 would exhibit if elongated somewhat radially. The two radial and tangential sections now appear to harmonize without difficulty, and, unless I am seriously mistaken, they combine to sustain the genetic hypothesis which I have ventured to advance. This hypothesis is not without interest when viewed in connexion with various similar phenomena described in my previous memoirs; all these examples exhibit the innermost cellular bark approximating to the conditions which it ultimately attained in the cambium of exogenous stems.

In the examples of the root which I have hitherto described, the maximum diameter of the vascular axis is not more than ·075. But I possess others of larger dimensions, especially one for which I am indebted to Mr. Nield, of Oldham, in which the ligneous axis, deprived of its bark, measures fully an inch in diameter. All these specimens more or less resemble Plate IX. fig. 56, which represents a transverse section of a vascular axis, enlarged nine diameters. They exhibit numerous very strongly defined lines of circumferential growth, which are always irregularly concentric, each layer of vascular tissue being of unequal thickness on opposite sides of the stem, even when it forms a complete circle; but they very frequently only form crescentic segments of circles. We have already seen that the second exogenous layer of Plate VII. fig. 45 is of this character; and several of the thin outermost ones at the lower part of Plate IX. fig. 56 exhibit the same aspect. Had the deficiency in these growths always been on the same side of the axis, we might have supposed that it was due to some peculiarity in the position of the growing plant relatively to light or moisture; but this is not the case. The strongly marked boundaries of these concentric layers render this one of the most remarkable examples of exogenous growth in a Cryptogamic plant that has hitherto come under my notice.

I have as yet brought forward no evidence to prove the correlation of these roots with the stems of *Asterophyllites*; but such evidence is not lacking. At an early stage of my inquiries I observed in several of my sections of these roots indications that some peculiarity existed in what ought to have been their medullary centres. In some of these there was only a triangular cleft; in others I observed some clusters of vessels, of larger

size than those constituting the exogenous layers, occupying the same central position, and suggesting the idea of a primary vascular axis whose arrangements had been disturbed during the subsequent growth of the root. All these appearances received their explanation on my obtaining some additional sections, one of which is represented in Plate IX. fig. 57; I am indebted for this fine example to Mr. J. WHITTAKER, of Oldham. Both its general structure and its peculiar bark at once identify this as being identical with Plate VII. fig. 46.

Plate IX. fig. 58 is an enlarged view of the central portion of fig. 57, in which we find the triangular axis of *Asterophyllites* in its most definite form. The identity of this structure in the two conditions of stem and root, both in its contour and in the arrangement of its vessels, is most obvious. Its diameter, too, corresponds closely with that of the axes of the larger aërial stems; the only difference that I can detect appears in the larger size of its vessels, compared, for instance, with those of Plate II. fig. 9. It will be observed also that the laminæ of the young exogenous growth curve away from each of the angles of the central bundle in exactly the same fan-shaped manner as is seen in *Asterophyllites*—an arrangement to which I have already alluded when I said that the axis of *Calamostachys Binneyana* (Plate VI. fig. 38) exhibited some Asterophyllitean features in its exogenous layer. The vessels of the exogenous layers of the root-structures are much smaller than in the aërial stems described. This difference may be merely characteristic of a distinction between the roots and stems, or it may be due to the circumstance that the roots belong to a different species from those of which I have described the stems. The latter view is sustained by the fact that I possess several examples of undoubted amyeloid* roots having the structure seen in Plate IX. fig. 59. Some similar ones discovered since this figure was drawn are not only much larger and more perfect, but are invested by the characteristic bark of these amyeloid roots, removing all doubt as to the generic identity of the entire series of specimens. Plate IX. fig. 59 is drawn to the same scale as the corresponding sections of the aërial stems. We thus see that the vessels of the former correspond closely, both in size and arrangement, with what we see in Plate II. fig. 10, allowance being made for the non-existence in Plate IX. fig. 59 of the central triangle. The closeness of these general as well as detailed resemblances renders it probable that, whatever may be the case with Plate VII. fig. 46, Plate IX. fig. 59 belongs to the same species as Plate II. fig. 10 and the allied sections. We see in Plate IX. fig. 59 the same tendency to the formation of crescentic exogenous growths as appeared in Plate VII. fig. 46 & Plate IX. fig. 56.

There are a few physiological features in the roots now described meriting attention.

* As my investigations progressed, I have found it necessary to assign provisional generic names to certain well-marked types of structure on the same principle that the Lepidodendroid and Sigillarian roots were designated Stigmariæ before their real nature was known. Nor is this process wholly without a permanent use; even after the true correlation of the objects with well-known and previously named plants is discovered, such names serve as convenient descriptive adjectives. Thus we now speak of sternbergian piths and diploxyloid stems. So the term amyeloid conveniently designates these pithless roots of the Coal-measures.

I have already referred to the fact that the ligneous reticulations are deposited on the inner surfaces of the walls of the vessels of *Asterophyllites*, and that these deposits are more distinct on those sides that are in contact with the medullary rays than on those which are tangential to the circumference of the stem. In the axis of the root this difference becomes yet greater, since on the tangential surfaces the reticulations disappear altogether. When we remember that precisely similar conditions exist in the distribution of the circular disks on the corresponding portions of the fibres of Coniferous stems, we may suspect that some similar influence regulates that distribution in both cases. Not that I mean to infer the existence of any systematic relationship between *Asterophyllites* and the Conifera, but of some influence common to both which has produced differences between the deposits and secondary growths of the radial and tangential surfaces of their fibro-vascular elementary tissues. That some relations subsist between the intervascular disks of Coniferous fibres and the ligneous deposits within the same fibres is well known*.

The most determined opponent of exogenous development must admit that we have here circumferential growth, as unmistakable as that of an oak-tree both in the stems and in the roots. Not only have we the lines of growth, so distinctly seen in Plate IX. fig. 56, but the newer laminæ not unfrequently split off from the older ones, as in Plate IX. fig. 59. In both stems and roots these new vascular layers can only have originated in the inner portions of the inner bark, *g*. This unity of function leads us to the conclusion that the parenchymatous cells of the inner bark of the stem (Plate II. fig. 10, *g*) are homologous with the non-radiating parenchyma of the roots (Plate VIII. figs. 47 & 49, *g*), and that in both cases this layer has been the chief seat of genetic activity. I have already shown that in the case of the roots the *outer* layer of the bark has clearly been derived from the outer surface of the *inner* parenchyma by the development of new and parallel tangential divisions within the interior of its constituent cells; so that, in this organ at least, the parenchyma has been subject to a double waste of its cell-supply—an inner one to furnish new vessels, and an outer one thickening the outer bark-layer. This exhaustion, whether it was continuous or periodic, must necessarily have been antagonized by a corresponding activity of cell-reproduction. It thus appears that this inner parenchyma was a meristem layer in a more or less constant state of intense genetic activity. I have not been able to satisfy myself in the same conclusive manner respecting the origin of the prosenchymatous outer bark of the aërial stems. How far the latter is the altered homologue of the exterior of the outer bark of the roots is also an open question. It is not improbable that both these tissues may be genetically identical, but modified in their form to suit the aërial and subterranean conditions under which they had to fulfil their respective functions.

The most difficult question of all yet remains to be considered—What is *Asterophyllites*? In the first place, it has the closest possible affinity with *Sphenophyllum*. I

* See on this point my memoir "On the Structure and Affinities of some Exogenous Stems from the Coal-measures," Monthly Microscopical Journal, August 1, 1869.

assume this, being convinced that the observations of Professor Renault at Autun are accurate; that able observer is much too cautious to speak so decisively as he has done respecting the stems of *Sphenophyllum* for his evidence to be doubted. He has kindly sent me examples of his stems, and they agree closely with my own; so that the near affinity of *Asterophyllites* and *Sphenophyllum* must now be accepted, not as a probable hypothesis, but as a determined fact*. It is equally clear that these stems have no relationship to *Calamites*. M. Grand'Eury some time ago arrived at this conclusion at St. Etienne (see 'Comptes Rendus,' tome lxviii., "Observations sur les *Calamites* et les *Astérophyllites*") from the study of their external forms; Dr. Dawson has insisted upon the same distinction in the case of his Canadian specimens. Professor Renault writes to me respecting the stems of *Sphenophyllum*:—"Il serait difficile de vous donner la description des détails de structure de ces tiges curieuses, *qui n'ont jamais rien eu de commun avec les* Calamites." I am convinced that the same conclusion is true in the case of *Asterophyllites*. To continue to affirm that these plants were the branches and foliage of *Calamites* in the face of this combined testimony of independent observers, whose positions have given them opportunities for forming a judgment on the question, will surely be to stultify all scientific testimony.

But, unfortunately, determining what these plants are not does not tell us what they are. Whether or not the so-called *Calamites verticillatus* was the arborescent stem of *Asterophyllites*, as I have deemed probable, my large specimens of amyeloid roots prove that these stems must have attained to a comparatively arborescent form. This conclusion has also been arrived at by M. Grand'Eury.

In the disposition of the vessels of their primary triangular vascular bundle they seem to me to approach the *Lycopodiaceæ*. In Professor Renault's specimens the vessels forming the extreme apex of each arm of the triangle appear to be more barred than reticulated; and both in his specimens and mine their small size, compared with that of the more central vessels, as well as the entire exclusion of all cellular elements from the interior of the bundle, are strongly indicative of a genesis like that of the similar bundles of the Lycopods. The nearest approach to this structure that I am acquainted with amongst the living *Lycopodiaceæ* is seen in the five- or six-angled bundle of *Psilotum triquetrum*; but in this example the central procambium is not converted into vessels, but into a compact rod of thick-walled, permanent prosenchyma. Some of the vessels of *Psilotum* exhibit a remarkable tendency to abandon the scalariform, and assume an irregularly reticulate type of structure. The bark of this recent plant affords an instructive instance of the differences between the several parts of a continuous tissue destined to live under different conditions and to perform different functions. In the aërial stem its outer layer consists of long narrow prosenchymatous cells with parallel sides. The same tissue in the subterranean axis, now generally regarded as an underground stem rather than as a root, is composed of very large, coarse, thick-walled, irregular

* I have already (page 42, note) called attention to the fact that some species of *Sphenophyllum*, at least, exhibit the three superficial grooves of the bark so characteristic of young twigs of *Asterophyllites*.

parenchyma. The difference between these two tissues is quite as great as that which I have shown to exist between the corresponding ones of *Asterophyllites*. In both cases the narrow prosenchyma of the aërial bark is converted into the modified parenchyma of the subterranean one.

These resemblances to *Lycopodiaceæ* are further sustained by the nature of the fructification. A German critic of my memoir on *Volkmannia Dawsoni* objected to my determination that it was a fruit of *Asterophyllites* on the ground that it appeared to be Lycopodiaceous in its aspect, and I have already pointed out the resemblance of its spores to the macrospores of *Selaginella inæquifolia*. At the first glance a verticillate jointed Lycopod appears an anomaly, but the jointed character of *Asterophyllites is more apparent than real*. In this respect the plant differs altogether from *Calamites* and *Equisetum*. In *Asterophyllites* the node is a mere centrifugal expansion of the bark, since the central vascular axis pursues its course through the node altogether unaffected by its presence. In the two latter genera, on the other hand, the node affects the entire structure of the stem from its circumference to its innermost centre, altering equally the arrangements of the tissues in the bark, in the ligneous zone, and in the medulla. Such being the case, the merely verticillate arrangement of the leaves does not constitute a difficulty of much importance. In reference to this latter point, Professor Sachs writes to me:—"With regard to the question concerning the possibility of verticillate leaves in Lycopods, I believe that *Lycopodium complanatum* with its apparently decussate pairs, and *Lycopodium selago* with from three to five-membered apparent verticils, present analogies which are not to be entirely rejected." And, after giving me further evidence in the same direction, he adds:—"I should accordingly find in the existence of verticillate leaves no reason for excluding a plant from the *Lycopodiaceæ*." Cordially agreeing with these statements, I am at present of opinion that *Asterophyllites* and *Sphenophyllum* are plants allied in the closest possible manner, if they do not belong to one genus, and that they are to be regarded as somewhat aberrant members of the Lycopodiaceous family. In 1865 Dr. Dawson wrote concerning *Sphenophyllum*:—"A beautiful specimen of *Sphenophyllum emarginatum* from New Brunswick, in the collection of Sir W. E. Logan, has enabled me to ascertain that its stem had a simple axis of one bundle of reticulato-scalariform vessels, like those of *Tmesipteris* as figured by Brongniart. These curious plants were no doubt cryptogamous, having a habit of growth like that of *Equisetaceæ*, leaves like those of ferns or *Marsiliaceæ*, and fructification and structure like those of *Lycopodiaceæ*. They were closely allied to *Asterophyllites* and *Annularia*"*. I need scarcely point out how this significant paragraph foreshadows part of what has since been worked out with more elaborate detail by Professor Renault and myself.

I have received from Professor Renault a copy of the "Rapport" of MM. Daubrée and Brongniart upon his two memoirs, extracted from the 'Comptes Rendus,' t. lxxvi., which contains several important conclusions agreeing with those embodied in my

* Quarterly Journal of the Geological Society, vol. xxii. p. 135.

memoir. Thus, speaking of *Sphenophyllum*, the "Commissaires" say:—"La structure interne de ces parties indique les rapports de ces végétaux avec les Lycopodiacées et les Marsiléacées." The report then proceeds to examine certain spikes of fructification, including those described by M. Ludwig and Mr. Binney. Speaking of the fruit associated by the latter author with *Calamodendron commune*, the report remarks:—"Il reste cependant encore bien des doutes à éclaircir; si les épis de fructification d'Autun appartient, sans aucun doute aux *Annularia*, ceux décrits par MM. Ludwig et Binney sont-ils réellement des fructifications, soit de *Calamites*, soit de *Calamodendron*? Ne se rapporteraient-ils pas plutôt à des *Sphenophyllum*, dont ils se rapprochent par le nombre moindre des organes verticillés, et d'après les figures de M. Binney, par leur axe plus grêle et vasculaire dans son centre?" In making the above and other comparisons contained in the "Rapport," M. Brongniart has overlooked my still more significant *Volkmannia Dawsoni*. I can only conclude that the disturbances occasioned by the war have prevented him from receiving the copy of the memoir which I sent to him.

I have again to acknowledge my obligations to Messrs. Butterworth, Whittaker, and Nield for their valuable assistance in procuring specimens for my investigation from the Oldham district, and to G. Grieve, Esq., for aiding me in like manner with examples from those Burntisland beds which the right of discovery has made so peculiarly his own. I may further observe, in reference to the Plates illustrating this memoir, that having made every drawing myself with the most scrupulous care, upon lithographic paper, so that they reappear in the Plates exactly as they left my hands, they may be relied upon as faithful transcripts of the objects represented.

[As this memoir is passing through the press I have received from M. Renault a copy of his valuable memoir entitled "Recherches sur l'organisation des *Sphenophyllum* et des *Annularia*," the publication of which has been delayed through the disturbances occasioned by the war. It demonstrates most clearly the very close general resemblances existing between his specimens of *Sphenophyllum* and mine of *Asterophyllites*. But there is one singular difference. In the French specimens, the zone corresponding with my exogenous zone, and of which the *transverse* sections correspond in the most minute manner with my figures 3, 4, & 9, chiefly consists not of long vessels, but of cubical cells arranged in vertical piles. Like my vessels, the walls of these cells are beautifully reticulated. It appears as if these cells were arranged in vertical lines preparatory to their coalescence and conversion into vessels, but that this double process had never taken place. This substitution of remarkably distinct reticulated cells for equally distinct reticulated vessels is a curious physiological fact. It is also perfectly clear that the vessels given off to supply the leaves in the French examples spring from the central triangular axis, and not from the exogenous layers, as I have already pointed out in the note on page 45, the former having been the primary vascular bundle of the young shoot. On the other hand, the vascular bundles given off to the *rootlets* evidently spring, not from the triangular axes of the roots, but from the exogenous layer. These arrange-

ments are exactly identical with those found respectively in Lepidodendroid stems and in their Stigmarian roots, as I pointed out in my third memoir*. The close relationship between *Asterophyllites* and *Sphenophyllum* being thus established upon the sound basis of closely identical organizations, the inquiry which suggests itself is, What are the real distinctions in their foliage? It appears to me that typically there are none. The leaf of *Asterophyllites* has a single midrib; that of *Sphenophyllum* has three or more. May not each cuneiform leaf of *Sphenophyllum* be regarded as composed of three or more of the divergent leaves of *Asterophyllites*, which have coalesced throughout the greater part of their length, but whose typically separate origin is indicated by the number of the teeth usually seen at their broad extremities? This explanation further accounts for the small number of the leaves usually composing each foliar verticil of *Sphenophyllum* compared with those of *Asterophyllites*.

M. Renault's specimens demonstrate very clearly the existence of a layer of ordinary delicate parenchyma within the radiating lines of parenchyma seen in my figures 4, 9, 10, & 15. As mentioned in the preceding memoir, I have only caught a glimpse of this layer in one or two of my specimens, it having almost invariably disappeared from them, leaving a vacant space as in fig. 16, *g*. This demonstration materially simplifies the identification of the layers of the bark in the stems and roots. Thus the layer *g* in figures 47, 49, & 57 is identical with the innermost parenchyma of Professor Renault; it corresponds with the layer *g* in figures 9, 10, 11, & 21. In the same way the layer *h* in these latter figures is identical with *h* in figures 47, 48, 49, & 57. If these conclusions are correct, the roots of *Asterophyllites* have a bark like that of the stems, but almost or altogether deprived of the superficial prosenchyma seen in the latter organs; this layer, however, may be regarded as reappearing in the outermost parenchyma (fig. 53, *o*) of the rootlets.

I may add that a second copy of my memoir on *Volkmannia Dawsoni*, which I sent to M. Brongniart, *has* reached him. In a note to M. Renault's paper, M. Brongniart, acknowledging this memoir, recognizes the important points in which the organization of that fruit agrees with that of the structures described by M. Renault, as well as those on which it differs, "au moins spécifiquement."—Manchester, February 20, 1874.]

Description of the Plates.

The letters of reference in the accompanying memoir have been used to indicate as nearly as possible the same structures as in the previous memoirs of the series. The letters are not alphabetically continuous, because some of the structures existing in other plants are wanting in *Asterophyllites*; and I have thought it preferable to omit the letters referring to these absent structures, rather than to introduce confusion by employing the same letter to represent organs and tissues that were not homologous. The only exceptions to this rule will be found in a few of the final letters of the alphabet, which have been used for special purposes.

* Philosophical Transactions, 1872, Part II. p. 307.

c. Medullary vessels.
d. Ligneous zone.
f. Medullary rays.
g. Inner bark.
h. Middle parenchymatous bark.
k. Prosenchymatous outer bark.
l. Leaves.
n. Root-bundles of vessels.
o. Rootlets.
r. Cicatrices of strobili.
s. Strobilus-axis.
t. Bracts of strobilus.
u. Sporangia.
v. Sporangiophores.
w. Spores.
x. Lateral branch.

PLATE I.

Fig. 1. Transverse section of a very young twig from Oldham, showing the primitive vascular bundle, the equivalent of the medullary vessels of the *Lycopodiaceæ*, magnified 30 diameters.

Fig. 2. Similar section of a twig, with one exogenous layer of the ligneous or vascular zone surrounding the primitive bundle, enlarged 30 diameters.

Fig. 3. Similar section of a young branch with two exogenous layers, magnified 30 diameters.

Fig. 4. Similar section of a young branch, with double bark-furrows and three exogenous layers, magnified 30 diameters.

Fig. 5. Longitudinal section of a young branch, with three exogenous layers, magnified 30 diameters.

Fig. 6. One of the vessels of the ligneous zone of fig. 5, seen on the side parallel with the medullary rays, magnified 420 diameters.

Fig. 7. Cells of the inner bark, from a longitudinal section like fig. 5, magnified 200 diameters.

PLATE II.

Fig. 8. Longitudinal section of some of the prosenchymatous cells of the outer bark, magnified 130 diameters.

Fig. 9. Transverse section of the vascular axis and inner bark of a more matured stem, magnified 30 diameters.

Fig. 10. Similar section to the last, but of a more fully developed stem and with a thicker inner bark, magnified 30 diameters.

Fig. 11. Transverse section of a yet further developed stem, exhibiting a secondary series of concentric exogenous growths, magnified 30 diameters.

Fig. 12. Similar section to fig. 11, with very large vessels in the inner exogenous laminæ, magnified 30 diameters.

Fig. 13. Tangential section of two vessels of the exogenous layers of the same specimen as fig. 9, showing the medullary rays, magnified 125 diameters.

PLATE III.

Fig. 14. Tangential section of the peripheral margin of a nodal cortical disk with sections of leaves, magnified 30 diameters.

Fig. 15. Longitudinal but slightly tangential section of part of the outer prosenchymatous bark, exhibiting a longitudinal section of a leaf prolonged from the margin of the nodal disk, magnified 30 diameters.

Fig. 16. Slightly oblique transverse section of a branch passing through a nodal foliar disk, so as to intersect the verticil of leaves on the left of the section, magnified 30 diameters.

Fig. 17. Series of transverse sections of leaves, magnified 30 diameters.

Fig. 18. Transverse section of a young twig of *Asterophyllites* or *Sphenophyllum* from Burntisland, corresponding to fig. 1, magnified 30 diameters.

Fig. 19. Similar section to fig. 18, but slightly oblique, and exhibiting two of the three superficial cortical grooves, magnified 30 diameters.

Fig. 20. Transverse section of a branch from Burntisland, with three or four exogenous layers, magnified 30 diameters.

PLATE IV.

Fig. 21. Transverse section of a more matured stem from Burntisland, magnified 30 diameters.

Fig. 22. Tangential section of part of the exogenous zone of fig. 21, showing the intersected extremities of the medullary rays, magnified 130 diameters.

Fig. 23. Radial longitudinal section of part of the same zone as fig. 22, made in the plane of the medullary rays, magnified 130 diameters.

Figs. 24 & 25. Portions of the barred vessels of fig. 23, magnified 420 diameters.

PLATE V.

Fig. 26. Fragment of *Sphenophyllum Schlotheimi*. *k'*, lower portion of the internode *k*, with its lateral groove and the bases of a verticil of leaves.

Fig. 27. Transverse section of part of a stem from Burntisland, giving off a branch, magnified 30 diameters.

Fig. 28. Transverse section of *Volkmannia Dawsoni*, a fruit of *Asterophyllites* or *Sphenophyllum*, magnified 7½ diameters.

Fig. 29. Central vascular axis of fig. 28, magnified 22 diameters.

Fig. 30. Three spores like those of fig. 28, magnified 140 diameters.

Fig. 31. Single strobilus of *Asterophyllites* from a bed of shale at Brooksbottom, magnified 2 diameters.

Fig. 32. Fruiting stem of an *Asterophyllites* from the Lancashire Coal-measures. Natural size.

PLATE VI.

Fig. 33. External aspect of the upper part of a small strobilus of *Calamostachys Binneyana* from Oldham, enlarged 6 diameters.

Fig. 34. Transverse section of part of a sterile foliar disk of *Calamostachys Binneyana*, magnified 24 diameters.

Fig. 35. Part of the periphery of a similar section to fig. 34.

Fig. 36. Transverse section of *Calamostachys Binneyana* made in the plane of a verticil of sporangiophores, magnified 27 diameters.

Fig. 37. Transverse section of the same specimen as fig. 36, but made in the plane of an internode, intermediate between a fertile and a barren verticil, magnified 12 diameters.

Fig. 38. Central vascular axis of fig. 37, enlarged 80 diameters.

Fig. 39. Part of the outer surface of a sporangium, magnified 98 diameters.

PLATE VII.

Fig. 40. Section of the wall of a sporangium of *Calamostachys Binneyana* made transversely to the longer axes of its cells, magnified 98 diameters.

Fig. 41. Section of a cell of the wall of a sporangium of the same, made parallel with its longer axis, and showing one of its barred lateral walls, magnified 98 diameters.

Fig. 42. Inner surface of part of a sporangium-wall of the same, magnified 98 diameters.

Fig. 43. Cluster of spores of *C. Binneyana*, showing the spores enclosed in mother-cells, magnified 140 diameters.

Fig. 44. Fertile stem of an *Asterophyllites*, from Brooksbottom, from which strobili have been detached, enlarged 2 diameters.

Fig. 45. "*Calamites*" *verticillatus*, reduced to two thirds of the natural size.

Fig. 46. Transverse section of a young root with two clusters of rootlets of *Asterophyllites*, from near Oldham, magnified 12 diameters.

PLATE VIII.

Fig. 47. Part of the bark of fig. 46, enlarged 80 diameters.

Fig. 48. Vertical section of part of the same root with one cluster of rootlets, enlarged 18 diameters.

Fig. 49. Portion of a similar section to fig. 48, showing the two layers of bark, enlarged 80 diameters.

Fig. 50. Tangential section of the vascular axis of fig. 46, enlarged 75 diameters.

Fig. 51. Part of a radial longitudinal section of the same vascular axis, made in the plane of a medullary ray, enlarged 100 diameters.

Fig. 52. Tangential section like fig. 50, but passing transversely through a vascular bundle going to one of the clusters of rootlets, enlarged 75 diameters.

PLATE IX.

Fig. 53. A longitudinal section of part of a rootlet of *Asterophyllites*, enlarged 30 diameters.

Fig. 54. Obliquely tangential section of the outer bark of fig. 46, enlarged 90 diameters.

Fig. 55. Tangential section of part of the outer bark of fig. 46, enlarged 90 diameters.

Fig. 56. Transverse section of the vascular axis of a large root, exhibiting the successive lines of concentric growth, enlarged 9 diameters.

Fig. 57. Transverse section of a similar root to fig. 46, but exhibiting the central prismatic axis seen in the aërial stems of *Asterophyllites* and *Sphenophyllum*, enlarged 12 diameters.

Fig. 58. Central part of fig. 57, showing the central triangular axis, enlarged 30 diameters.

Fig. 59. Transverse section of the vascular axis of another species of amyeloid root with very large vessels, corresponding closely with the stems figs. 1 to 16, enlarged 30 diameters.

IV. *On the Action of Electricity on Gases.*—II. *On the Electric Decomposition of Carbonic-acid Gas.*

By Sir B. C. Brodie, *Bart., D.C.L., F.R.S., late Waynflete Professor of Chemistry in the University of Oxford.*

Received June 19,—Read June 19, 1873.

At a very early period of the investigation on the action of electricity upon oxygen, which formed the subject of my previous memoir*, the idea occurred to me that although but a small and limited proportion of the total oxygen passed through the induction-tube was converted into ozone (which proportion could not be exceeded by any modification I had been able to effect in the conditions of the experiment), it might be practicable to replace that portion of the oxygen which was unaffected by the action of electricity by an indifferent gas, and thus effect the total conversion of oxygen into ozone, or even the actual isolation of the ozone by the subsequent removal of the gas by which it was diluted. Thus, for example, by the passage of 100 cub. centims. of oxygen through the induction-tube, a gas can readily be obtained of which the iodine-titre is 5 cub. centims. This corresponds, according to my previous experiments, to an absorption by hyposulphite of soda of 10 cub. centims. of a gas containing in that space the matter of 15 cub. centims. of oxygen. If, therefore, we were to mix 15 cub. centims. of oxygen with 85 cub. centims. of an indifferent gas which should be unaffected by the action of the electricity, and pass the mixed gases through the induction-tube (assuming the same proportion of ozone to be still formed as in the case of the passage through the tube of pure oxygen), the total amount of oxygen in the gas would be converted into ozone, and be removed in that form by passing the gas though a solution of hyposulphite of soda. With the view of testing this idea by a critical experiment, I passed such a mixture of carbonic-acid gas and oxygen through the induction-tube. The formation of ozone was at once apparent, and was rendered evident by the action of the gas issuing from the induction-tube upon a neutral solution of iodide of potassium. But on examining the composition of the gas I soon discovered that the proportion of oxygen in it had actually increased, owing to the decomposition in the induction-tube of the carbonic-acid gas into oxygen and carbonic oxide. I did not publish this experiment; but the apparatus by which it was effected long stood upon my laboratory table, and, together with the results, has repeatedly been explained by me to other chemists. Since that time, as appears from the pages of the 'Comptes Rendus,' this decomposition of carbonic-acid gas under the influence of electricity has been cited as a novel discovery. So far as the bare fact of the decomposition of carbonic-acid gas, under the influence

* Philosophical Transactions, Part II. 1872, p. 435.

of electricity, in the induction-tube into oxygen and carbonic oxide is concerned, the results of this experiment might, even at the time it was made by me, have reasonably been anticipated, not only from the circumstance that carbonic-acid gas is, as is well known, decomposed by the passage of the electric spark, but also that PLÜCKER* had already observed (although I was unaware of the observation) that when the electric discharge was passed through rarefied carbonic-acid gas, the spectrum of the gas after a short time changed into the spectrum of carbonic oxide, and from this circumstance had inferred the decomposition of the gas. But the observations of this eminent investigator were made under very different circumstances to mine; and he was not cognizant of the formation of ozone, which was the critical point of my experiment, and a result which could not have been ascertained by his method of observation.

The experiment having failed in its immediate object was for a time laid aside by me. Subsequently, however, I reverted to it under a somewhat modified form. It occurred to me, instead of mixing oxygen with carbonic acid, to endeavour to generate in the very atmosphere of the carbonic acid itself, by the electric decomposition of the gas, the requisite amount of oxygen. I passed, therefore, pure and dry carbonic acid through the induction-tube, and examined the gases resulting from its decomposition, estimating the ozone by the titre of the gas, and also the oxygen and carbonic oxide formed. This examination at once convinced me of the importance of the experiment in reference to the problem of the isolation of ozone, and became the foundation of the following research, which has gradually been extended in more than one direction.

SECTION I.

The subject will be conveniently considered in two divisions.

I will first lay before the Society the results of four experiments, in which the quantities were carefully measured, which may serve to define the general features of the decomposition and the methods of operation.

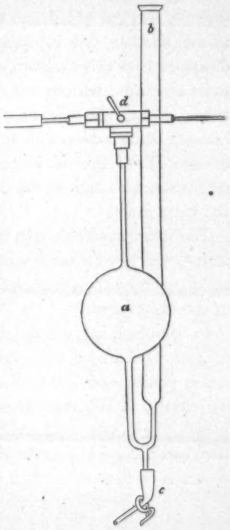
A measured volume of carbonic-acid gas was passed through the induction-tube and there submitted to the electric action. On quitting the tube the gas traversed a bulb containing a solution of neutral iodide of potassium, the decomposition in which afforded a measure of the ozone present in the gas. The gas was then passed into a bulb containing a strong solution of caustic potash, which absorbed the carbonic acid, while the oxygen and carbonic oxide were collected in it.

The carbonic acid was made, in the usual way, from pure marble and dilute hydrochloric acid in a (so called) KIPP's apparatus. Having been washed with water and dried by passing through a system of tubes containing sulphuric acid and pumice imbued with sulphuric acid, the gas was collected in a sulphuric-acid gas-holder. A description of this gas-holder and of the other apparatus here employed for the collection of and measurement of gases is given in my previous paper, to which the reader is referred for information on the subject†. The carbonic acid before its passage through the induction-

* Pogg. Ann. vol. cv. p. 82.

† Philosophical Transactions, *loc. cit.* p. 437.

tube was transferred to a large gas-pipette and there measured, the pipette being filled with the gas at atmospheric pressure and a known temperature. The arrangement of the induction-tube was precisely as in my previous experiments with oxygen, the tube being in all cases placed between two sets of small bulbs permanently attached to it containing anhydrous phosphoric acid. The bulb also containing the solution of potassium iodide was of the form previously employed. A sketch is here annexed of the absorption-bulb, from which this part of the arrangement will be easily understood.



The stopcock (*d*) was a three-way stopcock of steel similar to that attached to the aspirator. Before the experiment the bulb *a* was filled with mercury by means of the tube *b*, the stopcock being open to the air; it was then closed, and a partial vacuum created in the bulb by running out a portion of mercury at *c*, where an india-rubber tube closed by a clamp was attached. The solution of caustic potash was introduced by pouring the solution into the absorption-bulb through the tube *b*. The upper part of the bulb and the capillary tube were filled with the solution to within about half an inch of the stopcock, which should not be allowed to come in contact with the alkali.

At the commencement of the experiment the apparatus was thoroughly swept out with carbonic acid, the connexion with the absorption-bulb being shut off and a suitable channel provided for the gas by means of the three-way stopcock. The pipette having been filled with the gas and the requisite observations made for its measurement, the gas was drawn through the induction-tube, which may readily be effected at any required rate by opening the communication with the interior of the bulb and running mercury from it. The absorption at first is very rapid, and some management is required not to expose at once too great a surface of the alkaline solution to the action of the gas. The connexion having been made with the coil, iodine speedily appears in the titre-bulb, and after a short time a sensible quantity of permanent gas is collected in the absorption-apparatus. After the first pipette of gas had thus been passed through the tube, the coil was temporarily disconnected, a second pipette filled and drawn over as before. The experiment was continued until in each case five pipettes of carbonic-acid gas had been thus acted on.

This part of the experiment having been completed, the absorption-bulb was disconnected, and having been attached to the aspirator described in my former memoir, the total gas was transferred to it and measured. A small quantity of an alkaline solution of pyrogallic acid was now introduced into the absorption-bulb and the gas drawn

back from the aspirator into the bulb, in which it was allowed to remain for some time until no further change in the volume of the gas occurred; the gas was then retransferred to the aspirator and again measured: the difference between the two measurements gives the volume of oxygen in the gas. The total volume of oxygen due to the changes occurring in the induction-tube is represented by the sum of this difference and the titre of the gas. The whole gas, or a portion of it, was transferred to a eudiometer of known capacity, oxygen added, and the mixed gases detonated. The contraction which occurs affords another and independent measure of the oxygen formed, the oxygen which disappears on this contraction being the volume of oxygen required to restore the gas to its original condition and reconstitute the state of things broken up in the induction-tube. I shall not now enter upon the further analysis of the gas, which presents peculiarities better reserved for future consideration; but evidence will hereafter be given, in Section II., of the substantial identity of the results afforded by these two modes of estimation, although of the two measures that afforded by the contraction is undoubtedly the more exact.

The following Table contains the experimental results obtained in these four experiments, which were made with the same coil, the same battery-power, and under the same general circumstances, no intentional variation being made in any of the conditions of the experiment.

In column I. is given the volume of carbonic acid passed through the induction-tube, estimated (as are all other volumes of gas in these experiments) in cub. centims. at 0° C. and a pressure of 760 millims. In column II. is given the volume of gas unabsorbed by potash; in III. the titre of the gas; in IV. the oxygen absorbed by pyrogallate of potash; in V. the total oxygen found by actual experiment—namely, the sum of the titre and the gas absorbed by pyrogallate. In column VI. the contraction is given which occurred on detonation with oxygen of the unabsorbed gas.

Experiment.	I. Carbonic acid.	II. Unabsorbed gas.	III. Titre.	IV. Gas absorbed by pyrogallate.	V. Total oxygen.	VI. Contraction.
1.	1413	29·89	2·44	7·98	10·42	10·49
2.	1385	26·67	1·55	7·08	8·63	8·88
3.	1347	37·66	1·61	10·99	12·6	13·47
4.	1390	54·99	3·38	15·6	18·98	19·8

Now if we assume two thirds of the sum of the titre and unabsorbed gas in the several experiments to be carbonic oxide, which is nearly although not quite the case, we have for the amount of carbonic oxide formed, and therefore of carbonic acid decomposed, in the induction-tube in the four experiments respectively

(1) . . . . . 21·56
(2) . . . . . 18·81
(3) . . . . . 26·18
(4) . . . . . 38·92

and for the amount per cent. of carbonic acid decomposed

(1) . . . . . 1·52
(2) . . . . . 1·35
(3) . . . . . 1·94
(4) . . . . . 2·68

Also the iodine-titre on 100 volumes of carbonic acid passed through the induction-tube in the four experiments is

(1) . . . . . 0·17
(2) . . . . . 0·11
(3) . . . . . 0·12
(4) . . . . . 0·12

Similarly, the iodine-titre on 100 volumes of the sum of the titre and unabsorbed gas is

(1) . . . . . 7·54
(2) . . . . . 5·44
(3) . . . . . 4·1
(4) . . . . . 5·79

And the iodine-titre on 100 volumes of oxygen formed in the induction-tube, as represented by the contraction given in the last column, is

(1) . . . . . 23·29
(2) . . . . . 17·47
(3) . . . . . 11·97
(4) . . . . . 17·05

The titre on 100 volumes of the sum of the titre and unabsorbed gas, although somewhat higher than the titre on 100 volumes of pure oxygen as arrived at by my previous experiments, in which the maximum value of this titre was 6·52 per cent., is nevertheless not far removed from that titre; but the titre on 100 volumes of the oxygen thus generated by the decomposition of carbonic acid in the induction-tube is fully three times as great as that titre; and the proportion of 100 parts of oxygen converted into ozone is in the four experiments severally

(1) . . . . . 69·87
(2) . . . . . 52·41
(3) . . . . . 35·91
(4) . . . . . 51·15

It is assumed in these remarks not only that the ozone thus produced in the decomposition of carbonic acid is an ozone essentially the same in kind as that formed by the action of electricity on pure oxygen, but also that no other substance is associated with it capable of decomposing iodide of potassium. But these assumptions might, for aught we know to the contrary, be incorrect: there may be more than one variety of ozone itself; or, again, some higher oxide of carbon might, in such an experiment, be formed by the

oxidation of carbonic acid, analogous in properties to the organic peroxides of the bibasic acids and capable of decomposing iodide of potassium. No definite conclusion could be drawn from these experiments until this point was ascertained.

In my previous investigation certain definite quantitative reactions of ozone have been established, which may be submitted to exact measurement, by which this gas is discriminated from all other known gases, and by which it may be effectually recognized and estimated. Of these, the following reaction is perhaps the most certain and characteristic*.

When oxygen which has been submitted in the induction-tube to the electric action is passed through a solution of neutral hyposulphite of soda a diminution of bulk occurs, and a volume of gas is absorbed by the hyposulphite of soda equal to twice the volume which would be occupied by the quantity of oxygen absorbed, under similar circumstances, when an equal volume of the same gas is passed through a solution of neutral iodide of potassium. This critical test was applied in the present instance as follows.

Five pipettes of the electrized carbonic acid (the capacity of the pipette being 290·8 cub. centims.) were successively passed through a bulb containing a solution of neutral iodide of potassium, by which the "titre" of the gas was estimated. After the experiment the carbonic acid was, as before, absorbed in the absorption-bulb, and the unabsorbed gas transferred to the aspirator and measured. It was demonstrated in my previous experiments that the volume of electrized oxygen is not affected by the passage of the gas through a solution of neutral iodide of potassium.

In the first column of the following Table is given the "titre" of the gas, T; in column II. the volume of the gas unabsorbed by potash after the passage of the five pipettes of gas through the solution of iodide of potassium, V. In column III. is given the volume of unabsorbed gas after the passage of an equal volume of the electrized gas through the solution of hyposulphite of soda, V_1; in column. IV. the difference of these volumes, $V-V_1$; in column V. the ratio of this difference to the "titre" of the gas, $R=\frac{V-V_1}{T}$.

The strength of the solution of hyposulphite of soda employed in the several experiments was greatly varied, but, as in the case of my previous experiments with electrized oxygen, without producing any appreciable effect on the result.

I. T.	II. V.	III. V_1.	IV. $V-V_1$.	V. $R=\frac{V-V_1}{T}$.
2·44	29·89	26·37	3·52	1·44
1·55	26·66	23·56	3·10	2·00
3·69	67·93	62·17	5·76	1·55
3·69	67·93	60·22	7·71	2·08
3·47	64·94	56·78	8·16	2·35
3·47	64·94	57·49	7·45	2·14
1·61	37·66	34·51	3·15	1·95
3·38	54·99	48·94	6·05	1·79
1·94	43·50	39·46	4·04	2·08
1·06	39·04	36·84	2·20	2·07
4·34	59·49	52·96	6·53	1·50
3·47	47·33	40·86	6·47	1·86

* Philosophical Transactions, *loc. cit.* p. 473.

In order to form a just estimate of the value to be attached to these experiments, which exhibit considerable divergences from the mean, I have calculated the probable error of the result, as in the Tables given in my previous paper, by the method of least squares. The data for the calculation are given below.

$R=\frac{V-V_1}{T}$.	Differences from the mean.	Squares of the differences.
1·44	−·46	·2116
2·00	+·1	·01
1·55	−·35	·1225
2·08	+·18	·0324
2·35	+·45	·2025
2·14	+·24	·0576
1·95	+·05	·0025
1·79	+·11	·0121
2·08	+·18	·0324
2·07	+·17	·0289
1·50	−·4	·16
1·86	−·04	·0016
Mean = 1·90		Sum = ·8741

The number of these experiments is 12.

The probable error of the result $=0{\cdot}6745\sqrt{\frac{8741}{12\times 11}}$

$= {\cdot}054$ cub. centim.;

and also

the probable error of a single experiment $=\sqrt{12}\times{\cdot}054$

$={\cdot}19$ cub. centim.

It appears, therefore, from these experiments that it is an equal chance that the true value of the ratio $\frac{V-V_1}{T}$ lies between the limits 1·95 and 1·85; also, from the calculated value of the probable error of a single experiment ·19, half the values of this ratio given in the preceding Table might theoretically be expected to be found within the limits 2·09 and 1·71. Of the twelve experiments, seven experiments are within these limits, and five experiments outside these limits.

As might be expected from the comparatively rough method of experimentation and the small quantities of gas operated upon, the probable error in these experiments is much greater than in the case of the similar experiments made with the electrized oxygen, where, in the case of a set of seventeen such experiments, the probable error amounted only to ·017 cub. centim. In the case of those experiments the limits of probable error were 2 and 2·04, which agree with theory; whereas in the case of the present experiments the theoretical value 2 is not contained within the limits of probable error, and indeed lies considerably outside those limits. These experiments, therefore, are not in themselves adequate to sustain this theory, and we should not be justified in basing it upon them; but the theory has been conclusively demonstrated by perfectly independent

methods; and, taken in conjunction with the previous experiments referred to, we may confidently infer that the fundamental reaction is the same as in the case of the experiment with pure oxygen, but that the experiments are affected by the operation of a constant cause which depresses the mean value of the ratio R.

Such a cause is to be found in the absorption of oxygen by the strongly alkaline solution employed in the absorption-bulb for the removal of the carbonic acid from the mixed gases. The gas passed through the hyposulphite of soda and the gas passed through iodide of potassium have, it is true, been subjected to a similar treatment in this respect; and if the same quantity of oxygen were absorbed by the alkaline solution in the two cases, the value of the difference between the volumes of the gases, which is the numerator of the fraction $\frac{V-V_1}{T}$ in the preceding calculation, would not be affected by this circumstance. But this cannot on the average be the case; for the relative volume of oxygen present in the latter gas being considerably greater than that present in the former, a larger quantity of it will be absorbed by the solution. Now we have (as has been explained) in the contraction which the unabsorbed gas undergoes on detonation with oxygen an independent measure of the total oxygen formed in the induction-tube; and, in fact, if we compare the average values thus actually obtained for the total oxygen in the gas, it will be found that the sum of the "titre" and the oxygen absorbed by pyrogallate of potash is sensibly less than the contraction. This, however, only appears when we consider the mean result, as there are many causes of error which affect the individual experiments. The comparison of these values in the case of fourteen experiments is given below. In column I. is given the sum of the oxygen absorbed by pyrogallate of potash and the "titre" of the gas; in column II. the contraction which the same gas underwent when detonated with oxygen—the results of the experiments being calculated on equal volumes of gas in the two experiments respectively.

I.	II.
15·76	16·17
14·49	14·62
11·20	10·78
10·53	10·84
9·21	9·75
10·42	10·49
8·63	8·89
12·60	13·47
16·83	17·62
16·10	17·68
11·80	12·05
17·34	18·42
3·20	3·61
14·96	15·42
Sum =172·17	Sum =179·81

It hence appears that for every 100 volumes of oxygen found in the induction-tube as estimated by the contraction, only 95·8 volumes are found by the other method,

showing a diminution in the amount of oxygen, due to the absorption of that gas by the alkaline solution in the absorption-bulb or to some equivalent cause, of somewhat over 4 per cent. Another cause operating in the same direction as the preceding is the absorption of oxygen by the alkaline solution of iodide of potassium, which (as appears from my previous experiments) although very small in amount is yet an appreciable quantity.

As the question of the identity of the ozone thus formed by the action of electricity on carbonic acid with the ozone similarly formed from oxygen is of great importance in relation to the present inquiry, and as the comparison of the diminution in volume which the electrized gas undergoes when passed through neutral hyposulphite of soda with the "titre" of the gas affords the most satisfactory evidence we can apply to establish or negative this identity, I do not hesitate to lay before the Society the results of a set of experiments in which the diminution in volume which the electrized gas undergoes when passed through hyposulphite of soda is calculated on both hypotheses:—(1) on the hypothesis that the total oxygen formed in the induction-tube is equal to the "titre" of the gas together with the gas absorbed by pyrogallate of potash, and (2) on the hypothesis that the total oxygen is equal to the contraction which the gas after its passage through hyposulphite of soda undergoes on detonation with oxygen. These results are given in the two following Tables; the experiments to which the same number is attached are comparable with each other, having been made with different portions of the same gas.

In column I. of the first Table given below is given the "titre" of the gas; in column II. V_1, the volume absorbed by pyrogallate of potash in the case of the experiment with hyposulphite of soda—that is to say, the oxygen present in the gas after its passage through the solution of hyposulphite by this mode of estimation. In column III. is given the sum of the "titre" of the gas and the oxygen absorbed by pyrogallate in the case of the experiment with iodide of potassium—that is, the total oxygen formed in the induction-tube by this method of estimation. In column IV. is given the difference of these volumes—that is to say, the gas absorbed by hyposulphite of soda according to this mode of estimation, $V-V_1$; and in column V. is given the ratio of this difference to the "titre" of the gas, $r=\frac{V-V_1}{T}$.

I.

Experiment.	I. T.	II. V_1.	III. V.	IV. $V-V_1$.	V. $r=\frac{V-V_1}{T}$.
1.	1·61	7·91	12·60	4·69	2·91
2.	3·38	9·74	18·98	9·24	2·71
3.	3·43	8·87	17·34	9·47	2·76
4.	1·94	9·53	14·96	5·43	2·80
5.	4·34	9·91	20·41	10·50	2·42
					Mean=2·72

The probable error in these experiments is ·055, and we have for the limits of probable error

$$2{\cdot}72+0{\cdot}05=2{\cdot}77,$$
$$2{\cdot}72-0{\cdot}05=2{\cdot}67,$$

the theoretical number 3 being external to these limits.

In the second Table the value C, given in the third column, is the contraction which the gas after its passage through hyposulphite of soda underwent on detonation with oxygen, which is taken as representing the total oxygen formed in the induction-tube*, the value $r=\frac{C-V_1}{T}$ in the last column being calculated on that assumption.

II.

Experiment.	I. T.	II. V_1.	III. C.	IV. $C-V_1$.	V. $r=\frac{C-V_1}{T}$.
1.	1·61	7·91	13·50	5·59	3·41
2.	3·38	9·74	20·32	10·58	3·10
3.	3·43	8·87	18·59	9·72	2·83
4.	1·94	9·53	15·07	5·54	2·85
5.	4·34	9·91	22·23	12·32	2·84
					Mean=3·01

Now, proceeding to calculate the value of the probable error in these experiments, we have as the data for this calculation

Differences from the mean.	Squares of differences.
+·4	·16
+·09	·0081
−·18	·0324
−·16	·0256
−·17	·0309
	Sum=·2570

whence the probable error $=0{\cdot}6745\sqrt{\frac{\cdot257}{5\times4}}$
$=0{\cdot}076;$

and we have for the limits of probable error

$$3{\cdot}01+0{\cdot}08=3{\cdot}09,$$
$$3{\cdot}01-0{\cdot}08=2{\cdot}93.$$

* [Since the above was written I have ascertained that the unabsorbed gas contains a very small proportion of hydrogen. This proportion, under the circumstances under which these experiments were made, is excessively minute, and would not affect the contraction to an extent to interfere with the conclusions as here given. The subject will be fully discussed in my next memoir.—B. C. B., March 16, 1874.]

We have also for the probable error of a single experiment:—

$$\text{probable error of a single experiment} = \sqrt{5} \times \text{p. e. of result}$$
$$= \sqrt{5} \times 0{\cdot}076$$
$$= 0{\cdot}17,$$

$$3{\cdot}01 + 0{\cdot}17 = 3{\cdot}18,$$
$$3{\cdot}01 - 0{\cdot}17 = 2{\cdot}84.$$

Three experiments are within and four experiments outside these limits.

If in the case of these five experiments we compare the total oxygen in the gas as estimated by these two methods (that is to say, the sum of the quantities given in column III. in the two Tables respectively), it appears that this total oxygen, estimated by the contraction, amounts to 89·71 cub. centims. against 84·21 cub. centims. as the same quantity estimated by the other method. These quantities are proportional to the numbers 100 and 93·95. The number 93·95 is below the average arrived at from the fourteen experiments before given, namely 95·8; and it may be noticed that the value of the ratio r in the first Table is below the average value of the same ratio in the former system of experiments, 2·9. But the result, in the case of so few experiments, is less trustworthy than the preceding.

These experiments supply an important link in the chain of evidence absent in my previous experiments, and demonstrate that not only does the gas absorbed by hyposulphite occupy twice the space occupied by the volume of oxygen absorbed by neutral iodide of potassium, but also that it consists of three times that quantity of oxygen and of oxygen alone.

I may here mention that when the electrized carbonic acid is passed through a solution of hyposulphite of soda rendered strongly alkaline with carbonate of soda, the diminution in volume is the same as when the gas is passed through neutral hyposulphite—a result quite different to that observed in the case of electrized oxygen*, and due doubtless to the presence of the large proportion of carbonic acid, which, as will be seen, modifies other reactions also.

In my previous memoir I have investigated the action of ozone upon a solution of protochloride of tin†, and I have shown that the oxidation there effected by the ozone is equal in value to three times the "titre" of the gas, and also that a diminution in the volume of the gas occurs equal to twice that "titre." The evidence which I have given of these facts is, I believe, perfectly valid; but the result is not easily ascertained, owing to the difficulty of disengaging and separating the effects of the ozone from the effects of the oxygen with which it is associated; but in the present form of experiment, in which the ozone is so largely diluted with carbonic-acid gas, and in which the proportion of oxygen present in relation to the total volume of gas is so small, the influence of the associated oxygen is reduced to a minimum, and the oxidation and diminution in volume are those due to the ozone alone. This point is demonstrated by the concordance of the

* Philosophical Transactions, *loc. cit.* p. 468.

† Ibid. *loc. cit.* p. 479.

results of various experiments made under different conditions. The oxidation effected by the passage of the ozone was estimated as follows.

Before the experiment a volume of the tin solution equal to that employed in it was run from a pipette into a measured volume of a standard solution of iodine, and the excess of iodine was estimated by titration with hyposulphite of soda. A similar estimation was made after the experiment with the tin solution through which the gas had been passed; the oxidation effected is measured by the difference between the two titrations. One pipette of the electrized carbonic-acid gas was employed for each experiment, and was passed through 10 cub. centims. of the tin solution. The experiments were made successively; and no appreciable change in temperature or barometric pressure occurred during the course of the experiments, so that the same volume of gas was always operated upon.

In the first column of the following Table the strength of the tin solution as measured by the volume of oxygen required to effect the complete oxidation of 1 cub. centim. of the solution is given. Column I. contains the "titre" of the gas, T*; column II. the oxidation effected in the tin solution, S; column III. the ratio of this oxidation to the "titre" of the solution, $r=\frac{S}{T}$.

In experiment 5 the bulb containing the tin solution was placed in ice; the other experiments were made at the temperature of the air, about 16° C.

Experiment.	Strength of the tin solution.	I. T.	II. S.	III. $r=\frac{S}{T}$
1.	15·4	2·8	7·59	2·71
2.	15·4	2·8	7·87	2·81
3.	4·2	2·8	7·9	2·83
4.	4·2	4·73	13·56	2·87
5.	4·2	4·73	12·77	2·70
6.	0·84	4·73	12·66	2·72
				Mean=2·77

In the two following experiments, besides the oxidation effected, the diminution in volume also which the gas underwent by its passage through the tin solution was determined, the diminution being estimated, as in the case of the previous experiments, with hyposulphite of soda. In column IV. is given the total gas unabsorbed by potash after the passage of the electrized gas through the solution of iodide of potassium, V. In column V. is given the volume of gas similarly unabsorbed after the passage of the electrized gas through the tin solution, V_1; in VI. the difference of these volumes, $V-V_1$, being the volume of gas absorbed by the tin solution; in VII. the ratio of this difference to the "titre" of the gas, $R=\frac{V-V_1}{T}$. Two pipettes of gas were employed in these two experiments.

* The titre and oxidation in columns I. and II. are given in terms of the cub. centims. of hyposulphite of soda employed for their estimation.

Experiment.	Strength of the tin solution.	I. T.	II. S.	III. $r=\frac{S}{T}$.	IV. V.	V. V_1.	VI. $V-V_1$.	VII. $R=\frac{V-V_1}{T}$.
1.	4·2	3·22	8·87	2·75	56·07	49·94	6·13	1·9
2.	2·1	3·22	8·68	2·69	56·07	50·25	5·82	1·8

The density of the gas thus absorbed by the protochloride of tin may be calculated from these data. Putting Δ as this density, the density of oxygen being the unit of comparison, we have for the value of Δ, in the two experiments respectively,

$$(1) \qquad \Delta=\frac{8\cdot87}{6\cdot13}=1\cdot45,$$

$$(2) \qquad \Delta=\frac{8\cdot68}{5\cdot82}=1\cdot49;$$

that is, once and a half the density of oxygen. It is to be observed that the value of Δ here given is quite independent of the "titre" of the gas and unaffected by any errors in its estimation.

The volumes of gas operated upon in these experiments were altogether too small to bring out very sharp results, and the agreement between theory and experiment is perhaps as close as could be anticipated. I am, however, inclined to believe the somewhat low value of the mean, 2·77, to be due to a real although slight diminution in the oxidation, owing to the dilution of the ozone with carbonic-acid gas. In the two following experiments, in which the solution of the tin salt was excessively dilute, a much lower number for the comparative oxidation was obtained.

Strength of the tin solution.	I. T.	II. S.	III. $r=\frac{S}{T}$.
·42	3·24	8·18	2·51
·21	3·24	7·98	2·45

The following experiments were made with an electrized gas obtained by passing a mixture of one volume of oxygen and four volumes of carbonic-acid gas through the induction-tube, S and T being the weight in grammes of the oxygen found by titration.

Experiment.	I. T.	II. S.	III. $r=\frac{S}{T}$
1.	·0084	0273	3·25
2.	·0079	0241	3·05
3.	·0079	0257	3·26
4.	·0073	0242	3·29

In experiment 3 the concentration of the solution of the tin salt was twice that of the solution employed in the two previous experiments. In experiment 4 that solution was

diluted with $\frac{1}{4}$ of its bulk of a solution of strong hydrochloric acid. The reaction was unaffected by these circumstances. But in the case of similar experiments made with electrized oxygen very different results were obtained. In three experiments thus made, in which the strength of the solution of the tin salt was the same as in experiments 1 and 2, 5·89, 6·49, 7·14 were severally obtained for the value of the ratio $\frac{S}{T}$. The conclusion to be drawn from these experiments, taken in connexion with those on the same subject given in my previous paper, is that the oxidation effected in a solution of protochloride of tin by oxygen gas is greatly diminished and retarded when the oxygen is diluted with a large proportion of carbonic-acid gas, whereas the oxidation effected in the protochloride of tin by passing ozone through the solution is unaffected by such dilution. The influence of the carbonic-acid gas in thus preventing the combination of oxygen may be compared to the influence of even a small proportion of the same gas in extinguishing combustion.

When ozone prepared from pure oxygen is passed through a solution of hydriodic acid of such a degree of concentration that not less than one gramme of iodine is contained in 8 cub. centims. of the solution, the average oxidation effected was found to be equal to twice that effected by the passage of an equal volume of the same gas through a solution of neutral iodide of potassium*. But this oxidation is considerably reduced when the electrized oxygen is largely diluted with carbonic-acid gas, as is shown by the following experiments, which were made with an electrized gas obtained by passing through the induction-tube a mixture of oxygen and carbonic acid in the proportion of one volume of the former to nine volumes of the latter gas. A pipette of the capacity of about 100 cub. centims. was employed to measure the gas; the temperature and pressure were constant throughout the several experiments.

In column I. is given the degree of concentration of the hydriodic acid employed. In column II. is given the titre of a pipette of the gas with neutral iodide of potassium, T, as represented by the cub. centims. of hyposulphite of soda necessary for the titration. In column III. is given the titre of a pipette of the same gas with hydriodic acid, S, similarly represented. In column IV. is given the ratio of the oxidation effected in the hydriodic acid to the oxidation effected in the neutral iodide of potassium, $R=\frac{S}{T}$.

The independent oxidation effected under these circumstances by the oxygen mixed with the ozone must be very small, since in an experiment similarly conducted, in which a bulb containing the solution of hydriodic acid was placed after the bulb of neutral iodide employed for the titration of the gas, no oxidation whatever was effected in the second bulb by the passage of the oxygen. I have therefore not attempted to make any allowance for this oxidation; in other respects the experiments were conducted as those before referred to. In the case of the last four experiments in the Table I have no memorandum of the degree of concentration of the hydriodic acid employed.

* Philosophical Transactions, *loc. cit.* p. 462.

I. Degree of concentration of the solution of hydriodic acid employed.	II. T.	III. S.	IV. $R=\frac{S}{T}$
1 gramme of Iodine in 3 cub. centims. ...	26·2	41·5	1·54
1 gramme of Iodine in 6 cub. centims. ...	26·2	38·8	1·47
" " ...	26·2	39·2	1·48
" " ...	26·2	40	1·52
1 gramme of Iodine in 12 cub. centims. ...	26·2	38·7	1·47
" " ...	26·2	37·5	1·42
	20	28·7	1·4
	14	21·5	1·53
	14	22·5	1·6
	27·7	42	1·51
			Mean=1·49

The oxidation, therefore, of the hydriodic acid effected by the ozone, when thus largely diluted with carbonic acid, is exactly intermediate between the oxidation effected in the case of neutral iodide of potassium and the oxidation effected in hydriodic acid of the same concentration by undiluted ozone. When, in the case of the experiments with electrized oxygen, the concentration of the solution of hydriodic acid is less than that indicated by one gramme of iodine in 16 cub. centims. of the solution, the oxidation effected is very appreciably diminished, and appears (for the experiments do not enable us to pronounce with certainty on the point) to pass continuously through all the intermediate stages until an oxidation is reached closely approximating to (but still in no case reaching) the oxidation effected in neutral iodide of potassium, the smallest oxidation obtained in my experiments by an excessive dilution being $1\frac{1}{10}$ that oxidation; but in the present case the reaction is of a different character, being perfectly definite and not subject to variation, with the variation of those conditions by which it is affected in the other instance referred to. We need not conclude that these conditions are here altogether inoperative, but only that the influence of the dilution of the ozone is so predominant as to render these causes insignificant as regards the total result. The experiments were made with a gas of which $\frac{1}{10}$ consisted of oxygen, the rest being carbonic acid; and this proportion was constant in the several experiments. In the case, however, of two experiments with gas derived from the decomposition of pure carbonic acid, 1·71 and 1·79 were obtained as the value of the ratio $\frac{S}{T}$; and it is quite likely that the reaction may vary with the degree of dilution of the ozone and the proportion of the several gases present, just as it varies with the degree of dilution of the hydriodic acid. Using the notation employed in my previous memoir* (to which the reader is referred for explanation) in addition to the two forms of the decomposition of ozone by hydriodic acid there mentioned, namely, $r=\frac{5}{3}$, in which the unit of ozone is

* Philosophical Transactions, *loc. cit.* p. 482, and 1866, pp. 792, 805, 810. Journ. Chem. Soc. 1868, vol. vi. p. 367.

distributed according to the equation

$$4\xi^3=\xi^3+10[\xi],$$

and $r=2$, in which

$$2\xi^3=\xi^3+4[\xi],$$

we have here an example of a third form of decomposition, $r=\frac{3}{2}$, in which

$$4\xi^3=3\xi^3+6[\xi].$$

These various experiments lead to one and the same conclusion, and demonstrate that the ozone produced in the electric decomposition of carbonic-acid gas in the induction-tube is the same in kind as that produced by the action of electricity upon pure oxygen, but that its quantitative reactions are in certain cases, and to a limited extent, modified by the circumstances peculiar to the experiment.

It was of special importance, in reference to the primary object of this investigation, to ascertain the combination of conditions most favourable to the production in the unabsorbed gas of a high percentage of ozone. From the numerous circumstances of the experiment, and the variety of ways in which these circumstances may be varied and combined, this question is of a very complex order, and can only be fully and satisfactorily answered by a careful special investigation, which I have not attempted.

The following experiments, however, throw considerable light upon the problem.

I. The question which most obviously comes before us is, as to the result of the prolonged action upon the gas of the electricity generated by a coil of high intensity. This question is answered by the following experiment, although it did not happen to have been instituted in reference to it. A very slow current of pure carbonic-acid gas, carefully dried, was passed through the induction-tube and there submitted to the action of the electricity generated by a coil of the above description; the gas was collected in a sulphuric-acid gas-holder. A pipette of this gas was drawn over into the absorption-bulb, and there allowed to stand for some time over a strong solution of caustic potash. The ozone in this case undergoes an expansion in presence of the alkali similar to that which it undergoes by the action of heat. I may observe that I had first tried to expand the gas, as in the case of electrized oxygen, by passing it through a heated tube, but had found reason to believe that a small portion of the oxygen present in the gas was removed by this operation. The gas was then transferred from the aspirator to a eudiometer and there detonated (without any addition of oxygen), the contraction noted, and the carbonic acid determined by removal with potash. Two experiments thus conducted gave the following results:—

Volume of gas in pipette.	Titre.	Volume of unabsorbed gas.
249·14	2·52	
250·5	.....	64·81
249·6	.....	63·77

The 63·77 cub. centims. of gas from the last experiment, when detonated in the

manner mentioned, underwent a diminution in volume of 19·58 cub. centims., and 40·14 cub. centims. of carbonic acid were formed, leaving a residue of 4·05 cub. centims.; the theoretical volume of the oxygen is 19·9 cub. centims. We may hence infer that 230 cub. centims. of carbonic acid were passed through the induction-tube. Further, out of every 100 volumes of carbonic acid passed through the induction-tube, 17·4 volumes were decomposed. The iodine-titre on the same 100 volumes was ·11, and the iodine-titre on 100 volumes of oxygen formed was 12·8.

Now if these numbers be compared with those found in the previous experiments given on page 86, say with experiment (3) there given, it will be seen that although the proportion of carbonic acid decomposed is increased tenfold, the iodine-titre, as estimated on 100 volumes of oxygen, is nearly the same in the two experiments—that is to say, the result of the electric action, after a certain point has been reached in the decomposition, is simply to decompose the carbonic-acid gas without effecting any corresponding increase in the proportion of ozone. It is therefore to be inferred that the prolonged action of a powerful coil is by no means favourable to the object of these experiments.

II. I now proceeded to ascertain the result of repeating the electric action upon the same gas. This was effected by placing the induction-tube between two sulphuric-acid gas-holders, and passing the electrized gas forwards and backwards between the gas-holders in the manner elsewhere described in the case of an analogous experiment with electrized oxygen*. The gas was thus passed ten times through the induction-tube. An undetermined quantity of the electrized gas was then passed through a solution of iodide of potassium, and collected as usual in the absorption-bulb. When the absorption of carbonic acid was complete, the volume of unabsorbed gas was measured. The oxygen was then absorbed by pyrogallate of potash, the gas again measured, and lastly detonated in the eudiometer with excess of oxygen.

The second experiment, given below, was made in a similar manner, all the conditions being preserved as nearly as possible the same as in that just described; but the carbonic acid was passed once only through the induction-tube immediately from the vessel in which it was generated. The coil employed in these experiments was of high intensity.

In column I. of the Table below the titre of the gas is given; in II. the volume of gas after absorption of the carbonic acid; in III. the volume absorbed by pyrogallate of potash; in IV. the sum of the gas absorbed by pyrogallate and the titre of the gas; in V. the contraction on detonation with oxygen.

Experiment.	I.	II.	III.	IV.	V.
1.	3·18	58·03	16·89	20·07	20·27
2.	3·55	44·58	12·21	15·76	16·18

From these data the titre calculated on 100 volumes of the oxygen formed in the two experiments respectively is

(1) . . 15·6, (2) . . 21·9.

* Philosophical Transactions, *loc. cit.* p. 447.

These experiments confirm the result last obtained, and indicate that the frequent repetition of the electric action on the same gas operates in the same direction as the prolongation of that action by passing the gas very slowly through the induction-tube—namely, to diminish the proportion of ozone in the gas*. They afford, moreover, a satisfactory explanation of the very different amount of decomposition and very different proportion of ozone formed in experiments made under conditions apparently the same.

III. Another important question is, as to the effect upon the electric decomposition of the presence of moisture in the gas. The following three experiments were instituted with this object; and although not made in immediate connexion with the preceding experiments, but from another point of view, may find a place here. In experiment 1 the carbonic-acid gas was saturated with moisture immediately before entering the induction-tube. In experiment 2 the gas was not dried in the usual manner by sulphuric acid, but was only partially dried by a short tube containing chloride of calcium, by which desiccating agent a large proportion, it is true, but by no means the whole moisture in a gas, is removed. In experiment 3 the gas was dried first by sulphuric acid, and subsequently by passing it through a tube about 12 inches long, containing anhydrous phosphoric acid. The induction-tube was in every case kept at nearly the same temperature by immersion in water containing ice. In experiments 2 and 3 five pipettes of the electrized gas were employed for the titration and subsequent estimations. In experiment 1, where the volume of carbonic acid decomposed was comparatively small, eight pipettes of the gas were employed with a view of securing greater accuracy; but the results of that experiment, for the sake of comparison, are calculated on five pipettes of the gas—that is to say, the numbers are $\frac{5}{8}$ of those actually obtained.

Experiment.	I.	II.	III.	IV.	V.
1.	·48	21·95	6·89	7·37	7·53
2.	2·75	48·88	14·09	16·83	17·62
3.	3·43	51·47	13·91	17·34	18·42

From these data the titre calculated on 100 volumes of oxygen as measured by the contraction given in column V. is, in the three experiments respectively,

(1) . . 6·38, (2) . . 15·6, (3) . . 18·62.

These quantities are proportional to the numbers 1, 2·45, 3·76, indicating a progressive increase in the proportion of ozone formed corresponding to the more perfect desiccation of the gas. In experiment 3, where the gas was most completely deprived of moisture, 55·86 per cent. of the total oxygen formed was converted into ozone.

The conclusion at which I arrived from the preceding experiments was, that the mode of conducting the experiment most favourable to the production of a large proportion of ozone in the electrized gas would be (1) to operate upon the gas by means of electricity of feeble tension, (2) to submit the gas for as short a time as possible to the

* For the account of similar experiments in the electrized oxygen, conf. Philosophical Transactions, *loc. cit.* p. 447.

electric action, (3) rigorously to dry the carbonic acid employed, (4) to keep the induction-tube at a low temperature. These conditions I endeavoured to realize in the following manner.

The carbonic-acid gas was generated in a KIPP's apparatus, and having been washed and dried, as in experiment 3 in the last Table, was passed immediately through the induction-tube, which was placed in a glass cylinder, surrounded with flannel, containing a mixture of ice and salt, by which means the temperature could readily be kept during the experiment at from $-10°$ C. to $-14°$ C. On leaving the induction-tube the gas traversed three bulbs containing anhydrous phosphoric acid (which were permanently attached to the tube to preclude the entrance by diffusion of aqueous vapour), and, passing through the solution of iodide of potassium contained in the titre-bulb, entered the absorption-apparatus, where the carbonic acid was absorbed by a strong solution of caustic potash. The gas was passed through the induction-tube in as rapid a current as possible consistent with the absorption of the carbonic acid. When a sufficient quantity of gas for analysis had been collected in the absorption-bulb, for which several hours were required, the experiment was stopped, the iodine separated in the titre-bulb was estimated with hyposulphite of soda, the gas contained in the absorption-bulb was measured, and the whole or a portion of it transferred to a eudiometer and there detonated with oxygen. The oxygen originally present in the gas was assumed to be equal to the contraction occurring on detonation. These data supply all the elements necessary for the calculation of the proportion of that oxygen converted into ozone in the induction-tube.

It is not difficult, by this mode of experiment, to effect the conversion of as much as 75 per cent. of the total oxygen eliminated in the induction-tube into ozone, in which case the ratio $\frac{C}{T}=4$; and the gases formed are constituted of ozone and oxygen in the proportion of two units of the former gas to one of the latter, the matter of the oxygen being thus distributed, $2\xi^3+\xi^2$; but it is difficult to increase the proportion of the ozone beyond this limit; however, I have made several experiments, conducted with all possible care, in which this limit has been exceeded. The results of these experiments are given in the following Table. In column I. is given the total gas unabsorbed by potash; in column II. the titre, T; in III. the contraction on detonation (that is to say, the total oxygen), C; in IV. the ratio, $R=\frac{C}{T}$; and in V. the proportion of the matter of oxygen converted into ozone calculated on 100 parts of the total oxygen, as measured by the quantity of the matter of oxygen which would be absorbed by a solution of neutral hyposulphite of soda were the gas passed through such a solution, which has been proved to be a quantity equal to three times the titre of the gas.

I. Unabsorbed gas.	II. T.	III. C.	IV. $R=\frac{C}{T}$	V. Ozone per cent.
22·6	2·06	7·63	3·7	81·3
32·73	3·01	10·63	3·53	85·5
15·92	1·46	5·48	3·75	79·8
31·42	3·0	10·78	3·59	83·5
28·14	2·55	9·76	3·82	78·3
11·03	1·13	4·32	3·82	78·3
16·57	1·58	6·19	3·91	76·6
9·43	0·97	3·46	3·57	84·1
10·51	1·09	3·88	3·56	84·3

The maximum result attained in these experiments is that in which 85·5 per cent. of the matter of oxygen is converted into ozone, in which case the value of the ratio $\frac{C}{T}$ is 3·53. The value of this ratio, 3·5, corresponds to a gas in which 85·7 per cent. of that matter has been thus converted, and which is constituted of ozone and oxygen in the proportion of four units of the former to one of the latter gas, thus $4\xi^3+\xi^2$. The numbers, it is to be observed, given in the last column are undoubtedly somewhat too low, from the circumstance, already mentioned, that the unabsorbed gas contains a minute quantity of hydrogen*, and also possibly from the removal of traces of iodine from the absorption-bulb by the rapid current of carbonic acid, so that the proportion of ozone found is certainly not less than is there indicated.

These experiments, taken in connexion with those described in my previous investigation, leave no room for doubt as to the true nature of the unit of ozone, the composition of which must henceforth be regarded as established on evidence hardly less conclusive than that on which our knowledge rests of the composition of the unit of water. Nevertheless it is not to be anticipated that this result will be equally clear to all minds. As in the analogous case of the dual nature of oxygen, it is not a point to be demonstrated by any single experiment, but is a conclusion derived from various trains of reasoning, each of which has to be separately mastered and appreciated. In interpreting the experiments on which this conclusion is based, I have made frequent use of the well-known method of the science of probabilities, the method of least squares, which (so far as I am aware) has not hitherto found any serious application in chemistry; but in such cases as the present, where the experiments are complicated and where an exact result can only be obtained by the successful performance of numerous operations, each of which is liable to error, this method is of essential service. We cannot, indeed, get rid of accidental errors from our experiments; but instead we eliminate from our conclusion the result of those errors considered individually. It has, moreover, the very great advantage of enabling us to estimate the numerical result of our experiments at neither more nor less than its true value; and removing that value from the uncertainties incidental to the appraisement of individuals, assigns to it its true position according to an external standard.

* Conf. note, p. 92.

No discovery has ever, perhaps, been made more calculated to throw light upon the nature of the elemental bodies than the discovery of the chemical constitution of ozone. Since the time when the molecular constitution of the elements first seriously engaged the attention of chemists, the opinion has prevailed that the units of oxygen and chlorine are of the same class, being each constituted of two similar atoms, between which we cannot discriminate. This view, notwithstanding the real and cogent arguments by which it is supported, had for many years to struggle against the prepossessions of the electro-chemical theory. It is, however, now paramount, and the opinion that we have no alternative in the matter is almost universal. I myself have, however, distinctly proved* that there is such an alternative, and that chlorine may at least with equal probability be regarded as a triad element constituted of three "simple weights," of which the unit is to be symbolized as $\alpha\chi^2$. Now in ozone, ξ^2_ξ, we have actually before us an element of this peculiar triad class, to which not only is the unit of chlorine analogous in form, but to which it is also analogous in properties. The formation of this triad element places beyond a doubt the possibility of the existence of such a class. Another chemical substance which stands in the most intimate relation to ozone is the binoxide of hydrogen, the artificial element "hydroxyl," $\alpha\xi^2$, which is connected on the one hand with the unit of ozone, on the other with the unit of chlorine†, being derived from the unit of ozone by the substitution in that unit of the simple weight α for one of the simple weight ξ, and derived from the unit of chlorine by the substitution in that unit of the simple weight ξ for the simple weight χ, the three substances being also connected by the closest analogy in their chemical properties.

The complete analysis of the electrized gas will be considered in another communication.

* Philosophical Transactions, 1866, vol. clvi. p. 818.

† In relation to the analogies of chlorine and the binoxide of hydrogen, see Chem. Soc. Q. J. vol. xvii. (1864) p. 281, "The Organic Peroxides theoretically considered," by the author.

V. *On the Anatomy and Histology of the Land-Planarians of Ceylon, with some Account of their Habits, and a Description of two new Species, and with Notes on the Anatomy of some European Aquatic Species.* *By* H. N. Moseley, *M.A. Oxon. Communicated by* G. Rolleston, *M.D., Professor of Anatomy and Physiology in the University of Oxford.*

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Preface.

At the outset I would desire to express my deep obligations to Professor Rolleston in the matter of this paper. I was first informed of the existence of Land-Planarians in Ceylon by Professor Rolleston, and of the importance of investigating the correctness or incorrectness of Schmarda's description of a ganglionated nerve-cord in *Sphyrocephalus*, to which Professor Rolleston has referred in his 'Forms of Animal Life,' as have also many other authors. Professor Rolleston at first agreed that the paper should be a joint one, and himself prepared a large number of sections of *Rhynchodemus*, one of which is figured, but subsequently decided that my name only should appear in the matter. I have to thank him for suggestions and assistance rendered during the whole of the investigation, which took more than two months' constant work, and also for help in the getting up of the bibliography of the subject. The work was done in the Anatomical Department of the Oxford Museum.

The Land-Planarians the anatomy and histology of which are described in the present memoir were obtained in Ceylon during the months of January and February last year (1872) by the author, in the Royal Botanic Gardens, Peradeniya, Ceylon, through the kind assistance of G. H. K. Thwaites, Esq., F.R.S., the distinguished Curator of those Gardens.

The Planarians are forms of very great interest, as lying in the stem of the family-tree of the animal kingdom at a point where very many branches are given off by it. *Bipalium* and *Rhynchodemus*, as being the largest of known Planarians, offer especial advantages for an accurate and complete investigation of their anatomy; and a thorough knowledge of the anatomy of these larger forms cannot fail to throw great light on that of their smaller congeners. It is also of extreme interest to see how far largeness of size and difference of habit is accompanied by corresponding modifications of structure in forms such as these. The published accounts of the anatomy of *Bipalium* and *Rhynchodemus* are, as will be seen in the sequel, imperfect and, in many very important particulars,

erroneous. On the whole, then, no apology is required for the present memoir. A list of all the works referred to is given further on; when more than one memoir by the same author is cited the successive memoirs are numbered.

Land-Planarians were first discovered by O. F. Müller in Denmark in 1773 (*loc. cit.** *infrà* 1, p. 68), and the species he discovered was called by him *Fasciola terrestris*. Dugès discovered the same species in Languedoc in 1830 (*loc. cit.* 2), and called it *Planaria terrestris*; Fritz Müller discovered it at Greifswald in Germany; Noll found it in Switzerland, at St. Goar, in 1862 (*loc. cit.*); Grube in Silesia in 1866 (*loc. cit.*). It was first discovered in England by Mr. Jenyns (Observations in Nat. Hist. p. 315, 1846) at Bottisham Hall; then by Sir John Lubbock, Bart., in Kent (*loc. cit.*), in 1868; and, thirdly, by the Rev. W. Houghton, M.A., F.L.S., in Shropshire. During this period and subsequently a large number of Land-Planarians have been discovered in various parts of the world, and have been referred to several genera, and indeed to different families.

All the Land-Planarians as yet known belong to the Dendrocœlous group of these animals, which group is thus split up by Diesing (*loc. cit.*), here quoted only as far as regards Land-Planarians:—

Dendrocœla.

Family I. Planaridea (eyes two).
. . . . Genus *Rhynchodemus*.
Family II. Polycelidea (eyes many).
Phalanx I. *Polycelidea apoda*.
Phalanx II. *Polycelidea gasteropoda*.
Subfamily III. *Geoplanidea*.
Genera. *Geoplana*.
Bipalium.

The genus *Rhynchodemus* is thus characterized† by its founder; and under this genus are included by Diesing the following Land-Planarians:—

Rhynchodemus	
terrestris	Denmark. O. F. Müller.
	England. Sir J. Lubbock (Kent), 1868; Houghton (Shrops.), Ann. & Mag. 1870, vi. p. 256; and first by Jenyns, Observations in Nat. Hist. p. 315.
	France. Dugès, 1830.
	Germany, Greifswald. Fritz Müller.
	Germany, Silesia. Grube, 1866.
	Switzerland, St. Goar. Noll, 1862.

* See pp. 113–115 *infrà* for Bibliography.

† "Corpus elongatum, subdepressum, antrorsum attenuatum, utrinque obtusum. Ocelli duo subterminales."—Dr. Leidy, Proc. Acad. Nat. Sci. Philad. vol. v. 1851.

RHYNCHODEMUS *sylvaticus*	N. America. LEIDY, Proc. Acad. Phil. 1851, pp. 241, 289; 1858, p. 172. Said by SCHMARDA (Neue wirbellose Thiere, 1859) to be very like *terrestris*.
bistriatus	Samoan Islands (Navigator Islands, Schiffer-Inseln). GRUBE.
quadristriatus . . .	
Nietneri	Ceylon. HUMBERT, Mém. Soc. Phys. Genève, 1861, p. 306.

And there may now be added

tannagi	Brazil. FÉRUSSAC, Ann. Gén. Sci. Phys. vol. viii. 1821, pp. 90–92.
bilineatus (*Geodesmus*)	Giessen? MECZNIKOW, Bull. Acad. St. Pétersb. 1865, vol. ix. p. 433.
Thwaitesii	Ceylon. *Mihi*, 1872.

In the absence of any accurate description of the anatomy of most of the animals included under this genus *Rhynchodemus*, the genus cannot yet be said to be very satisfactory. Observations on the anatomy of *R. terrestris* and *R. sylvaticus* are very much wanted. It is uncertain whether MECZNIKOW'S *Geodesmus*, which, as GRUBE (Jahresbericht, *loc. cit.* p. 64) remarks, was probably not European, but introduced with foreign plants, should be considered a *Rhynchodemus* on account of its elongated body and single pair of eyes. At all events, as far as *R. Thwaitesii* and therefore *R. Nietneri* (which is evidently closely allied) are concerned, DIESING'S wide separation of the genera *Rhynchodemus* and *Bipalium* is unfortunate. The two genera are evidently closely allied in the presence in both of an ambulacral line, the absence of ramifications on the inner side of the posterior prolongation of the digestive tract, and in the arrangement of the muscular structures and viscera, as also in general facies, colouring, and habit. STIMPSON (*loc. cit.* 1) is quite right in placing together the genera *Geoplana*, *Bipalium*, and *Rhynchodemus*; but in his description of the subfamily the statement concerning the mouth will not apply to that of *Rhynchodemus*, and the discovery of eyes all down the body of *Bipalium* necessitates a further change in the definitions given both by DIESING and STIMPSON.

Altering slightly from STIMPSON (*loc. cit.* 1, p. 24), the definition of the subfamily Geoplanidæ will stand thus:—

GEOPLANIDÆ.—Corpus elongatum depressum vel depressiusculum subtus pede sat distincto; caput continuum vel discretum. Ocelli duo vel plurimi in capite solum aut etiam passim in corpore dispositi. Os postmediale. Œsophagus protractilis campanulatus margine sæpius sinuoso aut doliiformis. Apertura genitalis pone os.

Under this subfamily there stand the three genera *Geoplana*, *Bipalium*, and *Rhynchodemus*. The genus *Bipalium* was formed by STIMPSON (*loc. cit.*) to include four species of remarkable Land-Planarians which he obtained on the U. S. A. Expedition under Captain RODGERS. The genus is an extremely well-marked one, being distinguished by

the extraordinary cheese-knife-shaped anterior extremity or head. The following is Stimpson's diagnosis of the genus with the slight alteration concerning the eyes:—Corpus lineare, depressiusculum; caput discretum lunatum transversum, auriculis longis retrorsum tendentibus. Ocelli numerosi, minuti, in capite plerumque in ejus marginibus, et etiam nonnunquam in corpore usque ad extremitatem posteriorem sparsim dispositi. Os centrale vel postcentrale. Apertura genitalis inter os et extremitatem posteriorem, sæpius ad dimidiam distantiam.

The first to describe a species of this genus was Dr. Gray, of the British Museum, who in 1835 named a *Bipalium* from Bengal *Planaria lunata*. Cantor next (in 1835) evidently speaks of a *Bipalium* as occurring in China, but he does not name the animal. The Rev. W. Houghton, M.A., F.L.S. (*loc. cit.*), describes and figures two Land-Planarians from Borneo, the shapes of the heads of which evidently show them to belong to the genus *Bipalium*. As he gives no name to them, I propose to call No. 1 *Bipalium Everetti*, after Mr. Alfred Everett, who discovered them in Borneo, and the other *B. Houghtoni*.

The following is as perfect a list as I have been able to form of the species of *Bipalium* at present known to exist:—

Bipalium

Phœbe	Ceylon.	Humbert, Mém. Soc. Phys. Gen. 1861, p. 302.
Diana		
Proserpina		
Ceres	Ceylon.	*Mihi*, 1871–72.
dendrophilus . . .	Ceylon.	Schmarda, Neue wirbellose Thiere, 1859, p. 36.
lunata	Bengal.	Gray, Zool. Misc. p. 5, 1835, cit. Silliman's Journ. 1861, p. 135.
ferudpoorensis . . .	Bengal.	Wright, Ann. & Mag. Nat. Hist. 1860, vi. p. 54; see also Cantor, Naga Hills.
„ ? . . .	Naga Hills.	Ann. & Mag. Nat. Hist. 1842, ix. p. 277.
Cantoria	China.	
Grayia	Chusan.	
Stimpsoni	China, Hong Kong.	See Silliman's Journal, 1861, pp. 134 & 135.
virgatum	Loochoo.	U. S. A. Expedition under Captain Rodgers. Stimpson's Prodromus, Proc. Acad. Philad. 1857, pp. 30, 31.
maculatum	Ousimon.	
fuscatum	Simodu.	
trilineatum	Jesso.	
	Madras.	Novara Exped. Band ii. Abth. 3, p. 45. Grube.
Everetti	Borneo.	Rev. W. Houghton, M.A., F.L.S., Ann. & Mag. Nat. Hist. 1870, vi. p. 255.
Houghtoni	Borneo.	Rev. W. Houghton, M.A., F.L.S., *ibid.*

Professor SEMPER, of Würzburg, has told me that he has a number of species of Land-Planarians to describe from the Philippines. No doubt some of these will belong to our present genus. The species referred to by Dr. CANTOR as having been found under stones in the Naga Hills by Mr. GRIFFITH in 1836 may possibly be the same as the species from Ferudpoor described by Dr. PERCEVAL WRIGHT in 1860 (*loc. cit.* p. 54), the two localities being at no very great distance apart. It is also possible that *B. Cantoria*, discovered by Mr. FORTUNE in China, may be the same as *B. Stimpsoni* discovered by Sir J. BOWRING in Hong Kong.

A map may readily be made to show the distribution of the genus *Bipalium*, in a manner which cannot fail to be of interest in the case of a genus so well marked as this. The range will be seen to be a wide one, extending from Jesso to Ceylon, but still to be much more confined than that of either *Rhynchodemus* or *Geoplana*. GRUBE (*loc. cit.*) remarks that the Land-Planarians correspond in their distribution with the Land-Leeches, in so far as neither animals are found in Western Asia or Africa.

Mr. LAYARD noticed his specimens at St. Pedra. In Ceylon itself the genus *Bipalium* seems to be widely spread. HUMBERT found his specimens at Kandy, and also *Rhynchodemus*, on a coffee-estate further up amongst the hills. SCHMARDA found his at Bellingham; and I obtained one specimen at Trincomalee like *B. Proserpina*, but it unfortunately perished.

Of the genus *Geoplana* twenty-one species are enumerated by DIESING (Akad. Wiss. Wien, 1861, pp. 509, 513). These species have been observed by DARWIN, FR. MÜLLER (Halle Abhandlung. *loc. cit.* p. 25), DIESING (Sitz. Akad. Wiss. Wien, *loc. cit.* p. 496), *Polycladus*, in the Andes, DIESING (*loc. cit.* p. 495), SCHMARDA (Neue wirb. Thiere, Taf. ii. fig. 31), and *Limacopsis*, DIESING (*loc. cit.* p. 519), SCHMARDA (*loc. cit.* Taf. vi. fig. 69).

The Land-Planarians of Ceylon were first observed by Mr. LAYARD (*loc. cit.*); but he mistook the anterior extremity of the animal for its tail, and did not name the species. SCHMARDA (*loc. cit.* p. 36) describes a *Bipalium* from Ceylon, but being apparently unacquainted with STIMPSON's genus, makes a genus *Sphyrocephalus* for it. He figures his species (Taf. viii. fig. 83, Band i.) under the name of *Sphyrocephalus dendrophilus*. ALOIS HUMBERT (*loc. cit.*) describes three new species of *Bipalium* from Ceylon, and gives beautiful figures of them drawn from life; he names them *B. Diana*, *Proserpina*, and *Phœbe*. Another Planarian which he obtained he refers to the genus *Rhynchodemus*, *R. Nietneri*. I obtained abundance of specimens of *B. Diana* and *B. Proserpina*, but none of *B. Phœbe* or *B. Nietneri*; but I found a new species of *Bipalium*, and also a new *Rhynchodemus*.

The following are the characteristics of the new species of *Bipalium*:—

B. Ceres, Plate X. figs. 1 & 2. Body rather more convex superiorly, and less broad in proportion to its length, than in the other Ceylon species. Upper or dorsal surface divisible into five bands or stripes, a median and two pair of lateral. The median is light yellow in colour; the two bands which lie on either side of it are pale brownish and of little more than linear width. The external lateral bands are in every part of

the animal's length of at least the width of the central one, from which they differ by being of a slightly browner yellow colour. The whole of the animal's dorsal aspect is irregularly dotted with black specks. The semicircular anterior border of the head is limited by a dark violet line, immediately anterior to which a broader but similarly semilunar band of flesh-colour is to be seen on the dorsal surface. The central dorsal band is in some specimens prolonged up so as to join this band; in others it falls short of this; in all it swells out into a sort of lozenge-shaped termination, sharply defined on each side by a dark violet patch, which shades off gradually into the dusky yellow of the lateral bands of the body. On the under surface of the body, on each side where the convex surface of the dorsal aspect meets the nearly flat ambulacral surface, a *slight ridge* is formed extending the whole length of the body. This ridge is highly characteristic of this species, and contains peculiar glandular bodies not present in the other two species of *Bipalium* examined.

Dimensions of an average specimen after contraction in spirit:—

	millims.
From anterior extremity to mouth	52
From mouth to generative orifice	12
From generative orifice to posterior extremity	15
Entire length . . .	79

Habitat. The Royal Botanic Gardens, Peradeniya, Ceylon, in company with *B. Proserpina* and *B. Diana*.

The possession of a large series of specimens allowed the difference between young animals and adults to be studied. In *Bipalium Ceres* and also *B. Diana* the very young specimens are much more definitely marked than the adults. The animals gradually lose their definite striping as they grow older. The case is paralleled in many instances amongst higher animals. It would appear here as if the species were endeavouring to escape detection by enemies by getting rid of a somewhat conspicuous colouring which at present survives only in the young condition.

Fig. 2, Plate X., represents a very young specimen of *B. Ceres*. The lateral bands are in young specimens much browner, and the central band of a brighter yellow; the two linear intermediate stripes are jet-black.

Fig. 3, Plate X., shows a very young specimen of *B. Diana*. A comparison of this drawing only with that of M. Humbert's of the entire animal would lead to the conclusion that the present must belong to another species, but the examination of a large series of specimens has shown that this is undoubtedly the young of *B. Diana*. A very narrow light-coloured line is seen in the midst of the broad black line on the head, and this light-coloured line extends a short distance along the animal's back; it represents the broad median light-coloured line of *B. Ceres*, and shows *B. Diana* to be five-striped in the early condition, as are nearly all the Ceylon Land-Planarians. The longest specimen of *B. Diana* obtained measured 5½ inches in length and ¼ inch in breadth; it was used for

dissection. *B. Proserpina* in the young condition is marked exactly like the adults of the same species, but in this case the adults are conspicuously banded.

The following are the characteristics of a new species of *Rhynchodemus*. In ascribing the species to the genus *Rhynchodemus* I have followed M. HUMBERT, this animal being evidently of the same genus as his *R. Nietneri*.

Rhynchodemus Thwaitesii, sp. nov., Plate X. fig. 4. The dorsal surface of the animal may be considered divisible into three bands, a median and two lateral, of about equal width: the median band light brownish yellow with a black line down its centre; the two lateral bands violet-grey, very dark at the edges which bound the median band, but shading off into a light tint at their outer margin. The animal is thus five-striped. Ventral surface marked by a median white band, the ambulacral line bordered by darkish violet-grey.

Dimensions of an average specimen after contraction in spirit:—

	millims.
From tip of tail to generative orifice	10
From generative orifice to mouth	12
From mouth to tip of anterior extremity	22
Entire length . . .	44

Habitat. The Royal Botanic Gardens, Peradeniya, Ceylon, occurring, together with three species of *Bipalium*, amongst fallen leaves &c. in plantations, especially abundant in Banana and Manilla-hemp plantations. The species I have named after my friend Mr. G. H. K. THWAITES, F.R.S., the distinguished Curator of the Peradeniya Gardens.

On the Habits of Land-Planarians.

Land-Planarians are probably all of them nocturnal in habit. DARWIN (*loc. cit.* p. 249) remarks on the avoidance of light by *Geoplana*, and Dr. LEIDY (*loc. cit.* 2, p. 172) observed *Rhynchodemus sylvaticus* to be a nocturnal animal crawling about on fences at night. *Geoplana subterranea* (*Geobia*, DIESING), according to F. MÜLLER, lives underground in the holes of *Lumbricus corethrurus* (Halle Abh. *loc. cit.* pp. 26, 27). The Ceylon Land-Planarians appear to avoid light in the same manner as the *Geoplanæ*; they are to be found in dark places, such as under large fallen leaves, and in confinement they coil themselves up away from the light.

This avoidance of light appears to be common to nearly all Planarians, and not confined to terrestrial forms. MAX SCHULTZE (*loc. cit.* 1, p. 17) refers to the fact that aquatic Planarians always choose the darkest side of any vessel in which they may be placed to rest upon; and I have myself observed this fact in the case of *Planaria torva* and *Dendrocœlum lacteum*. Green chlorophyl-containing Planarians, such as *Mesostomum viridatum*, form an exception to the general rule, since, as MAX SCHULTZE informs us (*loc. cit.* 1, p. 17), they always place themselves on the light side of a vessel: they nevertheless die when exposed to direct sunlight. It may have been due to the fact that I did not give them sufficient shade that all my attempts to keep the Ceylon Land-

Planarians alive in confinement failed. I never preserved them for more than a few days. Mr. THWAITES, who has also tried the experiment several times, has had a like result. Mr. DARWIN seems to have found no difficulty in keeping *Geoplanæ* alive: he kept some alive in a box twenty-one days, and they increased in size during that time.

The Planarians which I obtained were almost all procured in a Manilla-hemp plantation in the Royal Botanic Gardens, Peradeniya, Ceylon, under fallen leaves and under the leaf-sheathes of the growing plants. I found some in these situations myself; but the larger number were procured by one of Mr. THWAITES's coolies, trained by him as a collector. The coolie's plan was to lay a large fresh plantain-leaf on the grass near the plantation, and in a few hours he found the Planarians adhering to the under surface. Three species of *Bipalium* and one of *Rhynchodemus* were obtained all together in this manner. Occurring with the Planarians is found, as remarked by M. HUMBERT (*loc. cit.* pp. 302–3), the mollusk *Vaginulus*, which was also found associated with *Geoplana* in South America by Mr. DARWIN (*loc. cit.* p. 241). Mr. DARWIN was led to believe that *Geoplana* fed on rotten wood; but this is most probably not the case. All Planarians appear to be carnivorous, like their congeners the Nemertines (M'INTOSH, *loc. cit.* p. 338); and it is possible that the increase in size observed in his specimens by Mr. DARWIN was due to cannibalism on their part. MAX SCHULTZE searched carefully in the digestive tract of *Geoplana* (Halle Abhandl. *loc. cit.*) and found no trace of vegetable tissue in it; but he did find the palate and jaws of a snail.

FR. MÜLLER (Halle Abhandl. *loc. cit.* p. 27) says that *Geobia* sucks out the juices of its host *Lumbricus corethrurus*. LEIDY (Proc. Acad. Sci. Phil. 1858, p. 172) fed *Rhynchodemus sylvaticus* with crushed house-flies; and although I have examined microscopically sections of twenty or thirty individuals of four species of Ceylon Land-Planarians, I have never seen a trace of vegetable tissue in their intestines. As far as regards aquatic Planarians, Von BAER, who was the first to give an account of the anatomy of Planarians and separate them from the Leeches, with which they had been confounded by SHAW and KIRBY long ago, remarked (Nova Acta, tom. xiii.) on the carnivorous propensities of these animals; and on Professor ROLLESTON placing an earthworm, killed by immersion in warm water, in a dish in which were a number of living *Planariæ torvæ* and *Dendrocœla lactea*, these animals crowded on to the worm's body and soon sucked all the hæmoglobin out of it, leaving it white and pulpy.

Dr. LEIDY remarked that from the tail end of *Rhynchodemus sylvaticus* is secreted a delicate mucous thread; and Sir J. DALYELL (*loc. cit.* vol. ii. p. 113) observed of *Planaria Arethusa* that it makes threads of mucus, by means of which it suspends itself in the water. *Bipalium* in the same manner uses a thread of its tough investing slime for suspension in air; and I have frequently seen it let itself down in this manner from a twig held at a short distance from the ground. The cellar-slug makes use of a slime-thread for suspension in the same manner. This fact in the habits of *Bipalium* does not seem to have been noticed by M. HUMBERT, although he gives a very interesting account of that animal's mode of life in his memoir (Mém. Soc. Phys. de Genève, xvi.

1861, p. 294). *Bipalium*, as is observed by this author, when moving carries its head slightly elevated, and moves it from side to side, evidently investigating with it any obstacles which occur in the line of movement. When the semilunar head is thus made use of, there are projected from its narrow anterior border tentacular-like eminences, which appear to be used as feelers. The tentacles are evidently not localized, but may be formed at any spot on the border of the head by contraction of surrounding tissue. M. Humbert searched for some corresponding permanent papillæ or sense-organs on the border of the head without success; but I have been more fortunate, and have found a peculiar narrow line of delicate papillæ in this region, evidently connected with the sensory function of this part of the head, and which will be described in the sequel. See Plate XIII. figs. 16 & 17, and their accomanying descriptions.

The following is a list of the works and memoirs concerning Planarians and allied forms which have been consulted, and to which reference is made in the present paper.

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Dugès ..... 1. Recherches sur l'organisation et les mœurs des Planariés. Ann. Sci. Nat. xv. (1828), p. 139.

2. Aperçu de quelques observations nouvelles sur les Planaires et plusieurs genres voisins. Art. 3. Planaires. Ann. Sci. Nat. xxi. (1830) pp. 81–90.

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2. Bericht über einige im Herbst 1853 an der Küste des Mittelmeeres angestellte zootomische Untersuchungen. Verhandl. phys.-med. Ges. Würzburg, iv. (1853) p. 222.

3. Zoologische Skizzen. Zeits. f. wiss. Zoologie, iv. (1853) p. 178.

4. Beiträge zur Kenntniss der Land-Planarien nach Mittheilungen des Dr. Fritz Müller in Brasilien und nach eigenen Untersuchungen. Halle Abhandl. 1856, iv. p. 20.

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2. On the genus *Bipalium*. Silliman's Journal of Science, May 1861, 2nd ser. xxxi. p. 134.

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W. Carmichael M'Intosh, M.D., F.L.S. On the Structure of the British Nemertians and some new British Annelids. Edinb. Roy. Soc. Trans. xxv. 1869, p. 305.

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Tafeln zur vergleichenden Anatomie. Tübingen, 1864.

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Ed. Grube ..... Ueber Land-Planarien.
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2. Reise der österreichischen Fregatte 'Novara' um die Erde. Zoologischer Theil, 2. Band, 3. Abth. p. 45. Anneliden.

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Rev. W. Houghton ..... On two species of Land-Planarians from Borneo. Ann. & Mag. Nat. Hist. 1870, vol. vi. p. 255. See also pages 347 & 495 *ibidem*.

Gegenbaur ..... Grundzüge der vergleich. Anatomie. Zweite Auflage. Leipzig, 1870.

Dr. F. Sommer und Dr. L. Landois. Beiträge zur Anatomie der Plattwürmer, von Dr. F. Sommer und Dr. L. Landois. Erstes Heft. Ueber den Bau der geschlechtsreifen Glieder von *Bothriocephalus latus* (Bremser). Leipzig: W. Engelmann, 1872.

As regards the foregoing bibliography of the anatomy and zoology of the Dendrocœlous Turbellaria, it may be sufficient to remark that the two memoirs by Dugès deserve perusal and attention even at the present day, though written with reference to freshwater species exclusively (with the single exception of the marine species which I have myself used in this paper for purposes of comparison), and at a period when the microscope was a very different instrument from the one I have employed. As regards the correctness with which the organs in relation with the generative outlet were identified by Dugès and Von Baer respectively, there is no doubt that the French naturalist has the advantage. Dugès's anatomical description (*loc. cit.* xv. p. 163, 1828) of the water-vascular system is, speaking roughly, correct; he would have done well, however, to have adhered to his original view of its intimate connexion with the nervous system; and the demonstration of the large share of truth which this view embodies, as established definitely by Quatrefages (Ann. Sci. Nat. 1845, iv. pp. 172–177), marks an epoch of advance in our interpretation of the anatomy of the entire order. Schultze, writing in 1852 (Zeits. f. wiss. Zool. iv. p. 187), is a little too summary in his condemnation of the method of injection as employed by Blanchard (Ann. Sci. Nat. 1847, sér. 3, tom. viii. p. 146). On the other hand, Schultze would, judging from the analogy furnished by our own Land-Planarians and by all freshwater species, appear to be right (Halle Abhandl. 1856, p. 33) in accusing Blanchard, in his description of *Polycladus*, of having mistaken the tail of the animal for its head. The *Geoplanæ* dissected by Schultze (*loc. cit.* p. 33) appear to have had their reproductive organs in a state of quiescence.

I shall now proceed to describe in detail the anatomy of the Ceylon Land-Planarians, and also that of certain structures of some freshwater and marine species, after a short account of the methods employed in the investigation.

Methods.

The methods employed in the investigation of the anatomy of Planarians were the following:—

In Ceylon the animals were placed, whilst still living, either in a large quantity of strong alcohol, this alcohol being changed after the lapse of twenty-four hours, or they were put into a weak solution of chromic acid, the solution being gradually strengthened as they became rigid, when, after remaining in the chromic-acid solution for about a week, they were transferred to strong alcohol. As soon as the specimens reached England they were placed in absolute alcohol. Portions of the bodies of the animals thus prepared were imbedded in the usual manner in a mixture of sweet oil and white wax. Sections were made in various directions with razors wetted in absolute alcohol*; they were stained with a simple solution made by boiling carmine in water with a few drops of ammonia solution, and leaving the resulting fluid exposed to the air in order to obtain as neutral a solution as possible. The sections, after being stained and washed with water, were treated with absolute alcohol, rendered transparent with oil of cloves and mounted in dammar varnish. All the operations had, in the majority of cases, to be performed on the glass slide, the sections being too fragile to allow of transference. Some sections were also mounted at once in dammar varnish unstained, especially those prepared from specimens hardened in chromic acid. Some of the minuter details of structure were best to be observed in the chromic-acid preparations, and the nervous system especially was only to be seen, with any clearness, in sections made from specimens thus prepared; but it is almost impossible to prepare sections of large area from such specimens, owing to the brittleness of their tissue. Such sections were therefore made from animals hardened in alcohol, and the relations of the various organs to one another thus determined. The sections of *Dendrocœlum lacteum* figured were prepared from specimens which had remained about a year in strong glycerine, and which were transferred thence to absolute alcohol before cutting. Specimens thus prepared afford far better preparations than specimens placed in alcohol directly.

Lastly, dissections with scalpels and scissors of spirit specimens of *Bipalium Diana* were made under alcohol in the ordinary manner.

The sections of the Sea-Planarian, *Leptoplana tremellaris*, employed were made from a specimen which had been preserved in ordinary spirit for a number of years, and not with a special view to histological examination. The sections were stained and treated in the same manner as those of the Land-Planarians.

* Though it is expensive to use absolute alcohol for this purpose it is much better to do so. Absolute alcohol thoroughly wets a razor, forms an even film all over its surface, and allows a large section to be made with much greater ease than is the case if ordinary alcohol be employed.

Keferstein (*loc. cit.* xiv. 1868) employed similar methods to those here described for the investigation of *Leptoplana tremellaris* and other Sea-Planarians; but he does not mention having used carmine, whence probably his somewhat imperfect account of the structure of the cerebral ganglia, and his failure to recognize the water-vascular system in *Leptoplana tremellaris*. There is no doubt that the use of carmine in such cases as this is of great assistance; and several important anatomical details described in the present paper would have been missed had the preparations made not been stained: the water-vascular system, *e. g.*, would probably not have been recognized at all.

Max Schultze recommends a solution of chromate of potash for the preservation of Planarians for microscopic purposes.

The investigation of the anatomy of such animals as Planarians, except in cases in which they are perfectly transparent, is only to be carried out, with any certainty as to results, by such methods as those here described. Schmarda and Blanchard, trusting to ordinary dissection with a scalpel, described series of ganglia in *Bipalium* (*Sphyrocephalus*), Schmarda (*loc. cit.*), and *Polycladus Gayi*, Blanchard (*loc. cit.* p. 147), respectively, which were probably in each case only the ovaries and testes, and certainly not ganglia, and made a number of other blunders. And Claparède's account of the anatomy of *Bipalium*, apparently derived from the employment of similar methods, is very meagre and in part incorrect. The method of sections is, however, rather tedious, and in the course of this investigation more than forty dozen microscopic slides of sections of Planarians had to be prepared and preserved for careful comparison.

Anatomy.

For a general preliminary notion of the broader features of the anatomical arrangement of *Bipalium* and *Rhynchodemus*, reference should be made to Plate XII. figs. 1, 2, & 3, and their accompanying descriptions. The various structural parts will here be considered *seriatim*.

Tegumentary System.—The outermost investment of *Bipalium* and *Rhynchodemus* consists of a well-defined epidermic layer (Plate X. figs. 5, 6, & 7). This layer is seen in the figure first cited to be comparatively thin on the median line of the dorsal surface and that of the ambulacral line, whilst it is thicker in the lateral regions of the body. When the vertical section in a simple stained preparation mounted in dammar varnish is examined with a high power, the epidermic layer appears to be made up of a number of elongated elements lying closely packed together and arranged with their longer axes at right angles to the surface of the body. In these preparations it is impossible to make out any definite form amongst these elements, or any thing like a nucleus in them (Plate XI. figs. 1 & 2, and Plate XV. fig. 9). The cuticle of the common freshwater Planarian, *Dendrocœlum lacteum*, presents in similarly prepared specimens a similar separation into elements by lines passing at right angles to the surface (Plate XIV. fig. 7, E). Occurring in this epidermic tissue are very numerous gland-cells (Plate X.

fig. 10) and rod-like bodies (Plate X. fig. 9), and also in great abundance bodies like A (Plate X. fig. 10), which are irregular elongated masses of finely granular material which stain deeply with carmine; and as they are often seen to be in connexion with the glands beneath the skin, are probably masses of slime hardened by the spirit in the act of their extrusion by those glands. The gland-cells and rod-like bodies are usually far more numerous than they appear in fig. 1, Plate XI.: they present various curious forms in spirit preparations; those seen in Plate X. figs. 9 & 10 are common; but these forms are unnatural and due to shrinking.

If a vertical section of the epidermis be treated with a solution of caustic potash the contracted tissue swells up, and it presents the appearance represented in fig. 4, Plate XI., which is probably very nearly that which exists in the living state. The epidermis here is seen to be made up of large gland-cells (G) and cells containing rod-like bodies (R) and a certain amount of vertical filaments. The gland-cells are large oval sacs with granular contents, and are very like those which may be seen in a living *Planaria torva*. From the appearance presented by gland-cells altered by the action of spirit, as in B, fig. 10, Plate X. (which is the form most usually to be met with), it would appear that these cells burst at the extremity and discharge their contents. The cell appears to have a double wall, for an irregular crumpled membrane is to be seen often within it, evidently shrivelled up by the action of spirit. The rod-like bodies ("Stäbchen-Körperchen"), when acted on by potash, are seen to be of an elongated cylindrical form, with rounded ends and with a closely fitting investing membrane, which appears to be usually attached below by a sort of peduncle to the basement membrane. The irregular filaments which fill up the interspaces between the gland-cells and rod-like bodies appear to be the remains of the cell-walls of gland-cells and rod-like bodies, a violent discharge of rod-like bodies and mucus having probably taken place when the animals were put into spirit or chromic acid. The entire substance of the epidermis is probably made up, in the living condition, of cells resembling the gland-cells described, but of various dimensions, and of cells containing rod-like bodies. I was unable to detect cilia on other parts of the body, except in the region of the papillary line on the head, which will be described with the special sense-organs, and on the living membrane of the mouth and its cavity. Cilia are, however, probably present all over the body-surface, although they must be much weaker and more easily destroyed than those which are situated on each side of the ambulacral line, which were found to be invariably present in all specimens, and which require further study in the fresh state for their perfect elucidation. Max Schultze (*loc. cit.* 4, p. 34) found cuticular cells present in *Geoplana*, though they are absent in the smaller aquatic Planarians. *Geoplana* had, however, remarkably enough, no "Stäbchen-Körperchen," which is remarkable; they perhaps escaped observation, as only one specimen was available for examination.

Cilia.—On each side of the ambulacral line in *Bipalium*, and also in *Rhynchodemus*, the epidermis alters its character. It becomes far thicker and apparently less definite in structure; and it is here clothed with long and stout cilia (Plate X. fig. 5), which

probably assist the muscular movements in the act of progression. Max Schultze (*loc. cit.* 4, p. 34), though he did not see the cilia in *Geoplana*, yet concludes that it must be covered all over with them from the experiments of Fr. Müller (p. 23), who covered the body of one of these Planarians with arrowroot, and observed a motion of the particles which seemed to show the presence of cilia. Darwin came to the same result from the observation of the motion of air-globules in the slime of *Geoplana*. Mecznikow found the skin of *Geodesmus bilineatus* covered with cilia. The alteration in structure of the epidermic layer where it carries the strong ambulacral cilia is remarkable; there are no rod-like bodies or gland-cells at all in this region, which seems to be entirely devoted to the production and working of the cilia.

Basement Membrane.—The epidermic elements in *Bipalium* rest on a very fine basement membrane, homogeneous in structure. The basement membrane is, in transverse sections of the body, with difficulty to be seen as a structure separate from the external circular muscular coat which lies immediately beneath it; but in favourable preparations it is easily distinguished by its much deeper staining with carmine (Plate XI. fig. 1, B). In longitudinal sections the external circular muscular coat is seen in section, and then the basement membrane stands out in considerable relief (Plate XV. fig. 9, B). This basement membrane is not to be confounded with the thick membrane often described as such in Nemertines and Planarians, and which, as will be seen further on, is the homologue of the external circular muscular coat. The basement membrane is perforated for the discharge of the secretion of the subcutaneous glands and the passage of the rod-like bodies, the parent cells of which are situated beneath it. A definite basement membrane could not be detected with certainty in *Rhynchodemus*.

Subcuticular Region.—Immediately beneath the basement membrane is the external muscular system, consisting of a circular and longitudinal series of fibres, which will be considered under the heading "Muscular System." Beneath this system, again, is the zone of loose tissue before referred to, which is occupied by fibres continuous with the radiating muscular fibres of the body, which form a loose stroma, in which are situated the parent cells of the rod-like bodies, a large quantity of glands, the pigment, and also the eye-spots, which latter will be most conveniently considered with the nervous system. The parent cells of the rod-like bodies (Plate XI. fig. 1, R G) are arranged beneath the external longitudinal muscular layer at a tolerably even depth; they are, in spirit specimens, of an elongated ovul form, with the upper extremity drawn out to a point or long filament, which in some cases may be seen to reach up to the basement membrane. In spirit specimens they assume various forms (Plate X. figs. 11 & 12); and I was at first led to believe that they actually contained dart-like bodies like those described by Mecznikow in *Geodesmus bilineatus* (*loc. cit.*), which he calls "Nessel-Organe," which he found in vacuoles in the animal's skin, which seemed to be shot out when the animal was pressed under the covering-glass, and which were developed in glands beneath the epithelial layer. But after more careful observation, and especially comparison of the corresponding organs in *Rhynchodemus* with those of *Bipalium*, I came to the con-

clusion that the curiously and often spirally contorted appearance of these parent cells was merely due to the action of spirit on a highly elastic investing membrane, and the bodies shown in fig. 8 were merely débris of such a membrane. In *Rhynchodemus* these cells may often be observed uncontracted and of an oval form, and containing two or three rod-like bodies (Plate XI. fig. 2), in fact in every way resembling the rod-cells described from ordinary aquatic Planarians. On treatment with potash, the cells of *Bipalium* swell up (Plate XI. fig. 4), are seen to contain rod-like bodies, and the fine filament at the upper extremity appears like a duct leading to the surface of the basement membrane. In sections of the integument taken parallel to the surface, the parent cells of the rod-like bodies are seen to occupy positions opposite the interval between the stout external longitudinal muscular fibres (Plate XI. fig. 5); and when cut through transversely (Plate X. fig. 11), they prove to be divided into two or three compartments, and to be provided with a very stout horny-looking cell-wall. In vertical sections they are usually seen to contain more than one rod-like body, often three, in apparently different stages of development. The cells have usually a nucleus-like body at their inferior extremity. The rod-like bodies and their parent glands are distributed all over the body, except on the ambulacral line and the special sense-line on the anterior margin of the head; they are far less numerous on the under surface of the body than on the upper. There can be very little doubt that the organs here described are the homologues of the well-known "Stäbchen-Organe" of aquatic Planarians; and it is almost certain that the organs described by Mecznikow as existing in *Geodesmus bilineatus* come within the same category. He makes a great point of saying that *Geodesmus* has no "Stäbchen-Körperchen," but has "Nessel-Organe." The only difference is that his rod-like bodies are pointed instead of blunt. It may fairly be concluded that these peculiar skin elements in all Planarians, and probably in all Turbellaria, are homologous, though they differ more or less in details of structure. Whether these bodies are also homologous with the nettle-cells of Cœlenterata is another question, and one which will not here be discussed; it has been fairly gone into by Gegenbaur (*loc. cit.* p. 171) and by Max Schultze (*loc. cit.* 1, p. 15).

I much regret that I did not carefully examine the skin-organs of *Bipalium* in the fresh state whilst I was in Ceylon; but I am not certain that I should have derived much benefit, for I am more and more convinced that the study of tissue in the fresh condition should succeed and not precede that of sections of the hardened structures in all histological investigations of soft parts. It is only when a thorough knowledge of the relations and relative sizes &c. of the various elements has been gained in preparations in which they may be observed *in situ*, that we are able to derive much information from the investigation of tissues in the recent state.

Pigment.—The pigment of the body is entirely confined to this, the subcuticular, region, if we except a certain small amount of liver-like pigment to be found in some diverticula of the digestive organs. The pigment, as in the medicinal leech (Leuckart, *loc. cit.* p. 638) and in other Planarians (Max Schultze, *loc. cit.* 1), is of three colours,

yellow, black, and brown, and exists as minute rounded particles embedded more or less thickly in multiramified, transparent, and homogeneous protoplasmic masses: these protoplasmic elements (Plate XI. fig. 1) have usually a larger or main mass, which lies almost immediately beneath the external muscular system, in the same position as that occupied by the parent cells of the rod-like bodies; and from this main mass as a centre they send out fine branched thread-like processes, which ramify amongst the loose radiating muscular elements and the gland-tissue, and sometimes penetrate as far as between the fibres of the internal muscular system, and occasionally pass outwards in an opposite direction a short distance between the epidermic elements. These masses are sometimes densely crowded with pigment-granules, which are very dark and well defined; sometimes they contain hardly any at all; and occasionally the pigment-granules are absent from a circumscribed spot on the principal mass, which gives the spot the appearance of a nucleus. Keferstein (*loc. cit.* p. 15) speaks of the pigment of Planarians as being soluble in alcohol; such is certainly not the case with that of *Bipalium* or *Rhynchodemus*, nor with the eye-pigment, at least, of *Leptoplana*. A *Leptoplana* which had been preserved in spirit for several years had the eye-pigment in perfect condition. The pigment-masses occur more or less irregularly all over the body in *Bipalium Diana* and *B. Ceres*, except on the ambulacral line, which is quite free from them. Where there are well-defined stripes on an animal's body the pigment is arranged accordingly; and thus in the transverse section of *Rhynchodemus* (Plate X. fig. 7) the dark spots are seen to be gathered up into three lines, one median and two lateral, corresponding to the three dark stripes on the animal's back.

Glandular Tissue.—The same zone which contains the pigment-cells and parent cells of the rod-like bodies is also thickly beset with elongated, irregular, and more or less branched masses, which stain themselves an intense colour with carmine, and are filled with coarsely granular contents. It is these bodies which were said to be observed to be continuous with similar irregular projecting masses found amongst the epidermis (Plate X. fig. 10), and which were considered to be slime hardened by the action of alcohol in the act of its ejection by the subcutaneous glandular bodies just described. These glandular bodies ramify often in an arborescent manner, but do not run into such fine threads as the pigment-masses; there is no nucleus to be detected in them. They are not confined to the zone here under consideration; they are also present in greater or less quantity all over the body, even in the septa between the intestinal diverticula, and especially abundant in a region just exterior on each side of the body to the testes. In *Rhynchodemus* they are developed internally to a much greater extent than in *Bipalium*, as will be seen by a comparison of figs. 5 & 7, Plate X.; they are also very conspicuous in preparations, from their being so deeply stained with carmine.

The internal masses of this gland-substance present slight differences from those which are external and subcutaneous. Thus their contents are on the whole less coarsely granulated, and their processes finer. In the main water-vascular canal, which

is occupied by a fine network of connective tissue, the fine connective-tissue threads spring from masses exactly like these latter in appearance (Plate XIV. fig. 6, X).

In Planarians the tissue-elements are morphologically in a rudimentary condition. In the lower Planarians there is a large amount of slimy protoplasmic undifferentiated, or sparingly differentiated, tissue; in the Land-Planarians before us the differentiation is more perfect, but, at the same time, elements which probably perform very different functions are still, in many cases, hardly to be distinguished morphologically. The gland-cells, connective-tissue threads, and pigment-bodies are all of them more or less perfect differentiations of a primitive protoplasmic substance, which may be considered to represent the connective tissue of higher animals. LEUCKART (*loc. cit.* p. 639) in the same manner says of the pigment-bodies of the medicinal leech that they belong to the connective-tissue elements, and resemble the rest of this substance found in the body, and (p. 640) refers to the great resemblance of the gland-cells in this animal to connective tissue. In *Bipalium* and *Rhynchodemus* the differentiation of these several structures has not gone so far as it has in the leech. The subcuticular glands of these Planarians probably represent the superficial glands of the leech called by LEYDIG "einzellige Hautdrüsen" (*loc. cit.* Taf. i. fig. 6); whilst those situate deeper in the substance of the body represent the deeper set of glands of that animal, which, singularly enough, are described by LEUCKART as being more like transparent vesicles, and not so granulated in appearance as the superficial set, which exactly corresponds to what is found to be the case in *Bipalium* and *Rhynchodemus*. The resemblance in many points of structure between the leech and these Planarians is most striking; and if sections of the skin treated in the same manner be examined side by side, the similarity in appearance is most remarkable, the pigment and glandular bodies being most alike, and the absence of "Stäbchen-Organe" being the most striking difference. The glandular and connective-tissue masses appear to be represented in the lower Planarians by the structures described and figured (*loc. cit.* 1, fig. 24) by MAX SCHULTZE as "sehr zarte Fädchen mit Ausschwellungen;" but here there was yet no distinction between glandular and connective tissue. The "Binde-Substance" observed by KEFERSTEIN in *Leptoplana* is of the same nature; and so are probably also the "plasmatische Kanäle" described and figured by SOMMER and LANDOIS (*loc. cit.* p. 10), which in the figure have the same granulated appearance as the glands of *Rhynchodemus* and *Bipalium*, and of which they state that their finest twigs form connexions with processes of connective-tissue-like bodies. As reference will not again be made specially to the connective tissue of the Planarians under consideration, it should be stated that an irregular network of slimy connective tissue is to be found all over the body between the muscles and around the various organs; it is often seen to be connected with the large irregular masses which so closely resemble the glandular substance.

Those portions of it which are specially developed to form capsules for the generative organs and the network in the water-vascular canals will be described with those organs. The basement membrane of the skin has already received description. The subcutaneous

glandular masses are well developed all over the body, except on the ambulacral line: at the sides of this organ, where the long cilia are, they are quite absent, but a very few are occasionally to be seen along its median line. In the head of *Bipalium* there is an unusual development of the internal glandular system, as will be seen by reference to Plate XIV. fig. 3. In longitudinal sections of the body taken in a direction parallel to its ambulacral surface, there may be observed in the region of the mouth an irregular tree-like arrangement of the internal glandular masses, but no definite duct or termination of this glandular tree was to be discovered.

Remarkable Glandular Masses in Bipalium Ceres.—In *Bipalium Ceres* a remarkable structure was met with which could not be found in either of the other species of *Bipalium* examined.

In this species, on the ventral surface, is a pair of peculiar ridges which run longitudinally along the entire length of the body. The ridges are situate at the line of junction of the convex superior surface of the animal's body with its flatter ventral surface. When viewed with a lens, these ridges merely appear as elevations or pinchings up of the integument. They were present in all specimens of *Bipalium Ceres* examined. In vertical sections of the body of this species, such as that figured in Plate X. fig. 6, these ridges are seen to contain peculiar oval masses (A, A). These masses presented a fine granular structure, very like that which has been described as characterizing the glandular tissue of *Bipalium Diana* and *Rhynchodemus*. No further structure could be made out in them; but from these oval masses an irregular tract of similar glandular matter, disposed in elongated masses as in the skin-glands, could be traced up to a point a little above the testes. The oval masses and their accompanying glandular tracts were not to be seen in every successive vertical section, but occurred at slight intervals. But a satisfactory longitudinal section of the ridges could not be obtained in order to show the amount of separation between the gland-masses in it, or whether there was any regularity in their arrangement, the tissue of the specimens of *B. Ceres* being somewhat soft. No special opening corresponding to the gland-masses could be detected along the line exteriorly. These organs in *B. Ceres* are of great interest, and need further investigation, which I hope that the expected arrival of a large number of specimens from Ceylon will permit of. It is possible that these gland-masses, with their accompanying tract of glandular matter, may be a foreshadowing of the segment-organs in Annelids, which, in the leech at least, as is well known, make their appearance in development as solid masses of tissue, and subsequently become hollow (Leuckart, *loc. cit.* p. 704).

Muscular System.—Great stress has been laid by various authors on the supposed fact that whilst in Annelids, Nematoids, Trematodes, and, in fact, all higher worms, the external coat of the body was arranged circularly, and the internal longitudinally, in Turbellarians the reverse was the case; and the statement has been made in such a form that it really appeared as if an inversion of the muscular coats must be supposed in order to get at the proper homological relations of the muscular structures. And

hence there appeared to be a wide gulf fixed between the Planarians and their really very near allies the Leeches. A study, however, of the muscular arrangement of *Bipalium* and *Rhynchodemus* shows that in these animals the muscular arrangement is almost identical with that in the leech; and, further, that that which exists in the lower Planarians and also in the Nemertines is easily reducible to the same type.

In *Bipalium* and *Rhynchodemus* the muscular arrangements are of very great complexity, and may be regarded as belonging to two systems, superficial and deep. The general arrangement of each system will first be considered, and then the special arrangement found to exist in the ambulacral line, this being a purely muscular organ, and therefore properly described in this place. The special muscular arrangements which hold in the generative and digestive organs will be described under the headings of these systems.

Superficial Muscular System.—Immediately beneath the thin basement membrane of the epidermis of *Bipalium* and *Rhynchodemus* is a layer of closely apposed muscular fibres. This layer is clearly to be distinguished all over the body; but it varies in thickness in different regions, and also in the arrangement of its fibres, although the general trend of these latter is always circular. This layer may be seen in Plate XI. fig. 1 (E. C. M.) or, with its fibres seen in section, in Plate XV. fig. 9 (E. C. M.). It is thickest on the dorsal region, and inferiorly on each side of the ambulacral line. In these regions also the arrangement of its fibres is most complex: this arrangement is displayed diagrammatically in Plate X. fig. 13; a sort of basketwork of fibres running in three directions is formed; and an almost exactly similar arrangement of fibres is described as existing in the external muscular coat of higher worms, in the leech and Nematodes, by Leuckart (*loc. cit.* pp. 459, 645). When this muscular coat is viewed from above, the fibres are seen crossing one another diagonally, whilst others take a directly transverse or circular course (Plate XI. fig. 5).

The decussating fibres are thus doubly oblique in their direction. At the sides of the body this muscular coat is almost entirely absent, and especially in *Rhynchodemus* (Plate XI. fig. 2). The muscular coat appears to be almost homogeneous and structureless; in fact it exactly resembles the external coat of aquatic Planarians, such as *Leptoplana tremellaris* (Plate XIV. fig. 1, E. C. M.) or *Dendrocœlum lacteum* (Plate XIV. fig. 7, E. C. M.), though in this latter instance the external coat is more evidently muscular. Now the remarkable arrangement of fibres which is common to both groups being taken into consideration, there can be little doubt that the external circular muscular coat here described in *Bipalium* and *Rhynchodemus* is the homologue of the similar external coat of the leech; and it is evident that the external coat of *Dendrocœlum lacteum* answers to that of *Bipalium*, and that of *Leptoplana tremellaris* to that of *Dendrocœlum lacteum*. The body investments are essentially homologous; but muscular elements are developed in them more perfectly in some forms than in others, and in some parts of some forms more perfectly than in other parts. In higher worms the development of fibres is almost constantly perfect in all parts of the body. Keferstein

and other observers have called the external tissue of the Planarians they have examined a basement membrane, and have therefore described longitudinal muscles as being external in that animal—which no doubt is correct from one point of view, since it is probably impossible to detect any muscular fibres in the external tissue of the body; but still it is apt to lead to an erroneous conclusion. The great fact to be borne in mind is that, whatever this external tunic may be called, it is the homologue of the external circular muscular coat of higher worms and *Bipalium*, and that therefore the distinction between the arrangements of the muscular system in the two groups is of very little importance. M'Intosh (*loc. cit.* p. 310) lays great stress on the fact that in *Ommatoplea alba* the circular muscles are external and the longitudinal internal, whilst in *Borlasia* the reverse is the case; and accordingly he regards these two worms as belonging to very different types indeed. Now it would be difficult to overestimate the wide gulf which would exist between these two forms if there were really any inversion of the muscular coats here; but the external circular coat of *Ommatoplea* is evidently the homologue of the thick external tunic of *Borlasia*, called by M'Intosh the basement membrane, since in *Ommatoplea* there is said to be no basement membrane, and the external circular muscular coat lies immediately beneath the epidermis, as does the so-called basement membrane of *Borlasia*. It will be noted that the extremely thin and delicate basement membrane which intervenes between the external circular muscular coat in *Bipalium* and the epidermis has nothing to do with the thick tunics (as I believe, improperly termed basement membranes) of *Borlasia* and *Leptoplana*. These latter are probably contractile, and perform the part of muscular tunics, although no definite fibrillar arrangement has been detected in them. Immediately beneath the external circular muscular layers are the longitudinal muscles of the superficial muscular system. These muscles do not form a continuous tunic to the body as the circular muscles, but occur as isolated bundles of fibres. The fibres composing these bundles are remarkably stout, and the bundles themselves are arranged beneath the external circular layer at tolerably regular distances from one another, the intervals between them being the situation occupied by eye-spots and the fine upper extremities of the parent cells of the rod-like bodies (see below, p. 144 *seqq.*), and also by terminations of the radiating muscular fibres. These longitudinal fibres are well seen in section in Plate XI. figs. 1, 2, & 9 and Plate X. fig. 13, and as viewed from the surface of the body in Plate XI. fig. 5. In *Rhynchodemus* the fibres do not form such definite isolated bundles as in *Bipalium*, as may be seen by a comparison of figs. 1 & 2 in Plate XI. The muscular system generally in *Bipalium* is far more highly specialized than it is in *Rhynchodemus*. This longitudinal system of muscles is most fully developed in the middle of the dorsal surface of the body and in the infero-lateral regions, in correspondence with the greater elaboration of the external muscular coat in those regions already described.

This superficial longitudinal muscular system is evidently represented in the leech by the few longitudinal fibres which, according to Leuckart (*loc. cit.* p. 645), are to be

found sparingly present between the layers of the external circular muscular coat of that animal. The superficial muscles, both circular and longitudinal, are to be found all over the body-surface of *Bipalium* and *Rhynchodemus*, except on the under surface of the head of *Bipalium*, where they appear to be absent, being nevertheless well developed on its superior aspect. In the ambulacral line their arrangement is greatly modified; but the description of this modification will be given when the general muscular structure of the ambulacral line is considered. Max Schultze, in describing *Geoplana* (*loc. cit.* p. 35), says that the longitudinal muscles are external as in other Turbellarians; but as he only had one specimen to work at, and apparently did not examine that by means of sections, it is possible that a delicate external circular coat was overlooked.

Deep Muscular System.—A broad zone, already described, occupied by loose radiating muscular fibres and various skin-organs, intervenes between the superficial and deep muscular systems in *Bipalium* and *Rhynchodemus*. Internally to this zone the whole of the body may be regarded as made up of a dense mass of muscular fibres, in which are hollowed out the digestive tract, water-vascular canals, and space for the generative system, and the interstices of which are more or less filled up with glandular matter and connective tissue. The muscular mass consists of fibres which in their general arrangement may be described as longitudinal, circular, and radiating.

In a transverse section of *Bipalium* or *Rhynchodemus*, longitudinal fibres are seen in cross section, dotted all over the central mass of the body, but in certain regions they are much larger and more numerous than in others. Thus they form a conspicuous zone on the periphery of the central gastro-intestinal tube near the commencement of the intestinal diverticula, as may be seen in Plate X. fig. 5; and in this zone the fibres are especially stout and closely aggregated infero-laterally, where they appear in section as irregular masses separated from one another by fibres of the radiating system. A similar aggregation of fibres forms a pair of longitudinal bands which run the entire length of the body, one on each side of the ambulacral line. A further special development of longitudinal fibres is constituted by a group of small fibres around the oviduct. Over the remainder of the central body-mass the longitudinal fibres are scattered pretty evenly; they are entirely absent in the zone of loose radiating fibres intervening between the central muscular mass and the skin-muscles, and in the main water-vascular trunks.

Circular and radiating Muscular Fibres.—There is no separate and distinct circular muscular layer in the inner muscular mass, though fibres more or less circular in their direction are dispersed all over the body-mass; an especial number of these is to be found at the lateral regions of the body, just externally to the extremities of the intestinal diverticula. In *Rhynchodemus* the circular fibres in the corresponding region are more highly developed, and form a tolerably well-defined layer, lying just externally to the internal mass of longitudinal fibres, as may be seen from Plate X. fig. 7 and Plate XI. fig. 2; and this layer appears to be homologous with that which exists in *Lepto-*

planâ tremellaris, Plate XIV. fig. 1, E. C. M. In *Bipalium* and *Rhynchodemus*, as will be seen from Plate X. fig. 5, the arrangement of the muscular fibres is extremely complex; and although a general distinction may be drawn between fibres which take a circular course and those which take a radial one, and are prolonged outwards to form the zone of radial fibres, yet the two sets of fibres run into one another, and no sharp line of distinction can be drawn between them. In Planarians with a less highly specialized muscular system, such as *Dendrocœlum* or *Leptoplana*, the case is different, and here the muscles are sharply divided into systems. The radial fibres of *Bipalium* and *Rhynchodemus* appear to correspond with the vertical or dorso-ventral fibres of these aquatic species, whilst the irregularly disposed circular fibres are homologous with the definite circular layer of the same animals.

Radial fibres pass outwards in *Bipalium* and *Rhynchodemus* in every direction, to form the clear zone already described as existing beneath the skin-muscles. Specially stout muscular fibres, derived from the circular system, pass transversely immediately beneath the digestive tract (Plate X. figs. 5 & 6), and a series of transverse fibres in the region of the ambulacral line is to be found about the level of the inferior boundaries of the main water-vascular canals (Plate X. fig. 5). Stout vertical fibres form the lateral walls of the central digestive tube, and mingled with these are finer fibres which bend over to form a sort of circular muscular layer to the intestinal tube. The septa between the diverticula of the intestine are formed by very fine fibres, which are continuous with fibres belonging to the body-mass, and which may be seen in Plate X. fig. 5 (where portions of the septa remain *in situ*) to have a decussating arrangement. A similar arrangement may be seen in *Rhynchodemus* (fig. 7). Max Schultze (*loc. cit.* 4) remarks on a difference between the special muscular fibres of the intestine in *Geoplana* and the other motor body-muscles; and the extreme fineness of the muscular fibres of the intestinal septa above mentioned points to a similar histological differentiation in *Bipalium* and *Rhynchodemus*.

When viewed in longitudinal sections, the circular muscular fibres of the body-mass are seen to take the same oblique direction with regard to the long axis of the body which is to be observed in the external circular muscles. The fibres decussate in a similar manner, and take then a more or less spiral course. In the head, bundles of fibres may be seen crossing one another along the middle line, and spreading out right and left towards the margins of that curious semilunar structure.

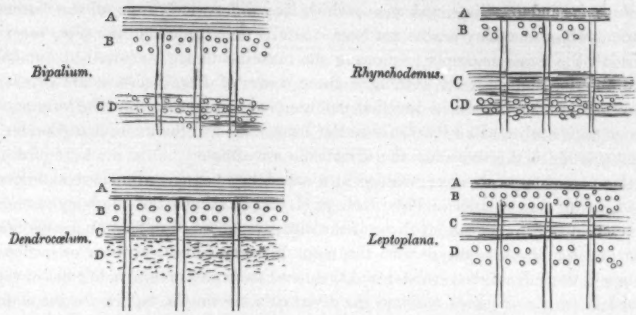
Ambulacral line.—In *Bipalium*, just before the contour of the under surface of the body begins to swell out to form the projecting ambulacral line, the external circular muscular layer splits into two parts (Plate XI. fig. 3)—one external, which, preserving the same direction which it takes in the other regions of the body, forms a thin layer immediately beneath the basement membrane of the skin; the other passes directly inwards, towards the median muscular mass of the ambulacral line, and its fibres then separating spread out fanwise and become lost in this mass. In so doing it separates off a series of smaller longitudinal muscular bands from the larger one, forming the

regular external layer of longitudinal muscles. This latter layer, after being continued a short distance inwards, is lost. The series of smaller longitudinal muscular bands is continued all over the ambulacral line, as may be seen by reference to the figure referred to above. The stout vertical muscles of the line swell out as they pass between these longitudinal bundles to end at the external transverse or circular muscular layer. In *Rhynchodemus* the ambulacral line is merely marked by an absence of pigment, and a slight increase in strength of the vertical fibres.

In *Dendrocœlum lacteum*, as may be seen from Plate XIV. fig. 7, a well-defined external circular muscular coat exists, to which are attached the stout bundles of vertical or dorso-ventral fibres. The dorso-ventral fibres pass between bundles of longitudinal fibres, just as do the radiating fibres of *Bipalium* and *Rhynchodemus* between the superficial longitudinal muscular bundles. The cells of the rod-like bodies lie just beneath and between the bundles of longitudinal fibres, in fact bear the same relation to them as in *Bipalium* and *Rhynchodemus*. Internally to these well-defined muscular elements, and between these and the digestive cavity, is an interval filled up with a pulpy tissue, which is not so perfectly differentiated histologically. In this tissue are embedded the thread-cells and connective-tissue elements, and in cavities excavated in its substance the generative organs and water-vascular system. It evidently corresponds to the internal muscular mass of *Bipalium*. On its outer margin can be traced fibres having a circular arrangement; but I was unable to detect a second series of definite longitudinal fibres in it.

In *Leptoplana tremellaris* (Plate XIV. fig. 1) the external circular muscular coat is reduced to a mere membrane, and this is succeeded by a layer of longitudinal fibres. In their histological development the elements of the body-mass lying internally to this are much more perfectly differentiated than in *Dendrocœlum lacteum*. The internal circular muscular layer is well defined, and is succeeded in most regions of the body by a region occupied by numerous longitudinal fibres. It appears, then, on the whole, that the arrangement of the muscular fibres in the bodies of Planarians, and indeed all Turbellarians, is essentially the same as that in other worms. The external muscular coat is circular, the internal longitudinal, though in some cases the external coat becomes rudimentary, and appears as a simple membrane; and this may occur in different parts of the same animal. The external longitudinal muscles are succeeded by internal circular muscles, distinctly marked as a separate layer in *Leptoplana*, and just to be made out as such in *Dendrocœlum*, interspersed between the internal longitudinal fibres of the body-mass in *Bipalium* and *Rhynchodemus*, but in this latter forming in some parts of the body a distinct layer, as in *Leptoplana* (see Plate X. fig. 7). The interval forming the zone occupied by radiating fibres in *Bipalium* and *Rhynchodemus* is absent in the flattened aquatic species.

The following diagrams represent the arrangements and homologies of the various muscular layers in *Bipalium*, *Rhynchodemus*, *Leptoplana*, and *Dendrocœlum* (A, external circular layer; B, longitudinal; C, D, internal circular and longitudinal muscular systems respectively):—



Digestive System.—As is usually the case in Planarians, the digestive canal is single anteriorly and double posteriorly in both *Bipalium* and *Rhynchodemus*; and a similar condition exists in *Geoplana* (Max Schultze, *loc. cit.* 4, p. 35). From the opening of the pharynx into the digestive tract forwards there is a single straight and broad digestive tube, which gives off diverticula or cæca on each side, and breaks up into ramifications in the head. At the opening of the pharynx this single tube becomes split into two branches, which diverge widely to pass one on each side of the sheath which contains the pharynx and which forms their inner wall; they approach one another somewhat more closely as they pass on each side of the sheath containing the generative organs, which lies immediately behind that of the pharynx and is continuous with it. Posteriorly to this point a stout septum runs down the median line of the body to the very tip of the tail, and separates the two tubes from one another, forming their inner wall on each side. This septum is continuous with the sheath of the generative organs superiorly; and, indeed, these organs and the pharynx might justly be said to be contained in spaces hollowed out in the septum itself. These points are displayed in Plate XII. fig. 2.

There is no perforation in the septum, and thus the two posterior canals have no communication with one another behind the point at which they branch off from the single anterior tube. From the outer surface of the lateral walls of the digestive canals arise stout transverse septa, which pass outwards to join the lateral region of the muscular body-mass. These septa are parallel both in *Rhynchodemus* and *Bipalium*, excepting at the immediate anterior and posterior extremities of the animal. The intervals between them are diverticula or cæca, which communicate with the intestinal tube by means of apertures in its wall. These diverticula are present in the posterior part of the body only, on the outer side of the two intestinal tubes, as is also the case in *Geoplana* (Darwin, *loc. cit.* p. 243). The parallelism, approximation, and enormous number of digestive diverticula form the characteristic point of resemblance between

Rhynchodemus and *Bipalium*, and one in which they differ widely from all the flattened Planarians whose anatomy has as yet been carefully described; and the septa seem to foreshadow the transverse septa connecting the intestine to the body-wall in Annelids. In the anterior and posterior extremity of the bodies of *Rhynchodemus* and *Bipalium* the diverticula take an oblique direction, and are given off fanwise from the termination of the digestive tube. In *Rhynchodemus* the arrangement at the anterior and posterior extremity is much the same and the diverticula are simple; but in the head of *Bipalium* these secondary tubes are ramified to a considerable extent without retaining any parallelism, as may be seen in Plate XII. fig. 1, and at this part of the body resemble very much in their ramifications those of the digestive tract of the flat aquatic species.

The diverticula communicate with the main digestive tubes by means of oval perforations in the walls of those tubes; and these oval apertures are often, but not always, disposed in pairs. In *Rhynchodemus* the diverticula are simple, but in *Bipalium* they are partially split up into two or four compartments by means of secondary and tertiary septa, which extend from their outer extremities through a portion of their length. The septa, when cut transversely in a longitudinal section of the animal's body, are found, both in *Rhynchodemus* and *Bipalium*, to consist of two muscular walls, between which is a quantity of the peculiar glandular matter already described as existing in the other parts of the body. The muscular fibres of the septa are, as has been also stated, finer than those of the general body-mass; but occasional stout fibres, taking a more or less vertical direction, are to be seen in transverse sections of the septa. In the neighbourhood of the pharynx, the wall of the main anterior digestive canal is strengthened by special longitudinal muscular fibres. The pharynx, or muscular prehensile apparatus forming the entrance to the digestive tract, and by means of which the food is taken into the body, is contained in a receptacle or sheath formed of the same material as the median septum of the posterior portion of the digestive tract and continuous with it. This sheath has an extremely elongated oval form in *Bipalium*, and a shorter oval one in *Rhynchodemus*; and it communicates with the exterior in each case by means of a circular aperture, which has been termed the mouth. In *Geoplana* the mouth is described by Mr. Darwin as being a transverse slit, not circular, as is here the case. The pharynx, which, when the animal is in search of food, is protruded through the mouth, lies ordinarily entirely out of sight within its sheath, to the dorsal surface or wall of which it is attached by strong muscular roots. Claparède was mistaken in supposing that a folded pharynx was characteristic of Land-Planarians. Though the pharynx of *Bipalium* is thus constructed, that of *Geodesmus bilineatus* (Mecznikow, *loc. cit.*) and of *Rhynchodemus* is cask-shaped or cylindrical. The pharynx of *Bipalium* has been described and figured by Claparède. It is (Plate XII. fig. 3) of an elongated form, and consists of a pair of closely apposed tumid lips, thrown by contraction into a series of transverse folds. Between the lips, somewhat anteriorly to the centre of the organ, is the aperture which leads, by means of a short canal, to the opening (O, Plate XII. fig. 3) into the digestive tract, which opening lies just in front of the anterior

termination of the sheath of the pharynx. The muscular attachment of the pharynx to the roof of its sheath is confined to the region immediately surrounding the tube by which it leads into the digestive tract, and it is thus anterior to the middle of the organ, and forms a sort of pedicle. The oral aperture lies immediately over the centre of the pharynx. In *Rhynchodemus* the pharynx has the form of a cylinder directed antero-posteriorly, and narrowed in front and behind, and which may thus be said to be cask-shaped ("*doliiformis*," Stimps.). The pharynx is perforated in the direction of its length by a tube which runs directly into the digestive tract, entering it by an aperture having the same relation to the sheath of the pharynx as in *Bipalium*. The pharynx is attached to its sheath only at its anterior extremity. The cylindrical form of pharynx is evidently the older and simpler one, and is that invariably possessed by the lower Rhabdocœle Planarians. The folded mouth of *Bipalium* is to be regarded as a modification of that of *Rhynchodemus*, the mouth of the cylinder having been gradually spread out until the whole organ became a flattened sucker, which, when retracted and packed away, assumes the form shown in the figure. Regarding this as being the case, and considering therefore the long irregular slit between the lips of the pharynx of *Bipalium* to be homologous with the rounded aperture at the extremity of the cylindrical pharynx of *Rhynchodemus*, the muscular arrangement in the two cases will be found to be identical. The pharynx is built up of an internal and an external set of longitudinal and circular muscular fibres, which muscular masses are connected by transverse fibres, in the interstices of which is a quantity of glandular matter. The outer muscular wall consists of external fibres, which run from the point of attachment of the pharynx towards the margin of its aperture or lips, and which must therefore be considered longitudinal. These longitudinal fibres, as will be seen from Plate XII. fig. 9, are extremely stout, and are succeeded inwardly by a zone of circular fibres, internally again to which are transverse or radiating fibres crossed by scattered circular fibres, and with stout longitudinal ones in their interstices. A wide zone, occupied by loosely packed radiating fibres, only intervenes between the outer muscular tissue of the pharynx and its inner longitudinal and circular muscles (Plate XII. fig. 8); and in this wide zone is a considerable quantity of glandular tissue, which is more abundant in *Bipalium* than *Rhynchodemus*. The inner circular muscular coat which clothes the tubular cavity of the pharynx immediately beneath its epithelium is composed of densely packed fibres, and stains, as do the corresponding inner circular tissues in the penis, vagina, &c., very deeply with carmine. The exterior of the pharynx and the walls of its sheath are lined with an epithelium, in which no definite cell-structure could be observed; but it appeared transparent and marked by vertical lines, which might represent separation into cellular elements. In the interior of the pharynx the epithelium is, especially towards the margin of the lips or aperture, composed of long pear-shaped elements, which were ascertained from chromic-acid preparations to be in *Bipalium* covered with cilia, and which are probably ciliated in *Rhynchodemus* also, although no ciliation was discernible in spirit preparations.

The interior of the digestive tract is clothed with a thick glandular investment, which varies slightly in its structure in the various regions of the body. In the region of the mouth, the lining of the main digestive tract consists of peculiar rounded bodies arranged irregularly in rows at right angles to the surface, and gathered into elongated groups, so as to have a certain resemblance to the gastric glands of vertebrates (Plate XV. fig. 15). These rounded bodies are imbedded in a finely granular matrix. The glandular lining of the diverticula is made up of rounded or pear-shaped elements, with finely granular contents and occasional nuclear bodies; and in places these cells are distended into large transparent sacs with nuclei, and they then closely resemble those which form the glandular cells of the digestive tract of *Planaria torva*, which are figured in Plate XV. fig. 14. The lining of the cæca is much thinner than that of the main antero-posterior digestive tubes. The lining of these latter always partakes more or less of the form displayed in Plate XV. fig. 15, although that structure is best marked in the neighbourhood of the mouth. The glandular lining in the diverticula is often, especially in the head, tinged with a brown pigment; and it is highly probable that these diverticula discharge a function somewhat like that of the hepatic tubes of Annelids, whilst the glandular lining of the main tract has a gastric function. I found no traces of vegetable matter in the digestive tracts of any of the many specimens of *Bipalium* and *Rhynchodemus* examined by me, nor, indeed, any distinctly recognizable foreign body at all. The digestive tube close to the mouth, in one specimen of *B. Ceres*, contained a mass of apparently animal matter; but it was so far decomposed or digested that I could not determine its exact nature. The diverticula of both *Bipalium* and *Rhynchodemus* contain numerous gregariniform parasites, which are also to be found imbedded in the neighbouring tissue: hardly any diverticulum is free from them, and they are usually massed together at the blind ends of the diverticula.

Water-vascular System.—In vertical sections of the body of *Bipalium* and *Rhynchodemus*, such as those figured on Plate X., there are to be seen a pair of rounded spaces lying one on each side of a region immediately above the ambulacral line, and thrown out into relief by the fact that they are always very little tinged with the carmine-staining fluid. In *Bipalium* these spaces are separate, but in *Rhynchodemus* they are connected by a broad transverse tract; and when sections of this animal are viewed under the microscope, the resulting figure, resembling somewhat a pair of spectacles in shape, is the most striking feature of the preparation. The spaces are irregularly oval in form, with the long axis of their figure directed transversely; and throughout the entire length of the body they present nearly the same figure on section, except in the region of the pharynx and generative organs, where they are necessarily somewhat contracted. The spaces bear a constant relation in position to the oviduct and testis in the anterior portion of the body. The oviducts lie, in *Bipalium Diana*, *B. Proserpina*, and *Rhynchodemus Thwaitesii*, just above the space on each side, somewhat exteriorly to its middle line; whilst in *Bipalium Ceres* they lie just within the space itself, but in the same region as in *B. Diana*. The testes in all these species lie just externally to the spaces on each

side, at the same level with them, and closely abutting on them. In *Dendrocœlum lacteum* similar spaces, situate at equal distances from the median line of the body and towards its ventral surface, are to be seen in a transverse section of the animal's body (Plate X. fig. 8, W, W). These spaces are, as in the Land-Planarians, little stained with carmine; and the oviduct (OD) also has exactly the same relation to them. There can be no doubt as to the homology of these spaces with one another; and, as will be seen further on, they are remarkably similar in their minute structure. The spaces (W, W) in *D. lacteum* are the main water-vascular trunks seen in section, and those of the Land-Planarians, as homologous with them, must receive the same appellation. The water-vascular trunks of *Leptoplana tremellaris* are found similarly situated and constructed on section (Plate XIV. fig. 1, W). Part of one of the drawings of Sommer and Landois's paper on the sexually mature joints of *Bothriocephalus latus* has been reproduced in Plate XIV. fig. 2, in order to show the very close resemblance in structure between the lateral vessels of that worm and those of the Planarians. In *Bipalium* and *Rhynchodemus* the pair of main vascular trunks extend throughout the entire length of the body, preserving their relative position to the ambulacral line and body-structures generally. In *Rhynchodemus* they terminate abruptly with rounded ends at both extremities of the body, as may be seen, as far as the anterior extremity is concerned, in Plate XIV. fig. 5. In *Bipalium*, on the other hand, the vascular trunks, though terminating posteriorly as in *Rhynchodemus*, spread out in the anterior extremity in an irregularly ramified manner (Plate XIV. fig. 4), the ramifications being imperfectly defined by rows of vertical muscular fibres. The wide expansion of the vascular trunks in the head of *Bipalium* is seen in vertical section in Plate XIV. fig. 2. Frequent branches are given off from the vascular trunks in both *Rhynchodemus* and *Bipalium*; and in some preparations they are conspicuous by their not being stained with carmine. These branches usually take a transverse course, and are in reality irregular lacunæ, having no more definite wall than their main trunks. The large transverse trunks connecting the main trunks in *Rhynchodemus* have already been spoken of: these are seen in longitudinal sections to be a series of irregular transverse channels hollowed out in the intervening muscular mass, and separated from one another by irregular intervals. Other finer transverse channels may be traced passing off from the vascular trunks (Plate XIII. fig. 15), being especially well seen in *Rhynchodemus*. As will be seen from Plate XIV. fig. 4, the large vascular space in the head of *Bipalium* does not extend to the verge of that structure, but is separated from this by a zone of solid tissue. Through this zone of tissue proceeds in straight lines a series of fine branches or tubular spaces, which are directed outwards to the margin of the head, and appear to pass directly to the peculiar ciliated sacs which there exist, and possibly to communicate through these with the exterior. These branches are figured, as seen in a longitudinal and horizontal section, Plate XV. fig. 3; and one may be sometimes traced passing from the lateral extremity of the vascular space to the papillary line in the vertical section, Plate XIV. fig. 3. The relation of these branches to the ciliated sacs will

be more carefully considered when these latter organs are treated under the head of special sense-organs. The structure of the spongy tissue with which the water-vascular spaces are filled is very remarkable. In *Bothriocephalus latus*, as in *Leptoplana*, *Dendrocœlum*, *Bipalium*, and *Rhynchodemus*, it is characterized by being very little stained with carmine (Sommer and Landois, *loc. cit.* p. 13). The tissue forms a fine reticulation, in which are pierced oval or rounded openings. A comparison of figs. 1, 2, & 7, Plate XIV., will show the remarkable resemblance of structure in the various worms. In *Bipalium* and *Rhynchodemus* the main water-vascular trunks are traversed by muscular fibres of the body-mass, which give a characteristic appearance to these structures, from the circumstance that they form two parallel sets, which preserve a constant direction throughout the body and cross one another at a constant angle. One set is nearly vertical in *Bipalium Ceres* (Plate X. fig. 6), slightly inclined inwards (*i. e.* towards the middle line) inferiorly in *B. Diana* (Plate X. fig. 5), and inclined rather outwards in *Rhynchodemus Thwaitesii*. The other set has about the same inclination in all these species, and will be seen to slope downwards and outwards on each side at an angle of about 60° with the vertical. The two sets of fibres thus crossing include between each other rhomboidal spaces. The fibres are here said to be muscular, because they are continuous with undoubted muscular fibres of the body-mass; but, as will be seen in Plate XIV. fig. 6 (where a portion of one of the main vascular trunks of *Bipalium Diana* is shown greatly magnified), these fibres give off ramifications which are histologically continuous with the fine connective-tissue network occupying their interspaces, and this, again, in direct connexion with the peculiar protoplasmic elements (X, X) which have been before treated of.

I observed no cilia within the vascular canals of any of the Planarians which I examined.

The term "water-vascular system" has here been given to the peculiar canals or spaces in *Bipalium* and *Rhynchodemus*, because they are most evidently of the same nature as the lateral vessels of *Tænia* and *Bothriocephalus*, to which that name was given by V. Siebold, and because they are distinctly homologous with the longitudinal canals of *Dendrocœlum* and *Leptoplana*, which usually receive that appellation. But the term would seem to be rather unfortunate, because a "water-vascular system" has come to be regarded more or less as necessarily an excretory organ ("Excretions-Organ," Keferstein, *loc. cit.*), and to have necessarily some communication with the exterior. The term "primitive vascular system" would seem to be more appropriate; for the case would seem to be as follows. In primitive animal forms of more or less homogeneous constitution, as advancement in organization proceeds, a circulation of the body-fluids becomes a necessity, and vascular spaces become gradually developed in certain parts and along certain lines of the body-mass. These spaces become more and more clearly defined, and assume at length the form described as existing in *Bipalium*, where the spaces or canals are by no means as yet open channels, but merely tracts where the body-tissue has become extremely porous and permeable to fluids, and which are still traversed

by stout muscular fibres. These canals, however, subserve in this animal all the purposes of imperfect circulation required, and even, by means of their branches, may effect the erection of the penis, and perhaps also the distention of the pharynx (KEFERSTEIN, *loc. cit.* p. 21), as do the blood-vessels of higher forms.

This primitive vascular system, which in *Tænia* and *Bothriocephalus* assumes a very definite and tubular form, though still occupied internally by spongy tissue, is directly homologous with the body or perivisceral cavity, which is persistent throughout life in *Branchiobdella*, and present in all leeches at some period of development, and in all adult leeches, in a rudimentary condition at least (LEUCKART, Die menschl. Paras. p. 666). The true blood-vessels are, when present, developed within and partitioned off from this primitive vascular system. In *Tænia* and Planarians no such further development has taken place. The nervous system lies within the primitive vascular system; but when blood-vessels are developed, the nervous system is often included within the latter. In some animals there are further developed, from without inwards, ciliated tubes, sacs, or pores, which communicate with the primitive vascular system, by means of which excretion from the vascular system takes place; and such an arrangement reaches its highest development in such forms as the Trematodes. It is, however, erroneous to consider the main and necessary function of the primitive vascular system as excretory, since in such forms as *Bipalium* it obviously performs many other circulatory functions, although here, as in Trematodes, it may also subserve an excretory function by means of the ciliated sacs in the region of the head. MERTENS (*loc. cit.* p. 12, 1833) describes the ganglia of *Leptoplana* as hearts, and the nerves given off from them as vessels. DUGÈS held a like opinion (*loc. cit.* 1, 1828). It seems highly probable that this is to be explained by the fact that the nerve-ganglia of this Planarian, and probably of all others where such exist, lie within a vascular sinus, a part of the primitive vascular system, and in continuation with it, the sinus giving off branches in which the nerve-branches lie. I cannot agree with KEFERSTEIN in supposing that BLANCHARD'S injection of the sinus round the ganglia of *Leptoplana* and its branches resulted from unskilful manipulation, and does not represent the true state of the case (KEFERSTEIN, *loc. cit.* p. 21). I had only one spirit-specimen of *Leptoplana* available for examination; but in this the ganglia were seen on section to be surrounded by a space occupied by loose spongy tissue, very little stained by carmine and exactly resembling that seen in the vessels of *Bipalium*, and the nerves occupied broad tracts of similar appearance; and on a vertical transverse section being made of the body, the two canals filled with spongy tissue were cut across, exactly resembling those of *Bothriocephalus* in structure, and coinciding in position with the large pair of longitudinal body-nerves. QUATREFAGES (Sur les Planaires, p. 172) says the brain is placed in a cavity or lacuna, prolongations of which accompany the viscera. He figures no regular water-vascular system apart from this. It is especially to be remarked that, almost invariably, observers who figure the nervous system of Planarians distinctly, do not in the same animal give the vascular system, and *vice versâ*. Thus OSCAR SCHMIDT (Zeitschrift für wiss. Zool. x. 1859, p. 29) speaks of seeing in a

Planarian the water-vascular system very imperfectly, but two long and stout "Seiten-Nerven" very plainly. Every line of evidence seems to point to the fact that in all Planarians, and indeed all worms in which a special blood-system is not differentiated off from the primitive vascular system or body-cavity, the nervous system lies within the vascular canals and spaces; and in *Bipalium*, as will be seen in the sequel, what was believed to be the nervous system was found to occupy such a position.

It is an interesting fact, and I believe new to science, that there exist Planarians which contain in their body-fluids hæmoglobin. I detected this substance by means of the spectroscope in a small Planarian, apparently a species of *Derostomum* (Schmarda, Neue wirb. Thiere, Band i. Hälfte i. Taf. i. fig. 8), which I found infesting in considerable abundance the surface of the integument of an echinoderm, one of the *Clypeastridæ*, Ag., which abounds at Suez. It is possible that the red colour of many other Planarians is due to the presence of hæmoglobin. A red tinge may be seen at the base of the eye of *Dendrocœlum lacteum*, but I found it too faint to give any absorption-spectrum. The reddish colour of the ganglia of Nemertines would be well worth testing.

Generative System.—The generative organs of Land-Planarians have hitherto been very imperfectly described, these animals having apparently never before been studied by means of sections, which is the only way of arriving at a satisfactory result in the case of opaque and solid worms such as these. Blanchard, speaking of *Polycladus Gayi* (*loc. cit.* p. 149), says that the generative organs are not well preserved in spirit; but there can be no doubt that what he calls a nervous system were the testes and ovaries. Schmarda made the same mistake in the case of *Bipalium*. Max Schultze could only find a penis and seminal receptacle in *Geoplana*. Claparède figures the general appearance of the mass containing the intromittent organs in *Bipalium*; but he calls the uterus the penis, and failed altogether in his description of the organs, probably from want of adequate material. He found no testes or ovaries. Schmarda, by some unaccountable mistake, describes his *Sphyrocephalus* (=*Bipalium*) as having two generative orifices.

The general arrangement of the generative organs will best be understood by reference to Plates XII. & XIII. and their description. We shall consider in order the ovaries and their duct, the testes with their duct, and the receptive, anal, and intromittent organs.

Ovaries.—The single pair of ovaries (Plate XII. figs. 1 & 3, OV.) is placed, in both *Bipalium* and *Rhynchodemus*, in the anterior extremity of the body or head at an enormous distance from the uterus, with which they communicate by means of a long and slender duct. The ovaries themselves are simple sacs, pear-shaped in *Bipalium*, spherical in *Rhynchodemus*, and they lie imbedded in the stout longitudinal muscles of the body-mass, which separate from one another and form cavities for their reception. The ovaries have a distinct but delicate membranous capsule, on the inner surface of which are to be seen marked out a series of irregular spaces (Plate XIII. fig. 8), which may represent a cellular lining. Externally to the capsule is a wide space occupied by an irregular meshwork of connective tissue, with large interspaces, which is probably in connexion with the primitive vascular system and supplies nutritive fluids to the organ; a similar

arrangement exists in the case of the testes. The ovary was filled, in all the specimens which I examined, with ova in all stages of development, the riper ova occupying the central and lower regions of its cavity, and the less mature the peripheral.

A meshwork of connective-tissue fibres with spindle-cells upon them passes from the walls of the ovary between the ova, and apparently furnishes the capsules in which the mature ova are seen to be contained. This meshwork is best seen in *Rhynchodemus* (Plate XIII. fig. 13). The successive stages in the development of the ova are given in Plate XIII. fig. 12. In its earliest stage the ovum is indistinguishable from the cellular lining of the sac of the ovary: it then apparently becomes rounded and increases in size, the germinal vesicle as well as the surrounding yelk remaining finely granulated up to a certain stage; then the germinal vesicle clears up and becomes transparent, and the ovum becomes enclosed in a capsule with a transparent area between it and the capsular wall. The final stage consists in the development of transparent fatty looking vesicles or globules within the fine granulated yelk-area. I am uncertain whether the capsules here described as enclosing the ova in their latest stages descend with them into the uterus, or whether they are merely ovarian follicles which open in the ovary and allow of the escape of the ripe ova. From a study of the ovary of *Rhynchodemus*, in which these capsules are not so well marked, and when present more apparently connected with the stroma of the ovary, I was led to consider that such was probably the case. If it be so, then it is quite possible that several ova may subsequently be enclosed within one true egg-capsule formed in the uterus, as in *Dendrocœlum lacteum*. At the base of the cavity occupied by the ovary of *Bipalium* is a mass of small rounded spindle-cells, which may represent an accessory gland in a rudimentary condition. In one specimen of *Bipalium* which I examined there was present on each side, just externally to the lower extremities of the ovaries, a small mass of large nucleated cells (Plate XIII. fig. 8, *g*) connected by a pedicle with the ovary itself. This mass, though extremely well defined in this one specimen, was absent entirely in many others, and present only as a trace in some few. It may represent a yelk-gland in a rudimentary condition, or possibly, as all specimens which I obtained had their testes and ovaries filled with ova and spermatozoa, this accessory gland had already performed its function for the season in the formation of ova, and had shrunk in consequence. The former hypothesis is, however, most probably correct, since the female organs generally in *Bipalium* seem to be undergoing a process of simplification. No corresponding glandular mass was seen in *Rhynchodemus*. The oviduct leaves the ovary on its outer side in *Bipalium*, on its inner in *Rhynchodemus*. The duct consists in both animals of an external well-marked basement membrane, on which rest internally a series of well-defined nucleated cells, which are rectangular in longitudinal section, truncated cuneiform in transverse section. These cells bear long hairs or cilia, which are inclined in a direction from the ovary towards the uterus, and in transverse sections of the oviduct show a spiral twist (Plate XIII. fig. 9). The hairs are in many parts of the oviduct so long that it seems possible that they have no vibrating motion, but more probably act merely so as to prevent the

ova, which are driven forwards by muscular pressure, from making a retrograde motion. The exact connexion of the oviduct with the cavity of the ovary in *Bipalium* could not be determined; it was only traced as far as is figured (Plate XIII. fig. 8), where the oviduct is seen to expand as it passes about halfway up the outer side of the ovary. In *Rhynchodemus* the oviduct was found to take origin from a papilla on the upper and inner side of the oviduct, and projecting into its cavity (Plate XIII. fig. 13). The papilla is formed of spindle-cells, and a number of similar cells are to be found scattered in the loose tissue around its base. The oviduct passes up through the loose external capsule of the ovary, and lies externally to the membranous internal capsule. The oviducts are directed in straight lines down the body, and passing just externally to the sheath of the pharynx reach the region of the uterus, to which organ they are directed by a sharp turn inwards (Plate XII. figs. 1 & 6). In *Rhynchodemus* the ducts are directed at first a little obliquely outwards to reach their position immediately internal to that of the vasa deferentia. In both *Bipalium* and *Rhynchodemus* the oviducts are crossed in the lower part of their course by the vasa deferentia, which pass to the dorsal aspect of the ducts. In *Bipalium* this crossing takes place about the region of the oral aperture; in *Rhynchodemus* lower down, opposite the base of the penis. In *Bipalium*, in the region occupied by the posterior portion of the pharynx, and from thence to the uterus, the oviducts have connected with their external aspect on each side peculiar branches or diverticula. These diverticula are very short, and are all directed at a slight angle backwards, *i. e.* towards the animal's tail. There are about six of them on each side, and they are situated at tolerably regular intervals. One of these diverticula is seen highly magnified in Plate XIII. fig. 9. A short branch is seen to enter the oviduct at an angle. The branch is constructed in the same manner as the oviduct itself, and has its cilia directed inwards towards the cavity of the duct. It ends abruptly, as far as definite oviducal structure is concerned, but from its abrupt termination is prolonged an interlacement of connective-tissue fibres, forming a sort of tube or cavity. There can be no doubt that these branches here described are the rudiments of the branched ovary possessed by lower Planarians, such as *Dendrocœlum lacteum*. In *Bipalium* a single one of the branches, the most anterior or terminal one, has become enlarged and differentiated, and has taken on the whole of the ovarian duties: the remaining branches are present in a rudimentary condition, like the yelk-glands: in *Rhynchodemus* both diverticula and yelk-gland have disappeared. Our Land-Planarians are, in fact, in their ovary just like *Polycelis cornuta*, O. Schmidt (Dendrocœlen Strudelwürmer, Taf. iii. fig. 1). The position of the oviduct, as seen in transverse vertical sections, is of great importance: it is always in close relation with the main trunk of the primitive vascular system, lying immediately above it or just within it in *B. Ceres*, and slightly externally to its median vertical line. This position holds good throughout the course of the oviduct in *Bipalium*, *Rhynchodemus*, and, singularly enough, in *Dendrocœlum lacteum* also (Plate XIV. fig. 7), showing how closely related are all the forms; and I was surprised to find in *Dendrocœlum lacteum* the oviduct so far histologically differ-

entiated that it exhibited a well-formed cellular lining, just like that in *Bipalium*. The testes in *Bipalium* (Plate XII. fig. 1) are arranged in pairs along a line just external to that occupied by the oviduct. They commence a short distance behind the ovaries, and extend for about half the interval between the ovaries and the oral aperture. There are about twenty-four or twenty-five pairs; they are separated from one another by short intervals, and communicate by means of lateral apertures with the vas deferens, which passes straight down the body along their inner side to reach the region of the penis. In transverse vertical sections of the body the testes are seen to occupy a position immediately external to the main primitive vascular trunks in both *Bipalium* and *Rhynchodemus*, and the vas deferens to lie immediately between the vascular trunks and the testis (Plate X. fig. 5, V. D.). The separate testes are in *Bipalium* irregularly spherical; they consist (Plate XIII. fig. 4) of a sac or wall and contents. The sac is formed, as in the ovary, of a fine but dense inner membrane, and an outer loose investment full of irregularly oval spaces, and probably in communication with the vascular system. The whole testis lies imbedded in a cavity amongst the longitudinal muscular fibres, as does the ovary. On the inner side of the spherical testicular sac is an opening, by means of which it communicates with the cavity of the vas deferens; and it would appear that the basement membrane of the vas deferens becomes continuous with the inner membrane of the sac. There is a bulging out of the wall of the vas deferens opposite the spot where the testicular cavity communicates with it, and a sort of sinus is thus formed in it at this place. The vas deferens (Plate XIII. fig. 7) consists of a basement membrane lined with nearly rectangular nucleated cells; these cells are much smaller than those of the oviduct; and the vas deferens is oval in section, not circular, and has a wide lumen. Opposite the opening into the testis the epithelial lining of the vas deferens is thickened in a remarkable manner. The epithelium is not ciliated, as is that of the oviduct. The cavity of the testis is always divided into two regions—an outer broad zone full of immature cells, and a central cavity in which are ripe parent cells of spermatozoa with spermatozoa attached to them in process of development and also in the free condition. The outer zone is so well defined from the inner space that it would almost appear as if a thin membrane were reflected back from the point of union of the vas deferens with the testicular sac-wall, and separated the two regions. I could not, however, make sure of the existence of such a membrane. It appears that the cells, as they become more and more developed within the outer zone, pass gradually outwards away from the opening into the duct till they reach a point opposite this opening, by which time they have reached a high state of maturity; hence they pass into the inner space of the testis, and here give birth to the spermatozoa. It appeared, from the study of a large number of sections, that the large parent cells of the spermatozoa were formed by the aggregation of a number of smaller rounded cells; but such a point could not, of course, be determined with certainty from preserved specimens. The spermatozoa are formed at the periphery of the large parent cells as in Annelids, and at first are provided with pear-shaped heads, which, however, are absent in mature spermatozoa, since some of them were present in the spermatozoa crowding the vas deferens.

In *Rhynchodemus* the testes are not of a definite spherical form as in *Bipalium*, but oval, with one end of the oval drawn out. They are very numerous, packed tightly together (Plate XIII. fig. 15), and they extend along each side of the body as far down as the base of the penis, instead of stopping short halfway as in *Bipalium*. I was unable to find the vas deferens in the anterior part of the testes of *Rhynchodemus*; and it is possible that the successive testicular sacs may here communicate directly with one another, or that thus the whole testis may here be one long moniliform or contorted tube. I had not sufficient material to determine this point. The arrangement of the parent and immature cells is the same as in *Bipalium*. The spermatozoa are linear, and resemble those of *Bipalium*. The losing of the bead-like swelling by mature spermatozoa has been noted by Max Schultze in other Planarians (Max Schultze, *loc. cit.* 4, p. 31). The position of the vas deferens in *Bipalium* and *Rhynchodemus* is remarkable as being to the inner sides of the testes, since in the leech it is to the outer sides of those organs.

The male and female intromittent organs form a small mass placed just posteriorly to the pharynx in an excavation into the median septum between the two posterior digestive tubes (Plate XII. fig. 2, G). This mass is seen enlarged in Plate XI. fig. 8. It consists in *Bipalium* of a posterior portion of an inverted flask-shape, with an ovoid body attached to it at a slight inclination anteriorly. The narrow neck of the flask-shaped body stands vertically, immediately over the external generative aperture. At its base anteriorly the body is entered by the oviduct, whilst the ovoid anterior mass is pierced by the vas deferens, which first passes backwards beyond its point of entrance, and then returns upon itself a short distance with a tortuous course to its destination. Plate XI. fig. 6 shows the arrangement of parts within the bodies just described. The external generative orifice opens into a short vestibular cavity, into which superiorly opens the mouth of the flask-shaped mass; leading from the mouth of this mass is a straight vertical vaginal tube, which at its summit bends sharply towards the anterior extremity of the body, and forms a small space, into which open the oviducts, hence termed uterus. In the anterior aspect of the vaginal tube close to its orifice opens the cavity of the ovoid portion of the generative mass, which consists of the sheath containing the penis and its contents: the cavity of the penis-sheath is of the same form as that of the sheath. To its internal surface, at its upper and anterior extremity, is attached the penis, which in its shrunken preserved state does not reach as far as the opening into the vagina.

The external generative orifice is provided with a circular sphincter muscle, and is clothed internally, as is the whole of the vestibule, vagina, and uterus, with ciliated epithelium. The vestibule is provided with a muscular wall, consisting of circular and longitudinal fibres, with radiating fibres interspersed between them, and serving to hold the tubular cavity in place by attaching it to surrounding tissue. The vagina (Plate XII. fig. 6 & Plate XI. fig. 9) has a very dense internal circular coat, external to which is a thick mass of longitudinal and radiating muscular fibres.

The vagina is lined with a densely ciliated epithelium, which rests on a stroma of connective tissue; the lumen of the tube in transverse section is, when the lining of the tube is in a contracted condition as here depicted, cruciform. A quantity of glandular matter of a branched thread-like form passes to the muscular mass containing the vagina from all sides. The irregular threads composing the gland break up into finer branches, which appear to become lost between the longitudinal muscular fibres of the vagina. I at first took these thread-like masses for nerves, but they are evidently homologous with the shell-making glands described by KEFERSTEIN (*loc. cit.* p. 28) in *Leptoplana tremellaris*, and figured originally in the same species by MERTENS. The threads of the gland appeared to become continuous in places with the general glandular matter of the body, which is especially abundantly present in the neighbourhood of the generative organs. It may be that this shell-gland, so highly developed in *Leptoplana*, is here rudimentary and nearly functionless, or possibly it may be in a more active condition at a different period of the year from that at which I gathered my specimens of *Bipalium*. The uterus is provided, like the vagina, with strong circular longitudinal and radiating fibres; it terminates (Plate XII. fig. 4) in a heart-shaped cavity, clothed with a very thick epithelial layer and densely ciliated. At the apex of the heart, *i. e.* inferiorly, there is a papilla, on each side of which opens one of the oviducts, which may be seen in the figure curving inwards to meet at that point, but remaining distinct from one another to the very last. It is probable that the aspect of both vagina and uterus becomes much altered when the egg-capsule is formed, if such be formed. The cavity is probably greatly enlarged, its muscular walls proportionally thinned, and very probably the uterine and vaginal tubes are run into one.

The penis in *Bipalium* is more or less conical in shape. In the base of the cone is a large glandular cavity with a muscular wall, the prostate, into the anterior extremity of which open the vasa deferentia, and from which a tube is continued down the centre of the penis to its tip. The penis has an elaborate muscular structure, for which reference should be made to Plate XII. fig. 5 & Plate XIII. fig. 3. The cavity of its central tube is lined with a glandular epithelium, which rests on spongy tissue: this is succeeded outwards by a dense circular muscular layer; then follows a space occupied by radiating fibres, more or less intertwined, and having canal-like spaces in their interstices and also bundles of strong longitudinal fibres. Externally is a row of delicate longitudinal fibres and a thin layer of circular ones. Finally, the penis is covered with an epithelium, consisting of rounded vesicular elements. The cavity containing the penis is lined with a simple even layer of epithelium, divided by vertical lines into irregular elements, which are apparently without nuclei. The penis is attached to its sheath by means of its radiating and longitudinal fibres, which spread out and invest the large glandular cavity at its base or prostate, and thus form an ovoid muscular bulb. The cavity in this bulb or prostate is lined with a glandular epithelium disposed in a series of follicles (Plate XIII. fig. 2). The epithelium is of the same nature as that found lower down in the central tube of the penis, but is here especially developed: it is very conspicuous

in carmine preparations, from the fact that it remains almost unstained. The glandular elements are transparent and devoid of nuclei; at the mouths of the follicles they are long and tumid, and form finger-like processes, whilst in the cavities of the follicle they are smaller and angular in outline from mutual appressure.

In longitudinal sections of the bulb of the penis there are to be seen running inwards from its periphery to its glandular cavity peculiar wavy bands, which in carmine preparations stand out into relief unstained amongst the surrounding deeply stained muscular tissue (Plate XIII. fig. 2, *u*). Similar structures are to be seen running down the penis longitudinally (Plate XIII. fig. 1, *u*), at pretty regular intervals from one another. When these wavy bands are seen in section, they appear as spaces filled with a fine areolar network, like that exhibited by the primitive vascular system (Plate XIV. fig. 6). I once held that these bands represented a series of tubular canals in connexion with the primitive vascular space, and serving for distending and erecting the penis, in the same manner as, according to KEFERSTEIN'S conjecture, the proboscis of *Leptoplana* is distended by injection of body-fluid (KEFERSTEIN, *loc. cit.* p. 21). In highly magnified transverse sections of the penis these wavy bands are seen (Plate XIII. fig. 3) passing inwards between the masses of longitudinal muscular fibres, making their way through the dense zone of internal circular fibres, and breaking up at the inner verge of this zone into a series of fine branches. These branches, as seen in the drawing, which is accurately made with the camera lucida, pass through the loose tissue which intervenes between the epithelium of the glandular prostatic follicles and the internal circular muscular layer, and appear as if they became, in some instances, continuous with the glandular epithelial elements themselves. It seemed possible that this should be the case, and that when the penis was distended with fluid, a liquid derived from that fluid should be poured out by these glands which should serve to dilute the semen. The structure, as shown in the drawing, is very remarkable; but after the examination of an oceanic* Planarian I have come to see that the structure in question is a system of muscles retracting the penis. Running along the centre of the wavy bands may usually be seen fine thread-like structures, and such are indicated in Plate XIII. fig. 3. It is very probable that these are nerves following, as in *Leptoplana*, the vascular canals.

In *Rhynchodemus* the generative organs are somewhat different from those of *Bipalium*, as may be seen from Plate XII. fig. 3 & Plate XI. fig. 7. The penis is larger and longer, both actually and still more so in proportion to the size of the body. There is no large swelling or bulb at the base of the penis, but a strongly muscular straight canal, of larger bore than that which pierces the penis, extends from the base of that organ towards the mouth in the middle line, and terminates blindly just opposite the termination of the testes. This canal is not provided with any special glandular epithelium, and is probably solely ejaculatory in function. The vasa deferentia are wide and tortuous; they turn twice upon themselves before they enter the ejaculatory tube, which they join at a

* This animal came into my hands whilst I was upon H.M.S. 'Challenger,' subsequently to the reading of this paper before the Royal Society.

short distance posteriorly from the anterior fluid extremity. In the female portion of the generative organs there is no definite vaginal tube as in *Bipalium*. The uterus forms a much larger cavity, and the oviducts here unite into a single tube instead of remaining distinct like those of *Bipalium*, and they enter the posterior wall of the organ instead of the anterior.

Looking at the testes and ovaries as shown in *Bipalium* (Plate XII. fig. 1), there can be little doubt that these are the "ganglia" which SCHMARDA described, the ovaries being the first pair. Moreover, BLANCHARD'S similar ganglia in *Polycladus* are almost certainly due to the same cause: the especially large ganglia in what he calls the tail of his animal, but which in reality, as explained by MAX SCHULTZE, was the head, is evidently the ovary, and the remainder of the ganglionic chain the testes. *Polycladus* must therefore be closely related to *Bipalium* in the arrangement of its generative organs.

Nervous System.—There is no trace of the series of ganglia described as existing in *Bipalium* by SCHMARDA, other than the ovaries and testes. After most careful examination I could not discover any thing like a ganglion-cell in the whole body of either *Bipalium* or *Rhynchodemus*. I believe that the nervous system, which is in these Planarians very indistinctly differentiated histologically, forms a meshwork within the primitive vascular canals. In the head such a meshwork is to be observed, in sections made from specimens hardened in chromic acid, occupying the same region and having the same form as the vascular ramifications in the head. A portion of this meshwork is figured, Plate XV. fig. 5. The dark matter at the points of union of the bundles of fibres is merely finely granular in structure and has no cell-structure. Similar tissue may be traced along the whole length of the body in the primitive vascular canals, both in *Bipalium* and *Rhynchodemus*. After very careful examination I have been able to discover no more specialized nervous system in these Planarians than this. The fine threads within the vascular canals in the penis of *Bipalium* are probably nerves; and in *Rhynchodemus* an undoubted and distinct nervous filament is given off from the inner extremity of the eye (Plate XV. fig. 8), but it cannot be traced to connexion with any definite nervous structure; it passes to the main vascular trunk and is there lost. In order to make certain that ganglia such as those known to exist in other Planarians were really absent in *Bipalium* and *Rhynchodemus*, and had not merely been destroyed by the method of treatment, I prepared sections from a specimen of *Leptoplana tremellaris* hardened in spirit in the same manner as my Land-Planarians, so as to display the structure of the nervous ganglia. Figs. 1, 2, 3, & 4, Plate XV., represent four longitudinal and horizontal sections from this specimen drawn with the camera lucida, and are given here to show the remarkable complexity of the structure of the ganglia in this Planarian, and the very great distinctness of the ganglion-cells. Most certainly no such structure as this exists in *Bipalium* or *Rhynchodemus*.

It is constantly asserted by older and even by modern writers that ganglion-cells do not exist in Planarians and Nemertines (FREY and LEUCKART, *loc. cit.* p. 92; LEYDIG, Vom Bau des thierischen Körpers, p. 124). QUATREFAGES (Sur les Planaires, p. 172)

says, "Les deux lobes de cette espèce de cerveau sont composés d'une substance entièrement diaphane et homogène." KEFERSTEIN is the only investigator, so far as I am aware, who has made sections of these organs in Planarians and recognized their true structure, whilst M'INTOSH (*loc. cit.* pl. vii. fig. 2) has figured nerve-cells from the ganglia of Nemertines. KEFERSTEIN gives a figure of a section of the ganglia of *Leptoplana tremellaris*, but the wonderful complexity of the nervous structures is not so minutely treated as in the present drawings.

Special Sense-organs.—The only special sense-organs observed in *Bipalium* were the eyes and the peculiar organs on the anterior margin of the head. In *Rhynchodemus* a single pair of eyes was all that could be found.

The eye-spots which appear when *Bipalium* is viewed with a hand-lens as black specks, are thickly set all over the upper surface of the flat semilunar head, except along its median line, a small space (broader anteriorly) on each side of this, and along a narrow band bordering the actual anterior margin of the head. The eyes are especially densely packed at the tips of the corona of the head, and are thickly set along all its margin, except in the region of the band of special sense-papillæ. Besides this, eyes are present sparingly scattered over the entire length of the body to the very tail. It has hitherto been supposed by all writers on the subject that the eyes were confined to the anterior extremity in *Bipalium*; but the spots are constantly being met with in sections in the region of the mouth and generative organs. CLAPARÈDE, examining the eye-spots of *B. Phœbe*, could not make certain whether they were sense-organs at all. His specimens probably were not in sufficiently good preservation. In *B. Diana*, *B. Ceres*, and *B. Proserpina* the eye-spots are usually of the form shown in Plate XV. figs. 6 & 7, though they are often less elongated and indeed nearly spherical. They consist of a simple sac or cell, the anterior portion of which, or that turned to the light, is transparent and lens-like, whilst the posterior and larger portion of the sac is darkened and rendered opaque by the presence of brown pigment-granules imbedded thickly in its wall. An unpigmented dot, often present in the posterior part of the eye-spots and represented in Plate XV. fig. 6, seems to show that these eye-spots are to be regarded as modifications of single nucleated cells.

In the interior of the eye-spot, when seen in section, is to be observed, under favourable circumstances in deeply stained sections, a lens-like body (Plate X. fig. 7), but this body is very little differentiated from the general cell-contents, and is hard to see. Between the lens-like body and the interior of the pigmented back of the eye-spot is a highly refracting substance. No nervous structures were observed in connexion with the eye-spots. The eyes are arranged beneath the external circular muscular coat of the body, and in the intervals between the external longitudinal muscular bands (Plate XV. fig. 9). The light which reaches them must penetrate the external circular muscles first; but these are not thick in the regions where the eyes are most numerous.

Rhynchodemus possesses only a single pair of eyes, but these are very much larger than those of *Bipalium*; they are elongate, and somewhat like those of the leech in

form; they have a transparent cornea in front, which projects amongst the epithelium of the skin, and a posterior pigmented sac. From the pointed extremity of the sac a nerve-fibre can be traced a short distance. I had not sufficient material to allow of the examination of the internal structure of the eye. Leidy has given some account of the eyes of *Planaria sylvatica* (=*Rhynchodemus sylvaticus*, Diesing). He says the eyes consist of a vitreous humour, two thirds covered with pigment, and $\frac{1}{100}$ of an inch in diameter. Mecznikow (*loc. cit.*) describes the eyes of *Geodesmus bilineatus* as very complex. Their pigment-skin is composed of clearly definite hexagonal cells, and the eye contained a red-coloured crystalline body consisting of rod-like elements. The crystalline body evidently is homologous with the lens-like body in the eye of *Bipalium*, the similarly complex body in *Leptoplana*, and the vitreous body in *Dendrocœlum lacteum*, which has usually a reddish tinge. The eye of *Rhynchodemus Thwaitesii* probably contains a corresponding structure. Mecznikow considers that the great complexity of eye of *Geodesmus* has been brought about by the animal's terrestrial habits, it requiring to use its sight more on land than in water; but the eye of *Leptoplana* is as complex as that of *Geodesmus*, and it is possible that the aquatic ancestor of *Geodesmus* was already provided with highly developed ciliated sacs.

In describing the habits of *Bipalium*, I described the manner in which that animal throws out tentacular-like projections from the anterior margin of its semilunar head when in motion, and evidently uses these temporary tentacles as sense-organs. In reading M. Humbert's interesting account of *Bipalium*, I found that he had observed this habit of the animal as well as I, and had been led by his observation to seek for sense-organs or tentacular structures on the margin of the head. He was not successful in finding any; but on very careful examination of well-hardened specimens I was more fortunate, and discovered a narrow band extending along the whole anterior margin of the head, entirely free from pigment, and occupied by a row of cylindrical rounded papillæ placed vertically side by side, and with small oval openings between their superior extremities (Plate XIII. fig. 16). This row of papillæ is in the upper part of the lower fifth of the margin of the head, so that it lies close to the ground when the animal's head is lowered. The papillæ are covered with short cilia; but I could find no special structure in them, except that in their region, and that of the ciliated pits, there is a large quantity of tissue formed of small spindle-cells. The oval apertures between the papillæ lead to ciliated pits, the appearance presented by which is shown in figs. 11, 12, & 13, Plate XV. In longitudinal and horizontal sections the appearance presented in fig. 13 is seen. The light bands, which appear to pass to the bottoms of the ciliated pits, are continuous with the vascular network of the head. Whether they represent tubes in communication here with the exterior I cannot say. They may convey nerves to the sacs. From the manner in which the animal uses the front of its head, there can be little doubt that the papillary line discharges some special sense-function; but it is possible that this function is discharged by the papillæ, whilst the ciliated pits with their communicating vascular stems act as excretory organs. The papillary line with its pits was found in

all the species of *Bipalium* examined. The ciliated sacs in Nemertines come at once, of course, into one's mind in connexion with these curious structures. Careful examination may perhaps give evidence of the existence of similar ciliated sacs in *Geoplana* and other Planarians. Nothing of the kind was found in *Rhynchodemus*.

Summary.

The writer commences by expressing his great obligations to Professor Rolleston, whose pupil he formerly was. Professor Rolleston first informed him of the existence of Land-Planarians in Ceylon, and of the importance of investigating them. The paper was at first intended to be a joint one; and Professor Rolleston himself made a number of preparations of *Rhynchodemus*, one of which is figured. He likewise rendered great aid in the bibliography, and by constant suggestions and assistance during the progress of the work.

Two new species of Land-Planarians from Ceylon are described:—one belonging to the genus *Bipalium* (Stimpson), *B. Ceres*; the other to that of *Rhynchodemus*, *R. Thwaitesii*, so called after Mr. G. H. K. Thwaites, F.R.S., the illustrious curator of the Peradeniya Gardens, by whose assistance the specimens made use of were procured.

Lists are given of all the known species of *Bipalium* and *Rhynchodemus*.

With regard to the habits of *Bipalium*, the most interesting facts noted are that these animals use a thread of their body-slime for suspension in air, as aquatic Planarians were observed to do for their suspension in water by Sir J. Dalyell, and the cellar-slug does for its suspension in air. The projection of small portions of the anterior margin of the head in the form of tentacles, originally observed by M. Humbert, becomes interesting in connexion with the discovery of a row of papillæ and ciliated pits in that region. The anatomy of the Planarians was studied by means of vertical and longitudinal sections from hardened specimens. The skin in *Bipalium* and *Rhynchodemus* closely conforms to the Planarian type, but is more perfectly differentiated histologically than in aquatic species, and approaches that of the leech in the distribution, colour, and structure of its pigment, and especially in the arrangement of the glandular system. The superficial and deep glandular system of the leech are both here represented. In *B. Ceres* peculiar glandular structures exist, which may foreshadow the segmental organs of Annelids, it being remembered that these segmental organs are solid in an early stage of development. Rod-like bodies (Stäbchen-Körperchen) are present in abundance, though, singularly enough, Max Schultze failed to find any in *Geoplana*. These Stäbchen-Körperchen are probably homologous with the nail-like bodies of Nemertines; and it is possible that the setæ of Annelids are modifications of them. No light is thrown by the structure of these bodies in *Bipalium* on the question whether they are homologous with the urticating organs of Cœlenterata.

The muscular arrangement in *Bipalium*, which is very complex, throws great light on the homologies between the muscular layers of *Turbellaria* and other Vermes. It is commonly said that whilst in all other Vermes the external muscular layer is circular,

and the longitudinal internal, in Turbellarians the reverse is the case. A wide gulf is thus apparently placed between these groups. In *Bipalium* there is an external circular muscular coat, which even presents the same imbricate structure which is found in it in leeches and other worms. In *Dendrocœlum lacteum* there is also an external circular coat. In cases where a distinct external circular muscular coat is absent, it is represented by a thick membrane, which is very probably contractile. The question resolves itself simply into a more or less perfect fibrillar differentiation of that membrane. All Turbellarians are built on the same essential type, as regards muscular arrangement, as are other worms. The general muscular arrangements in the bodies of *Bipalium* and *Rhynchodemus* have become much modified from those of flat Planarians by the pinching together and condensation of the body; but they are nevertheless referable to the same type.

The digestive tract consists of three tubes (one anterior, two posterior), as in other Planarians, and as in the embryo leech before the formation of the anus. Characteristic of Land-Planarians, and consequent on the condensation of the body, is the absence of all diverticula from the inner aspects of the two posterior digestive tubes. This is found to be the case in *Geoplana*, *Bipalium*, *Rhynchodemus*, and *Geodesmus*. The close approximation of the intestinal diverticula in *Bipalium* and *Rhynchodemus*, and the reduction of the intervening tissue to a mere membranous septum, is very striking, and seems to foreshadow the condition of things in Annelids. The great difference in the form of the mouth in *Rhynchodemus* and *Bipalium* is also remarkable, considering the many points in which these forms are closely allied.

A pair of large water-vascular trunks, or, as they are here termed, primitive vascular trunks, are conspicuous objects in transverse sections of the bodies of *Bipalium* and *Rhynchodemus*. A peculiar network of connective tissue is characteristic of these vascular canals on section, and is shown to present exactly similar features in *Leptoplana tremellaris*, *Dendrocœlum lacteum*, and *Bothriocephalus latus*. The close agreement in the relative position of the oviducts to the vascular canals in *Dendrocœlum* and our Land-Planarians is very remarkable. This primitive vascular system is homologous with the body-cavity present in the embryo leech and in *Branchiobdella* throughout life. It is not necessarily an excretory system, though the term water-vascular system has been generally considered to imply such a function for it. The nerves and ganglia of Planarians lie within the primitive vascular system, as do the corresponding structures within the primitive body-cavity of the leech.

Branches from the primitive vascular system in *Bipalium* possibly proceed to the ciliated sacs in the head, and perform an excretory function. A small marine Planarian was found to contain hæmoglobin. In *Bipalium* there are a series of separate testes disposed in pairs as in the leech. In *Rhynchodemus* the testicular cavities are more closely packed, and follow no such definite arrangement. The ovaries are simple sacs in both *Bipalium* and *Rhynchodemus*, and are placed very far forwards in the head, a long distance from the uterus. In *Bipalium* short branches given off from the

posterior portions of the oviduct are the rudiments of a ramified ovary, such as exists in *Dendrocœlum lacteum*. There are also glands present, which probably represent the yelk-glands and shell-making glands of aquatic Planarians in a more or less rudimentary condition. There is a comparatively simple penis and female receptive cavity in both *Bipalium* and *Rhynchodemus*. In *Bipalium* there is, further, a glandular cavity at the base of the penis (prostate). The organs described as nervous ganglia by BLANCHARD in *Polycladus* are almost certainly its testes and ovaries; and therefore the arrangement of these bodies in *Polycladus* is the same as that in *Bipalium*.

The chain of nervous ganglia described as existing in *Bipalium* (*Sphyrocephalus*) by SCHMARDA, and which has been referred to by so many authors, does not exist. There is no doubt that SCHMARDA mistook the ovaries and testes for ganglia. The real nervous system is ill-defined, but appears to consist of a network of fibres without ganglion-cells, which lies within the primitive vascular canals. In *Leptoplana tremellaris* the structure of the ganglionic masses is remarkably complex in the arrangement of the fibres; and well-defined ganglion-cells of various sizes are present and have a definite arrangement.

Numerous eye-spots are present in *Bipalium*, most of them being grouped in certain regions in the head, but some few being found all over the upper surface of the body, even down to the tail. The eye-spots appear to be formed by modification of single cells. In *Rhynchodemus* two eyes only are present. All gradations would appear to exist, between the simple unicellular eye-spot of *Bipalium* and the more complex eye of *Leptoplana* or *Geodesmus*, where the lens is split up into a series of rod-like bodies, forming apparently a stage towards the compound eyes of Articulata. It is quite probable that these compound eyes have arisen by such a splitting-up into separate elements of a single eye, and not by fusion of a group of unicellular eyes such as those of *Bipalium*. A peculiar papillary band runs along the lower portion of the margin of the head of *Bipalium*. The delicate papillæ are in the form of half cylinders, ranged vertically side by side. Between the upper extremities of the papillæ are the apertures of peculiar ciliated sacs. The papillæ, from the mode in which the animal makes use of them, are probably endowed with a special sense-function. The sacs may have a similar office, or they may be in connexion with the primitive vascular system, and have an excretory function; they may further be homologous with the ciliated tubes in Nemertines.

In considering the general anatomy of *Bipalium*, it is impossible to help being struck by the many points of resemblance between this animal and a leech. Mr. HERBERT SPENCER has, in his 'Principles of Biology,' placed a gulf between Planarians and Leeches by denoting the former as secondary, the latter as tertiary aggregates*. It is obvious, however, that a single leech is directly comparable to a single *Bipalium*. The successive pairs of testes, the position of the intromittent generative organs, the septa of the digestive tract, and, most of all, the pair of posterior cæca are evidently homolo-

* The idea is that an Annelid represents a series of Planarians, or corresponding secondary aggregates.

gous in the two animals. Further, were leeches really tertiary aggregates, the fact would surely come out in their development, or at least some indication of the mode of their genesis would survive in the development of some Annelid. Such, however, is not the case. The young worm or leech is at first unsegmented, like a Planarian; and the traces of segmentation appear subsequently in it, just as do the protovertebræ in vertebrates, which Mr. SPENCER calls secondary aggregates. If Mr. SPENCER'S hypothesis were correct, we should expect to find at least some Annelid developing its segments in the egg as a series of buds. It is not, of course, here meant to be concluded that Annelids are not sometimes in a condition of tertiary aggregation, as *Nais* certainly is when in a budding condition, but that ordinarily they are secondary and not tertiary aggregates; and if so, then so also are Arthropoda.

Much more information concerning the anatomy of Planarians will be required before it will be possible to trace the line of descent of *Bipalium* and *Rhynchodemus*, and determine what was the form of their aquatic ancestors. In the absence of accurate accounts of the structure of the American Land-Planarians, and even of the European *Rhynchodemus terrestris*, the question is very puzzling. The formation of either one of the two forms *Bipalium* or *Rhynchodemus* might be accounted for with comparative ease, from the arrangement of parts in the flat head of *Bipalium*. From the tree-like branching of the digestive tract in that region, the corresponding ramification of the vascular system, and general muscular arrangement, it might be imagined that *Bipalium* had come from a flattened parent of the common Planarian form, and that all the body except the head had become rounded and endowed with an ambulacral line. In nearly all points, except the eyes and the absence of branches to the oviduct, *Bipalium* seems more highly specialized than *Rhynchodemus*. We might imagine that *Rhynchodemus* and *Bipalium* had a common parent, and that when an ambulacral line was just beginning to be developed the two forms took different lines—*Rhynchodemus* losing all traces of the original flatness of its ancestor, and never developing any ciliated sacs or papillæ, but cherishing a single pair of large eyes at the expense of all the rest which it possessed, its testes, moreover, remaining in a comparatively primitive condition. But then comes the difficulty about the great difference in shape in the pharynxes of the two forms; and if it be suggested that, as is highly probable, several or many aquatic Planarians have taken to terrestrial habits, and that *Bipalium* has been derived from a form like *Leptoplana*, with a folded pharynx, whilst *Rhynchodemus* came from an ancestor with a tubular one, it is difficult to account for the many points of close resemblance between these two forms, and especially their similarity in external colouring, though this latter may perhaps be explained by mimicry. On the whole, it is evident that a close study of the anatomy of Land-Planarians cannot fail to lead to interesting results; and it is hoped that this memoir may lead to further work of the same kind. It would be of especial value to have a good account of the anatomy of *Geodesmus* and *Rhynchodemus sylvaticus*.

Description of the Plates.

PLATE X.

Fig. 1. *Bipalium Ceres*, sp. nov., of the natural size, from specimens preserved in absolute alcohol.

Fig. 2. Young of the same, from the same source.

Fig. 3. Young of *Bipalium Diana*, from specimens preserved in absolute alcohol, and of the natural size.

Fig. 4. *Rhynchodemus Thwaitesii*, sp. nov.

All four specimens from the Royal Botanic Gardens, Peradeniya, Ceylon; drawn of the exact dimensions.

Fig. 5. Vertical section of *Bipalium Diana*, taken in a direction transverse to the long axis of the body, at a spot about half an inch distant from the anterior margin of the animal's head. Drawn with a camera lucida.

The letter D, seen a little above the middle of the figure, is placed in the central digestive cavity, which in that portion of the body which is anterior to the entrance of the pharynx is single, as is ordinarily the case in Dendrocœlous Turbellaria. The cavities lettered D′ on either side of it represent the lateral diverticula which it gives off: as they are not given off quite at right angles to the central stem, they are not exposed in their entire length in a transverse section such as this. Between the letters D and D′ are seen portions of the septa which separate the successive diverticula and their branches from each other, and show decussating muscular fibres very plainly. The clear spaces (W) on either side the middle line, inferiorly to the origins of the lateral diverticula, are the two chief trunks of the water-vascular system, running antero-posteriorly, and are less stained with carmine than the rest of the section. The glandular masses (T) lying immediately exteriorly to them are the testes; in the interval between each testis and the water-vascular trunk is seen the *vas deferens* (V.D.) in section, and in the upper and outer angle of the water-vascular trunk is seen the oviduct (OD). The inferior surface of the body is flatter than the superior; but a considerable projection is formed along the middle line of the surface by an "ambulacral line" or "sole," whence these Turbellaria are sometimes termed "gasteropodous" (Diesing, *loc. cit.* p. 509). Where the ambulacral line rises above the level of the rest of the inferior surface of the body, cilia of large size are seen to clothe it. A zone of tissue, contrasting by its greater clearness with the cuticle and its basement membranes and muscles, runs round the whole body of the animal immediately internally to those structures. This greater clearness is due to the absence in this zone of the longitudinal layer of muscular fibres, which is largely developed both internally and externally to it. The area of the zone itself is

mainly occupied by radiating muscular fibres, prolongations of the circular muscles surrounding the viscera. It contains, interspersed in its substance, gland-cells containing rod-like bodies and pigment-cells. The longitudinal muscles, having been exposed in section to the action of carmine, are readily recognizable by the deep tint which they have taken, and may be seen to be specially developed (I. L. M.) along a line reaching to the region of the testes and water-vascular trunk of either side, from a point a little way internally to the lateral border of the animal. Opposite the lateral borders of the animal's body these muscles are sparingly developed; inferiorly, again, on either side, to the commencement of the intestinal diverticula, they are aggregated in considerable masses. The superficial muscular layers are largely developed along the median dorsal and infero-lateral lines. A collection of glandular tissue is seen on the outer side of either testis at X.

D. Central gastro-intestinal canal.
D'. Lateral diverticula given off from central canal, D.
E. Epidermis.
P. Pigment.
E. C. M. Exterior muscular layer, consisting of circular and decussating fibres.
E. L. M. External longitudinal muscular layer.
I. L. M. Internal longitudinal muscular layer.
L. M. External longitudinal muscular layer in the ambulacral line.
R. M. Zone occupied by radiating muscular fibres, and containing also glands, *Stäbchen-Organe*, and pigment-cells.
OD. Oviduct.
V. D. Vas deferens.
T. Testis.
W. Water-vascular trunk.
X. Aggregation of glandular cells.

Fig. 6. Vertical section of *Bipalium Ceres*, taken in a direction transverse to the long axis of the body, at a spot about half an inch distant from the anterior margin of the animal's head. Drawn with the camera lucida. The form of the body is seen to be very different from that of *B. Diana* (see fig. 5, Plate X.), and in some respects to approach that of *Rhynchodemus*. The points A, A correspond to the ridges characteristic of this species, which run along the body inferiorly on each side of the ambulacral line, and which contain masses of finely granular material, A, A, from which tracts of glandular matter lead to a point just above the testes. The ambulacral line projects less than in *Bipalium Diana*; the oviducts are lower in position than in that species, lying within the water-vascular trunks. The external circular and longitudinal muscular systems are greatly developed in the dorsal region.

A, A. Lateral glandular masses.
E. C. M. External circular muscular layer.
E. L. M. External longitudinal layer.
R. M. Radiating muscular fibres.
I. L. M. Internal longitudinal muscles.
E. Epidermis.
T. Testes.
OD. Oviduct.
W. Water-vascular space.
D. Central or gastro-intestinal canal.
D'. Lateral diverticula given off from central canal, D.

Fig. 7. Section transverse to the longer axis of the body of *Rhynchodemus Thwaitesii*, at a spot a little below the commencement of the series of testes. The ambulacral line does not in this case form a projection as in *B. Diana*; indeed the organ is here in a comparatively rudimentary condition, and does not possess the peculiar muscular arrangement described as existing in *Bipalium*; but its situation is marked by an entire absence of pigment in the part of the animal's body corresponding to it. These spots are seen on the superior surface (one median, two lateral), where there is a special concentration of dark pigment. These form the longitudinal dark stripes with which the animal's body is marked. A special development of the most external muscles may be observed here, as in *Bipalium Diana*, about the median dorsal and infero-lateral region, with a corresponding development of internal longitudinal fibres.

Only one of the diverticula (D) of the intestine is laid open, viz. that on the left side of the section. On the right side the septum is entire, and shows its decussating fibres. The water-vascular trunks (W) are seen to be connected by a transverse tract. The peculiar elongated glandular bodies, deeply stained, are seen to be abundant over an irregular arc around the testes on each side, and also just above the central digestive canal; at the lateral margins of the section some are seen passing towards the epidermis.

E. Epidermis.
E. L. M. External longitudinal muscles with external circular epidermis.
P. Pigment.
R. M. Radiating muscular fibres.
I. L. M. Internal longitudinal muscles.
T. Testis.
OD. Oviduct.
W. Water-vascular trunks.
X. Peculiar protoplasmic bodies.

Fig. 8. Central region of vertical section of *Dendrocœlum lacteum*, taken in a direction

transverse to the long axis of the body, at a spot just anterior to the mouth. Drawn with the camera lucida.

The section contrasts strongly with those displayed in figs. 5, 6, & 7, in being lengthened out from side to side, and having its superior and inferior surfaces irregularly parallel, owing to the flattened-out form of the Planarian from which it was prepared. The central digestive cavity D, with its diverticula D', is more irregular in outline than in *Bipalium* and *Rhynchodemus*. The dark glandular masses (T, T) are seen scattered over the section, but in this particular one are exposed in greater abundance on the left side. The main trunks of the water-vascular system (W, W) are here, as in *Bipalium* and *Rhynchodemus*, rendered conspicuous by their being but slightly tinted with carmine. The oviducts are situate just above them, as also is the case in *Bipalium* and *Rhynchodemus*. A circular muscular coat succeeds the epithelial layer; and closely opposed to this are the longitudinal muscles of the body, in this animal not divisible into two systems, as in *Bipalium* and *Rhynchodemus*. The stout vertical fibres which run from one surface of the body correspond to the internal circular and radiating muscular fibres of the two Land-Planarians already figured. The small darkly stained masses partly internal to and partly mixed up with the longitudinal muscular fibres consist of urticating organs and small glandular masses, and some of them represent longitudinal muscular fibres exceptionally darkly stained.

D. Central gastro-intestinal canal.
D'. Lateral diverticula given off from central canal, D.
E. Epidermis.
E. C. M. External circular muscular layer.
L. M. Longitudinal muscular layer.
V. M. Vertical muscles.
OD. Oviducts.
T. Testis.
W. Water-vascular trunks.
X. Glandular masses.

Fig. 9. Three rod-like bodies from *Bipalium Diana* as they appear in sections from spirit-specimens, stained with carmine and mounted in dammar varnish.

Fig. 10. Epidermic structures from similar preparations.

A. Elongated irregularly shaped body often seen in the epidermis, deeply stained with carmine, and frequently in continuity with the glandular masses (G) represented in fig. 1, Plate XI.; probably masses of mucus hardened by alcohol in the act of their ejection from these glands.

B. One of the gland-cells (G. C.), fig. 4, Plate XI.

Fig. 11. From similar preparations, two parent cells of rod-like bodies; on the right a cell in transverse section, showing three chambers. Fig. 12. Fragments of similar cells.

Fig. 13. Small portion of a longitudinal section in the plane of the body of *Bipalium Diana* in the dorsal region, to show the muscular arrangement.

The appearance here represented is to be observed when a longitudinal section of *Bipalium* in the superficial dorsal region is viewed from above.

A. Superficial external circular and decussating muscular layer.

B. Stout longitudinal muscular bands seen beneath the external layer (A) by alteration of the focus of the microscope.

C. Urticating cells seen in section.

PLATE XI.

Fig. 1. Vertical section, taken in a direction transverse to the long axis of the body, of the skin of *Bipalium Diana* with the adjoining tissues, from the lateral margin of the animal in the region of the external generative opening. Drawn with the camera lucida from a specimen hardened in spirit. Highly magnified.

Superiorly is seen the epidermis (E), the structure of which is not well shown in this preparation; imbedded in it, however, are three of the peculiar rod-like bodies—Stäbchen-Körperchen. Beneath the epidermis lies the external circular muscular layer (E. C. M.), in this region (lateral) of the body not displaying that complex interlacement of fibres which is to be observed in it in the dorsal and inferior regions. A dark line separates this muscular layer from the epidermis, and indicates a narrow line deeply stained with carmine, which represents the basement membrane (B). Though this section is taken in the region of the external generative opening, an eye (O) is seen on the right-hand side. E. L. M. points to one of a row of longitudinal muscular bundles seen in section. Between the external circular muscular layer and the internal longitudinal muscles (I. L. M.) a layer of loose tissue intervenes, composed of fibres passing from the radiating muscles (R. M.) to the external circular muscular layer. In this loose tissue are imbedded the glandular masses (G), the branching pigment-bodies (P), and the parent glands of the rod-like bodies (RG).

E. Epidermis.

B. Basement membrane.

E. C. M. External circular muscular layer.

O. Eye.

E. L. M. External longitudinal muscles.

I. L. M. Internal longitudinal muscles.

R. M. Radiating muscles.

G. Glandular masses.

P. Pigment-bodies.

RG. Parent glands of the rod-like bodies.

Fig. 2. Corresponding section to that shown in fig. 1, from *Rhynchodemus Thwaitesii*; also drawn with the camera lucida.

The epidermis (E) shows a vertical striation, and contains four cells with rod-like bodies in them. The external muscular coat (E. C. M.) is very thin; between the external (E. L. M.) and the internal longitudinal muscles (I. L. M.), and closely opposed to these latter, a band of internal circular muscular fibres is developed, being formed of a special development of the radiating muscles (R. M.) in this region of the body (see fig. 7, Plate X.). The parent glands of the rod-like bodies are less shrivelled by the action of spirit than in the foregoing preparation.

E. Epithelial layer.
R. Cell containing rod-like bodies.
E. C. M. External circular muscular layer.
E. L. M. External longitudinal muscles.
G. Glandular mass.
RG. Parent glands of rod-like bodies.
I. C. M. Internal circular muscular band.
I. L. M. Internal longitudinal muscles.
R. M. Radiating muscles.

Fig. 3. Vertical section transverse to the longer axis of the body through half the ambulacral line of *Bipalium Diana*, with the immediately adjoining region included. The side of the ambulacral line is seen to be clothed with long and strong cilia, which, however, fade off and almost disappear to the right and left in the direction of the general body-integument and in that of the actual inferior surface of the ambulacral line severally. The epithelial layer is seen to change its character entirely as it approaches the region where it bears the strong cilia, being there thicker and entirely free from intermixture with rod-like bodies; on the inferior surface of the ambulacral line it is very thin indeed. The external circular and decussating muscular layer is seen to split into two portions—one of which is continued as a thin layer over the ambulacral line immediately externally to the external longitudinal muscles of that organ; whilst the other, passing inwards horizontally, separates off a series of smaller muscular bundles from the main body of external longitudinal muscles, and then spreads its fibres out fanwise to become lost among the general muscular mass of the ambulacral line. Strong vertical fibres are seen to descend and end in club-shaped extremities between the external longitudinal muscular bundles of the ambulacral line. These are specially developed radiating fibres.

C. Cilia.
E. Epithelial layer.
E. C. M. External circular muscles.

x 2

E. L. M. External longitudinal muscles.

L. M. External longitudinal muscles of the ambulacral line.

R. M. Radiating muscles.

V. M. Vertical muscles of the ambulacral line.

I. L. M. Internal longitudinal muscles.

Fig. 4. Portion of the epidermis of *Bipalium Diana* in vertical section. From a section of a specimen hardened in spirit, treated with caustic-potash solution.

The epidermic structures in a state of contraction from the action of the spirit have been swollen out again by the potash. The rod-like bodies (R, R) are here of an elongated oval form; each is enclosed with a capsule. They vary in size, and are placed at different altitudes in the epidermis. The large irregularly oval cells (G., G. C.), filled with granular contents, are glandular elements of the epidermis. The remainder of the epidermis is made up of stout fibres, probably remnants of empty capsules of rod-like bodies. Beneath the epidermis and external circular muscular coat (E. C. M.) is seen a gland-capsule (RG) with a rod-like body contained in it. The capsule sends a process up to the epidermis.

Fig. 5. Schematic representation of the arrangement of the fibres of the external circular and decussating muscular layer of *Bipalium Diana* in vertical section, as seen in its extreme development in the dorsal region.

A. Decussating fibres.

B. Circular fibres.

C. Longitudinal muscular bands in section.

Fig. 6. Diagrammatic representation of the generative organs of *Bipalium Diana*, as seen when laid open by longitudinal section. The external generative orifice (EX.) leads into a ciliated cavity: into this cavity opens superiorly the common internal generative aperture (CG.). A straight ciliated tube, the vagina, passes vertically from this aperture, and turning sharply at right angles terminates at the entrance of the oviducts (OD.). The horizontal portion may be considered to be the uterus. Into the lower extremity of the vagina posteriorly opens the cavity containing the penis (C. P.), at an angle of about 45°. The dark lines radiating from the muscular body containing the uterus and vagina are accessory glands.

EX. External generative orifice.

CG. Common internal orifice.

V. Vagina.

UT. Uterus.

OD. Oviduct.

C. M. Circular muscular coat.

L. M. Longitudinal muscular coat.

C. P. Cavity containing penis.

P. Penis.
PR. Prostate.
V. D. Vas deferens.

Fig. 7. Diagrammatic section of the generative organs of *Rhynchodemus Thwaitesii*. The penis is seen contained in its spacious special cavity communicating by a narrow channel with a small cavity common to itself and the uterus, which cavity again communicates by a narrow opening with a third cavity, which opens into the external generative orifice. The pair of oviducts unite into a single tube, which passes up the back of the uterus to open near its summit. The muscular fibres and epithelium of the uterus are greatly thickened at its summit, forming a square-shaped projection into its cavity; this probably disappears when the uterus is distended either by ova or (to judge from the analogy of freshwater Planarians) by a capsule containing several ova, and is most likely due to the action of spirit.

EX. External generative orifice.
V. Vagina.
C. P. Canal for penis.
UT. Uterus.
D. E. Ductus ejaculatorius.
P. Penis.
A. R. Azygos receptaculum seminis.
V. D. Vas deferens.
OD. Oviduct.
T. Testis.

Fig. 8. Terminal segment of generative organs of *Bipalium Diana*, enlarged five diameters. Beneath are seen the organs intact, above the same opened by vertical section. The rounded mass on the left contains the vagina and uterus, the elongated oval body on the right the penis. The capsules of the two bodies are intimately united, and the cavity containing the penis communicates exteriorly with an aperture common to it and the vagina.

U. Uterus.
Pr. Prostate.
P. Penis.
ex. External generative aperture.
OD. Oviduct.
V. D. Vas deferens.

Fig. 9. Portion of a vertical section transverse to the long axis of the body of *Bipalium Diana*, passing through the external generative orifice. The cavity containing the penis is seen to open by a small aperture into the vagina. Above is the uterus, which turns off at right angles from the vagina.

EX. External generative orifice.

CG. Common internal generative opening.
V. Vagina.
C. P. Minute opening of the cavity containing the penis.
U. Uterus.
C. M. Its circular muscular coat.
L. M. Its longitudinal muscles.
A, A. Accessory glands.

PLATE XII.

Fig. 1. Diagrammatic representation of the positions and relative proportions of the organs of *Bipalium Diana* in the anterior portion of the body, as seen from beneath, enlarged two diameters. The digestive tube (D) is represented only in front of the pharynx, and its diverticula only at the most anterior portion of the body. In the peculiar cheese-knife-shaped extremity the digestive tube is seen to ramify in an arborescent manner. The ovaries (OV.) lie just behind the angle formed by the union of the broad head with the body, and their long ducts (OD.), attached to them on their exterior aspect, pass down the body, coming into close contact with the sheath of the pharynx, and crossing the vasa deferentia (V. D.), and making a sharp turn inwards, pass into the uterus (UT.). Between the angle formed by this sharp turn and the pharynx some short branches, which slope backwards and outwards, join the oviducts.

The testes (T, T) commence a short distance behind the ovaries, and are slightly external to them in position. There are about twenty-four pairs of them. They communicate with a pair of ducts (V. D.), which, skirting the sheath of the pharynx and here coming into close relation with the oviduct, pass to the inner side of the latter, and turning sharply backwards upon these lines, enter the sheath of the penis (S. P.).

D. Digestive tube.
M. Mouth.
O. Opening of pharynx into digestive tube.
OV. Ovaries.
OD. Oviduct.
T, T. Testes.
V. D. Vas deferens.
S. P. Sheath of penis.
UT. Uterus.
EX. External generative aperture.

Fig. 2. Posterior extremity of *Bipalium Diana* as seen when dissected under spirit, viewed from beneath, magnified two diameters. At the part of the body which lies in front of the oral apparatus the digestive tube (*d*) is seen to be single. At

the point (O) it divides into two, which pass one on each side of the sheath of the pharynx, and from this point backwards to the very extremity of the body the digestive tube is double. A stout median septum intervenes between the two tubes in the posterior part of the body, and being prolonged upwards splits, as it were, into two to form the investing sheath of the generative organs (G) and the pharynx (PH.). Just above the pharynx and below the generative organs, on the left-hand side of the drawing, are seen indicated the openings of the lateral diverticula into the main digestive tubes. The peculiarly elongated pharynx (PH.) communicates with the tube by the small aperture (O). The small oval mass consists of the penis and uterus with their immediate investments.

d. Digestive tube.
PH. Pharynx.
O. Opening of pharynx into digestive tube.
G. Small mass containing penis and uterus.

Fig. 3. Diagram representing the various organs, their relative positions and dimensions, of *Rhynchodemus Thwaitesii*, supposed to be seen from beneath, enlarged four diameters. Though the anterior extremity of the body is considerably broader than the posterior, the body is seen to run more abruptly to a point at the anterior than at the posterior extremity, a sort of shoulder being formed at the point where the sides of the body bend inwards to form its apex.

This difference of form between the two extremities is always marked in specimens preserved in spirits. The digestive tract in the anterior part of the body exists as a single median tube, with a series of lateral cæca or diverticula opening into it on either side; these are indicated by the faint transverse lines, as is also their arrangement at the anterior and posterior extremities of the body. The median tube divides into two, to pass on each side of the capsule of the tubular pharynx, and remains double henceforth, embracing the capsule of the generative organs in like manner. Anteriorly is seen the single pair of eyes. The penis is represented as opened by horizontal section. The oviducts are observed to pass from the inner sides of the ovaries.

E. Eyes.
OV. Ovaries.
D. Digestive tract.
T. Testis.
OD. Oviduct.
O. Opening of pharynx into digestive tract.
M. Indicates position of mouth.
PH. Pharynx.
A. R. Azygous seminal reservoir.

V. D. Contorted vas deferens.
P. Penis, with seminal canal displayed.
C. P. Cavity through which the penis is protruded.
EX. External generative aperture.
UT. Uterus.

Fig. 4. Blind extremity of the uterus of *Bipalium Diana*, and entrance into it of the oviducts, as seen in a vertical section transverse to the long axis of the body, and slightly anterior in position to the one from which fig. 6 is taken.

The internal muscular coat here does not preserve its circular disposition. The fibres composing it pass inwards with the oviducts, inferiorly mingling with some fibres accompanying these ducts. The oviducts pass inwards to meet one another in the middle line of the body; they, however, do not anastomose, but open into the cavity of the uterus by separate orifices (O), which are divided from one another by a slight ridge or projection. The blind termination of the uterine cavity (U) is seen here in the contracted state. The epithelial lining of the uterus is very thick, and probably glandular in function; it is covered with long cilia.

R. M. Radiating muscular fibres.
L. M. Longitudinal muscular fibres seen in section.
C. M. Circular muscular fibres.
OD. Oviduct.
O. Opening of oviduct into uterus.
U. Blind extremity of uterus.
E. Epithelium.

Fig. 5. Vertical section, transverse to the long axis of the body, of the penis and its immediate surroundings in its basal region, from *Bipalium Diana*. Drawn with the camera. The base of the penis is here seen in section; superiorly it is seen attached to the body by a mass of muscular fibres, which converge towards it and unite with its proper muscular system. This mass of muscular fibres is part of that which is seen passing into the penis in fig. 1, Plate XIII.: the fibres composing it radiate towards the penis in all directions, and are thus exposed in both longitudinal and vertical sections of the body; they become in great part longitudinal muscles of the penis (L.P.), which are especially developed on its dorsum, as seen in the section, but partly also join the radiating and circular muscular systems of that organ. On its inferior aspect the penis, where free in its cavity or sheath, is provided with delicate special longitudinal (L. M.) and circular (C. M.) muscular fibres. The penis, as here seen in section, is seen to consist of from without inwards, first, a layer of epithelium (*a*), which appears as if composed of small rounded transparent vesicles; then a thin layer of external circular muscular fibres (E.C.), which superiorly are lost amongst the vertical

fibres entering the organ; then longitudinal fibres only just visible inferiorly as small black dots (Ep.), but largely developed on the dorsum of the penis (L. P.); then a series of stout radiating fibres (R. M.), with what resembles water-vascular spaces (S) in their interstices, and which are sections of muscles similar to those (W) seen to pass into the penis superiorly between the masses of longitudinal muscles (L. P.). Succeeding these radiating fibres we find a stout and compact ring of internal circular muscular fibres (I. C.), succeeded by the prostatic glandular tissue (Pr.). Compare figs. 1 & 2, Plate XIII.

C. Sheath of penis.
E. Epithelium of cavity of sheath of penis.
C. M. Its circular muscular coat.
L. M. Its longitudinal muscles.
R. f. Radiating fibres attaching it to the surrounding tissue, *i. e.* here the septum between the two intestinal canals.
a. Epithelium of penis.
E. C. External circular muscular coat of that organ.
L. P. Longitudinal muscles of same in section.
R. M. Its radiating muscles.
S. Retractor muscles in section.
W. Retractor muscles in longitudinal section.
I. C. Internal circular muscular coat.
Pr. Prostate gland.

Fig. 6. Transverse section of the uterus of *Bipalium Diana*. The external longitudinal muscular coat is omitted. Externally is seen the circular muscular coat (C. M.), formed of densely interlaced fibres; then internally to this the stroma (S), composed of small spindle-cells and fibres, succeeded by the ciliated epithelium (E). The line of demarcation between the epithelium and the stroma is, however, often not well marked. The cavity of the uterus in its contracted state is cruciform in transverse section.

C. M. Circular muscles.
S. Stroma.
E. Epithelium.

Fig. 7. Portion of the longitudinal muscles of the vagina from the same preparation, highly magnified, to show how the accessory gland-tissue breaks up into fine twigs which ramify amongst the muscular fibres.

A. Accessory gland-tissue.
L. M. Longitudinal muscular fibres of the vagina.

Fig. 8. Section of the pharynx of *Rhynchodemus Thwaitesii*, taken in a direction transverse to its longer axis. Drawn with the camera lucida.

In the centre is the tubular cavity of the pharynx, clothed with a layer of epithelium, which appears as a narrow light zone. Immediately externally

to this zone is a broader zone darkly shaded, as being deeply stained with carmine, and consisting of dense circular muscular fibres. Externally to this the black dots represent a zone of longitudinal muscular fibres seen in section. Following on this is a broad zone occupied by loose radiating fibres, then two irregular darkish lines, corresponding with a large quantity of glandular matter present in this region, and some slight circular fibres, succeeded by a zone (A) occupied by fine dots, representing longitudinal muscular fibres in section, which fibres lie in meshes formed by radiating and circular fibres. The structure of the extreme verge will best be comprehended by reference to fig. 9, in which this is represented much enlarged.

A. Muscular zone referred to in the description of fig. 9.

Fig. 9. Portion of the periphery of the foregoing section, much enlarged. Drawn with the camera. To the right hand is seen the external epithelium of the pharynx. A light line (A) follows this to the left, representing an apparently structureless membrane, succeeded by a single row of very stout muscular fibres (L. m). The remainder of the drawing represents the broad muscular zone (A) of the last figure, which is here seen to consist of a meshwork of radiating and circular fibres, in the interstices of which are stout longitudinal fibres in section. The circular fibres are more densely aggregated at the outer margin of the zone, and form a sort of special zone (*i. c. m.*).

e. Epithelium.

A. Structureless layer.

L. m. Longitudinal muscular layer.

i. c. m. Circular muscular layer.

PLATE XIII.

Fig. 1. Penis of *Bipalium Diana* within its proper cavity or sheath, as exposed in a longitudinal section and in the plane of the body; drawn with the camera. The penis (*p*) is seen to be conical in form; it is bent upon itself. Towards its pointed extremity may be seen the termination of its central canal, lettered *d*. Strong muscular fibres are seen passing into the organ from above to form its longitudinal muscular system; between the interlacement of these fibres are spaces (*s*) which are lightly stained in carmine preparations, and are transverse sections of branched retractor muscles of the penis.

The external circular fibres of the penis are faintly indicated.

C. Cavity of sheath of penis.

p. Penis.

d. Spermatic duct.

s. Branched retractor muscles seen in transverse section.

u. The same muscles seen in their longitudinal section.

a. Vesicular epithelium clothing the base of the penis.
c. m. Circular muscles of the sheath of the penis.
e. Its irregular epithelial lining.

Fig. 2. Longitudinal section in the plane of the body, through the muscular bulb at the base of the penis. Drawn with the camera.

The rounded muscular mass here seen is that from which the penis takes its origin, and the fibres seen passing into the penis in fig. 1, Plate XIII., are portions of this muscular bulb. The fibres of the bulb are continuous externally with those of the sheath of the penis, internally with those of the penis itself. The oval fissure in the middle, which stands out in relief as very slightly tinged with carmine, is the prostate cavity nearly filled with glandular substance. The retractor muscles (*u*) are seen passing inwards, and appear to join the septa between the glandular crypts of the prostate, as in fig. 3.

c. Cavity of sheath of penis.
e. Epithelium of the sheath.
em. Its circular muscular coat.
m. Muscular mass of bulb of penis.
u. Retractor muscles.
I. C. Internal circular muscular coat.
Po. Glandular substance of prostate.

Fig. 3. Small portion of the innermost region of a transverse section of the base of the penis of *Bipalium Diana*, magnified. Drawn with the camera.

The retractor muscles (W, W) are seen passing inwards between the bundles of longitudinal muscular fibres (L.P., L.P., L.P.), traversing the dense internal circular muscular ring (I. C.), and forming a network in the space which intervenes between the internal circular muscles and the glandular prostatic tissue (Pr.). The actual termination of the retractor muscles was not determined, but they often have the appearance of becoming continuous with the glandular bodies (Pr.).

I. C. Internal muscular layer of penis.
Pr. Glandular prostatic tissue.
L. P. Longitudinal muscles of the penis.
W, W. Branched retractor muscles.
C. M. Scattered circular muscular fibres.

Fig. 4. Testis of *Bipalium Diana* in a plane parallel to the inferior surface of the body. A section of the organ is seen to be circular in outline. The organ is enclosed in a capsule consisting of an outer layer of loose tissue with large open spaces in it, and an inner layer of compact tissue. There is an opening in the capsule laterally, where the interior cavity of the gland becomes continuous with that of its ducts, and its epithelium is thickened opposite the point of entrance into it of the duct from each testis. The duct is narrower

immediately above the junction with the testis. The contents of the testis-cavity divide themselves into two regions—an outer, where the formation of the larger spermatic cells takes place, and an inner, where the cells ripen and the spermatozoa are formed. The tails of the spermatozoa are seen turned towards the outlet.

E. External layer of capsule.
I. Internal ditto.
D. Duct.
G. External region with smaller cells.
C. Internal with larger cells and spermatozoa in process of formation.
m. Longitudinal muscular fibres of the body.

Fig. 5. Testis of same animal, which has discharged its contents and collapsed, lowly magnified. An internal membrane has parted from the capsule, and is seen folded up in the anterior.

Fig. 6. Spermatozoa of *R. Thwaitesii* in process of development.

Fig. 7. Sections of vas deferens of *Bipalium Diana*, highly magnified.

Fig. 8. Longitudinal section in the plane of the body of the ovary of *Bipalium Diana*, from various chromic acid and alcohol preparations. The ovary, which is thus seen in section, has an oval outline, and is crowded with ova in various stages of development. The oviduct, with its funnel-shaped expansion and ciliated epithelium, is seen passing up on the right-hand side of the ovary in the drawing. The exact manner of its connexion with the ovary was not ascertained in this species, but is probably similar to that in *Rhynchodemus* (see fig. 13). In *Bipalium Diana* the duct was not traced further than is represented in the drawing It enters on the most external side of the ovary—that is, the side which is furthest from the median line of the body. Immediately exteriorly to the oviduct is seen the small yelk-gland attached to the ovary by a pedicle, which is probably its duct. This gland was present in the condition here represented only in one specimen examined; in the remainder it was quite rudimentary. In the ovary there is a special aggregation of immature ova at the summit. Some stroma-fibres with fusiform cells are seen to pass between the more mature ova. The organ has a compact inner tunic, which is succeeded externally by a loose fibrous investment. The whole lies imbedded between the internal longitudinal muscles, some fibres of which are seen on each side of it in the drawing.

a. Outer loose fibrous investment of ovary.
b. Inner denser ditto.
c. Immature ova.
d. Mature ova.
e. Stroma-cells.
f. Longitudinal muscular fibres.

g. Yelk-gland.
h. Mass of small cells.
od. Oviduct.

Fig. 9. Longitudinal section of oviduct of *Bipalium Diana* in the region of the external generative organs. A short lateral branch tube is seen to enter the duct at an inclination which slopes towards the posterior extremity of the body. The short branch is clothed with cilia, which are directed towards the opening into the oviduct. Attached to the extremity of the branch are stroma-fibres and fusiform cells, which appear to form branched spaces in the body-substance. The branch enters at the external side of the oviduct. The cilia of the oviduct are seen here, as elsewhere, to be direced towards the termination of the duct. The duct has a distinct basement membrane.

a. Anterior extremity of segment.
d. Stroma and fusiform cells.
e. Basement membrane of branch entering oviduct.

Fig. 10. Transverse section of oviduct of *B. Diana*, magnified about twice as much as in foregoing. The basement membrane is well seen. The cilia appear to have a spiral arrangement.

Fig. 11. Epithelial lining of ovary.

Fig. 12. Series of stages in the development of ova. The youngest ova cannot be distinguished from the lining cells of the ovary. The germinal vesicle enlarges rapidly to its full size; it then loses its finely granular appearance and becomes quite clear and transparent. The investing yelk-mass continues to increase in size after the germinal vesicle has ceased growing. The last stage in its development is the appearance of large oil-globules in its substance. The external capsule, which is seen to invest the more mature ova, is derived from the stroma of the ovary. A clear space is seen between the ovum and this capsule.

Fig. 13. Longitudinal section of ovary of *Rhynchodemus Thwaitesii* in the plane of the body. From several preparations. The ovary is seen to be circular in outline. The oviduct passes up on the right-hand side of the figure, which corresponds to the internal side of the ovary. Where the duct joins the ovary there is a mass of fusiform cells, which forms a papillary projection into the cavity of that organ. From this projection stroma-fibres radiate out between the ova. The ovary has two investments, as in *Bipalium Diana*. Immature ova are seen all round the margin of the cavity. The mature ova occupy a central position.

a. Outer loose fibrous investment of ovary.
b. Inner denser ditto.
c. Immature ova.
d. Mature ova.
e. Stroma.

h. Mass of cells and terminal papilla.

o. oviduct.

Fig. 14. Transverse section of oviduct of *R. Thwaitesii*.

Fig. 15. Longitudinal section in a plane parallel to the ambulacral surface of *Rhynchodemus Thwaitesii* in the region of the testes. The section passes through the main water-vascular space and the testes. These organs stand out in relief, being not so deeply stained as the surrounding tissue. The dark tissue on the right-hand side of the drawing consists of muscles; that on the left of glandular matter, which is present in great quantity in this situation, immediately externally to the testes, all through the anterior part of the body. The testes (T) are seen as irregularly oval bodies closely packed against one another. The water-vascular trunk, in which some very faint longitudinal fibres may be traced, gives off transverse branches, which are seen as white lines on a dark ground: these are very numerous on the right side amongst the muscular tissue; on the left side beyond the testes, amongst the glandular tissue, only three are to be seen.

Fig. 16. Edge of the anterior semilunar extremity of *Bipalium Diana*, as seen by reflected light, magnified. The edge of the anterior extremity or head is seen to be traversed vertically by deep indentations or sulci; these are caused by contraction, due to the action of spirit, but have been described as characteristic of certain species. This peculiar mode of contraction is probably, however, connected with the power which the animals possess during life of throwing out tentacular-like projections from this portion of their body. The edge of the head is seen to be marked out into several bands. The superior broad band (*a*) is somewhat darkly pigmented; it is succeeded inferiorly by a lighter unpigmented band marked by a fine line, *b*. At *d* is seen a row of semicylindrical papillary projections, between which, at their upper extremities, is seen a row of black dots (*c*), which indicate the apertures of a series of ciliated pits; the row of papillæ is succeeded by an unpigmented band.

f, *f*. Deep indentations of anterior margin.

a. Broad pigmented band containing eyes.

b. Fine line.

c. Row of openings of ciliated pits.

d. Row of papillary projections.

e. Unpigmented band.

Fig. 17. The papillæ of *Bipalium Diana*, shown in the preceding figure at *d*, as viewed from the side, and more highly magnified.

A. Position of the openings of the ciliated pits, lettered *c* in the preceding figure. Compare Plate XIV. fig. 8.

PLATE XIV.

Fig. 1. Portion of vertical section transverse to the long axis of the body of *Leptoplana tremellaris*. Drawn with the camera lucida. The section from which the drawing was made was cut nearly from the centre of the body of a specimen hardened in spirit. The portion here represented lies just to one side of the cavity containing the pharynx inferiorly. A portion of the water-vascular trunk (W) is seen in section. The external circular muscular coat (E. C. M.) is regarded by KEFERSTEIN (*l. c.* p. 17) merely as a basement membrane; it is, however, obviously homologous with the external circular muscular coat of *Bipalium* and *Rhynchodemus*.

E. C. M. External circular muscular coat.
E. L. M. External longitudinal muscular coat.
I. C. M. Internal circular muscles.
I. L. M. Internal longitudinal muscles.
V. M. Vertical muscles.

Fig. 2. Transverse section of main water-vascular trunk of *Bothriocephalus latus*, copied from 'Beiträge zur Anatomie der Plattwürmer,' SOMMER und LANDOIS (Leipzig, 1872), Taf. iv. fig. 1, *k*, querdurchschnittenes Seitengefäss, for comparison with fig. 1.

Fig. 3. Portion of a section taken in a direction transverse to the longer axis of the body of *Bipalium Diana* through the broadest part of the anterior semilunar extremity—that is, as it is from A to B in fig. 4. Since the digestive tract in this portion of the body spreads out in an arborescent manner, it is here seen cut through in a series of places (D, D, D). The spaces are arranged, as far as the drawing shows, symmetrically on each side of a central space (D′), which is the direct continuation of the main median digestive tube of the fore part of the body. The lateral offsets are cut more and more obliquely as they are more and more distant from D′, so that the outermost appears as an elongated space. Surrounding the median portion of the digestive tract is a large quantity of densely stained tissue, mainly glandular, which sends offsets downwards across the broad light space, which is the water-vascular space (W). The glandular tissue also spreads out laterally between the branches of the digestive tract and the water-vascular space. The water-vascular space stretches out to the margin of the head in the region of B, which points to two of the peculiar papilliform bodies which form a row along the whole anterior margin of the animal, and between which are the ciliated pits. The water-vascular space is traversed by a few stout vertical fibres and a number of fine horizontal fibres. A group of eyes is seen at E, on the margin of the animal's semilunar extremity. Superiorly a large number of eyes are seen lying just beneath the epidermis, and also some pigment spots, P. At A,

immediately above the median digestive space, is a region devoid of eyes and pigment, which corresponds to a light stripe in this situation in the living animal. The ambulacral line is not seen here, the section being taken in front of its termination. The muscular fibres are seen to decussate as they spread out from the median line towards the periphery, both above and below the region occupied by the digestive tract and water-vascular space.

A. Space in median line of body devoid of pigment and eyes.
B. Peculiar papillæ.
E. Groups of eyes.
P. Pigment.
W. Water-vascular space.

Fig. 4. Longitudinal section in the plane of the body of the anterior extremity of *Bipalium Ceres*, passing through the main water-vascular trunks. Drawn with the camera lucida.

The preparation from which the drawing is taken was made from a specimen hardened in alcohol, and the section was stained with carmine. The water-vascular spaces are very slightly stained, and thus stand out in relief. Between the two main water-vascular trunks (W, W) is seen a dark elongated mass, which terminates anteriorly in a pointed extremity. This mass is composed of muscles of the ambulacral line, and mainly of vertical fibres. The two water-vascular trunks pass up into the head, and there ramify and anastomose with one another in all directions, or rather form one large sinus divided up more or less into radiating channels by the vertical muscular fibres of the head, which appear in the drawing as dark dots.

A. Left lateral border of anterior extremity.
B. Right lateral border.
W, W. Main water-vascular trunks.
A. L. Muscles of ambulacral line.
E, E. Eye-spots.
D. Terminations of the diverticula of the digestive tract laid open, the specimen from which the section was taken having been slightly contorted by contraction in spirit.

Fig. 5. Longitudinal section in the plane of the body of *Rhynchodemus Thwaitesii*, passing through the main water-vascular trunks at their anterior extremity. The pair of water-vascular trunks (W, W) are separated by a dark mass composed of the muscles of the ambulacral line; they do not ramify at their extremities, but end bluntly. At the anterior extremity of the body are seen the eyes (E, E) *in situ*.

A. L. Ambulacral line.
W, W. Main water-vascular trunks.
E, E. Eyes.

Fig. 6. Small portion of the main water-vascular trunk of *Bipalium Diana*, as seen in vertical section. Drawn with the camera. Three muscular bands are seen traversing the portion of the water-vascular trunk here represented, and they are crossed diagonally by several finer muscular fibres. The interstices between these two sets of fibres are filled with a close network of very fine fibres, bearing here and there small nuclei. The darkly stained irregularly shaped masses (X, X) send out finely ramifying processes, which become lost amongst the fine fibres forming the general meshwork, and apparently anastomose with them.

V. M. Vertical muscular fibres.
D. M. Diagonal ditto.
X, X. Irregular protoplasmic bodies.

Fig. 7. Main trunk of the water-vascular system of *Dendrocœlum lacteum* in vertical section, with adjoining tissue, from a section corresponding to that represented in fig. 8, Plate X., but more highly magnified. Drawn with the camera.

The water-vascular trunk (W), which is here divided into two by a stout vertical muscular fibre, is seen to be filled with a network of extremely fine fibres, some of which have minute nuclei upon them. The open spaces in the meshwork are irregularly oval. The oviduct (OD.) immediately above the external portion of the water-vascular trunk is seen to be made up of definite nucleated cells, as in *Bipalium* and *Rhynchodemus*; it is imbedded in a mass of fusiform cells and stroma. The vertical muscular fibres (V. M.) pass downwards between the longitudinal muscles (L. M.), and unite with the circular muscles (C. M.).

V. M. Vertical muscular fibres.
OD. Oviduct.
W. Water-vascular system.
L. M. Longitudinal muscles.
C. M. Circular muscles.
E. Epithelium.

Fig. 8. The papillæ of *Bipalium Diana*, as seen in a thin section shaved off from the anterior margin of the animal's head, and viewed directly from the front, and magnified more highly still. Compare Plate XIII. fig. 17.

A. Openings of the ciliated pits between the upper extremities of the semicolumnar projections, of which four are here displayed.

PLATE XV.

Figs. 1, 2, 3, 4. Series of longitudinal sections in the plane of the body through the cephalic ganglionic masses of a Sea-Planarian, *Leptoplana tremellaris*, from a specimen preserved in spirit; drawn with the camera lucida. The series, as

numbered, commences from above. The distinctness and variety in size of the ganglionic cells is to be noted, also their disposition in a bilaterally symmetrical manner, and the complex arrangement of nerve-fibres passing between the several regions.

A pair of accumulations of especially large cells (A, A) is to be observed (posterior aspect of the ganglionic masses) in figs. 1 & 2. The nerve-cells are disposed mainly at the periphery of the ganglionic mass, the central region being occupied by commissural fibres; but a small aggregation of cells is seen amongst the commissural fibres at B. Well-defined transverse commissural fibres (C, C) connect the two halves of the ganglionic mass anteriorly and posteriorly. Connecting fibres run between the roots of the different nerves at the periphery of the central portion of the ganglionic mass. The central region is occupied by vertical fibres, here seen in section as dark specks: these vertical fibres are especially developed at D.

Fig. 5. Portion of the network of the nervous system from the head of *Bipalium Proserpina*, as seen in a longitudinal section in the plane of the body, from a specimen hardened in chromic acid. Drawn with the camera lucida.

The network here represented comes into view in sections prepared from specimens hardened in chromic acid when these sections are stained with carmine, and conforms in position and ramification with the water-vascular system. Masses of finely granular matter are seen imbedded amongst the fibres at the nodes of the network.

Fig. 6. One of the eye-spots of *Bipalium Diana*. The eye consists of a hollow cell of the shape here represented, the wall of which is pigmented posteriorly, but transparent anteriorly. There is a sharp line of demarcation between the transparent anterior and pigmented posterior portion. The pigment is present in the form of small rounded granules. An unpigmented spot appears to indicate a nucleus.

Fig. 7 shows a section of the eye. The cell-cavity is seen to contain a lens-like body, here dark as stained with carmine. A highly refracting substance exists between this lens-like body and the pigmented wall of the cell.

Fig. 8. Eye of *Rhynchodemus Thwaitesii*. The anterior transparent cornea projects through the epithelial layer of the animal's body. A short filament is seen attached to the proximal extremity of the eye.

Fig. 9. Small portion of the margin of a longitudinal section in the plane of the body of *Bipalium Diana*, from the region of the body just posterior to the semilunar anterior expansion. Drawn with the camera lucida. Superiorly is seen the epithelial layer (E), beneath which is the basement membrane (B) and the external circular muscular fibres (E. C. M.), here seen in transverse section; internally to these is the external longitudinal muscular layer (E. L. M.), here rather thin, and beneath these are the eyes (O).

This longitudinal section allows the basement membrane to be clearly distinguished from the external circular muscular layer. In transverse sections they nearly always appear fused.

Fig. 10. Ideal section of the eye of *Leptoplana tremellaris*.

C. Cornea.

L. Lens made up of rod-like bodies with rounded nucleus.

Ch. Choroid.

R. Retina.

V. Highly refracting transparent substance.

Fig. 11. The same preparation as the one figured Plate XIV. fig. 8, seen when focused, so as to render visible a deeper stratum. The mouths of the ciliated pits (A) are seen to be oval in form, and to have a very distinct lining, which is dotted with cilia.

Fig. 12. Papillæ from the anterior margin of the head of *Bipalium Proserpina*, from a section similar to that from which the preceding figure was drawn. The papillæ are of much the same form as those of *Bipalium Diana*, but the mouths of the ciliated pits (A) are of a more rounded form.

Fig. 13. Longitudinal section in the plane of the body of *Bipalium Diana* through the ciliated pits. Drawn with the camera. Slightly stained spaces appear to run to the pits; between the light spaces is a peculiar tissue formed of small spindle-shaped cells.

Fig. 14. Section of the glandular lining of the digestive tract of *Planaria torva*, from a transverse section of the animal's body. Drawn with the camera lucida. Made from an alcoholic preparation.

Fig. 15. Section of the glandular lining of the digestive tract of *Bipalium Diana*, from a transverse section of a specimen hardened in chromic acid. Drawn with the camera lucida. From the region of the mouth.

VI. *On a newly discovered Extinct Ungulate Mammal from Patagonia*, Homalodontotherium Cunninghami. *By* William Henry Flower, *F.R.S.*

Received May 30,—Read June 19, 1873.

The tertiary deposits of the east coast of Patagonia, which yielded to the researches of Mr. Darwin and Admiral Sulivan such interesting and aberrant mammals as *Macrauchenia*, *Nesodon*, and *Toxodon*, have again disclosed a new and remarkable form of extinct animal life. The evidence upon which the existence of this new genus rests consists of a nearly complete set of teeth and some fragments of bone, discovered on the bank of the River Gallegos, by Dr. Robert O. Cunningham*, Naturalist to H.M.S. 'Nassau,' during the voyage undertaken for the purpose of surveying in the Strait of Magellan and the west coast of Patagonia in the years 1866, 1867, 1868, and 1869. The spot was visited in conformity with instructions received before leaving England, "to institute a search for a deposit of fossil bones discovered by Admiral Sulivan and the present Hydrographer of the Navy, Rear-Admiral G. H. Richards, about twenty years previously, and which Mr. Darwin, Professor Huxley, and other distinguished naturalists were anxious should be carefully examined"†.

The conditions under which the specimens were found will be best understood from the following additional extract from Dr. Cunningham's narrative. "Accordingly, joined by the steamer, which again took us in tow, we proceeded onwards till we arrived opposite the first deposit of fallen blocks at the foot of the cliffs. The cutter was then anchored in the stream, while we pulled in towards the shore in the galley till she grounded, when we landed, armed with picks and geological hammers for our work. After examining the first accumulation of blocks, and finding in the soft yellow sandstone of which certain of them were composed some small fragments of bone, we proceeded to walk along the beach, carefully examining the surface of the cliffs and the piles of fragments which occurred here and there at their base. The height of the cliffs varied considerably, and the highest portions, averaging about 200 feet, extended for a distance of about ten miles, and were evidently undergoing a rapid process of disintegration, a perpetual shower of small pieces descending in many places, and numerous large masses being in process of detaching themselves from the parent bed. They were principally composed of strata of hard clay (sometimes almost homogeneous in its texture, and at others containing numerous rounded boulders); soft yellow sandstone; sandstone abounding in hard concretions; and, lastly, a kind of conglomerate, resembling

* Now Professor of Natural History in Queen's College, Belfast.

† Notes on the Natural History of the Strait of Magellan and West Coast of Patagonia, 8vo, 1871, p. 279.

solidified, rather fine gravel. The lowermost strata, as a rule, were formed of the sandstone with concretions; the middle, of the soft yellow sandstone, which alone appeared to contain organic remains; and the upper, of the gravelly conglomerate and hard clay. Nearly the whole of the lower portion of the cliffs, as well as all the principal deposits of fallen blocks, were examined by us in the course of the walk, and we met with numerous small fragments of bone; but very few specimens of any size or value occurred, and the generality of these were in such a state of decay as to crumble to pieces when we attempted, although with the utmost amount of care that we could bestow, to remove them from the surrounding mass. To add to this, the matrix in which they were imbedded was so exceedingly soft as not to permit of being split in any given direction. The first fossil of any size observed by us was a long bone, partially protruding from a mass, and dissolved into fragments in the course of my attempts to remove it. At some distance from this a portion of what appeared to be the scapula of a small quadruped, with some vertebræ, occurred; and further on one of the party (Mr. Vereker) directed my attention to a black piece of bone projecting from one side of a large block near its centre. This, which was carefully removed at the expense of a large amount of labour, with a considerable amount of the matrix surrounding it, by three of the officers, to whose zeal in rendering me most valuable assistance in my work I shall ever feel deeply indebted, afterwards proved to be a most valuable specimen; for on carefully removing more of the matrix when we returned to the ship, I found that it was the cranium of a quadruped of considerable size, with the dentition of both upper and lower jaws nearly complete. As no other specimens of importance were discovered, we reembarked towards the close of the afternoon."

The grey, soft, arenaceous matrix in which the specimens are embedded is very similar to that surrounding the remains of *Nesodon*, found in the same locality by Admirals Sulivan and Richards about twenty years previously, but is less indurated and of a paler colour. The exact geological age in both cases appears to be a matter of uncertainty; but they are probably of earlier date than the superficial deposits in which *Macrauchenia* and *Toxodon* have been found.

The specimens were placed by Dr. Cunningham for identification and description into the hands of Professor Huxley, who, having unfortunately been preoccupied by other engagements, has kindly deputed to me the duty of presenting an account of them to the Society. A brief mention was made of their existence in the Professor's Presidential Address to the Geological Society for 1870*, where they are alluded to under the very appropriate generic designation of *Homalodotherium*, which, with a slight modification, I gladly adopt.

However perfect the skull of the animal might have appeared when first discovered on the banks of the Gallegos, nothing of it remained when the specimens came into my hands, except fragments of alveoli around the roots of the teeth and a considerable portion of the rami of the lower jaw. The other specimens of bone received were in such a

* Quarterly Journal of the Geological Society, vol. xxvi. p. lvii.

very fragmentary condition, owing to the friable nature both of the bone itself and the matrix, that they afford no satisfactory evidence of the structure or affinities of the animal; on the other hand, the teeth are in a remarkably good state of preservation, and consist of the almost entire permanent series of an individual just arrived at maturity, and therefore in the best condition for affording such information as to the general characters of the animal as may be obtained from the dental structures of a single specimen. The teeth that are missing are certain of the inferior incisors and the posterior upper true molars. As nearly all the bone of the cranium had perished, and it has been necessary to adjust and cement the teeth artificially in place*, the exact number that were present cannot be ascertained with absolute certainty; but I have scarcely any doubt but that the animal possessed the complete typical number, forty-four, and that therefore four incisors are wanting from the lower jaw. In the upper series there were certainly eleven on each side. As in the specimens of *Nesodon* from the same locality, they have acquired a very dark, almost black colour.

All the teeth have crowns distinctly separated from the long and tapering roots, and with a well-marked cingulum around their base; but as compared with most Ungulates having distinct crowns to the teeth, they are decidedly "hypsodont," or long-crowned, contrasting especially in this respect with the early "brachyodont" forms, as *Palæotherium*, *Anoplotherium*, *Dichodon*, *Hyopotamus*, &c.

They are arranged in both jaws in a perfectly unbroken series, being, in fact, in some places (especially at the antero-lateral region of the mouth, where gaps are so frequent in recent Ungulata) so crowded as to be pressed out of the straight line and to overlap one another, as in the lower jaw of *Nesodon imbricatus*. The adaptation of the form of the teeth on both sides to this position, and the accurate adjustment of their contiguous surfaces, shows that it is a natural conformation. They are, moreover, of very nearly even height throughout the series, and in their configuration present a remarkable and gradual transition from the first incisor to the last molar, easily traced in both jaws, and more even and regular than in any other known heterodont mammal. Indeed it is only by the analogy of other forms that they can be separated into the groups, convenient for descriptive purposes, designated as incisors, canines, premolars, and molars. They are therefore most instructive in throwing light upon the homology of the component parts of the different teeth throughout the series.

The enamelled surface, especially of the molars, is not smooth and polished, but covered with fine intersecting reticulations, surrounding shallow pits or depressions. There is no distinct layer of coronal cement.

As the teeth are drawn in the accompanying figures of the exact natural size, I have not thought it necessary to give any detailed measurements of them.

Description of the upper teeth (Plate XVI. figs. 1 & 4).—The incisors increase slightly in size from the first to the third. Each has a single, long, tapering and slightly curved root, and at the base of the crown a well-marked cingulum, developing both on the outer and

* This was done by Mr. E. T. Newton, Assistant to Professor Huxley, at the Royal School of Mines.

the inner surface into a thick crescentic ridge, slightly notched on the free projecting border. The form of the crown of the first tooth (*i* 1) cannot be accurately described, as it is altogether lost in the left side, and its anterior* half has been broken off on the right. In the second tooth (*i* 2) the outer surface of the crown presents an irregular shape, as it consists of a vertical median convex ridge and a pair of lateral lamellar expansions of unequal extent, the posterior border of the tooth being much shorter than the anterior. The distal free margin of the tooth is compressed and the apex rounded. The base of the crown is nearly as thick from without inwards as from side to side. The inner surface is hollowed near the cutting-margin, and has a considerable rounded tubercle at the base, so that the crown of the tooth may be described as consisting of an outer, trenchant, or principal (*a e c*), and an inner, blunt, or accessory cusp (*a i c*), separated by a groove.

The third incisor (*i* 3) resembles the second in form, but the lateral lamellar expansions of the crown are somewhat more developed.

The canine tooth (*c*) only differs from the posterior incisor in being somewhat larger, and in trifling details of configuration. The apex is rather more pointed and conical, being supported by a median vertical ridge, not only on the outer, but also on the concave inner surface of the crown: the inner tubercle is relatively smaller to the principal cusp; the cingulum is much notched; the postero-internal margin of the crown is flattened, as if by the pressure of the succeeding tooth, a character also seen, though to a less degree, in the incisors.

The first premolar (*p* 1) has only one root on the outer side of the tooth, and apparently a second one on the inner side. It is displaced somewhat within the line of the teeth before and behind it. The crown is shorter, broader, and less pointed than that of the canine, but its inner cusp or lobe is considerably more developed.

The second premolar (*p* 2) assumes more the form of a true molar. It has two outer roots (anterior and posterior), and apparently a distinct root on the inner side. The external wall of the crown is oblong, nearly twice as high as it is broad from before backward, with a strongly marked crescentic cingulum, delicately ridged and tuberculated. Though the surface is in a general sense rounded or convex from before backwards, indications are seen upon it of two vertical ridges, with a concavity between them; the anterior ridge is the most conspicuous, and evidently corresponds with the single ridge developed in the preceding teeth. As in the incisors, canine, and first premolar, there is an inner lobe, but it is developed to a greater degree, and its sloping free surface shows two strong ridges or columns, which converge as they approach the grinding-surface of the tooth. These columns are separated from each other by a triangular depression, the base of which is crossed by the cingulum. The anterior and posterior surfaces of the crown are flattened, and the cingulum is continued all round

* To avoid confusion, in describing the incisors I apply the terms anterior and posterior to the parts corresponding to those occupying these positions in the molar series. "Outer" and "inner" always mean the labial and the lingual surfaces of the teeth respectively.

them, though developed to a much less extent than on the outer and inner faces of the crown. The grinding-surface of the tooth is a rather irregular four-sided area, broader externally than within; it is composed of a smoothly worn surface of dentine with a thin enamel margin, and is deeply excavated longitudinally, the outer and inner margins standing out prominently, especially the former. Near the middle of this area, but rather towards the anterior and inner angle, is a very deep oval fossa, formed by an inflection of the enamel-covered outer surface of the tooth, extending in depth almost to the base of the crown, placed obliquely, the long axis of the oval directed from before backwards and inwards. The enamel of the outer margin of this fossa is plicated.

The third and fourth premolars (*p* 3 and *p* 4) are formed on exactly the same principle as the one just described. They present respectively a slight increase in size, and the inner lobe of the crown becomes gradually rather broader, and its two pillars rather more widely separated from each other. The fourth, in addition to the principal oval fossa, has a second smaller one behind and to the outer side of it. All these teeth are wider from within outwards than from before backwards.

In the first true molar (*m* 1) a considerable increase of the size of the crown takes place, especially in the antero-posterior extent. It is, however, formed upon precisely the same pattern as the hinder premolars, but merely expanded in the direction just indicated.

Unfortunately only one of the upper true molars (the first of the left side) is preserved in a complete state, and that, as might be expected, is considerably worn. There is also a broken first molar of the right side, and fragments of the second and third of the left. The most perfect tooth has a subquadrate crown, with a broad, flattened, or slightly convex outer wall (*l*), presenting several shallow vertical elevations and depressions. The most marked ridge (*a e c*) is very near the anterior edge of the tooth, and corresponds with the anterior external ridge of the premolars, and with the principal cusp of the canine and incisor teeth. The second and more posterior elevation (*p e c*) is broader and far less salient. The inner wall has its two columns or ridges (*a i c* and *p i c*) as in the premolars, and still somewhat converging as they approach the grinding-surface; but they are wider apart and have a broad depression between them, which in the fragment of a nearly unworn more posterior tooth is seen to communicate with the central fossa. This fossa in the first molar is reduced to a very narrow but deep chink (*m s*), and there is no trace of the small second or posterior fossa of the last premolar. The outer wall of the crown is connected with the internal pillars by anterior and posterior transverse ridges (*a r* and *p r*), which pass one before and the other behind the median fossa or sinus.

The lower teeth (figs. 2 & 3).—Of the lower incisors but one pair are present; and as the symphysial portion of the jaw has completely perished, it is impossible to say what may have been the original number; but analogy would lead to the inference that there were three, as in the upper series. The crown (*i* 3) is of an elongated oval form, with sharp cutting-edges, a strongly developed cingulum, and on the inner surface, instende of a tubercle separated by a groove from the outer cusp, there is a vertical ridge

running from base to apex of the crown, rather nearer the anterior than the posterior edge of the tooth. The canine (*c*) is of similar form to the incisor, but of larger size. The first premolar (*p* 1) is also formed on the same plan, *i. e.* with a smooth and rounded external surface, and the internal surface with an anterior and posterior depression, separated by a vertical ridge, and bounded below by the cingulum. The second premolar (*p* 2) is wider from before backwards, the hinder part being more developed; its external surface has a deep groove passing from near the posterior part of the base upwards and forwards, with a gentle sigmoid curve to the apex, dividing the surface into an anterior and posterior area, of which the anterior is the larger. An indication of this groove exists in the first premolar of the right side. The inner surface has the vertical ridge increased in thickness, and a smaller ridge behind which isolates the posterior concavity. The cingulum is well developed and notched; on the outer side it sends up a row of small prominences from its border.

The transition from the second premolar (*p* 2) to the first true molar (*m* 1), through the third and fourth (*p* 3 and *p* 4) premolars, is very gradual, being effected, as in the upper teeth, chiefly by the lengthening out of the posterior part of the crown. The first and second true molars are almost exactly alike, but the second is slightly the larger; the difference in the appearance of their grinding-surface is simply the result of difference of wear. They are elongated from before backwards and much compressed. Their base is surrounded by a well-marked cingulum. The outer surface is, generally speaking, flat, but divided by a shallow vertical groove into an anterior and posterior area, each convex from before backwards, the anterior being about half the width of the posterior. The inner side is divided by a much deeper vertical groove (*m s*), running obliquely forwards as it penetrates the tooth so as nearly to meet the base of the external groove, and cutting the tooth into two lobes, each of which is further divided on the inner side by a much less marked depression (*a s* and *p s*). In other words, the inner side of the tooth may be described as consisting of two columns (*a i c* and *p i c*), each of which doubtless terminated in conical cusps in the unworn tooth, separated by a deep oblique indent (*m s*), and bounded before and behind by shallower indents (*a s* and *p s*), marking off anterior and posterior accessory columns (*a t* and *p t*) at each extremity of the tooth.

The posterior tooth is unfortunately injured; but the whole of the outer wall remains intact, with enough of the inner side to show that it is formed on precisely the same plan as the others; but its hinder part is slightly more elongated and compressed, though without any additional lobe as in many Ungulates.

Comparison of the teeth and taxonomic inferences.—There can be no doubt that the dental characters of *Homalodontotherium* warrant our placing it within the order Ungulata. Whether Perissodactyle or Artiodactyle may be at first sight less obvious. The evenness of height and unbroken continuity of the teeth is no test, as it is shared by the Artiodactyle *Anoplotherium* with the Perissodactyle *Macrauchenia*. A better criterion is the character of the premolars as compared with the molars. In every known Artio-

dactyle all the premolars, even the last, are structurely reduced, as compared with the true molars, so that a more or less obvious break in the continuity of the appearance of the teeth is seen between the last premolar and the first true molar. On the other hand, in many Perissodactyles, including all the existing representatives of the group, several of the posterior premolars are very close repetitions of the true molars in structure and even size; though it must be noted that this was not the case with the earliest known forms of the group, the Coryphodons, Lophiodons, and Hyracotheriums, and to a less extent the Palæotheriums, which so far approximate to the Artiodactyles, or perhaps rather to a more generalized Ungulate type, of which no representatives have as yet been discovered.

The similarity of the structure of the premolars and true molars of *Homalodontotherium* removes it from the vicinity of all known Artiodactyles. The special characters of the crowns of the molars of both jaws are yet more decisive; they fall into neither the Bunodont nor the Selenodont division of that group, and, except remotely to the somewhat aberrant *Anoplotherium*, present no resemblance to any known type of Artiodactyle.

On the other hand, as will be shown, they approach the Perissodactyle genus *Rhinoceros* more closely than to any other known mammal. In order to understand the nature and taxonomic value of this resemblance, a few preliminary remarks may be necessary.

As regards the upper molars, the essential character of the crown of Perissodactyle Ungulates, as is perhaps best exemplified in *Lophiodon*, is the presence of four principal cusps, arranged in pairs anterior (see fig. 4, *a i c*, *a e c*) and posterior (*p i c* and *p e c*), connected more or less by transverse ridges, also anterior and posterior (*a r* and *p r*). The outer cusps are joined together by an antero-posterior ridge, constituting the external wall or lamina (*l*), but there is no corresponding inner wall connecting the inner cusps. Between the two transverse ridges is a median sinus (*m s*), bounded externally by the outer wall, but open on the inner side. Behind the posterior transverse ridge is a smaller posterior sinus (*p s*), opening on the posterior surface of the tooth, or only enclosed by the cingulum, and in front of the anterior transverse ridge is a similar but less important anterior sinus (*a s*).

Furthermore, the transverse ridges are usually placed obliquely, their outer ends inclining forwards and running quite to the front edge of their respective external cusps, so that their posterior surface is more or less concave.

From this type there are three important deviations:—I. That in which the outer wall is undeveloped, and the transverse ridges become the prominent features of the crown of the tooth, as in the Tapirs. II. That in which the free edge of the outer wall acquires a strongly zigzag or bicrescentic character, being deviated inwards opposite each of the principal outer cusps, and outwards at the anterior and posterior angles of the tooth and in the middle between the cusps, as in *Palæotherium*, the Horses, and apparently (judging from Bravard's figure of the worn molars*) in *Macrauchenia*.

* Published in Burmeister's 'Anales del Museo Publico de Buenos Aires,' vol. i. pl. 1.

III. That in which the outer wall is greatly developed, and in the main flat or smoothly convex, though with slight elevations and depressions, corresponding with those so regular and well marked in the last section. To this group *Rhinoceros* (including its various modifications) and *Homalodontotherium* belong. If a single upper molar of the last-mentioned form had only been discovered, it might almost have been referred to the genus *Rhinoceros*, using the term in a wide sense. It would, however, have been found to differ from the most typical forms of that type in the more smooth and regular convexity of the outer wall, the fuller development of the cingulum, the more complete union of the two inner columns, which intercepts the floor of the inlet to the median sinus, the less oblique direction of the posterior transverse ridge and consequent smaller size of the posterior sinus, and the absence of the ridge projecting into the median sinus called "combing-plate."

The crowns of the lower molars of Perissodactyle Ungulates may, as is well known, be also derived from the same type as the upper teeth, *i. e.* two principal transverse ridges connecting two pairs of cusps; but there are only two fundamental modifications. I. That in which this primitive form is retained, the ridges remaining transverse and unconnected with each other, as in the Lophiodons and Tapirs. II. That in which the ridges assume a crescentic form, their outer extremities curving forwards, so that the hinder ridge abuts against the external surface of the ridge in front of it. This is the case in all the remaining animals of the group. An unworn lower molar of a Rhinoceros has thus externally two convex areas separated by a vertical groove, and internally two principal sinuses (see fig. 3, *a s* and *m s*) corresponding to the projections externally. The entrances to these sinuses are bordered by three conical pillars—the first (*a t*) of comparatively little importance, representing the anterior talon of the Tapir's tooth, the second (*a i c*) the largest, representing the antero-internal principal cusp, and the third (*p i c*) the postero-internal principal cusp. It is the large size and complex character of the last two, in addition to the excessive vertical lengthening of the crown, which distinguishes the Horse's lower molar from that of the *Palæotherium* and *Rhinoceros*.

On comparing a lower molar of *Homalodontotherium* with an equally worn tooth of *Rhinoceros*, it will be seen that they are formed on precisely the same type. The only important differences are that the outer surface of the former is rather more flattened, the posterior convex area is relatively more elongated, being produced backwards into a sort of heel (*p t*) separated by a groove (*p s*) on the inner side from the postero-internal column (*p i c*), and, as in the upper teeth, the cingulum is more completely developed. It differs from the regularly bicrescentic tooth of *Palæotherium* still more than from that of *Rhinoceros* or *Macrauchenia*; while in the lateral compression and flattening, and the complexity of the posterior column, it shows a slight approximation towards *Equus*.

The molar and premolar teeth of both upper and lower jaws thus without question show strongly marked Rhinocerotic characters; but on passing to the examination of the canines and incisors the resemblance completely fails, at least to the true Rhino-

ceroses, as all the latter have these teeth either quite rudimentary and deciduous, or, when functionally developed, greatly reduced in number and separated from the molars by a wide diastema. There is, however, an American genus from the Lower Miocene of Dakota, to which Leidy has given the name of *Hyracodon*, which, as proved by the sockets in the alveolar border, possessed the full complement of incisors and canines as in *Homalodontotherium**. Unfortunately the characters of these teeth are at present imperfectly known; but they appear to have been more differentiated than those of *Homalodontotherium*, and in fact to occupy an intermediate position between that genus and true *Rhinoceros*; so that, judging by the teeth alone, we may place *Homalodontotherium*, *Hyracodon*, and *Rhinoceros* as three terms of one series of modifications; and it is quite possible that as *Hyracodon* is of greater geological antiquity than *Rhinoceros*, so *Homalodontotherium* may be a still more primæval type†.

The discovery of this new form throws some light upon the affinities of the very enigmatical *Nesodon* and *Toxodon*. If, as observed by the first describer of those genera, "the interval between *Toxodon* and *Macrauchenia* is evidently partly filled by *Nesodon*"‡, *Homalodontotherium* is another link in the same chain connecting *Nesodon* with the true Perissodactyles. The modifications required to convert the molar teeth of *Homalodontotherium* into those of *Nesodon*, especially the lengthening of the crowns, if carried to a further degree would result in the rootless persistent-pulped teeth of *Toxodon*; and in the characters of the incisors and the canines *Nesodon* is obviously intermediate between the other two genera.

We have thus a new form of mammal, which, as far as the evidence of dental characters (by no means sufficient for deciding affinity, as the case of *Hyrax* shows) allows us to judge, is an extremely generalized type, related on the one hand to *Rhinoceros* through *Hyracodon*, also, though more remotely, to *Macrauchenia*, and apparently connecting these true perissodactyle forms with the more aberrant *Nesodon* and *Toxodon*.

To the fragments of bone sent with the teeth comparatively little importance can be attached, as it is quite uncertain whether they belonged to *Homalodontotherium* or to some other animal. The most characteristic among them are:—

1. The greater part of the right innominate bone of an animal about the size of the common American Tapir, and more resembling that species than any other with which it is comparable, though differing notably in many details, especially the form of the acetabulum.

2. A mutilated dorsal vertebra, which from its size and age (the epiphyses of the centrum being detached, as is that of the crest of the ilium in the last-mentioned specimen) probably belonged to the same individual. It is, however, far more equine or

* Extinct Mammalia of Dakota and Nebraska (1869), p. 232. Ancient Fauna of Nebraska (1853), p. 81, pls. xiv., xv.

† A more detailed comparison with the teeth of *Macrauchenia* would be desirable, but unfortunately the materials are not as yet forthcoming; judging from an unpublished drawing kindly lent me by Professor Gervais, the incisors and premolars of that genus have quite a different shape.

‡ Owen, British-Association Reports (1846), vol. xvi. p. 66.

rhinocerotic than tapiroid in character. These evidently belong to some hitherto unknown form; if *Homalodontotherium*, they are of smaller dimensions than would be inferred from the teeth, unless the animal had a disproportionately large and heavy head.

3. A fragment, apparently the inner half of the lower articular extremity of a tibia of an animal of much larger proportions, nearly equalling the Indian Rhinoceros. If I am right in its determination it is rather anomalous in its characters, somewhat resembling in general shape the corresponding part of the Tapir, but having a deep roughened depression notching the inner margin of the smooth articular surface, similar to that seen in the Hippopotamus alone among recent Ungulates.

4. A fragment of the shaft of a long bone, 10 inches in length, apparently the hinder border of an ulna of an animal as large as that to which the last-mentioned piece belonged.

These specimens will be deposited with the teeth in the British Museum, and will become of greater interest when compared with others that we may hope to obtain by future explorations in the same locality. It is possible that they may belong to one or other of the species of *Nesodon*, of which one, *N. magnus*, only known from a fragment of a molar tooth, equals the largest Rhinoceros in size*.

Description of the Plate.

PLATE XVI.

Fig. 1. Side view of the upper teeth of *Homalodontotherium Cunninghami*.
Fig. 2. Side view of the lower teeth.
Fig. 3. Grinding-surface of the left lower teeth.
Fig. 4. Grinding-surface of the left upper teeth.

All of the natural size.

a e c. Anterior external cusp.
a i c. Anterior internal cusp of the molar teeth; the corresponding parts of the incisors are marked with the same letters.
p e c. Posterior external cusp.
p i c. Posterior internal cusp.
a r. Anterior transverse ridge.
p r. Posterior transverse ridge.
a s. Anterior sinus.
m s. Median sinus.
p s. Posterior sinus.
a t. Anterior talon or accessory cusp.
p t. Posterior talon or accessory cusp.
l. External wall or lamina.

* See "Description of some species of the extinct genus *Nesodon* &c.," by Professor Owen, F.R.S., Phil. Trans. vol. cxliii. (1853) p. 291.

VII. *On the Atmosphere as a Vehicle of Sound.*

By JOHN TYNDALL, *D.C.L., LL.D., F.R.S.*

Received February 5,—Read February 12, 1874*.

§ 1. *Introduction.*

THE cloud produced by the puff of a locomotive can quench the rays of the noonday sun; it is not therefore surprising that in dense fogs our most powerful coast-lights, including even the electric light, should become useless to the mariner.

Disastrous shipwrecks are the consequence. During the last ten years no less than two hundred and seventy-three vessels have been reported as totally lost on our own coasts in fog or thick weather. The loss, I believe, has been far greater on the American seaboard, where trade is more eager and fogs more frequent than they are here. No wonder, then, that earnest efforts should have been made to find a substitute for light in sound-signals, powerful enough to give warning and guidance to mariners while still at a safe distance from the shore.

Such signals have been established to some extent upon our own coasts, and to a still greater extent along the coasts of Canada and the United States. But the evidence as to their value and performance is of the most conflicting character, and no investigation sufficiently thorough to clear up the uncertainty has hitherto been made. In fact, while the *velocity* of sound has formed the subject of refined and repeated experiment by the ablest philosophers, since the publication of Dr. DERHAM'S celebrated paper in the Philosophical Transactions for 1708, no systematic inquiry has, to my knowledge, been made into the causes which affect the *intensity* of sound in the atmosphere.

As an attempt to fill the blank here indicated, I beg to submit to the Royal Society some account of an investigation on fog-signals recently carried out at the instance of, and in conjunction with, the Elder Brethren of the Trinity House. Soon after my return from America I was requested, as the scientific adviser to the Corporation, to undertake the direction of this inquiry. I entered upon it inspired by duty rather than hope, for I feared that the observations would be tedious and the scientific results uncertain. But the study of any natural problem, if only steadfastly pursued, is sure in the end to reward the inquirer. And so in the present instance, after some preliminary groping,

* These dates apply to the chief points of the paper in its complete state, including the experiments on mixtures of gases and vapours and on differently heated air. A copious preliminary account, embracing the observations made at sea and the conclusions founded thereon, was read on January 15 (Proceedings, vol. xxii. p. 58). The experiments on the effect of fumes, &c. (§ 13) were described before the Society on May 21. In rearranging the paper with the view of throwing certain details into an appendix, these last experiments have been embodied in the original paper.

light began to dawn upon the subject, revealing many old errors and some novel truths. As the results had a scientific as well as a practical bearing, I requested permission to lay them before the Royal Society, the prompt and cordial consent of the Elder Brethren being the response.

§ 2. *Condition of the Question.*

A few extracts and references will suffice to show the state of the question when this inquiry began. "Derham," says Sir John Herschel, "found that fogs and falling rain, but more especially snow, tend powerfully to obstruct the propagation of sound, and that the same effect was produced by a coating of fresh-fallen snow on the ground, though when glazed and hardened at the surface by freezing it had no such influence"*.

In a letter addressed to the President of the Board of Trade in 1863†, Dr. Robinson, of Armagh, thus summarizes our knowledge of fog-signals:—"Nearly all that is known about fog-signals is to be found in the Report on Lights and Beacons; and of it much is little better than conjecture. Its substance is as follows:—

"Light is scarcely available for this purpose. Blue lights are used in the Hooghly; but it is not stated at what distance they are visible in fog: their glare may be seen further than their flame‡. It might, however, be desirable to ascertain how far the electric light or its flash can be traced§.

"Sound is the only known means really effective; but about it testimonies are conflicting, and there is scarcely one fact relating to its use as a signal which can be considered as established. Even the most important of all, the distance at which it ceases to be heard, is undecided.

"Up to the present time all signal-sounds have been made in air, though this medium has grave disadvantages: its own currents interfere with the sound-waves, so that a gun or bell which is heard several miles *down* the wind is inaudible more than a few furlongs *up* it. A still greater evil is that it is least effective when most needed; for fog is a powerful damper of sound."

Dr. Robinson here expresses the universally prevalent opinion, and he then assigns the theoretic cause. Fog, he says, "is a mixture of air and globules of water, and at each of the innumerable surfaces where these two touch, a portion of the vibration is reflected and lost‖. . . . . Snow produces a similar effect, and one still more injurious."

Reflection being thus considered to take place at the surfaces of the suspended particles, it followed that the greater the number of particles, or, in other words, the denser the fog, the more injurious would be its action upon sound. Hence optical transparency came to be considered as a measure of acoustic transparency. On this point Dr. Robinson, in the letter referred to, expresses himself thus:—"At the outset, it is obvious that, to

* Essay on Sound, par. 21.

† Report of the British Association for 1863, p. 105.

‡ A very sagacious remark: see letters in the Appendix.

§ Powerful electric lights have been since established and found ineffectual.

‖ This is also Sir John Herschel's way of regarding the subject. Essay on Sound, par. 38.

make experiments *comparable*, we must have some measure of the fog's power of stopping sound, without attending to which the most anomalous results may be expected. It seems probable that this will bear some simple relation to its opacity to light, and that the distance at which a given object, as a flag or pole, disappears may be taken as the measure." "Still clear air" is regarded in this letter as the best vehicle of sound, the alleged action of fogs, rain, and snow being ascribed to their rendering the atmosphere "a discontinuous medium."

To Mr. ALEXANDER BEAZELEY we are indebted for an extremely useful summary of existing knowledge regarding fog-signals*. He classifies the various instruments hitherto employed, and gives some account of their performance. As regards the action of fog upon sound, the statements made in the body of his papers agree with those just quoted from Dr. ROBINSON. "Fogs," he says, "have a remarkable power of deadening sound, and act in this respect so irregularly, that experiments made during clear weather have little or no practical value, except as mere competitive trials of different instruments."

In the discussion which followed the reading of Mr. BEAZELEY's paper at the Institution of Civil Engineers, Dr. GLADSTONE, who was a member of the Commission on Lights and Beacons, is reported to have said, "A difficulty in the use of sound was this, that fogs deadened sound very materially; but the evidence was very contradictory on that point. In a fog on land it was difficult to hear the passing of carriages or noises at a short distance; and so in a fog at sea these signals found a difficulty in penetrating the fog against which they are intended to be a protection."

On the same occasion Mr. JAMES N. DOUGLASS, the Engineer of the Trinity House, to whose ability as an observer I am able to bear strong testimony, stated that in his experience "he had found but little difference in the travelling of sound in foggy or in clear weather. He had distinctly heard in a fog, at the Small's Rock in the Bristol Channel, guns fired at Milford, twenty-five miles off." Mr. BEAZELEY had also heard the Lundy-Island gun "at Hartland Point, a distance of ten miles, during dense fog;" so that, in winding up his paper, he admitted "that the subject appeared to be very little known, and that the more it was looked into the more apparent became the fact that the evidence as to the action of fog upon sound is extremely conflicting."

In a paper presented to the Literary and Philosophical Society of Manchester on the 16th of December, 1873, Professor OSBORNE REYNOLDS affirms the prevalent doctrine with great distinctness, and makes a very ingenious attempt to explain it. "That sound," says Professor REYNOLDS, "does not readily penetrate a fog is a matter of common observation. The bells and horns of ships are not heard so far during fogs as when the weather is clear. In a London fog the noise of the wheels is much diminished, so that they seem to be at a distance when really close by."

My knowledge does not inform me of the existence of any other source for these

* Proceedings of the Institution of Civil Engineers, March 14, 1871; and Lecture at the United-Service Institution, May 24, 1872.

opinions regarding the deadening power of fog than the paper of DERHAM already referred to. In consequence of their *à priori* probability, his conclusions seem to have been transmitted unquestioned from generation to generation of scientific men.

§ 3. *Instruments and Observations.*

These extracts and references sufficiently indicate the uncertain state of the question when, on the 19th of May, 1873, this inquiry began. The South Foreland, near Dover, was chosen as the signal-station, steam-power having been already established there to work two powerful magneto-electric lights. The observations for the most part were made afloat, one of the yachts of the Trinity Corporation being usually employed for this purpose. Two stations had been established, the one at the top, the other at the bottom of the South-Foreland Cliff; and at each of them trumpets and air- and steam-whistles of great size were mounted. The whistles first employed were of English manufacture; but intelligence having been received regarding a large United-States whistle, and also a Canadian whistle, of great reputed power, the Elder Brethren had them subsequently added to the list.

On the 8th of October another instrument, which has played a specially important part in these observations, was introduced. During my recent visit to the United States, I was favoured by an introduction to General WOODRUFF by Professor JOSEPH HENRY, of Washington. Professor HENRY is Chairman of the Lighthouse Board, and General WOODRUFF is engineer in charge of two of the Lighthouse districts. I accompanied General WOODRUFF to the establishment at Staten Island, and afterwards to Sandy Hook, with the express intention of observing the performance of a steam-syren which, under the auspices of Professor HENRY, has been introduced into the lighthouse system of the United States. Such experiments as were possible to make under the circumstances were made; and I carried home with me a somewhat vivid remembrance of the mechanical effect of the sound upon my ears and body generally. Hence my desire to see the syren tried at the South Foreland. The formal expression of this desire was anticipated by the Elder Brethren, while their wishes were in turn anticipated by the courteous kindness of the Lighthouse Board at Washington. Informed by Major ELLIOTT, of the United States Army, that our experiments had begun, the Board forwarded to the Corporation, for trial, the instrument now mounted at the South Foreland.

In the steam-syren patented by Mr. BROWN, of New York, a fixed disk and a rotating disk are employed as in the ordinary syren, radial slits being cut in both disks instead of circular apertures. One disk is fixed vertically across the throat of a conical trumpet 16½ feet long, 5 inches in diameter where the disk crosses it, and gradually opening out till at the other extremity it reaches a diameter of 2 feet 3 inches. Behind the fixed disk is the rotating one, which is driven by separate mechanism. The trumpet is mounted on a boiler. In our experiments steam of 70 lbs. pressure has for the most part been employed. Just as in the ordinary syren, when the radial slits of the two disks coincide, and then only, a strong puff of steam escapes. Sound-waves of great intensity

are thus sent through the air, the pitch of the note produced depending on the velocity of rotation.

To the syren, trumpets, and whistles were added three guns—an 18-pounder, a 5½-inch howitzer, and a 13-inch mortar. In our summer experiments all three were fired; but the howitzer having shown itself superior to the other guns it was chosen in our autumn experiments as not only a fair but a favourable representative of this form of signal. The charges fired were for the most part those now employed at Holyhead, Lundy Island, and the Kish light-vessel—namely, 3 lbs. of powder. Gongs and bells were not included in this inquiry, because previous observations had clearly proved their inferiority to the trumpets and whistles.

A general knowledge of the instruments employed is thus imparted to the reader; while the Map on Plate XVII. will furnish him with all necessary information as to the position of the localities referred to in the paper.

On the 19th of May the instruments tested were:—

On the top of the cliff:

a. Two brass trumpets or horns, 11 feet 2 inches long, 2 inches in diameter at the mouthpiece, and opening out at the other end to a diameter of 22½ inches. They were provided with vibrating steel reeds 9 inches long, 2 inches wide, and ¼ inch thick, and were sounded by air of 18 lbs. pressure.

b. A whistle, shaped like that of a locomotive, 6 inches in diameter, also sounded by air of 18 lbs. pressure.

c. A steam-whistle, 12 inches in diameter, attached to a boiler, and sounded by steam of 64 lbs. pressure.

At the bottom of the cliff:

d. Two trumpets or horns, of the same size and arrangement as those above, and sounded by air of the same pressure.

e. A 6-inch air-whistle, similar to the one above, and sounded by the same means.

The upper instruments were 235 feet above high-water mark, the lower ones 40 feet. A vertical distance of 195 feet, therefore, separated the instruments. A shaft, provided with a series of twelve ladders, led from the one to the other.

The trumpets were constructed by that able mechanician, Mr. Holmes, who had them throughout under his personal superintendence. They were mounted vertically on the reservoir of compressed air; but within about 2 feet of their extremities they were bent at a right angle, so as to present their mouths to the sea (see sketch of horn on Plate XVIII., where the steel reeds are also shown). The aim of their constructor was to distribute the sound equably over an arc of 180°. To effect this, he placed the horizontal parts of the axes of the horns at right angles to each other, the one pointing S.W. by S., and the other S.E. by E., each horn being supposed to cover an arc of 90°.

The 12-inch steam-whistle was constructed by Mr. Baily, of Manchester.

Our first experiments with these instruments were a preliminary discipline rather than an organized effort at discovery. On May 19 we steamed round the Foreland and out

to sea in the axes of the horns. The maximum distance reached by the sound was about three and a half miles*. The wind, however, was high and the sea rough, so that local noises interfered to some extent with our appreciation of the sound.

Mariners express the strength of the wind by a series of numbers extending from 0 = calm to 12 = a hurricane, a little practice in common producing a remarkable unanimity between different observers as regards the force of the wind. Its force on May 19 was 6, and midway between the axes of the two trumpets it blew at right angles to the direction of the sound.

The same instruments on the 20th of May covered a greater range of sound; but not much greater, though the disturbance due to local noises was absent. At 4 miles distance in the axes of the horns they were barely heard, the air at the time being calm, the sea smooth, and all other circumstances exactly those which have been hitherto regarded as most favourable to the transmission of sound. We crept a little further away, and by stretched attention managed to hear at intervals, at a distance of 6 miles, the faintest hum of the horns. A little further out we again halted; but though local noises were absent, and though we listened intently, we heard nothing.

This position, clearly beyond the range of whistles and trumpets, was expressly chosen with the view of making what might be considered a decisive comparative experiment between horns and guns as instruments for fog-signalling. The distinct report of the 12 o'clock gun fired at Dover on the 19th suggested this comparison, and through the prompt courtesy of General Sir A. Horsford we were enabled to carry it out. At 12.30 precisely the puff of an 18-pounder, with a 3-lb. charge, was seen at Dover Castle, which was about a mile further off than the South Foreland. Thirty-six seconds afterwards the loud report of the gun was heard, its complete superiority over the trumpets being thus, to all appearance, demonstrated.

We clinched this observation by steaming out to a distance of 8½ miles, where the report of a second gun was well heard by all of us. At a distance of 10 miles the report of a third gun was heard by some of us, and at 9·7 miles the report of a fourth gun was heard by us all.

The result seemed perfectly decisive. Applying the law of inverse squares, the sound of the gun at a distance of 6 miles from the Foreland must have had more than two and a half times the intensity of the sound of the trumpets. It would hardly have been rash under the circumstances to have reported without qualification the superiority of the gun as a fog-signal. No single experiment is, to my knowledge, on record to prove that a sound once predominant would not be always predominant, or that the atmosphere on different days would show preferences to different sounds. On many subsequent occasions, however, the sound of the horn proved distinctly superior to that of the gun. This *selective* power of the atmosphere revealed itself more strikingly in our autumn experiments than in our summer ones; and it was sometimes illustrated within a few hours of the same day: of two sounds A and B for example, A would have the greatest range at 10 A.M. and B the greatest range at 2 P.M.

* In all cases nautical miles are meant.

In the experiments on the 19th and 20th of May, the superiority of the trumpets over the whistles was decided; and indeed, with few exceptions, this superiority was maintained throughout the inquiry. But there were exceptions. On June 2, for example, the sound of the whistles rose in several instances to full equality with, and on rare occasions subsequently even surpassed, that of the horns. The sounds were varied from day to day. On the date last mentioned a single horn was sounded, two were sounded, and three were sounded together; but the utmost range of the loudest sound, even with the paddles stopped, did not exceed six miles. With the view of concentrating their power, the axes of the horns had been pointed in the same direction, and, unless stated to the contrary, this in all subsequent experiments was the case.

On the 3rd of June the three guns already referred to were permanently mounted at the South Foreland. They were well served by gunners from Dover Castle.

On June 3 dense clouds quite covered the firmament, some of them particularly black and threatening, but a marked advance was observed in the transmissive power of the air. At a distance of 6 miles the horn-sounds were not quite quenched by the paddle-noises; at 8 miles the whistles were heard, and the horns better heard; while at 9 miles, with the paddles stopped, the horn-sounds alone were fairly audible. During the day's observations a remarkable and instructive phenomenon was observed. Over us rapidly passed a torrential shower of rain, which, according to DERHAM, is a potent damper of sound. I could, however, notice no subsidence of intensity as the shower passed. It is even probable that, had my mind been free from bias, I should have noticed an augmentation of the sound, such as occurred with the greatest distinctness on various subsequent occasions during violent rain.

The influence of "beats" was tried on June 3, by throwing the horns slightly out of unison; but though the beats rendered the sound characteristic, they did not seem to augment the range. At a distance from the station curious fluctuations of intensity were noticed. Not only did the different blasts vary in strength, but sudden swellings and fallings off, even of the same blast, were observed. This was not due to any variation on the part of the instruments, but purely to the changes of the medium traversed by the sound. What these changes were shall be indicated subsequently.

During the inquiry various shiftings of the horns and reeds were resorted to, with a view of bringing out their maximum power. The range of our best horns on June 10 was $8\frac{3}{4}$ miles. The guns at this distance were very feeble. That the loudness of the sound depends on the shape of the gun was proved by the fact that thus far the howitzer, with a 3-lb. charge, proved more effective than the other guns. In the axis of the horns the sound manifests its greatest strength, falling sensibly off as the angular distance from the axis is augmented. Now the whistles have no such axes, but send their sound-waves with equal strength in all directions. Hence, as the horns pointed seaward, near the line joining the Foreland and the South Sand Head light-vessel on the one hand, and that joining the Foreland and the Admiralty Pier on the other, the whistles were sometimes more than a match for the horns.

§ 4. *Influence of Sound-Shadow.*

On the 19th of May we noticed a phenomenon of grave import in connexion with the establishment of fog-signals. I refer to the rapid fall of intensity on both sides of the signal-station at the South Foreland. We had halted between the station and the South Sand Head light-ship, at a distance of $2\frac{1}{2}$ miles from the former. The trumpets and whistles were sounded, but they were quite unheard. We moved nearer; but even at a mile distance, with the instruments plainly in view, their sound failed to reach us. A light wind, however, was here opposed to the sound. Abreast of the signal-station the trumpets were very powerful; but on approaching the line joining the Foreland to the end of the Admiralty Pier the sound fell rapidly, though in this case the wind was favourable to the sound. Plainly, therefore, some other cause than the wind must be invoked to account for the phenomenon.

On the 10th of June the same effect was very strikingly manifested. After our day's work we steamed past the Foreland and towards the end of the Pier. At the distance of a mile the sounds fell with such rapidity that I thought something must have occurred to the whistles and horns. Happily the guns were there to test this surmise. At 2 miles distance we signalled for them. With a 3-lb. charge, though their puffs were clearly seen, not one of them was heard; with a 6-lb. charge the 18-pounder was barely heard, the howitzer was slightly better heard, while the mortar was quite unheard. No peculiarities of the horns or whistles could therefore account for the phenomenon.

On the 11th of June the effect was equally pronounced. On the line joining the Foreland and the Admiralty Pier, and at $\frac{3}{4}$ of a mile from the station, the sound rapidly sank in power, and soon afterwards became inaudible. At $1\frac{1}{4}$ mile distance we signalled for the guns; the report in each case was a low indistinct thud. A necessary requirement in fog-signals is stated to be that they should, under all circumstances, be heard to a distance of 4 miles. Now the gun was undoubtedly the signal of greatest range when this inquiry began, and here we find that conditions may exist which render even the gun ineffectual at less than half the distance deemed essential.

The Map on Plate XIX., which consists of a portion of Plate XVII. enlarged, will help us to an explanation of these observations. Near the fog-signal station a projecting chalk cliff at C receives the impact of the sonorous waves and disperses them by reflection. The whole sea space between the line A B and the cliffs under Dover Castle is in the sound-shadow. Within this line the instruments cannot be seen, without it they can; and we have to account for the fact that the enfeeblement of the sound occurs not only inside but immediately outside the boundary, and while the instruments are in sight. A sudden subsidence of the sound is always observed on crossing the boundary towards the shore, and a correspondingly sudden augmentation on crossing it towards the sea; but the stoppage of the sound on entering the shadow is by no means total. The whole of the shaded space is filled with sound of enfeebled intensity, produced in great part by the divergence into the shadow of the waves which abut against the boundary. Through this divergence the direct waves suffer, the portions nearest to

the shadow suffering most. (On the Map the condensations and rarefactions of the direct waves are shown by circular lines of varying closeness.) Here, then, we have one cause of the decay of the sound in the neighbourhood of the acoustic shadow. Another cause is the interference of the direct waves with those reflected from C and from other portions of the cliff. The remarks here applied to the sound-shadow west of the Foreland are also applicable to that upon the other side.

On July 25th a gradual improvement in the transmissive power of the air was observed from morning to evening; but at the last the maximum range was only moderate. The fluctuations in the strength of the sound were remarkable, sometimes sinking to inaudibility and then rising to loudness. A similar effect, due to a similar cause, is often noticed with church-bells. The acoustic transparency of the air was still further augmented on the 26th: at a distance of $9\frac{1}{4}$ miles from the station the whistles and horns were plainly heard against a wind with a force of 4; while on the 25th, with a favouring wind, the maximum range was only $6\frac{1}{2}$ miles. Plainly, therefore, something else than the wind must be influential in determining the range of the sound.

§ 5. *Rotation of Horn.*

Thus far I have confined myself to the salient point or points of each day's observations, omitting numerous details. The observations of July 1 and 3 are, I think, of a character to bear a fuller treatment. Tuesday, July 1, was devoted to the further investigation of the sound-range, and also to obtaining additional information as to how the sound diminishes in intensity as we depart from the axis of the horn. It is obvious that for this purpose, instead of carrying the ship round the horn, we may cause the horn to rotate round its vertical axis while the ship maintains a fixed position.

On this occasion the steam-whistle from the United States was mounted, and sounded at different times during the day. The whistle is 12 inches in diameter, and it was blown at 70 lbs. pressure. The resonant bell of the whistle cannot be moved, a fixed distance existing between the circular ring from which the steam issues and the cutting-edge.

We steamed to a point on the axis which bore S.S.W. from the station. Wind calm on sea; on shore N.N.W., with a force of 3. We halted at a distance of $5\frac{1}{4}$ miles, and received a very good sound from the horn when it was pointed towards us. Forty-five degrees right and left of this, the sound was also good, but it became feebler as the departure from the axis was augmented.

To attempt applying numerical estimates to the sounds would be mere guesswork; still it will fix the ideas if I give a sample of the method by which the day's observations were conducted. A circle was drawn, the circumference of which was divided at eight points into equal parts, the radius drawn from the centre to those points representing the direction of the horn. Opposite to each point was placed a number indicative of the strength of the corresponding sound. Thus, assuming the intensity when the

trumpet was pointed towards us to be represented by the number 10, the intensities when the trumpet was pointed in the other directions are represented by the other figures in the accompanying diagram*.

These are the actual numbers set down during a series of observations, but they are not to be taken as representing any thing further than the general decadence of the sounds as the trumpet was more and more turned away from us. Indeed there is every reason to believe that the decay is sometimes more rapid and considerable than is indicated by these numbers.

It may be remarked that a hoarding placed behind the trumpets contributed by reflection to augment the intensity of the sound when the trumpet was turned quite away from us.

In the annexed sketch the judgment of an observer is expressed in words instead of numbers. It may be stated that at different periods of the same day, the opinion of the same observer varied as to the relative intensities of the sounds in the several positions of the horn. The observation here recorded was made at a distance of $5\frac{1}{4}$ miles from the station.

For the better education of our ears we steamed closer to the Foreland, halting at $3\frac{3}{4}$ miles, and resting in this position during several rotations of the horn. The direct sound was very fine, the other sounds being expressed with approximate fairness by the numbers given in the first diagram. At $3\frac{6}{10}$ miles to windward (the wind, however, blowing only with a force of 2, and the water being smooth) the result was substantially the same. The day had become dark and lowering; the black and threatening clouds formed a kind of ceiling overhead, thereby possibly aiding the sound†, while haze was diffused between us and the station. For the purpose of making a signal, we steamed within 2 miles of the Foreland. On steaming out again the sounds were heard for some time through the paddle-noises; at about 4 miles distance they were lost, but they revived forcibly as soon as the paddles ceased moving; on resuming the motion they continued to be heard up to a distance of $6\frac{3}{4}$ miles.

At $7\frac{1}{2}$ miles the paddles were eased, and a strong sound was heard; at $9\frac{1}{2}$ miles the sound was good, but it appeared to be of lower pitch than at a less distance. The haze at this time quite hid the Foreland, a somewhat staggering result considering the acoustic clearness of the atmosphere. The wind S.W., force 2, appeared to be dead against the sound; but on shore it was N.W., with a force of 3, or nearly at right

* To Capt. Drew, I believe, we were indebted for this idea of a dial.

† This stands as it was written at the time; but I believe it is still to be proved that clouds, as such, have any sensible power of reflection.

angles to the direction of the sound. This variance in the records on shore and afloat is not uncommon at the Foreland, the formation of the land giving rise to local currents of air.

At a distance of 10 miles the horn once or twice yielded a plain sound, while the American whistle seemed to surpass the horn. We waited here for some time: at $10\frac{1}{4}$ miles occasional blasts of the horn came to us, but after a time all sound ceased to be audible; it seemed as if the air, after having been exceedingly transparent, had become gradually more opaque to the sound.

Returning along the same line, we halted and listened at a distance of $8\frac{1}{4}$ miles. The sounds were certainly much feebler than at 9 miles upon our journey out: it required attention and quiet on board to hear them at all. The haze continued, the cliffs of the Foreland being still hidden. At 6 miles we again halted: the sounds were indistinct, and not at all equal to those heard at $9\frac{1}{4}$ miles in coming out; they seemed, moreover, to have fallen in pitch: one of the observers described them as having degenerated to a kind of rumble. The sound as we approached the station became more and more unsatisfactory, and finally ceased. This we learned afterwards to be due to the breaking of the 2-inch reed of a wide horn. A narrower horn, which yielded a sound nearly equal to the large one, was used in the subsequent experiments.

At 4.45 P.M. we spoke the 'Triton,' and took the master of the Varne light-ship on board the 'Irené.' He and his company had heard the sound at intervals during the day, although he was dead to windward and distant $12\frac{3}{4}$ miles. All day long, however, the wind continued light, the force being only 2.

Here a word of reflection on our observations may be fitly introduced. It is, as already shown, an opinion entertained in high quarters that the waves of sound are reflected at the limiting surfaces of the minute particles which constitute haze and fog, the alleged waste of sound in fog being thus explained. If, however, this were an efficient practical cause of the stoppage of sound, and if clear calm air be, as alleged, the best vehicle, it would be impossible to understand how to-day, in a thick haze, the sound reached a distance of $12\frac{3}{4}$ miles, while on May 20, in a calm and hazeless atmosphere, the maximum range was only from 5 to 6 miles. Such facts foreshadow a revolution in our notions regarding the action of haze and fogs upon sound.

Steaming directly in towards the Foreland, at $4\frac{1}{2}$ miles, with the paddles going, the sounds were well heard; at $3\frac{3}{4}$ miles upon the same bearing the loudest sound was very full. We steamed towards Dover Pier end, and at $2\frac{5}{16}$ miles from the station, on the line between it and the pier, all sounds, as usual, fell considerably. At a distance of $1\frac{1}{2}$ mile on the same line the loudest sound emitted by the horn during its rotation was very feeble. Inasmuch as this extraordinary subsidence of the sound occurred when the horn was turned towards us, it cannot be referred to the deviation from the axis: as already explained (§ 4), it is in part due to the weakening of the direct waves by diffraction, and in part, doubtless, to interference.

Wishing to examine the condition of the sound at the other side of the Foreland, we

steamed abreast of it at a distance of ¼ a mile: the sound of the horn was here exceedingly powerful. At a distance of 3 miles from the station, and on the line between it and the South Sand Head light-ship, the sounds were stronger than at the other side; but we were here both nearer to the axis and further removed from the interfering influence of the shore. Close to the shore on both sides of the station the turning of the horn produced greater variations of the sound than in the open sea in front of the station: in one case, indeed, where the maximum sound was marked 10, the minimum was set down at 2.

An interval of 12 hours sufficed to change in a surprising degree the acoustic transparency of the air. On the 1st of July the sound had a range of nearly 13 miles; on the 2nd the range did not exceed 4 miles.

§ 6. *Contradictory Results.*

Thus far the investigation proceeded with hardly a gleam of a principle to connect the inconstant results. The distance reached by the sound on the 19th of May was 3½ miles; on the 20th it was 5½ miles; on the 2nd of June 6 miles; on the 3rd more than 9 miles; on the 10th it was also 9 miles; on the 25th it fell to 6½ miles; on the 26th it rose again to more than 9¼ miles; on the 1st of July, as we have just seen, it reached 12¾, whereas on the 2nd the range shrunk to 4 miles. None of the meteorological agents observed could be singled out as the cause of these fluctuations. The wind exerts an acknowledged power over sound, but it could not account for these phenomena. On the 25th of June, for example, when the range was only 6½ miles, the wind was favourable; on the 26th, when the range exceeded 9¼ miles, it was opposed to the sound. Nor could the varying optical clearness of the atmosphere be invoked as an explanation; for on July 1, when the range was 12¾ miles, a thick haze hid the white cliffs of the Foreland, while on many other days, when the acoustic range was not half so great, the atmosphere was optically clear. Up to July 3 all remained enigmatical; but on this date observations were made which seemed to me to displace surmise and perplexity by the clearer light of physical demonstration.

§ 7. *Aërial Reflection and its Causes; solution of contradictions.*

On July 3 we first steamed to a point 2·9 miles S.W. by W. of the signal-station. No sounds, not even the guns, were heard at this distance. At 2 miles they were equally inaudible. But this being the position in which the sounds, though strong in the axis, invariably subsided, we steamed to the exact bearing from which our observations had been made upon July 1. At 2.15 P.M., and at a distance of 3¾ miles from the station, with calm clear air and a smooth sea, the horns and whistle (American) were sounded, but they were inaudible. Surprised at this result, I signalled for the guns. They were all fired, but, though the smoke seemed at hand, no sound whatever reached us. On July 1, in this bearing, the observed range of both horns and guns was 10½ miles, while on the bearing of the Varne light-vessel it was nearly 13 miles. We steamed in to 3 miles,

paused, and listened with all attention; but neither horn nor whistle was heard. The guns were again signalled for; five of them were fired in succession, but not one of them was heard. We steamed in on the same bearing to 2 miles, and had the guns fired point-blank at us. The howitzer and the mortar, with 3-lb. charges, yielded a feeble thud, while the 18-pounder was wholly unheard. Applying the law of inverse squares, it follows that, with air and sea, according to accepted notions, in a far worse condition, the sound at 2 miles distance on July 1 must have had more than forty times the intensity which it possessed at the same distance at 3 P.M. on the 3rd.

"Over smooth water," says Sir JOHN HERSCHEL, "sound is propagated with remarkable clearness and strength." Here was the condition; still with the Foreland so close to us, the sea so smooth, and the air so transparent, it was difficult to realize that the guns had been fired or the trumpets blown at all. Had the sound been converted by internal friction into heat? or had it been wasted in partial reflections at the limiting surfaces of non-homogeneous masses of air? Sulphur in homogeneous crystals is exceedingly transparent to radiant heat, whereas the ordinary brimstone of commerce is highly impervious to it—the reason being that the brimstone of commerce does not possess the molecular continuity of the crystal, but is a mere aggregate of minute grains not in perfect optical contact with each other. Where this is the case, a portion of the heat is always reflected on entering and on quitting a grain: hence when the grains are minute and numerous this reflection is so often repeated that the heat is entirely wasted before it can plunge to any depth into the substance. The same remark applies to snow, foam, clouds, and common salt, indeed to all transparent substances in powder; they are all impervious to light, not through the immediate absorption or extinction of the light, but through repeated internal reflection.

HUMBOLDT, in his observations at the Falls of the Orinoco, is known to have applied these principles to sound. He found the noise of the Falls far louder by night than by day, though in that region the night is far noisier than the day. The plain between him and the Falls consisted of spaces of grass and rock intermingled. In the heat of the day he found the temperature of the rock to be considerably higher than that of the grass. Over every heated rock, he concluded, rose a column of air rarefied by the heat; and he ascribed the deadening of the sound to the reflections which it endured at the limiting surfaces of the rarer and the denser air. This philosophical explanation, which admits of experimental illustration in the laboratory, made it generally known that a non-homogeneous atmosphere is unfavourable to the transmission of sound.

But what on July 3, not with the variously heated plain of Antures, but with a calm sea as a basis for the atmosphere, could so destroy its homogeneity as to enable it to quench in so short a distance so vast a body of sound? I here submit to the judgment of scientific men my own course of thought regarding this question. As I stood upon the deck of the 'Irené' pondering it, I became conscious of the exceeding power of the sun beating against my back and heating the objects near me. Beams of equal power were falling on the sea, and must have produced copious evaporation. That the vapour

generated should so rise and mingle with the air as to form an absolutely homogeneous medium I considered in the highest degree improbable. It would be sure, I thought, to rise in streams, breaking through the superincumbent air now at one point now at another, thus rendering the air *flocculent* with wreaths and striæ, charged in different degrees with the buoyant vapour. At the limiting surfaces of these spaces, though invisible, we should have the conditions necessary to the production of partial echoes and the consequent waste of sound.

Curiously enough, the conditions necessary for the testing of this explanation immediately set in. At 3.15 P.M. a solitary cloud threw itself athwart the sun, and shaded the entire space between us and the South Foreland. The production of vapour was suddenly checked by the interposition of this screen; hence the probability of suddenly improved transmission. To test this inference the steamer was immediately turned and urged back to our last position of inaudibility. The sounds, as I expected, were distinctly though faintly heard. This was at 3 miles distance. At $3\frac{3}{4}$ miles the guns were fired, both point-blank and elevated. The faintest pop was all that we heard; but we did hear a pop, whereas we had previously heard nothing, either here or three quarters of a mile nearer. We steamed out to $4\frac{1}{4}$ miles, where the sounds were for a moment faintly heard; but they fell away as we waited; and though the greatest quietness reigned on board, and though the sea was without a ripple, we could hear nothing. We could plainly see the steam-puffs which announced the beginning and the end of a series of trumpet-blasts, but the blasts themselves were quite inaudible.

It was now 4 P.M., and my intention at first was to halt at this distance, which was beyond the sound-range, but not far beyond it, and see whether the lowering of the sun would not restore the power of the atmosphere to transmit the sound. But after waiting a little, the anchoring of a boat was suggested, so as to liberate the steamer for other work; and though loth to lose the anticipated revival of the sounds myself, I agreed to this arrangement. Two men were placed in the boat and requested to give all attention so as to hear the sound if possible. With perfect stillness around them they heard nothing. They were then instructed to hoist a signal if they should hear the sounds, and to keep it hoisted as long as the sounds continued.

At 4.45 we quitted them and steamed towards the South Sand Head light-ship. Precisely 15 minutes after we had separated from them the flag was hoisted: the sound had at length succeeded in piercing the body of air between the boat and the shore.

We continued our journey to the light-ship, went on board, and heard the report of the lightsmen. Returning towards the Foreland, in answer to a signal expressing a wish to communicate with us, we manned a boat and pulled to the shore. The exhaustion of the ammunition was reported, but the horns and whistle continued to sound. We steamed out to our anchored boat, and then learned that when the flag was hoisted the horn-sounds were heard, that they were succeeded after a little time by the whistle-sounds, and that both increased in intensity as the evening advanced. On our arrival, of course we heard the sounds ourselves.

Thus far, therefore, the explanation given above entirely agrees with the results of observation; but we pushed the test further by steaming further out. At $5\frac{1}{4}$ miles we halted and heard the sounds: at 6 miles we heard them distinctly, but so feebly that we thought we had reached the limit of the sound-range; but while we waited the sounds rose in power. We steamed to the Varne buoy, which is $7\frac{1}{4}$ miles from the signal-station, and heard the sounds there better than at 6 miles distance. We continued our course outwards to 10 miles, halted there, but heard nothing.

Steaming, however, on to the Varne light-ship, which is situated at the other end of the Varne shoal, we hailed the master, and were informed by him that up to 5 P.M. nothing had been heard, but that at that hour the sounds began to be audible. He described one of them as "very gross, resembling the bellowing of a bull," which very accurately characterizes the sound of the large American steam-whistle. At the Varne light-ship, therefore, the sounds had been heard towards the close of the day, though it is $12\frac{3}{4}$ miles from the signal-station. I think it probable that, at a point 2 miles from the Foreland, the sound at 5 P.M. possessed fifty times the intensity which it possessed at 2 P.M. On our return to Dover Bay at 10 P.M., we heard the sounds, not only distinct but loud, where nothing could be heard in the morning.

In consequence of the position of the promontory very curious winds and currents establish themselves round the South Foreland. Mr. HOLMES was, as usual, at the Foreland on July 3; and he informed me that from the motion of the smoke of some passing steamers, and from the sails of sailing-vessels, he could recognize a curious circulation of the air. A slight wind would sometimes hug the shore to the N.E., then bend round and move towards the South Sand Head light-ship. And, in point of fact, the wind at the light-vessel had been S.W., with a force of 3, nearly the whole of the day; whereas with us it had passed from S.W. by W. to a dead calm, and afterwards to S.E. On shore also it had shifted from S.W. to S.E. The atmospheric conditions between the light-vessel and the Foreland were therefore different from those between us and the Foreland; and the consequence was that at the time when we were becalmed and heard nothing, the light-keepers at South Sand Head heard the sounds plainly.

§ 8. *Aërial Echoes.*

But both the argument and the phenomena have a complementary side, which we have now to consider. A stratum of air less than 3 miles thick on a calm day has been proved competent to stifle both the cannonade and the horn-sounds employed at the South Foreland; while, according to the foregoing explanation, this result was due to the irregular admixture of air and aqueous vapour, which filled the atmosphere with an impervious *acoustic cloud* on a day of perfect *optical* transparency. But, granting this, it is incredible that so great a body of sound could utterly disappear in so short a distance without rendering any account of itself. Supposing, then, instead of placing ourselves behind the acoustic cloud we were to place ourselves in front of it, might we not, in accordance with the law of conservation, expect to receive by reflection the sound which

had failed to reach us by transmission? The case would then be strictly analogous to the reflection of light from an ordinary cloud to an observer placed between it and the sun.

My first care in the early part of the day in question was to assure myself that our inability to hear the sound did not arise from any derangement of the instruments on shore. Accompanied by Mr. EDWARDS, who was good enough on this and some other days to act as my amanuensis, at 1 P.M. I was rowed to the shore, and landed at the base of the South Foreland Cliff. The body of air which had already shown such extraordinary power to intercept the sound, and which manifested this power still more impressively later in the day, was now in front of us. On it the sonorous waves impinged, and from it they were sent back to us with astonishing intensity. The instruments, hidden from view, were on the summit of a cliff 235 feet above us, the sea was smooth and clear of ships, the atmosphere was without a cloud, and there was no object in sight which could possibly produce the observed effect. From the perfectly transparent air the echoes came, at first with a strength apparently but little less than that of the direct sound, and then dying gradually and continuously away. A remark made by my talented companion in his note-book at the time shows how the phenomenon affected him:—"Beyond saying that the echoes seemed to come from the expanse of ocean, it did not appear possible to indicate any more definite point of reflection." Indeed no such point was to be seen; the echoes reached us, as if by magic, from absolutely invisible walls.

Here, in my opinion, we have the key to many of the mysteries and discrepancies of evidence which beset this question. The foregoing observations show that there is no need to doubt either the veracity or capability of the conflicting witnesses, for the variations of the atmosphere are more than sufficient to account for theirs. The mistake indeed hitherto has been, not in reporting incorrectly, but in neglecting the monotonous operation of repeating the observations during a sufficient time. I shall have occasion to remark subsequently on the mischief likely to arise from giving instructions to mariners founded on observations of this incomplete character.

The question of aërial echoes has an historic interest. While cloud-echoes have been accepted as demonstrated by observation, it has been hitherto assumed that audible echoes never occur in optically clear air. We owe this opinion to the admirable report of ARAGO on the experiments made to determine the velocity of sound at Montlhery and Villejuif in 1822*. ARAGO'S account of the phenomenon observed by him and his

* Sir JOHN HERSCHEL gives the following account of ARAGO'S observation:—"The rolling of thunder has been attributed to echoes among the clouds; and if it is considered that a cloud is a collection of particles of water, however minute, in a liquid state, and therefore each individually capable of reflecting sound, there is no reason why very loud sounds should not be reverberated confusedly (like bright lights) from a cloud. And that such is the case has been ascertained by direct observation on the sound of cannon. Messrs. ARAGO, MATTHIEU, and PRONY, in their experiments on the velocity of sound, observed that under a perfectly clear sky the explosions of their guns were always single and sharp; whereas when the sky was overcast, and even when a cloud came

colleagues is as follows:—"Avant de terminer cette note, nous ajouterons seulement que tous les coups tirés à Montlhery y étaient accompagnés d'un roulement semblable à celui du tonnerre, et qui durait de 20″ à 25″. Rien de pareil n'avait lieu à Villejuif; il nous est arrivé seulement d'entendre, à moins d'une seconde d'intervalle, deux coups distincts du canon de Montlhery. Dans *deux* autres circonstances le bruit de ce canon a été accompagné d'un roulement prolongé. Ces phénomènes n'ont jamais eu lieu qu'au moment d'apparition de quelques nuages; par un ciel complétement serein le bruit était unique et instantané. Ne serait-il pas permis de conclure de là qu'à Villejuif les coups multiples du canon de Montlhery résultaient d'échos formés dans les nuages, et de tirer de ce fait un argument favorable à l'explication qu'ont donnée quelques physiciens du roulement du tonnerre?"*

It is not here stated that at Montlhery the clouds were seen when the echoes were heard. The explanation of the Montlhery echoes is, if I understand right, an inference from observations made at Villejuif. The inference I think requires qualification. Some hundreds of cannon-shots have been fired at the South Foreland, many of them when the heavens were completely free from clouds, and never in a single case has a "*roulement*" similar to that noticed at Montlhery been absent. It follows, moreover, so hot upon the direct sound as to present hardly a sensible breach of continuity between the sound and the echo. This could not be the case if the clouds were its origin. A reflecting cloud, even at the short distance of 1000 yards, would leave a silent interval of 5 seconds between sound and echo; and had such an interval been observed at Montlhery, it could hardly have escaped record by the philosophers stationed there.

However this may be, the foregoing observations prove that air of perfect visual transparency is competent to produce echoes of great intensity and long duration. The subject is worthy of additional illustration. On the 8th of October, as already stated, the syren was established at the South Foreland. I visited the station on that day, and listened to the syren-echoes. They were far more powerful than those of the horn. Like the others they were perfectly continuous, and faded, as if into distance, gradually away. The direct sound seemed rendered complex and multitudinous by its echoes, which resembled a band of trumpeters first responding close at hand, and then retreating rapidly towards the coast of France. The syren-echoes had 11 seconds, those of the horn 8 seconds duration.

I moved away from the station so as to lower the power of the direct without at the same time weakening the reflected sound. This was done by dropping into the sound-shadow behind an adjacent eminence. The echoes thus heard were still more wonderful

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in sight over any considerable part of the horizon, they were frequently accompanied by a long continued roll like thunder."—*Essay on Sound*, par. 38.

Observations and experiments recorded further on show the conclusion that clouds, as such, have any sensible power at all of reflecting sound, to be problematical, if not erroneous.

* Connaiss. des Temps, 1825, p. 361.

than before. In the case of the syren, moreover, the reinforcement of the direct sound by its echo was distinct. About a second after the commencement of the syren-blast, the echo struck in as a new sound. This first echo, therefore, must have been flung back by a body of air not more than 600 or 700 feet in thickness. The few detached clouds visible at the time were many miles away, and could clearly have had nothing to do with the effect.

This mingling of the echoes with the direct sound was much more distinctly heard in the case of the syren than in the case of the horn. With the horn, indeed, the echo was first plainly heard after the direct sound had ceased.

As we descended towards St. Margaret's Bay, where the horn and syren were well shaded by the hill, it was difficult to say when the direct sound ceased and the echoes began, so near in point of sensible intensity were the echoes to the direct sound.

On the 10th of October I was again at the Foreland listening to the echoes, with results similar to those just described. On the 15th I had an opportunity of remarking something new concerning them. To the late Mr. Daboll, of the United States, belongs the credit of bringing large trumpets into use as fog-signals. At Dungeness one of his horns had been erected under his own superintendence; and, wishing to make myself acquainted with its performance, we steamed thither on the 15th. The horn is worked by a caloric engine. Like the horns at the South Foreland it is vertically mounted, and like them bent near its extremity so as to present its mouth to the sea. It rotates automatically through an arc of 210°, halting at four different points on the arc and emitting a blast of 6 seconds duration, these blasts being separated from each other by intervals of silence of 20 seconds. The pressure in each case when the sound began was $8\frac{1}{2}$ lbs., and it sank during the blast to $5\frac{3}{4}$ lbs. per square inch.

I listened to the sound during several successive rotations of the horn: the augmentation was distinct when the axis of the horn was turned towards me. From the beach and from the lighthouse-tower I listened to the echoes: they were feeble compared with those of the syren, but were nevertheless very musical, and of 4 seconds duration. They were moreover purely aërial, as the heaven was quite free from clouds.

The 'Galatea' at first sent us a startling echo from her side. Being afterwards turned with her bow facing in, this echo almost entirely disappeared, leaving the continuous and gradually fading aërial echoes behind. Some ships were in sight, and when the horn accidentally pointed towards one of them, in the midst of the aërial echoes one would suddenly cease, thus breaking the uniform continuity of the sinking sound: the dying out would then continue. In the words of Admiral Collinson, who was at my side, the aërial sound, instead of ceasing suddenly and abruptly, "tapered away."

The new point observed was that as the horn rotated the echoes were always returned along the line in which the axis of the horn pointed. Standing either behind or in front of the lighthouse-tower, or closing the eyes so as to exclude all knowledge of the position of the horn, the direction of its axis when it sounded could always be inferred from the direction in which the aërial echoes reached the shore. Not only

therefore is knowledge of *direction* given by a sound, but it may also be given by the aërial echoes of the sound.

On various occasions, when the atmosphere was perfectly free from clouds, I have had myself rowed from the 'Galatea' to the Foreland Cliff to listen to the echoes. On the 16th of October, at half a mile from the shore, we stopped and found the syren-echoes at this distance very distinct. The sky being absolutely cloudless, the gig was manned, and we rowed in. As we approached the cliff, thus deepening the reflecting layer of air, the echoes augmented in intensity and duration. We did not quit the boat, but halted as near as possible to the water-mark. The echoes returned by the transparent and perfectly invisible atmosphere were of astonishing strength and sweetness.

On this day the whistles and syren were compared. The average duration of the syren-echoes was 11 seconds; that of the whistle-echoes 6 seconds. In all cases the sound with the longest echo had the greatest range.

With sounds of the same pitch the duration of the echoes might be taken as a measure of the penetrative power of the sound.

The earliest, strongest, and most trumpet-like tones are thrown back from the stratum of air less than a quarter of a mile in thickness, which is first pierced by the sound. After we had placed this air-stratum between us and the shore its echoes were withdrawn, the residual echoes being in consequence much feebler.

One additional illustration of this character will suffice. On the 17th of October, at about 5 P.M., the air being perfectly free from clouds, we rowed towards the base of the Foreland Cliff. The 'Galatea' being broadside on returned a loud echo to both syren and horns. It was a simple but wonderfully distinct copy of the original sound. At a certain moment it struck in, raised the intensity of the sound, then ceased, suddenly lowering the intensity, and leaving the long-drawn aërial echoes to pursue their course and die gradually into silence.

We landed, and passed over the seaweed to the base of the cliff. As I reached the base the position of the 'Galatea' was such that an echo of astonishing intensity was sent back from her side; it came as if from an independent source of sound established on board the steamer. As before, this echo ceased suddenly, leaving the aërial echoes to fade gradually away.

At the base of the cliff a series of concurrent observations gave the duration of the aërial syren-echoes from 13 to 14 seconds.

Lying on the shingle under a projecting roof of chalk, the somewhat enfeebled diffracted sound reached me, and I was able to hear with great distinctness, about a second after the starting of the syren-blast, the echoes striking in and reinforcing the direct sound. The first rush of echoed sound was very powerful, and it came, as usual, from a stratum of air 600 or 700 feet in thickness. On again testing the duration of the echoes, it was found to be from 14 to 15 seconds. The perfect clearness of the afternoon caused me to choose it for the examination of the echoes. It is worth remarking that this was our day of longest echoes, and it was also our day of greatest acoustic transparency,

this association suggesting that the duration of the echo is a measure of the atmospheric *depths* from which it comes. On no day, it is to be remembered, was the atmosphere free from invisible acoustic clouds; and on this day, and when their presence did not prevent the direct sound from reaching to a distance of 15 or 16 nautical miles, they were able to send us echoes of 15 seconds duration.

It remains to be added that on many occasions when the 'Galatea' was 2 miles and on some when she was 3 miles from the shore, with the Foreland bearing north, distinct and long-continued syren-echoes were sent back to us from the transparent *southern* air.

On various other occasions, when the atmosphere was exceptionally pure, we have fired the 'Galatea's' guns with a 1-lb. charge; echoes were always returned from that portion of the atmosphere towards which the gun was pointed.

Several times also during the prevalence of wind the ship has been turned across the wind and its two guns fired, the one to windward, the other to leeward. On all occasions the echoes of the leeward-pointed gun were distinctly more powerful and long-continued than those of the gun fired to windward.

To sum up this question of aërial echoes. The syren sounded three blasts a minute, each of 5 seconds duration. From the number of days and the number of hours per day during which the instrument was in action we can infer the number of blasts. They reached nearly twenty thousand. The blasts of the horns exceeded this number, while hundreds of shots were fired from the guns. Whatever might be the state of the weather, cloudy or serene, stormy or calm, the aërial echoes, though varying in strength and duration from day to day, were never absent; and on many days "under a perfectly clear sky" they reached, in the case of the syren, an astonishing intensity.

§ 9. *Experimental Demonstration of the stoppage of Sound by Aërial Reflection.*

The stoppage of sound by aërial reflection has never been experimentally demonstrated; it has been hitherto a matter of inference, and I wished to reduce it to experimental demonstration. A few preliminary experiments satisfied me that a carefully constructed apparatus would be necessary for the purpose. I therefore requested my assistant, Mr. Cottrell, who is eminently skilful in devising apparatus the object of which has been made clear to him, to make an arrangement by which alternate layers of carbonic acid and coal-gas, the one falling by its weight, the other rising by its lightness, should be obtained. After making in the first instance a rough model himself, he invoked the aid of an intelligent carpenter, and produced the instrument represented in section and plan in figs. 1, 2, & 3, Plate XVIII.

X Y (fig. 1) is the sectional elevation of a wooden tunnel with a glass front. B is a small bell enclosed in a carefully padded box with one opening, worked sometimes by electricity and sometimes by a small magneto-electric engine. The letters *a*, *b*, *c*, *d*, *e*, &c. indicate the front view of a series of pewter tubes about $\frac{3}{4}$ of an inch wide, bent over at the top, and inserted into a box behind X Y. The box is seen in plan at *r s*, fig. 2. Into it is inserted the tube *m n*, with an orifice and trunk-tube at *o*, this trunk-tube

being connected with an india-rubber bag filled with carbonic-acid gas. The apparatus looked at "end on" is shown in fig. 3. Here also is shown the section of a second box, b', in all respects similar to the first, furnished like the first with a tube behind, and connected by a trunk-tube with the gas-main of the Institution. The course pursued by the two gases when the cocks are turned on will be evident on inspecting fig. 3. The carbonic acid, entering from its box into the tunnel X Y, falls in layers across the tunnel, and escapes by a series of apertures placed vertically under its places of entrance; the lower arrows in fig. 1 mark the course of the carbonic acid. The coal-gas, on the other hand, entering from its box below rises in layers between the carbonic-acid layers, and escapes through a series of apertures at the top of the tunnel. The upper arrows in fig. 1 mark the course of the coal-gas. In this way five-and-twenty layers of the heavier gas, separated from each other by layers of the lighter one, are obtained, the limiting surfaces at which reflection takes place being therefore fifty in number.

With the ear properly defended and applied to the end of the tunnel, this apparatus proves effective; but an excellent objective test is furnished by one of the sensitive flames discovered by Lecomte, observed some years subsequently by Barrett, and described, with variations and extensions, in my sixth Lecture on Sound. Such a flame is repre sented at fig. 1 issuing from the burner S. It is protected from air-currents by a glass bulb surrounding its lower portion, and through the bulb passes the shank of a funnel intended to concentrate the sound.

The tube t leading to the sensitive flame being connected with a gas-holder, the bell B is set ringing, and the pressure on the gas is exalted until the flame F, acted upon by the sonorous waves from the bell, trembles violently and roars. A few preliminary trials instruct us as to the necessary pressure. The two gases are now turned on, and after a few seconds the light and heavy layers establish themselves within the tunnel; the sound is stopped, the roaring trembling flame becomes immediately tranquil and burns steadily at a multiple of its height when agitated. The stoppage of the sound by aërial reflections is thus demonstrated. When the tunnel is explored with the beam of the electric lamp, it is found perfectly clear optically. The cause which so powerfully affects the sound does not sensibly affect the light; and it will be shown subsequently that agents which powerfully affect light have no sensible influence upon sound. By cutting off the two gases their places are immediately taken by air, the homogeneity of the medium is restored, and the flame is again shortened to a fraction of its normal height and thrown into violent agitation. The experiment can be repeated at pleasure, with the same unfailing result.

Not only do gases of different densities act thus upon sound, but atmospheric air saturated in different degrees with the vapours of volatile liquids can be shown by experiment to produce the same effect. Introducing, for example, into the path pursued by the coal-gas in the last experiment one of the flasks which I have so frequently employed to charge air with vapour*, partially filling the flask with a volatile liquid,

* Philosophical Transactions, 1870, vol. clx. p. 337.

and permitting air to bubble through the liquid and enter the tunnel X Y, we divide the tunnel into spaces of air saturated with the vapour, and spaces of air in its ordinary condition. The action of such a medium upon the sound-waves issuing from the bell B is very energetic, instantly reducing the violently agitated flame to stillness and steadiness as long as it continues to be the vehicle of the sound. The removal of the heterogeneous medium instantly restores the noisy flaring of the flame.

That differences of density due to differences of temperature produce partial echoes and waste of sound is also capable of experimental demonstration. Across a tunnel resembling X Y sixty-six platinum wires were stretched, all of them being in metallic connexion. The bell B, in its padded box, was placed at one end of the tunnel, and the sensitive flame F, near its flaring-point, at the other. When the bell rang the flame flared. A current from 50 cells being sent through the platinum wires, layers of warm air rose from them through the tunnel, and immediately the agitation of the flame was stilled: on stopping the current the agitation recommenced. In this experiment the platinum wires were not heated to redness. Employing half the number and the same battery, they were raised to a red heat, the action in this case upon the sound-waves being also energetic. Employing one third of the number, with the 50 cells, the wires were raised to a white heat; and here also the flame was immediately rendered tranquil by the stoppage of the sound. A number of experiments executed by Mr. COTTRELL at my request showed that 66 wires at a dark heat act a little more strongly upon the sound-waves than 33 at a red heat or than 22 at a white heat*.

A few details of the experiments on the action of non-homogeneous atmospheres produced by the saturation of layers of air with the vapours of volatile liquids may follow here.

To secure a more uniform distribution of the layers of vapour in the tunnel X Y, the box *r s* (fig. 2, Plate XVIII.) was subsequently divided into three compartments, because, with the arrangement described in § 9, the air at the centre of the tunnel remained homogeneous after its two ends had become heterogeneous. In all cases the saturated air was forced in layers upwards, while common air was forced downwards; with the vapours of the liquids here mentioned the following results were obtained:—

Bisulphide of carbon.—Flame very sensitive; the action of the non-homogeneous atmosphere prompt and strong.

Chloroform.—Flame still very sensitive; action similar to the last.

Iodide of ethyl.—Action decided, but weak.

Iodide of methyl.—Action prompt and energetic.

Benzol.—Action decided, but very weak.

Amylene.—Very fine action, prompt and energetic. Rendered a short and violently agitated flame tall and quiescent.

Sulphuric ether.—Action prompt and energetic.

* Mr. COTTRELL himself has shown, in a very striking manner, the reflection of sound from a single layer of heated air (Proc. Roy. Soc. vol. xxii. p. 190). The action of a hot poker is also quite distinct.

Formic ether.—Action decided, but weak.
Alcohol.—No sensible action. A more delicate test is necessary*.
Acetic ether.—Action decided, but weak.
Nitrite of amyl.—Action uncertain.
Toluol.—No sensible action.
Valerianic ether.—No sensible action.
Butyric ether.—No sensible action.
Acetone.—Action very weak.

When the flasks containing iodide of ethyl, formic ether, and acetone were placed in warm water, their vapours exerted a strong action. The warming of valerianic ether, butyric ether, and alcohol, on the contrary, produced no sensible effect.

In all cases where an action was observable, the air-bags used to force air downwards might be suppressed, the saturated air only being forced across the tunnel; the result was the same as when common air was forced from bags in a direction opposed to that of the saturated air.

The action of non-homogeneous atmospheres is well shown by placing a ticking watch at 6 inches from the ear. The heated air-column from a Bunsen's rose-burner utterly stops the sound. I may add that all these results may be obtained with an apparatus only a fraction of the length of that first employed, and figured on Plate XVIII.

§ 10. *Action of Hail and Rain.*

The explanation here given is in harmony with other facts, while these facts are irreconcilable with prevalent notions. Derham, and after him all other writers, considered that falling rain tended powerfully to obstruct sound. I have already referred to an observation on June 3 which tended to throw doubt on this conclusion. Two other crucial instances will suffice to show its untenability. On the morning of October 8, at 7.45 A.M., a thunderstorm accompanied by heavy rain broke over Dover. But the clouds subsequently cleared away and the sun shone strongly on the sea. For a time the optical clearness of the atmosphere was extraordinary, the coast of France, the Grisnez lighthouse, and the Monument and Cathedral of Boulogne being clearly visible in positions from which they were generally quite hidden. The atmosphere at the same time was acoustically opaque. At 2.30 P.M. a densely black scowl again overspread the heavens to the W.S.W. At this hour, the distance being 6 miles, the horn was heard very feebly, the syren more distinctly. The howitzer was better than either, though not much superior to the syren. All was hushed on board during these observations.

A squall now approached us from the west. In the Alps or elsewhere I have rarely seen the heavens blacker. Vast cumuli floated in the N.E. and S.E.; vast streamers of rain were seen descending W.N.W.; huge scrolls of cloud to the N.; but spaces of blue were to be seen to the N.N.E.

* While this paper was passing through the press the action both of aqueous vapour and of the vapour of alcohol mixed with air upon sound was experimentally demonstrated.

At 7 miles distance the syren was not strong and the horn was very feeble.

At 3 P.M. the gun was fired, and it sent to us a very faint report, hardly equal to the sound of the syren. A dense shower now enveloped the Foreland.

The rain at length reached us; but although it was falling heavily all the way between us and the Foreland the sound, instead of being deadened, rose perceptibly in power. Hail was now added to the rain, and the shower reached a tropical violence. The deck was thickly covered with hailstones, which here and there floated upon the rain-water, the latter not having time to escape. We stopped. In the midst of this furious squall both the horns and the syren were distinctly heard; and as the shower lightened, thus lessening the local pattering, the sounds so rose in power that we heard them at a distance of $7\frac{1}{2}$ miles distinctly louder than they had been heard through the rainless atmosphere at 5 miles. This observation is entirely opposed to prevalent notions, but it harmonizes perfectly with the explanation of our experience on the 3rd of July, according to which water in the state of *vapour*, and so mixed with air as to form non-homogeneous parcels, acts powerfully in wasting sound. Under the action of a strong sun, prior to the rain, the air had been in this flocculent condition, but the descent of the shower restored in part the homogeneity of the atmosphere and augmented its transmissive power.

At 4 P.M. the rain had ceased and the sun shone clearly out: the air was calm afloat, but W., with a force of 2, ashore. At 9 miles distance the horn was heard feebly, the syren clearly, while the howitzer sent us a loud report. All the sounds were better at this distance than they had previously been at $5\frac{1}{2}$ miles; from which it follows that the intensity of the sound at $5\frac{1}{2}$ miles must have been augmented at least threefold by the descent of the rain.

The other instance to which I have to refer occurred on the 23rd of October. Our steamer had forsaken us for shelter, and I sought to turn the weather to account by making observations on the influence of the wind. Mr. DOUGLASS, the chief Engineer of the Trinity House, was good enough to undertake the observations N.E. of the Foreland; while Mr. AYRES, the Assistant Engineer, walked in the other direction. At 12.50 P.M. the wind blew a gale and broke into a thunderstorm with violent rain. Inside and outside the Cornhill Coastguard Station Mr. AYRES heard the sound of the syren distinctly through the storm, and after the rain had ceased all sounds were heard distinctly louder than before. Mr. DOUGLASS had sent a fly before him to Kingsdown; and the driver had been waiting for fifteen minutes before he arrived. During this time no sound had been heard, though 40 blasts had been blown; nor had the coastguard man on duty, who had been accustomed to observe the sounds, heard any of them throughout the day. During the thunderstorm, and while the rain was actually falling with a violence which Mr. DOUGLASS assures me was more like the descent of a water-spout than of ordinary rain, the sounds became audible and were heard by all. Prior to the thunderstorm the atmosphere, according to my explanation, was rendered acoustically flocculent by wreaths and streaks of mixed air and vapour. Through the precipitation of the

vapour during the storm the heterogeneity thus arising was in great part abolished, and a freer passage opened for the sound through the atmosphere.

To rain, finally, I have never been able to trace the slightest deadening influence upon sound. The reputed barrier offered by "thick weather" to the passage of sound was one of the causes which tended to produce hesitation in establishing sound-signals on our coasts. It is to be hoped that the removal of this error may redound to the advantage of coming generations of seafaring men.

§ 11. *Action of Snow.*

Falling snow, according to DERHAM, offers a more serious obstacle than any other meteorological agent to the transmission of sound. We have not extended our observations at the South Foreland into snowy weather; but I may be permitted to refer to an observation of my own which bears directly upon this point. On Christmas night, 1859, I arrived at Chamouni, through snow so deep as to obliterate the road-fences, and to render the labour of reaching the hamlet arduous in the extreme. On the 26th and 27th it fell heavily. On the 27th, during a lull in the storm, I reached the Montanvert, sometimes breast-deep in snow. On the 29th the entry in my journal is, "Snow, heavy snow; it must have descended through the entire night, the quantity freshly fallen is so great." Dr. DERHAM had referred to the deadening effect produced by a coating of fresh fallen snow upon the ground, alleging that when the surface was glazed by freezing the damping of the sound disappeared.

On December 29 I took up a position beside the Mer de Glace, with a view to determine its winter motion, and sent my assistants across the glacier with instructions to measure the displacement of a transverse line of stakes planted previously in the snow. I was standing at the time beside my theodolite, having waded to the position through snow which, being dry, reached nearly to my breast. A storm drifted up the valley, darkening the air as it approached. It reached us, the snow falling more heavily than ever I had seen it elsewhere. It soon formed a heap on the theodolite; still through the telescope I was able to pick up at intervals the retreating forms of the men. Here there was a combination of thick snow in the air, and of soft fresh snow on the ground such as DERHAM could hardly have enjoyed. Through such an atmosphere, however, I was able with my unaided voice to make my instructions audible for half a mile, while the experiment was rendered reciprocal by one of my assistants* making his voice audible to me.

Many years ago I mentioned this fact in the presence of Sir JOHN HERSCHEL, and I have a distinct recollection of the surprise with which he heard it. And, indeed, in relation to our previous convictions, it is simply astonishing to observe the facility with which sound makes its way among obstacles, and even penetrates solid bodies, so long as the continuity of the air in their interstices is preserved. The following experiments illustrate this.

* Mr. JOSEPH TAIRRAZ, now a photographer at Chamouni.

A piece of millboard or of glass, a plank of wood, or the hand placed across the open end Y of the tunnel X Y (Plate XVIII.) intercepts the sound of the bell B and stills the sensitive flame F.

An ordinary cambric pocket-handkerchief stretched across the tunnel end produced hardly an appreciable effect upon the sound, the flame being sensibly as much agitated when it was present as when it was absent. Sending the sound through two layers of the handkerchief, the flame continued to be much agitated; through four layers the flame was still agitated, while through six layers the flame, though nearly stilled, was not entirely so.

Dipping the same handkerchief in water, and stretching a single wetted layer across the tunnel, it stilled the flame as effectually as the millboard or the plank of wood. It is obvious therefore that the sound-waves in the first instance had passed through the interstices of the cambric.

Through a single layer of a thin silk handkerchief the sound passed without sensible interruption; through six layers of the same handkerchief the flame was strongly agitated; while through twelve layers the agitation was quite perceptible. Looking at the sun, a feeble luminosity was perceived through six layers of the silk, while twelve layers totally intercepted the light. It would be easy to multiply instances such as this of bodies opaque to light and transparent to sound.

A single layer of this silk, when wetted, stilled the flame.

A layer of soft lint produced but little effect upon the sound; a layer of thick flannel was almost equally ineffectual. Through four layers of the flannel the flame was perceptibly agitated by the sound. Through a single layer of green baize the sound passed almost as freely as through air; through four layers of the baize the action was still sensible. Through a layer of close hard felt, half an inch thick, the sound-waves passed with sufficient energy to sensibly agitate the flame.

Oiled silk has no sensible interstices; a single thin layer of this substance stopped the sound and stilled the flame. A single layer of goldbeater's skin did the same. A leaf of common note-paper, or even of foreign post, stopped the sound. A single column of heated air rising from a Bunsen's flame in front of the tunnel has been proved by Mr. Cottrell sufficient to still the sensitive flame.

The sensitive flame is not absolutely necessary for these experiments. Let a ticking watch be hung 6 inches from the ear, a cambric handkerchief dropt between it and the ear scarcely sensibly affects the ticking, a sheet of oil-skin or a heated gas column cuts it almost wholly off.

But though oiled silk, foreign post, and even goldbeater's skin can stop the sound, when the film becomes sufficiently thin to yield freely to the aërial pulses the sound is transmitted through the film. A thick soap-film produces a sensible effect upon the flame, a very thin one does not. The augmentation of the transmitted sound may be observed simultaneously with the generation and brightening of the colours of the film. A thin collodion-film acts in the same way.

Acquainted with the foregoing facts regarding the passage of sound through cambric, flannel, baize, and felt, the reader will be prepared for the statement that the sound-waves pass without sensible impediments through heavy artificial showers of rain, hail, and snow.

§ 12. *Action of Fog. Observations in London.*

But the mariner's greatest enemy, fog, is still to be dealt with; and here for a long time the proper conditions of experiment were absent. Up to the end of November we had had frequent days of haze, sufficiently thick to obscure the white cliffs of the Foreland, but no real fog. Still those days furnished demonstrative evidence that the notions entertained regarding the reflection of sound by suspended particles were wrong. On many days of the thickest haze the sound had twice the range that it attained in other days of perfect optical transparency. Such instances dissolved the association hitherto assumed between acoustic and optic transparency, but they left the action of dense fogs undetermined.

I conferred with the Elder Brethren as to the possibility of transferring the instruments to some portion of the coast more favoured by fogs than the South Foreland; but after due consideration the probability of spring fogs seemed so great, that it was decided to leave the instruments undisturbed and to await the expected appearance of the fog. Other duties called me to London, where on December 9th a memorable fog set in. I telegraphed to the Trinity House, suggesting some gun observations. A prompt reply informed me that such observations would be made in the afternoon at Blackwall or in its neighbourhood. I went to Greenwich in the hope of hearing the guns across the river; but, owing to the delay caused by the fog, the firing had ended before I arrived. Over the river the fog was very dense, and through it came various sounds with great distinctness. The signal-bell of an unseen barge rang clearly out at intervals, and I could hear the hammering at Cubittstown, on the opposite side of the river. So distinct were the strokes that they appeared close at hand, though the works were upwards of half a mile away; no deadening of the sound by the fog was apparent.

Captain Atkins, assisted by Mr. Edwards, took charge of the gun-observations. Up to a distance of 2 miles the report was good; at $2\frac{1}{4}$ miles, with much land intervening, the report was still heard. The firing was from a 12-pounder carronade with a charge of 1 lb. of powder; and this was heard through a dense fog distinctly better than the 18-pounder with a 3-lb. charge, and an optically clear atmosphere, on the 3rd of July.

The fog continuing unabated, on December 10 I sought to make some experiments upon a small scale over the Serpentine. I chose three organ-pipes of different lengths, a dog-whistle, a small bell struck by mechanism, and some percussion-caps. These were well heard across the Serpentine near the Watermen's Boathouse. At the same time I could converse with ease with my assistant across the water. The bell, the percussion-caps, and the shortest organ-pipe proving least effective, in subsequent experiments they were discarded in favour of the others.

In reply to my questions I was informed by two very intelligent policemen that they had

heard the bell of Westminster with great distinctness to-day, but that it had been heard still more loudly last night through exceedingly dense fog. On many clear days, they informed me, they fail to hear it at all. To-day the clangour at 12 o'clock was very loud.

At 3 P.M. I went again to the Serpentine; stationed my assistant, Mr. COTTRELL, with a whistle and organ-pipe on the walk below the south-west end of the bridge dividing Hyde Park from Kensington Gardens, while I went to the eastern end of the Serpentine. There I heard distinctly both the whistle and the pipe sounding Mi_3, which corresponds to 380 waves a second. The whistle was best heard. I then changed places with my assistant, and listening attentively at the bridge heard for a time the distinct blast of the whistle only. The organ-pipe at length sent its deeper note to me across the water; it sometimes rose to great distinctness, and sometimes fell to inaudibility. The whistle showed the same intermittence as to period, but in the opposite sense; for when the whistle was faint the pipe was strong, and *vice versâ*. To obtain the fundamental note of the pipe it had to be blown gently, and on the whole the whistle-sound proved itself the most efficient in piercing the fog.

There seemed to me to be an extraordinary amount of sound in the air on Dec. 10; it was filled with a resonant roar from the Bayswater and Knightsbridge roads. The railway-whistles, which were frequently blown, were extremely distinct, while the fog-signals exploded at the various metropolitan stations kept up a loud and almost constant cannonade. I could by no means reconcile this state of things with the statements so categorically made regarding the influence of fog on the sound of carriage-wheels and guns.

The Serpentine presented the instructive appearance to which I have already referred. The water was warmer than the air, and the ascending vapour was instantly in great part condensed, thus revealing its distribution. Instead of being uniformly diffused, it formed wreaths and striæ. I am pretty confident that had the vapour been able to maintain itself as such, the air to-day would have been more opaque to sound. In other words, I believe that the very cause which diminished the optical transparency of the atmosphere augmented its acoustic transparency.

On the 11th of December, the fog being denser than before, at 2.50 I stationed Mr. COTTRELL near the bridge, and took up a position myself near the end of the Serpentine. The whistle and the Mi_3 pipe were sounded in succession. I heard every blast of the whistle, and occasional blasts of the pipe. We reversed our positions, and I heard substantially the same, perhaps a little fainter. On joining my assistant at the bridge I heard the loud concussion of a gun, and was informed by a police-inspector that it came from Woolwich, and that he had heard several shots about 2 P.M. and previously. The fact, if a fact, was of the highest importance in relation to the present question; so I immediately telegraphed to my friend Professor ABEL, asking him for information. He was absent in Portsmouth when the telegram arrived, but on his return he found that guns had been fired at the proof-butts about the time mentioned in my telegram. On the following day he kindly furnished me with the following particulars:—

"The firing took place at 1.40 P.M. The guns proved were of comparatively small size—64-pounders, with 10-lb. charges of powder.

"The concussion experienced at my house and office, about three quarters of a mile from the butt, was decidedly more severe than that experienced when the heaviest guns are proved with charges of 110 to 120 lbs. of powder. There was a dense fog here at the time of firing."

These were the guns heard by the inspector; but on subsequent inquiry it was ascertained that two guns were fired at about 3 P.M. These were the guns heard by myself.

Professor ABEL also communicated to me the following fact:—"Our workmen's bell at the Arsenal Gates, which is of moderate size and any thing but clear in tone, is pretty distinctly heard by Professor BLOXAM *only* when the wind is *north-east*. During the whole of last week the bell was heard with great distinctness, the wind being *south-westerly*. The distance of the bell from BLOXAM's house is about three quarters of a mile as the crow flies."

Slowly but surely we thus master this question; and the further we advance the more we are assured that our reputed knowledge regarding it has been erroneous from beginning to end. Fogs have no such power to deaden sound as, since the time of DERHAM, has been universally ascribed to them.

The vane on St. George's Chapel, Albemarle Street, pointed west during these observations, but, judged by sensation, the air was dead calm. On the night of the 11th the vane veered, and on the morning of the 12th it pointed N.E. The fog now attained its maximum density. I was unable to read at my window, which fronted the open western sky. At 10.30 I sent an assistant to the bridge, and listened for his whistle and pipe at the eastern end of the Serpentine. The whistle rose to a shrillness far surpassing any thing that I had previously heard, but it sank sometimes almost to inaudibility. At these special times of subsidence the organ-pipe came out distinctly. Acoustic clouds were drifting through the fog, and exercising a selective action on the sound. A second pipe, which was quite inaudible yesterday, was heard this morning. The fog was very dense.

I exchanged places with my assistant and heard both the whistle and the pipe in my new position; they were generally on the verge of inaudibility, and the pipe was only occasionally heard. Sometimes, however, both of them rose to great distinctness. The rise and fall of the sound resembled the intermittent augmentation and diminution of light by the passage of clouds of varying density across the sun. We were able to discourse across the Serpentine to-day with much greater ease than yesterday.

On various occasions during our observations I had been able to fix the position of the Foreland in thick haze, and even in the acoustic shadow, by the direction of the sound. To-day I gave directions to Mr. COTTRELL to come up to the Watermen's Boathouse and sound his whistle as he came. He was entirely hidden by the fog; and I walked along the opposite side of the Serpentine, clearly appreciating for a time that the line joining us was oblique to the axis of the river. At length I came to a point

which I supposed to be exactly abreast of him, marked it, and on the following day, when the fog had cleared away, found that I was perfectly exact. If undisturbed by echoes, the ear, with a little practice, becomes capable of fixing with great sharpness the direction of a sound.

On reaching the Serpentine this morning the clangour of a peal of bells, which then began to ring, seemed so close at hand that it required some reflection to convince me that they were ringing to the north of Hyde Park. The sounds fluctuated wonderfully in power, sometimes pouring forth a wealth of sound and then sinking to sudden penury. They disclosed to the mind's eye the condition of the atmosphere through which these varying sounds were transmitted. Prior to the striking of eleven by the great bell of Westminster, a nearer bell struck with loud clangour. The first five strokes of the Westminster bell were afterwards heard, one of them being extremely loud; the six last strokes were inaudible. I subsequently stationed my second assistant to attend to the 12-o'clock bells. The clock which had struck so loudly at 11 was unheard at 12, while of the Westminster bell eight strokes out of twelve rendered themselves audible. To such astonishing changes is the atmosphere liable.

Wishing to test still further the acoustic fluctuations of the day, I sent Mr. COTTRELL and a younger assistant at 7 P.M. to the Serpentine. The Westminster bell striking seven was not at all heard, while the nearer bell already alluded to was heard distinctly. The fog had now cleared away, and the lamps on the bridge could be seen from the eastern end of the Serpentine burning brightly; but instead of the sound sharing the improvement of the light, what might be properly called an acoustic fog took the place of its predecessor. The whistle and organ-pipe were sounded, three blasts of each in succession, several times; one series only of the whistle was heard, all the other blasts being quite inaudible. Three series of the organ-pipe were heard, but exceedingly faintly. On reversing their positions and sounding as before, nothing whatever was heard.

At 8 o'clock the chimes and hour-bell of the Westminster clock were very loud. The "acoustic fog" had shifted its position or temporarily melted away.

An assistant placed at the end of the Serpentine sounded the whistle and pipe for fifteen minutes without interruption. An observer at the bridge noticed the fluctuations of the sound. Sometimes the whistle was heard alone, sometimes the organ-pipe. Sometimes both whistle and pipe began strongly and ended by sinking almost to inaudibility. Extraordinary fluctuations were also observed in the case of the bells to which reference has been already made: in a few seconds they would sink from a loudly ringing peal into utter silence, from which they would rapidly return to loud-tongued audibility. The intermittent drifting of fog over the sun's disk (by which his light is at times obscured, at times revealed) is, as already stated, the optical analogue of these acoustical effects. In fact, as regards such changes, the acoustic deportment of the atmosphere is a true transcript of its optical deportment.

At 9 P.M. three strokes only of the Westminster clock were heard; the others were inaudible. The air had in part relapsed into its condition at 7 P.M., when all the strokes

were unheard. The quiet of the park this evening, as contrasted with the resonant roar which filled the air on the two preceding days, was very remarkable. The sound, in fact, was stifled in the optically clear but acoustically more flocculent atmosphere.

On the 13th, the fog being displaced by a thin haze, I went with my assistants again to the Serpentine. We could plainly see from one bank to the other, and far into Hyde Park beyond. When I rose this morning the vane pointed S.E., but before starting it had moved on to N.W., and on our return it pointed W. The motion of the air, however, was of the lightest kind, at no time reaching a force of 1. The carriage-sounds were damped to an extraordinary degree. The roar of the Knightsbridge and Bayswater roads had subsided, the tread of troops which passed us a little way off was unheard, while at 11 A.M. both the chimes and the hour-bell of the Westminster clock were stifled. The air had become milder; the hoar-frost had disappeared from the grass, though a residue still overspread the walks. At the end of the Serpentine I listened for the sounds from the bridge, and heard them. The pipe was in general the best, but all sounds were exceedingly faint, and the whistle was often inaudible. We reversed our positions, and I at the bridge listened for the sounds excited at the end of the Serpentine. With the utmost stretch of attention I could hear nothing.

This failure of hearing occurred upon a day when the local noises were far less disturbing than they had previously been. The sounds of the carriage-wheels were low and muffled, the bells were for the most part inaudible, and, with the exception of one far faint sound, the locomotive-whistles were unheard. Subjectively considered all was favourable to auditory impressions; but the very cause that damped the local noises extinguished our experimental sounds. The voice across the Serpentine to-day, with my assistant plainly visible in front of me, was distinctly feebler than it had been when each of us was hidden from the other in the densest fog.

I sought to obtain a numerical estimate of the decay of the sound, and with this view stationed Mr. COTTRELL at the eastern end of the Serpentine, while I walked along its edge from the bridge towards the end. The distance between these two points is about 1000 paces. After I had stepped 500 of them the sound was not so distinct as it had been at the bridge on the day of densest fog: hence the optical cleansing of the air by the melting of the fog had so darkened it acoustically, that a sound generated at the end of the Serpentine was lowered to at most one fourth of its intensity at a point midway between the end and the bridge.

I add two or three more of these domestic observations, and then pass to others made with actual fog-signals at the South Foreland. On several of the first days of this year I placed myself beside the railing of St. James's Park, near Buckingham Palace, three quarters of a mile from the clock-tower. Not a single stroke of "Big Ben" was heard at noon. These days were moist and warm, the air was calm, and the clock-tower in sight. On January 19 fog and drizzling rain obscured the tower; still from the same position I heard not only the strokes of the bell but also the preceding chimes of the quarter bells. The air was calm at the time.

During the exceedingly dense and "dripping" fog of January 22 I placed myself near the same railings and heard every stroke of the bell. On the same day an assistant at the end of the Serpentine, when the fog was densest, heard the Westminster bell striking loudly eleven. Towards evening this fog began to melt away, and at 6 o'clock I went to the end of the Serpentine to observe the effect of the optical clearing of the atmosphere upon the sound. Not one of the strokes reached me. At 9 o'clock and at 10 o'clock my able assistant Mr. COTTRELL was in the same position, and on both occasions failed to hear a single stroke of the bell. It was a case precisely similar to that of December 13, when the dissolution of the fog was accompanied by a decided acoustic thickening of the air.

On the morning of the 5th of February a dense fog filled London. At 10.45 I went to the Green Park and heard the chimes of the great bell in a position where on many clear calm days no sound had been audible. I walked thence to the eastern end of the Serpentine and heard the chimes at 11 A.M. Eight of the subsequent strokes of the bell reached me with marked power, the ninth and tenth (doubtless through the passage of an acoustic cloud) sank almost to inaudibility, while in the eleventh the sonorous power was restored. The air seemed dead calm at the time; no observable motion could be seen among the lightest twigs of the adjacent trees, but my breath was wafted in a direction opposed to the sound. Such observations show what instructive results may be obtained from a mode of observation accessible to all.

§ 13. *Experiments on Artificial Fogs.*

The smoke from smouldering brown paper was allowed to stream upwards into the tunnel X Y, Plate XVIII.; the action upon the sound-waves was strong, rendering the short and agitated sensitive flame F tall and quiescent.

Instead of the smoke and heated air, the heated air alone from four red-hot pokers was permitted to stream upwards into the tunnel; the action on the sound-waves was very decided, though the tunnel was optically empty.

A thick fog of chloride of ammonium was sent into the tunnel; a candle placed at one end could not be seen at the other, still there was no appreciable action upon the waves of sound.

Air first passed through ammonia, and then through hydrochloric acid, was sent into the tunnel; the agitated flame was rendered immediately quiescent, indicating a very decided action on the sound-waves. The flask containing the hydrochloric acid was, however, very warm, suggesting that differences of air-temperature might have come into play.

Air passed through perchloride of tin and sent into the tunnel produced exceedingly dense fumes. The action on the sound-waves was very strong.

The dense smoke of resin, burnt before the open end of the tunnel and blown into it with a pair of bellows, had the effect of stopping the sound-waves, so as to still the agitated flame.

Were these results due to the fumes or to differences of temperature? To the latter. The flame of a candle was placed at the tunnel end, and the hot air just above its tip was blown into the tunnel; the action on the sensitive flame was decided.

A red-hot iron was placed in the same position, and the heated air blown into the tunnel; the action on the sound-waves was decided.

In both these cases the tunnel remained optically clear, while the same effect as that produced by resin, gunpowder, and phosphorus was observed. To differences of temperature, therefore, and not to the fumes, the stoppage of the sound-waves in these cases was probably due.

To arrive at certainty on this head, instead of the tunnel a cupboard with glass sides, constructed some years ago for experiments on the floating matter of the air, was employed. It was 3 feet long, 2 feet wide, and about 5 feet high. In the closed cupboard it was thought fumes might be generated and permitted to remain until differences of temperature had sensibly disappeared. Two apertures were made in two opposite panes of glass 3 feet asunder; in front of one aperture was placed the bell, and behind the other, and at some distance from it, the sensitive flame.

The flame being brought into that condition that a very slight action on the sound-waves sufficed to reduce it from agitation to quiescence, phosphorus placed in a cup floating on water was burnt within the closed cupboard. The fumes were very dense. Considerably less than the 3 feet traversed by the sound sufficed to extinguish totally a bright candle-flame. At first there was a slight action upon the sound; but though the cloud remained of the same sensible density, the action rapidly vanished, and the flame was affected, as if the sound passed through pure air.

The cupboard was next filled with the dense fumes of gunpowder. At first there was a slight action; but this disappeared more rapidly than in the case of the phosphorus, the sound passing as if no fumes were there. It required less than half a minute to abolish the action in the case of the phosphorus, but a few seconds sufficed in the case of the gunpowder. The fumes were far more than sufficient to quench the candle-flame.

The smoke of resin, very dense and white, was next introduced. With a density far more than sufficient to quench the candle, the action on the sound-pulse was sensibly *nil*.

In this case the smoke was produced by bringing hot irons into contact with the resin. The same experiment was adopted with gum-mastic, the fumes of which produced no effect upon the sensitive flame.

The fumes of the perchloride of tin, though of extraordinary density, exerted no sensible effect upon the sound.

Exceedingly dense fumes of chloride of ammonium next filled the cupboard. A fraction of the length of the 3-foot tube sufficed to quench the candle-flame. Soon after the cupboard was filled the sound passed without the least sensible deterioration. An aperture at the top of the cupboard was opened; but though a dense smoke-column

ascended through it, many minutes elapsed before the candle-flame could be seen through the attenuated fog.

Steam from a copper boiler was so copiously admitted into the cupboard as to fill it with a dense cloud. No real cloud was ever so dense, still the sound passed through it without the least sensible diminution. This being the case, cloud-echoes, I think, are not a likely phenomenon.

In any and all of these cases, when a couple of Bunsen's burners were ignited within the cupboard containing the fumes, less than a minute's action rendered the air so heterogeneous that the sensitive flame was completely stilled.

The same occurred when the air within the cupboard was optically clear.

The foregoing experiments were repeated, the density of the fumes being tested by their action on the electric light. The density of these acoustically inactive fogs was such as to cut off the light totally, even when concentrated by a lens. The fumes themselves were illuminated by diffused light, but the carbon-points could not be seen.

The action of rain, hail, and snow was imitated by dense showers of water, sand, seeds, and bran: their action, within the limits employed, was *nil*.

§ 14. *Observations at the South Foreland.*

Satisfactory and, indeed, conclusive as these results were, I desired exceedingly to test in fog the instruments actually employed at the South Foreland. On the 10th of February it gave me great pleasure to receive the following note and enclosures from the Deputy Master of the Trinity House:—

"My dear Tyndall,—The enclosed will show how accurately your views have been verified, and I send them on at once without waiting for the details. I think you will be glad to have them, and as soon as I get the report it shall be sent to you. I made up my mind ten days ago that there would be a chance in the light foggy-disposed weather at home, and therefore sent the 'Argus' off at an hour's notice, and requested the Fog Committee to keep one member on board. On Friday I was so satisfied that the fog would occur that I sent Edwards down to record the observations. . . . .

"Very truly yours,

"Fred. Arrow."

The enclosures referred to were notes from Capt. Atkins and Mr. Edwards. Capt. Atkins writes thus to the Chairman of the Fog-Signal Committee:—

"As arranged I came down here by the mail express, meeting Mr. Edwards at Cannon Street. We put up at the 'Dover Castle,' and next morning at 7 I was awoke by the sounds of the syren. On jumping up I discovered that the long-looked-for fog had arrived, and that the 'Argus' had left her moorings.

"However, had I been on board, the instructions I left with Troughton [the Master of the 'Argus'] could not have been better carried out. About noon the fog cleared up and the 'Argus' returned to her moorings, when I learned that they had taken both syren- and horn-sounds to a distance of 11 miles from the station, where they dropt a

buoy. This I know to be correct, as I have this morning recovered the buoy, and the distances both in and out agree with TROUGHTON'S statement. I have also been to the Varne light-ship (12¾ miles from the Foreland), and ascertained that during the fog of Saturday forenoon they 'distinctly' heard the sounds."

Mr. EDWARDS, who was constantly at my side during our summer and autumn observations, and who is thoroughly competent to form a comparative estimate of the strength of the sounds, informed the Deputy Master that the sounds were extraordinarily loud, both Capt. ATKINS and himself being awoke by them. He does not remember ever before hearing the sounds so loud in Dover; it seemed as though the observers were close to the instruments.

I here append Capt. ATKINS'S account of the observations made at the South Foreland during three days of fog culminating in the dense fog just referred to.

"Arrangements had been made for the 'Argus' remaining at Dover to wait for foggy weather, during which the instruments at the South Foreland might be tried.

"On Thursday, February 5, Mr. TROUGHTON, the master of the 'Argus,' reports that at 9 A.M. a dense fog-bank hung over the land, but that to seaward the atmosphere was clear. As the 'Argus' proceeded out to get on the S.S.W. line, the syren and horn were heard continually, the sounds being clear and distinct; but the noise of the paddles stifled the whistle-sound, which had been previously heard at Dover Pier while the vessel was still.

"At 9.52 reached the N.E. Varne buoy, 7¾ miles S.S.W. of the South Foreland and in the axis of the syren and horn. At this distance the syren- and horn-sounds were audible, although not strong; but as the tide carried the vessel towards the Varne light-ship the sounds diminished, and at 1½ mile S.W. by W. of the buoy all sounds were lost. This was 9 miles from the station.

"Proceeded on to the Varne light-ship, 12¾ miles, the master of which vessel reports 'horns feeble in the N.E.' from 8.5 until 10.15 A.M. At 10.30 the 'Argus' proceeded N.E. by E., and at 3 miles from the light-ship (*i. e.* 9¾ miles from South Foreland) picked up the sounds of syren and horn with paddles going, and carried them into the Varne buoy, where at 11.8 the fog had cleared and the instruments ceased sounding.

"On this morning the fog was quite local and did not extend to sea; it hung about the land, but the sea horizon was clear and well defined.

"On Friday, February 6, Mr. TROUGHTON reports that at 8.25 A.M. there was a dense fog inshore, but that it was clear to seaward, although, as the morning advanced, there seemed to be drifting patches of fog and alternate spaces of thick and clear atmosphere. The sea was calm.

"At 8.25, by Dover Pier, the horn and syren were plainly heard, but the whistle was inaudible. At 9 A.M. proceeded from pier to N.E. Varne buoy, and carried the sound of the syren only out to the buoy; but the sound was not useful. Stopped at the buoy, 7¾ miles, and heard feeble but distinct sounds from both syren and horn, the whistle being inaudible. Drifted with tide half a mile S.W. of the buoy, and there lost all

sounds. Returned and recovered faint sounds at the buoy. Fog cleared at 10.15 and instruments ceased."

On Saturday morning, February 7, a very dense fog prevailed, apparently thicker at sea than on land; but on shore objects were invisible at between fifty and a hundred yards distance.

"At 8.15 A.M. the three sounds through the fog were astonishingly powerful, the syren particularly filling all the air with a loud and full sound. There was not the slightest difficulty in at once indicating the exact direction from which the sound proceeded.

"By 9 A.M. the atmosphere had begun to clear in the west, and as the day advanced the fog rolled slowly eastward. At 10 A.M. the line of demarcation between fog and clear atmosphere was a little to the eastward of the pier; and there was a marked difference between the sounds at this period and those of the early morning, a very sensible diminution in power being noticeable.

"Mr. TROUGHTON'S report of this last day's observations is to the following effect:—At 8 A.M. dense fog all round; frosty: steamed out to get upon bearing of E.S.E. from lighthouse, *i. e.* at right angles to the axial line of the syren and horn.

"At 10.10, supposing that the vessel was not far from the E.S.E. line, and (allowing for the set of the tide and distance run as shown by the patent log) the distance from the lighthouse being apparently near 11 miles, stopped the vessel and heard syren and horn equally powerful, the sounds being thoroughly good and serviceable.

"Mr. TROUGHTON further states that he could locate the position from whence the sound proceeded with the greatest ease, and that he took his bearing from the South Foreland by the aid of the sound alone.

"At this position a wreck-buoy was dropped, the fog being at the time exceedingly thick.

"At 10.18 the fog began to break, and the sounds were not again heard.

"At the Varne light-ship, 12¾ miles from the lighthouse, the master reports that from 8 A.M. to 10.30 A.M. they heard the 'horns distinct but not loud.'

"At the South Sand Head light-ship the sounds were plainly heard on Thursday, Friday, and Saturday during the fog."

The results here recorded are of the highest importance, for they bring us face to face with a dense fog and an actual fog-signal, and confirm in the most satisfactory manner the previously recorded observations. The fact of Captain ATKINS and Mr. EDWARDS being awakened by the sound of the syren, proves, beyond all our previous experience, the power of the sound on the 7th of February.

It is important also to note that through the same fog the sounds were well heard at the South Sand Head light-vessel, which is in the opposite direction from the South Foreland, and, as will immediately appear, actually behind the syren.

It is exceedingly interesting to compare the transmission of sound on February 7

with its transmission on October 14, when the wind had the same strength and the same direction, namely N.N.W. with a force of 2. My notes of the observations show that the latter was throughout a day of extreme optical clearness. The range was 10 miles. On February 7 the 'Argus' heard the sound at 11 miles; and it was also heard through the densest fog at the Varne light-vessel, which is $12\frac{3}{4}$ miles from the Foreland.

But this important circumstance is also to be borne in mind: on February 7 the syren happened to be pointed not towards the 'Argus,' but towards Dover. Had the yacht been in the axis of the syren, it is highly probable that the sound might have been heard all the way across to the coast of France.

The fact that Mr. TROUGHTON was able through the fog to "locate" the Foreland, and that his fixing of the direction should be subsequently found correct, is also one of great importance.

Early in the morning of the 20th of February there was also a fog in the neighbourhood of the South Foreland. It was a patchy fog, at intervals very thick and then comparatively clear. The 'Argus' steamed out at 8 A.M., and carried the sounds to a distance of 10 miles along the axial line of the instruments. At that distance the horn-sounds were better than those of the syren, although at shorter distances the syren had greatly excelled the horn. At the Varne light-ship ($12\frac{3}{4}$ miles) the horn-sounds only had been heard during the fog. At the 10-miles position the atmosphere was beautifully clear, the sun shining strongly, and the fog-bank lying over the land clearly defined, extending seaward about 5 miles.

Mr. TROUGHTON states that the sounds of this day were very inferior to those of the much thicker and more homogeneous fog of February 7.

Since the publication of the first notices of this investigation various communications have reached me, to two of which I should like to refer. The Rev. GEORGE H. HETLING, of Fulham, has written to me with a circumstantiality which leaves no room for doubt that he has heard the Portland guns at a distance of 44 miles through a dense fog.

The Duke of ARGYLL has also favoured me with the following very interesting account of his own experience. Coming as it does from a disciplined scientific observer, it is particularly valuable. "This fact" (the permeability of fog by sound) "I have long known, from having lived a great part of my life within four miles of the town of Greenock, across the Frith. Ship-building goes on there to a great extent, and the hammering of the caulkers and builders is a sound which I have been in the habit of hearing with every variety of distinctness, or of not hearing at all, according to the state of the atmosphere; and I have always observed on days when the air was very clear, and every mast and spar was distinctly seen, hardly any sound was heard; whereas on thick and foggy days, sometimes so thick that nothing could be seen, every clink of every hammer was audible, and appeared sometimes as close at hand."

It is hardly necessary for me to say a word to guard myself against the misconception that I consider sound to be assisted by the fog itself. Fog I regard as the visible result

of an act of condensation, which renders homogeneous the acoustically flocculent or turbid air. The fog-particles appear to have no more influence upon the waves of sound than the suspended particles stirred up over the banks of Newfoundland have upon the waves of the Atlantic.

§ 15. *Atmospheric Selection.*

It has been stated in § 3 that the atmosphere on different days shows preferences to different sounds. This point is slightly touched upon in the record of the Serpentine observations; but it is worthy of further illustration.

After the violent shower which passed over us on October 18th, the sounds of all the instruments, as already stated, rose in power; but it was noticed that the horn-sound, which was of lower pitch than that of the syren, improved most, at times not only equalling, but surpassing the sound of its rival. From this it might be inferred that the atmospheric change produced by the rain favoured more especially the transmission of the longer sonorous waves.

But our programme enabled us to go further than mere inference. It had been arranged that up to 3.30 P.M. the syren should perform 2400 revolutions a minute, generating 480 waves a second. As long as this rate continued, the horn, after the shower, had the advantage. The rate of rotation was then changed to 2000 a minute, or 400 waves a second, when the syren-sound immediately surpassed that of the horn. A clear connexion was thus established between aërial reflection and wave-length.

The 10-inch Canadian whistle being capable of adjustment so as to produce sounds of different pitch, on the 10th of October I ran through a series of its sounds. The shrillest appeared to possess great intensity and penetrative power. The belief that a note of this character (which affects so powerfully, and even painfully, an observer close at hand) has also the greatest range is a common one. Mr. A. GORDON, in his examination before the Committee on Lighthouses in 1845, expressed himself thus:—"When you get a shrill sound, high in the scale, that sound is carried much further than a lower note in the scale." I have heard the same opinion expressed by other scientific men.

On the 14th of October the point was submitted to an experimental test. It had been arranged that up to 11.30 A.M. the Canadian whistle, which had been heard with such piercing intensity on the 10th, should sound its shrill note. At the hour just mentioned we were beside the Varne buoy, 7¾ miles from the Foreland. The syren, as we approached the buoy, was heard through the paddle-noises; the horns were also heard, but more feebly than the syren. We paused at the buoy and listened for the 11.30 gun. Its boom was heard by all. Neither before nor during the pause was the shrill-sounding Canadian whistle once heard. It was now adjusted to produce its ordinary low-pitched note, which was immediately heard. Still further out the low boom of the cannon continued audible after all the other sounds had ceased.

But it was during the early part of this day only that this preference for the longer

waves was manifested. At 3 P.M. the case was completely altered, for then the high-pitched syren was heard when all the other sounds were inaudible. On many other days we had illustrations of the varying comparative power of the syren and the guns. On the 9th of October sometimes the one, sometimes the other was predominant. On the morning of the 13th the syren was clearly heard on Shakespeare's Cliff, where two guns with their puffs perfectly visible were unheard. On October 16, 2 miles from the signal-station, the gun at 11 o'clock was inferior to the syren, but both were heard. At 12.30, the distance being 6 miles, the gun was quite unheard, while the syren continued faintly audible. Later on in the day the experiment was twice repeated. The puff of the gun was in each case seen, but nothing was heard; in the last experiment, when the gun was quenched, the syren sent forth a sound so strong as to maintain itself through the paddle-noises. The day was clearly hostile to the passage of the longer sonorous waves. I may anticipate matters so far as to say that on this day the syren heard at Shakespeare's Cliff surpassed the gun, while at the South Sand Head light-vessel the gun surpassed the syren. In the former case a light wind opposed, and in the latter case favoured the sound; and the opposition of the wind proves in all cases more damaging to the gun than to the syren.

October 17 began with a preference for the shorter waves. At 11.30 A.M. the mastery of the syren over the gun was pronounced; at 12.30 the gun slightly surpassed the syren; at 1, 2, and 2.30 P.M. the gun also asserted its mastery. This preference for the longer waves was continued on October 18. On October 20 the day began in favour of the gun, then both became equal, and finally the syren gained the mastery: but the day had become stormy, which is always a disadvantage to the momentary gun-sound. The same remark applies to the experiments of October 21. At 11 A.M., distance $6\frac{1}{2}$ miles, when the syren made itself heard through the noises of wind, sea, and paddles, the gun was fired; but, though listened for with all attention, no sound was heard. Half an hour later the result was the same. On October 24 five observers saw the flash of the gun at a distance of 5 miles, but heard nothing; all of them at this distance heard the syren distinctly: a second experiment on the same day yielded the same result. On the 27th also the syren was triumphant; and on three several occasions on the 29th its mastery over the gun was still more pronounced. Such experiments yield new conceptions as to the scattering of sound in the atmosphere. No sound here employed is a simple sound; in every case the fundamental note is accompanied by others, and the action of the atmosphere on these different groups of waves probably has its optical analogue in that scattering of the light-waves which produces the various shades and colours of the sky.

I have just glanced at the observations made on October 17; but, as this proved our day of maximum acoustic transparency, I will here introduce a fuller record of the day's proceedings. Mr. HOLMES having expressed a wish to sound his four horns together, the day was devoted to comparing them with the syren. The barometer had been rising during the whole of the previous day, the wind being W.S.W. This

morning we had a light air from the N.E., force 1. Haze over the sea; not deep, for the sun shone through it: water very smooth.

About a quarter of a mile from the pier end, and at 2·6 miles from the station, we waited, and heard distinctly the tuning of the horns. Never previously had they been heard so loud in this position—a result plainly due to the shifting of the wind.

At 10 A.M. the gun was good; the horn was also good, voluminous and musical; the syren was very loud, hard and penetrating. This was the result of repeated observations.

Steamed towards Dover Castle into the sound-shadow, on entering which all the sounds were greatly enfeebled; but the syren, though vastly fallen, remained distinctly superior to the four horns.

We manned the gig and rowed towards the shore. Halted near the end of East Cliff Terrace, where the horns and the syren were audible, but barely so. The sounds at times rose and sank in power, so that after one had been set down as barely audible, a succeeding blast rendered a qualification of the statement necessary. The sound appeared to be a little feebler at some distance from the beach than close to the beach itself.

It was rendered certain by these observations that the four horns, two of which were on the summit and two at the base of the cliff, had in the sound-shadow no advantage over the syren.

A gun was fired at 10.30; but we were not attending to it, and no one in the gig heard it. In the 'Galatea,' a quarter of a mile nearer the edge of the acoustic shadow, the report was heard as a feeble thud. Admiral COLLINSON, however, thought it had an advantage over the syren.

Steamed round the Foreland along an arc of 2 miles radius. I happening to be in the deck-cabin, heard there a sudden and powerful augmentation of the sound of both syren and horns; at the same moment Mr. EDWARDS came to me to announce that the 'Galatea' had just cleared the sound-shadow, the instruments being in sight. The shadow was therefore sharply defined. This sharpness was repeatedly observed on both sides of the Foreland.

Steamed towards the axis; but before we reached it the syren-sounds rose to an extraordinary degree of power, the horns yielded a full and mellow sound, far, however, below the syren in intensity; both were well heard through the noise of the paddles.

At 11 A.M. near the axis, distance 2 miles, the gun was fired, and yielded a very loud report. Steamed out along the axis; for a time both horns and syren were heard through the paddle-noises, but at 3 miles distance the horn-sounds vanished from the hearing of some, while a feeble murmur from them continued to be heard by others. To all, however, the syren was by far the clearest sound.

At 11.30, our distance being 5½ miles, the gun was fired, but no sound was heard. At this hour the gun was clearly mastered by the syren. At 7 miles distance, the paddles still going, the syren yielded a distinctly serviceable sound: to a sailing-vessel

it would be still more serviceable. At $7\frac{3}{4}$ miles the sound of the syren practically vanished.

Halted at 8 miles and listened: the syren was faintly heard, and after a little time seemed to rise in power. This has been an almost daily experience. After the stoppage of the paddles the ear appears to require a little time to recover its entire sensitiveness.

It may be questioned whether the change of intensity in passing from 7 to $7\frac{3}{4}$ miles was due to the increase of distance alone. It was probably due to the passage of an invisible acoustic cloud; as we waited, not only did the sound of the syren rise in power, but that of the horns became distinctly audible.

At 12.0 the gun was fired, and 46″ afterwards a faint but perfectly distinct report was heard.

Steamed on, the air being very light and the water very smooth. A thin haze overspread the whole surface of the sea, the horizon not being so sharp nor the coast of France so visible as they were yesterday. At 9 miles we stopped and listened. Both the French and English coasts were very dim with haze; the sun, however, was shining. From the bridge of the 'Galatea' occasional tones of the syren of the faintest character were heard; from the stern of the vessel the syren-sounds were also heard, but they were exceedingly faint. Once or twice the murmur of the horns was feebly heard; but on one occasion they rose to positive distinctness, this being entirely due to the varying state of the air.

At 12.30 we saw the puff of the gun, and 50″ afterwards, the distance being 9·2 miles, heard a faint but distinct report: the gun-sound here was better than that of the syren.

Stopped at 10 miles. From the stern of the vessel both syren and horns were heard as plainly as at 9 miles; if any thing, more plainly.

At 11 miles both syren and horns were faintly heard, a little more faintly than on the last occasion, but still quite distinct. Up to this point the sea had been of glassy smoothness; it now became very slightly ruffled. The horns on this day and at this distance seemed at times equal to the syren; at times, however, inferior to it.

At 1 P.M. a gun was fired. The report was fair, and much beyond the intensity of the syren; the day had evidently become favourable to the transmission of the longer sonorous waves.

Stopped at 15 miles. From the stern of the vessel Mr. Douglass and myself heard distinctly the very faint report of the 1.30 gun. A little afterwards guns fired at Dover were distinctly heard. We learned subsequently that they had been employed in target practice. We waited and listened for some time at 15 miles distance: both horns and syren were heard occasionally.

A little after 2 P.M. we heard a dull report, exactly answering to our time of firing; but half a minute afterwards we heard a second similar report from one of the guns at Dover. Distance about $16\frac{1}{2}$ miles, near Quenocs buoy, in front of Cape Blanc Nez. Besides the gun, nothing was here audible. This was the maximum acoustic range attained during this inquiry.

§ 16. *Action of Wind.*

The action of wind upon sound has been frequently observed, and a statement of Dr. ROBINSON's regarding it has been already quoted (§ 2). In stormy weather we were frequently forsaken by our steamer, which had to seek shelter in the Downs or Margate Roads, and on such occasions the opportunity was turned to account to determine the effect of the wind. On October 11, accompanied by Mr. DOUGLASS and Mr. EDWARDS, I walked along the cliff from Dover Castle towards the Foreland, the wind blowing strongly against the sound. On the Dover side of the Cornhill Coastguard Station (see Map, Plate XIX.), and at about a mile and a half from the Signal-Station, on the edge of a deep hollow or combe, we first heard the faint but distinct sound of the syren. The horn-sound was inaudible. A gun fired during our halt was also unheard.

Descending the combe the syren-sound vanished as we plunged more deeply into the acoustic shadow. On the eminence close to the Coastguard Station the wind was very violent and noisy; sheltering ourselves as well as we could behind some mounds, we listened for the sounds, but heard nothing. No sounds had been heard during the day by the coastguard men.

At the edge of the next combe we caught the sound of the syren, which continued to be heard as we walked round the combe. As we approached the station we saw the smoke of the gun. Mr. EDWARDS heard a faint crack, but neither Mr. DOUGLASS nor myself heard any thing. The sound of the syren was at the same time of piercing intensity.

We waited at this spot for ten minutes, when another gun was fired. I thought I heard a faint thud, but could not be certain. My companions heard nothing. On pacing the distance afterwards it was found to be only 550 yards. We were shaded at the time by an eminence from both the syren and the gun, and more deeply shaded from the latter than from the former; but the difference could not account for the utter extinction of the gun-sound at a time when the syren sent to us a note of great power. Subsequent experiments, moreover, confirmed the conclusion to which this one points, that an opposing wind affects the gun-sound far more seriously than that of the syren.

Requesting Mr. AYRES to walk to windward along the cliff and to note and report upon the sounds, and asking Mr. DOUGLASS to proceed to St. Margaret's Bay, during their absence I had 3 guns fired. Mr. AYRES heard only one of them. Favoured by the wind, Mr. DOUGLASS, at twice the distance, and far more deeply immersed in the sound-shadow, heard all three reports with the utmost distinctness.

Joining Mr. DOUGLASS, we continued our walk to a distance of $\frac{3}{4}$ of a mile beyond St. Margaret's Bay. Here, being dead to leeward, though the wind was as violent as it had been at the coastguard station, the sound of the syren was borne to us with extraordinary power*. In this position we also heard the gun loudly, and two other loud reports at the proper interval of ten minutes, as we returned to the Foreland.

To windward of the instruments, Mr. EDWARDS noticed a rapid and considerable falling

* The horn here was temporarily suspended, but doubtless would have been well heard.

off of the syren-sound. As far as the first combe the sound was fairly heard; he lost it in descending into the hollow, and recovered it in ascending the opposite slope. It accompanied him, being of variable strength and sometimes unheard, as far as the Coastguard Station. Mr. Edwards heard no guns. It is within the mark to say that the gun to-day was heard to leeward five times, and might have been heard fifteen times, as far as to windward.

In windy weather the shortness of its sound is a serious drawback to the use of the gun as a signal. In the case of the horn and syren, time is given for the attention to be fixed upon the sound; and a single puff, while cutting out a portion of the blast, does not obliterate it wholly. Such a puff, however, may be fatal to the momentary gun-sound.

The latter, moreover, is less distinguishable from the sound of the wind in the ears than is the sound of the syren. This action was well illustrated on October 22. We halted near the Cornhill Coastguard Station, and the wind blowing from W.S.W. being very noisy, I sheltered myself behind a bank and listened. The horn and the syren were about equal in power, neither of them being strong. The 3 P.M. gun was fired: the smoke was seen, but no sound was heard. We then sheltered ourselves behind the washhouse of the station, where the horn and the syren, which had been audible behind the bank, rose not only to great distinctness but to great power. The horn was sometimes particularly strong; the syren had the air of being more distant, which may in part arise from the admixture of its really distant echoes with the direct sound.

A gun fired while we were in the shelter of the washhouse produced a loud report; it was the first time during the day that the coastguardsman who was present had heard the gun. Accompanied by Mr. Edwards I returned to the shelter of the bank, where the report had previously escaped me. Corresponding to the known time of firing, I heard a faint thud; he heard nothing. The syren and horns were powerful at the time.

But the ears only required to be defended to render the gun effective. The 3.30 report was loud to Mr. Douglass and Mr. Edwards, who stood in shelter of the washhouse, whereas it was unheard by me who stood out in the wind. On changing places with Mr. Douglass, Mr. Edwards and I heard a loud report of the next gun; to Mr. Douglass it was barely audible. Mr. Edwards now went outside, while Mr. Douglass and I remained in shelter. We heard the 3.50 gun distinctly though not loudly; by Mr. Edwards it was unheard. These experiments illustrate the serious effect which local noises may have upon a signal-sound, particularly that of a gun.

At 4 P.M., near the edge of the combe beyond the coastguard station, we lay down in shelter of a furze bush. Here we heard the sounds of syren and horns distinctly but faintly; the gun was not heard. This position was 450 yards beyond the coastguard station. We then walked from the coastguard station through the fields towards the signal-station; both syren and horns sounded clear and loud. At 4.10 P.M. a gun was fired, but

it was not heard. At 4.20 we halted at the side of the combe nearest the signal-station. Here to me the report of the gun was fairly loud, but not loud to my companions.

Wishing to ascertain how the sound fared to leeward, we followed the line of coast to St. Margaret's Bay and reached the summit of the cliff above and a little beyond the coastguard station. Both the syren and the horn sent their sounds to us with extraordinary power. The gun fired at 4.50 was also very loud, though the distance far exceeded that at which the gun-shots had become entirely inaudible to windward.

As we returned we heard several shots which were of a totally different character from those heard at the same distance to windward of the signal-station; they were sharp and dense, while the others resembled the shock of a soft body against sheet iron.

On October 23 Mr. Douglass, at my request, was good enough to explore the atmosphere from the Foreland towards Deal, while Mr. Ayres undertook the observations on the Dover side of the Foreland. The wind, with a force of 6 to 8, blew from the W. and W.S.W. The instruments in use were the syren, the two upper horns, and the howitzer. They were all pointed southwards.

In the case of Mr. Ayres, at 600 paces from the station the syren was heard distinctly, the horns indistinctly and only occasionally; the gun was unheard. Between this point and another, 440 paces west of the Coastguard Station, or about $1\frac{1}{4}$ mile west of the instruments, eight observations were made at different points. In three cases only was the gun heard; in three only were the horns heard occasionally and faintly; in all cases the syren was heard. Against the wind the superiority of the syren over the horns, and more especially over the gun, is incontestable.

At Ringwold, on the other side of the Foreland, distant 3 miles, and at Walmer, distant $3\frac{3}{4}$ miles, the gun yielded a loud report; the syren and horns also yielded a good sound. Near Deal barracks, $4\frac{1}{2}$ miles, near the railway station, 5 miles, and near Sholden church, $5\frac{1}{2}$ miles, the gun yielded a fair report: the syren was heard, being about equal to the gun. At 6 miles distance the gun was heard, but very feebly; at $6\frac{3}{4}$ miles and at $7\frac{1}{2}$ miles neither gun nor horn was heard, while in both cases the syren was audible. On the Sandwich side of the Foreland, therefore, the sounds on this day were heard at least four times as far as on the Dover side, while in both directions the syren was furthest heard.

On the 24th the wind shifted to E.S.E., and the sounds, which when the wind was W.S.W. failed to reach Dover, were now heard in the streets through thick rain. On the 27th the wind was E.N.E. In our writing-room in the Lord Warden Hotel, in the bedrooms, and on the staircase the sound of the syren reached us with surprising power, piercing through the whistling and moaning of the wind, which blew through Dover towards Folkestone. The sounds were heard at 6 miles from the Foreland on the Folkestone road; and had the instruments not then ceased sounding they might have been heard much further. At the South Sand Head light-vessel, on the opposite side, no sound had been heard throughout the day. On the 28th, the wind being N. by E., the sounds

were heard in the middle of Folkestone, while in the opposite direction they failed to reach the South Sand Head light-vessel. On the 29th the limits of range were Eastware Bay on the one side and Kingsdown on the other; on the 30th the limits were Kingsdown on the one hand and Folkestone Pier on the other. With a wind having a force of 4 or 5 it was a very common observation to hear the sound in the one direction three times as far as in the other*.

It may be worthy of note that within twenty yards or so of the gun the sound was so intense as to render it necessary, prior to each shot, to remove a barometer which hung on the side of the wooden shed containing the horns of Mr. Holmes. On one occasion, when this precaution was neglected, the barometer was broken by the concussion. Neither horns nor syren appear to affect the instrument perceptibly. Still, notwithstanding this initial *vis viva*, the gun-sound is often overmatched at a distance by both syren and horns.

§ 17. *Influence of Pitch and Pressure.*

On October 18 experiments were made in which the steam-pressure was varied with a constant pitch, and others in which the pitch was varied with a constant pressure. At a distance of 3 miles from the shore the intensities corresponding to 40 lbs. and 80 lbs. pressure respectively did not differ from each other as much as might have been expected. With a pressure of 40 lbs. the sound was very fine; with a 50-lbs. pressure it was also very fine, and perceptibly *harder*; with a pressure of 60 lbs. it differed but little from that of 50; with a pressure of 70 lbs. the sound was harder and firmer than the last; the difference between 70 and 80 lbs. was scarcely perceptible.

* In vol. i. of the 'Annales de Chemie' for 1816, p. 176, Arago introduces a memoir by De la Roche, then recently deceased, in these words:—"L'auteur arrive à des conclusions, qui d'abord pourront paraître paradoxales, mais ceux qui savent combien il mettait de soins et d'exactitude dans toutes ses recherches se garderont sans doute d'opposer une opinion populaire à des expériences positives." De la Roche's paper was "On the Influence exerted by the Wind on the Propagation of Sound;" and the strangeness of his results consisted in his establishing, by quantitative measurements, not only that sound has a greater range in the direction of the wind than in the opposite direction, but that the range at right angles to the wind is the greatest of all.

The only attempt to account for De la Roche's results theoretically is due to Professor Stokes. In a short but exceedingly able communication presented to the British Association in 1857, this eminent physicist points out a true cause which, *if sufficient*, would account for the results referred to. The lower atmospheric strata are retarded by friction against the earth, and the upper ones by those immediately below them; the velocity therefore increases from the ground upwards. This difference of velocity throws the sound-wave upwards in a direction opposed to the wind, and downwards in a direction coincident with the wind. In this latter case the direct wave is reinforced by the wave reflected from the earth. Now the reinforcement is greatest in the direction in which the direct and reflected waves inclose the smallest angle—that is, at right angles to the direction of the wind; hence the greater range in this direction. It is not therefore, according to Professor Stokes, a stifling of the sound to windward, but a tilting of the sound-wave over the heads of the observers that defeats the propagation in that direction.

This explanation calls for verification, and I wished much to test it by means of a captive balloon rising high enough to catch the deflected wave; but on communicating with Mr. Coxwell, who has earned for himself so high a reputation as an aëronaut, I learned with regret that the experiment was too dangerous to be carried out.

In determining the influence of pitch, the rate of revolution was varied from 1500 to 2400 a minute. The sound in all cases was very fine, but that corresponding to the most rapid rate of revolution seemed the best and most penetrating. Though these experiments were made at distances of 2 and 3 miles from the shore, distinct and long-continued echoes followed every syren-blast, coming to us from a bearing directly opposed to that of the signal-station.

I have already mentioned October 17 as our day of maximum acoustic transparency; the transparency continued to some extent during the 18th; for on this day, while halting at a distance of 3 miles from the station, we heard the loud report, not of cannon, but apparently of musketry on shore. The sounds were afterwards traced to rifle practice on Kingsdown beach. The day was optically far less favourable than July 3, but each rifle made itself distinctly heard to twice the distance at which an 18-pounder failed to be heard on the 3rd of July.

Arrangements having been made to enable the syren to be pointed in different directions, November 21 and the two subsequent days were devoted to the investigation of both it and the gun, with reference to the direction of their axes. The sonorous waves surrounding the gun proved to be of almost equal intensity throughout, very little difference being observed between the sound when the gun was pointed at us, and when it was pointed at right angles to the line joining it and us. In the syren, however, the intensity in the axis was markedly superior to the intensity at right angles to the axis. This is what might be expected; for the syren-trumpet being expressly intended to project the sound in a certain direction, it can only do this by withdrawing it from other directions.

§ 18. *Concluding Remarks.*

A few additional remarks and suggestions will fitly wind up this paper. It has been proved that in some states of the weather the howitzer firing a 3-lb. charge commands a larger range than the whistles, trumpets, or syren. This was the case, for example, on the particular day, October 17, when the ranges of all the sounds reached their maximum.

On many other days, however, the inferiority of the gun to the syren was demonstrated in the clearest manner. The gun-puffs were seen with the utmost distinctness at the Foreland, but no sound was heard, the note of the syren at the same time reaching us with distinct and considerable power.

The disadvantages of the gun are these:—

a. The duration of the sound is so short that, unless the observer is prepared beforehand, the sound, through lack of attention rather than through its own powerlessness, is liable to be unheard.

b. Its liability to be quenched by a local sound is so great that it is sometimes obliterated by a puff of wind taking possession of the ears at the time of its arrival. This point was alluded to by Arago, in his report on the celebrated experiments of 1822. By such a puff a momentary gap is produced in the case of a continuous sound, but not entire extinction.

c. Its liability to be quenched or deflected by an opposing wind, so as to be practically useless at a very short distance to windward, is very remarkable. A case has been cited in which the gun failed to be heard against a violent wind at a distance of 550 yards from the place of firing, the sound of the syren at the same time reaching us with great intensity.

Still, notwithstanding these drawbacks, I think the gun is entitled to rank as a first-class signal. I have had occasion myself to observe its extreme utility at Holyhead and the Kish light-vessel. The commanders of the Holyhead boats, moreover, are unanimous in their commendation of the gun. An important addition in its favour is the fact that the flash often comes to the aid of the sound: on this point the evidence cited in the Appendix is quite conclusive.

There may be cases in which the combination of the gun with one of the other signals may be desirable. Where it is wished to confer an unmistakable individuality on a fog-signal station, such a combination might with advantage be resorted to.

If the gun be retained as one form of fog-signal (and I should be sorry, at present, to recommend its total abolition) it ought to be of the most suitable description. Our experiments prove the sound of the gun to be dependent on its shape; but we do not know that we have employed the best shape. This suggests the desirability of constructing a gun with special reference to the production of sound*.

An absolutely uniform superiority on all days cannot be conceded to any one of the instruments subjected to examination; still our observations have been so numerous and long-continued as to enable us to come to the sure conclusion that, on the whole, the steam-syren is, beyond question, the most powerful fog-signal which has hitherto been tried in England. It is specially powerful when local noises, such as those of wind, rigging, breaking waves, shore-surf, and the rattle of pebbles, have to be overcome. Its density, quality, pitch, and penetration render it dominant over such noises after all other signal-sounds have succumbed.

I have not, therefore, hesitated to recommend the introduction of the syren as a coast signal.

It will be desirable in each case to confer upon the instrument a power of rotation, so as to enable the person in charge of it to point its trumpet against the wind or in any other required direction. This arrangement has been made at the South Foreland, and it presents no mechanical difficulty. It is also desirable to mount the syren so as to permit of the depression of its trumpet fifteen or twenty degrees below the horizon.

In selecting the position at which a fog-signal is to be mounted, the possible influence of a sound-shadow, and the possible extinction of the sound by the interference of the direct waves with waves reflected from the shore, must form the subject of the gravest consideration. Preliminary trials may, in most cases, be necessary before fixing on the precise point at which the instrument is to be placed.

The syren, it will be remembered, has been hitherto worked with steam of 70 lbs.

* The Elder Brethren have already acted upon this suggestion, and have had plans of a new signal-gun laid before them by the constructors of the War Department.

pressure or thereabouts: the trumpets have been worked with compressed air; and our experiments have proved that a pressure of 20 lbs. yielded sensibly as loud a sound as higher pressures. The possibility of obtaining a serviceable sound with this low air-pressure may render the employment of caloric engines available with trumpets: if so, the establishment of trumpets on board light-vessels would be greatly facilitated. The signals at present existing on board such vessels are very inefficient, and may, I think, be immeasurably improved upon. There are, I am told, practical difficulties as to the introduction of steam on board light-ships; otherwise I should be strongly inclined to recommend the introduction among them of the Canadian whistle. The syren would probably be found too large and cumbrous for light-vessels.

The form of the syren which has been long known to scientific men is worked with air, and it would be worth while to try how the fog-syren would behave supposing compressed air to be substituted for steam. Compressed air might also be tried with the whistles. Such experiments, to render them comparable with our previous ones, ought to be made at the South Foreland.

No fog-signal hitherto tried is able to fulfil the condition laid down by Dr. ROBINSON, in the very able letter already quoted in § 2, namely, "*that all fog-signals should be distinctly audible for at least 4 miles, under every circumstance.*" Circumstances may exist to prevent the most powerful sounds from being heard at half this distance. What may with certainty be affirmed is, that in almost all cases the syren may certainly be relied on at a distance of 2 miles; in the great majority of cases it may be relied upon at a distance of 3 miles, and in the majority of cases to a distance greater than 3 miles.

Happily the experiments thus far made are perfectly concurrent in indicating that at the particular time when fog-signals are needed, that is during foggy weather, the air in which the fog is suspended is in a highly homogeneous condition; hence it is in the highest degree probable that in the case of fog we may rely upon these signals being effective at far greater distances than those just mentioned.

I say "probable," while the experiments seem to render this result certain. Before pronouncing it so, however, I should like to have some experience of warmer fogs than those in which the experiments have hitherto been made. That the fog-particles themselves are not sensibly injurious to the sound has been demonstrated; but it is just possible that in warm weather the air associated with the fog may not be homogeneous. It will probably be found so, but I would recommend the experiment to be made on some of the fogs of the early summer.

I am cautious not to inspire the mariner with a confidence which may prove delusive. When he hears a fog-signal he ought, as a general rule (at all events until extended experience justifies the contrary), to assume the source of sound to be not more than 2 or 3 miles distant, and to take precautions accordingly.

Once warned, he may, by the heaving of the lead or some other means, be enabled to check his position. But if he errs at all in his estimate of distance, it ought to be on the side of safety.

Unless strong practical reasons be adduced in its favour, I should deprecate a

len thened interval between the blasts of the trumpet or syren. My own small experience as a sailor has shown me how harassing to the mariner are some of our revolving lights with a long period of rotation. No light, in my opinion, ought to be obscured for a period exceeding 30 seconds; and the interval between two blasts of our fog-signal ought, in general, not to be longer.

With the instruments now at our disposal, wisely established along coasts, I venture to think that the saving of property in ten years will be an exceedingly large multiple of the outlay necessary for the establishment of such signals. The saving of life appeals to the higher motives of humanity.

In a Report written for the Trinity House on the subject of fog-signals, my excellent predecessor, Professor FARADAY, expresses the opinion that a false promise to the mariner would be worse than no promise at all. Casting our eyes back upon the observations here recorded, we find the sound-range on clear, calm days varying from $2\frac{1}{2}$ miles to $16\frac{1}{2}$ miles. It must be evident that an instruction founded on the latter observation would be fraught with peril in weather corresponding to the former. Not the maximum but the minimum sound-range should be impressed upon the mariner. Want of attention to this point may be followed by disastrous consequences.

This remark is not made without cause. I have before me a Notice to Mariners issued by the Board of Trade regarding a fog-whistle recently mounted at Cape Race, and which is reputed to have a range of 20 miles in calm weather, 30 miles with the wind, and in stormy weather or against the wind 7 to 10 miles. Now, considering the distance reached by sound in our observations, I should be willing to concede the possibility, in a more homogeneous atmosphere than ours, of a sound-range on *some* calm days of 20 miles, and on *some* light windy days of 30 miles to a powerful whistle; but I entertain a strong belief that the stating of these distances, or of the distance 7 to 10 miles against a storm, without any qualification, is simply calculated to inspire the mariner with a false confidence which may lead him to ruin. I would venture to affirm that at Cape Race calm days might be found in which the range of the sound will be less than one fourth of what this notice states it to be. Such publications do not fulfil their proper object if they are made the vehicle by which inventors vaunt the performance of their instruments; they ought to be without a trace of exaggeration, and furnish only data on which the mariner may with perfect confidence rely. The object which I had in view in extending these observations over so long a period was to make evident to all how fallacious it would be, and how mischievous it might be, to draw general conclusions from observations made in weather of great acoustic transparency. The mariner when he hears a fog-signal ought, as just stated, to assume the minimum rather than the maximum distance, and to take his measures accordingly.

Thus ends, for the present at all events, an inquiry which I trust will prove of some importance, scientific as well as practical. In conducting it I have had to congratulate myself on the unfailing aid of the Elder Brethren of the Trinity House. Captain DREW, Captain CLOSE, Captain WEBB, Captain ATKINS, and the Deputy Master have all from

time to time taken part in the inquiry. To the eminent arctic navigator Admiral Collinson, who showed throughout unflagging and, I would add, philosophic interest in the investigation, I am indebted for most important practical aid: he was almost always at my side, comparing opinions with me, placing the steamer in the required positions, and making with consummate skill and promptness the necessary sextant observations. I am also deeply sensible of the important services rendered by Mr. Douglass, the able and indefatigable Engineer, of Mr. Ayres, the Assistant Engineer, and of Mr. Price Edwards, the Private Secretary of the Deputy Master of the Trinity House.

The officers and gunners at the South Foreland also merit my best thanks, as also Mr. Holmes and Mr. Laidlaw, who had charge of the trumpets, whistles, and syren.

In the subsequent experimental treatment of the subject I have been most ably aided by my excellent assistant, Mr. John Cottrell.

Appendix.

On Gun-flashes as Fog-signals.

In crossing to Ireland to witness experiments instituted by the Board of Irish Lights, I have usually questioned the intelligent and courteous captains of the steamers plying between Holyhead and Kingstown regarding the coast-signals. I was distinctly informed by some of them that in fogs so thick as entirely to quench the powerful revolving light of the South Stack, the flash, or rather the glare upon the fog, produced by the gun at the North Stack was usually visible and of great utility. Bearing upon this remarkable point I have been favoured with the following letter from Captain Galway, which seems well worthy of insertion here:—

"November 27, 1873.

"Dear Sir,—Mr. Wigham having communicated to me a wish expressed by you to have an account of an observation of mine with respect to the fog-gun on Lundy Island, in the Bristol Channel, I have much pleasure in forwarding you a detailed account of the circumstance, and beg to say that I feel gratified that you should think the observation worthy of your notice. I was on board the Bristol Steam Navigation Company's steamer 'Inverna,' on a passage from Waterford to Bristol, on Sept. 9 last. We left Waterford at 8 A.M., and at 11.10 A.M. were abreast of the 'Conningbed' light-ship. When we took our departure and shaped a course S.E. by S. ½ S. to pass a short way to the westward of the Smalls, or failing seeing them (the weather being thick) to make Lundy Island, we put the patent log over at the same time. The weather was thick, wind W.S.W., with drizzling rain. At 3 P.M. we were keeping a look-out for the Smalls lighthouse, but were unable to see it owing to the thickness of the weather. At 8 P.M., estimating that we had run our distance, we hauled in the patent log and found that it registered 84 miles, the distance being 90 miles. We therefore thought we must be near Lundy Island, and were peering into the fog, looking out anxiously for the light. I suggested to the captain that he should keep away a little up the Channel in order to bring the gun more to windward. This we did, and about ten minutes afterwards we observed a flash as that of a gun bearing about south. I noted the time, 8.15 P.M., G.M.T., and remarked that I thought it must be the gun on Lundy, but could not understand our not being able to see the light. The weather at the time was thick, occasionally clearing and then thickening over again, with light showers of rain quite obscuring the horizon. I had remarked to the captain just before that I did not think he could see a first-class light for more than a mile. I watched the time, as I was not sure of the intervals at which the gun was fired and had no Admiralty List of Lights on board, and the chart the captain had did not give the required information. I felt certain, however, that it was either fifteen or seventeen minutes; and at 8.31 P.M. I again observed the flash: this time I was satisfied that I heard a slight concussion, but I am sure I should not have perceived it had not my attention been riveted to the spot by the flash. The vessel was also more to *leeward of the gun* and the *engines stopped*. At 8.45 I again saw the flash distinctly and caught a slight reverberation, and a few minutes afterwards (the fog lifting a little) we observed

the light very faintly and with a red appearance through the fog. However, having satisfied ourselves of our position and that the light was Lundy Island (by intervals of revolution), we steamed a course for the Nash Lights, which we made in due time, the night having become clear.

"In my narrative of the circumstance I say *we*, as although I was not in any way officially connected with the vessel, the captain, who had not been on this line for many years, knowing that I was in the habit of close coast navigation and that I had a good deal of experience in making lights, appealed to me for advice. I need scarcely add that I feel very much pleased that you think my observations worthy of your notice.

"I am, dear Sir,

"Yours faithfully,

"A. Knox Galway.

"N.B.—I am unable to say what position the gun is in from the lighthouse, as neither the Admiralty List of Lights or Chart gives it, but I feel quite sure that the Trinity House will give you that information."

The following letter, in reply to an inquiry instituted by Mr. Wigham at my request, is from the commander of the 'Ulster' Royal Mail Packet, and is dated Kingstown, Nov. 23, 1873:—

"Sir,—In answer to your inquiry I beg to say that I have found the guns stationed on the Kish Light and the North Stack lighthouse to be of great service to me in thick weather when approaching the harbours of Kingstown and Holyhead respectively. When the lights of the light-ship and of the Stack lighthouse have been quite invisible by reason of the density of the fog, I have distinctly seen the flash of the gun, making, as it were, an impression on the fog and indicating quite plainly the position in which the gun was placed. In some cases I have been able to see the flash when no sound from the gun has reached me, and I am therefore of opinion that the more brilliant such flashes could be made the better for maritime purposes.

"I am, yours truly,

"Richard S. Triphook,

"Commander 'Ulster' Royal Mail Packet."

From Captain Kendall, Commander of the 'Connaught,' the following letter has been received:—

"Kingstown, December 1, 1873.

"Sir,—In reply to your inquiry I beg to inform you that I have several times seen the flashes from the guns at the Kish light-ship and at the North Stack lighthouse, when, owing to dense fogs, the lights themselves have been invisible. These flashes have thus proved very useful to me by showing me clearly my position. On various occasions I have perceived the flash of the gun when I could not hear its sound. In my opinion these gun-flashes are very advantageous to seamen, and of course it is very desirable that they should be as vivid as possible.

"I am, Sir,

"Your obedient Servant,

"T. E. Kendall,

"Commander R.M.S. 'Connaught.'"

From the Commander of the 'Leinster' the following letter has been received:—

"December 1, 1873.

"Sir,—I beg to inform you that the gun at the Kish light-ship and also that at the North Stack lighthouse have been of very great advantage to me in foggy weather. I have on several occasions seen the flashes of these guns when the light in the lighthouse has been entirely obscured owing to the thickness of the weather, and thus their bearings have been clearly pointed out to me. Frequently I have seen the flash of the gun, although unable to hear its report. It is evident that the brighter these flashes can be made, the more useful they will prove to be.

"I am,

"Your obedient Servant,

(Signed) "Charles John Slaughter,

"Commanding R.M.S. 'Leinster.'"

The point referred to in these letters is, I think, one of practical importance. The intensity of the light is due to the concentration of the combustion of a considerable amount of matter into the fraction of a second of time. It is a question well worthy of consideration whether on board light-ships and elsewhere the combustion of a definite amount of gunpowder, or of gun-cotton, at definite intervals during fog may not turn out to be a simple and useful form of signal. The combustion, of course, would in this case be effected with a view to the production of the maximum light instead of the maximum sound.

In dealing with this subject the gas-gun proposed by Mr. Wigham would naturally be considered.

Remark added, May 27.—The more I think of it, and the more I experiment upon it, the more important does this question of flashes appear to me. In one of the sections of the foregoing paper experiments on artificial fogs are described. The densest of these were suddenly and strikingly illuminated throughout by the combustion of half a grain of gunpowder, and of a still smaller quantity of gun-cotton. The cutting off and restoration of the candle-light, or the electric light, used to test the density of the fog, produced a similar effect. It is its suddenness that renders the lightning-flash so startlingly vivid through a cloud. A revolving light like the South Stack does not fulfil the necessary conditions. Its revolution is slow, and the angular spaces between the beams being filled by laterally scattered light, the differential action is practically abolished. At a distance the luminosity, when uniform, may be so feeble as to be unseen, while its sudden extinction and revival would render it sensible.

Remarkable Instances of Acoustic Opacity.

In his excellent lecture entitled "Wirkungen aus der Ferne," Dove has collected some striking cases of the interception of sound. During the battle of Cassano on the Adda, between the Duc de Vendôme and the Prince Eugene, an army corps stationed under the Duke's brother five miles up the river failed to join the battle through not hearing the cannonade. In a river-valley, particularly on a warm day, it would, in my opinion, be perilous to place much dependence upon sound. Near Montereau on the Seine, during the battle between Napoleon I. and the King of Wurtemberg, which lasted seven hours, no sound of the conflict was heard by Prince Schwartzenberg 13 miles up the river. A Prussian officer sent thither at noon first heard the cannonade at a distance of 4½ miles from the field of battle. This happened on a day apparently resembling in point of mildness and serenity our 3rd of July. In the battle of Liegnitz, where Frederick the Great overthrew Laudon, the sound of the battle was unheard by Field-Marshal Daun, who was posted on a height 4½ miles from the battle-field. Dove himself recounts the fact of his having failed to catch a single shot of the battle of Katzbach at 4½ miles distance, while he plainly heard the cannonade of Bautzen 80 miles away.

The stoppage of the sound in the foregoing cases Dove referred, and doubtless correctly, to the non-homogeneous character of the air. He also notes the exceedingly interesting observation that in certain clear winter days, when the sun has already attained some power, the semaphore is difficult to decipher, the reason being that by the solar warmth upward currents of warm and downward currents of cold air (similar to those of Humboldt on the plain of Antures) are established, and that such days are also unfavourable to the transmission of sound. In another passage, however, he seems to indorse the prevalent notion that the optical transparency of the air and its power to transmit sound go hand in hand; whereas in our experiments days of the highest optical transparency proved themselves acoustically most opaque.

But nothing of this description that I have read equals in point of interest the following account of the battle of Gain's Farm, for which I am indebted to the Rector of the University of Virginia.

"Lynchburgh, Virginia,
March 19th, 1874.

"Sir,—I have just read with great interest your lecture of January 16th, copied by Littell's 'Living Age' from 'Nature,' on the acoustic transparency and opacity of the atmosphere. The remarkable facts you mention induce me to state to you a fact which I have occasionally mentioned, but always where I am not well known, with the apprehension that my veracity would be questioned. It made a strong impression on me at the time, but was an insoluble mystery until your discourse gave a possible solution.

"On the afternoon of June 28th, 1862, I rode in company with General G. W. Randolph, then Secretary of War of the Confederate States, to Price's house, about nine miles from Richmond; the evening before General Lee had begun his attack on M^cClellan's army, by crossing the Chickahominy about four miles above Price's, and driving in the right wing of M^cClellan's army. The battle of Gain's Farm was fought the afternoon to which I refer. The valley of the Chickahominy is about one and a half mile wide from hill top to hill top. Price's is on one hill top, that nearest to Richmond; Gain's Farm, just opposite, is on the other, reaching back in a plateau to Cold Harbour.

"Looking across the valley I saw a good deal of the battle, Lee's right resting in the valley, the Federal left wing the same. My line of vision was nearly in the line of the lines of battle. I saw the advance of the Confederates, their repulse two or three times, and in the grey of the evening the final retreat of the Federal forces.

"I distinctly saw the musket-fire of both lines, the smoke, individual discharges, the flash of the guns. I saw batteries of artillery on both sides come into action and fire rapidly. Several field-batteries on each side were plainly in sight. Many more were hid by the timber which bounded the range of vision.

"Yet looking for near two hours, from about 5 to 7 P.M. on a midsummer afternoon, at a battle in which at least 50,000 men were actually engaged, and doubtless at least 100 pieces of field-artillery, through an atmosphere optically as limpid as possible, *not a single sound of the battle* was audible to General Randolph and myself. I remarked it to him at the time as astonishing.

"The cannonade of that very battle was distinctly heard at Amhurst Court-house, 100 miles west of Richmond, as I have been most credibly informed.

"Between me and the battle was the deep broad valley of the Chickahominy, partly a swamp, shaded from the declining sun by the hills and forest in the west (my side).

"Part of the valley on each side of the swamp was cleared; some in cultivation, some not. Here were conditions capable of providing several belts of air, varying in the amount of watery vapour, arranged like lamina at right angles to the acoustic waves as they came from the battle-field to me. The direction of the valley is nearly due east and west. The part where the cannonade was heard 100 miles off was directly in the line of the valley, which, however, is a short one, less than 20 miles beyond the field.

"It occurred to me that this incident might interest you. I owe you thanks for the possible solution.

"Respectfully,

"Your obedient Servant,

"Professor John Tyndall." "R. G. H. Khan."

I learn from a subsequent letter that during the battle the air was still.—J. T.

Partial Summaries of Observations in Chronological Order.

May 19. *Partial Summary of Day's Work.*—Maximum range of horns, with paddles stopped, from 3 to 4 miles; wind almost directly across the direction of sound. On the South Sand Head side nothing heard at the distance of a mile; wind opposed. On the Dover side a rapid and considerable fall of intensity; wind coincident. Whistles markedly inferior to horns. Axes of horns perpendicular to each other.

	Afloat.	Ashore.
Wind	E.N.E., 6 to 7	—
Barometer	29·9	—
Wet bulb	51°	—
Dry „	52°	—

May 20. *Partial Summary of Day's Work.*—Maximum range of horns, with paddles stopped, 5 to 6 miles; wind force 2, the larger component in favour of sound; sea smooth, and no noise on board. Guns heard by all observers at 9·7 miles; clearly the best. At 4 miles, though the horns lingered on further, they were barely heard with paddles stopped. The whistles had failed previously. The atmospheric conditions, to all appearance, highly favourable to the transmission of the sound.

	Afloat.	Ashore.
Wind	N.E., 2	—
Barometer	30·3	—
Wet bulb	—	—
Dry „	—	—

June 2. *Partial Summary.*—Maximum range of horns to leeward, paddles stopped, about 6 miles. At 3 miles in the axis E.S.E. all sounds were at first heard, and then became inaudible. At 2 miles in the axis three horns sounded together were inaudible. Sound fluctuated in consequence of high wind; in one instance heard out of axis, with paddles going, 3 miles.

	Afloat.	Ashore.
Wind	E.N.E., 7	—
Barometer	30	—
Wet bulb	—	—
Dry „	—	—

June 3. *Partial Summary.*—Maximum range in axis of horn, paddles stopped, over 9 miles; heard with paddles going full speed at 3¾ miles, with wind (force 3) almost opposed to sound. At the maximum distance (air still, sea smooth, and sun shining) a single small horn proved equal to two of them sounded together; a short wide horn proved slightly superior to a long narrow one. The howitzer, with 3-lb. charge, proved to be the best of the guns.

	Afloat.	Ashore.
Wind	S.S.W. to E.S.E.N.	E.N.E., 2
Barometer	29·9	29·6
Wet bulb	—	59°
Dry „	—	62°

June 10. *Partial Summary.*—Maximum range of 3 horns (1 large and 2 small) sounded together 8¾ miles; might have been heard further. Guns heard also at this distance, but not better than horns. Wind, force 4, almost directly across sound, but slightly against it. At 5 miles distance the large horn yielded a serviceable sound with paddles going. The whistles proved inferior to two small horns, and the latter inferior to the large single horn. Near edge of phonic area whistles better than horns. Between Foreland and Dover Pier sound of horns suddenly subsided. Signalled for guns; they also were unheard.

	Afloat.	Ashore.
Wind	S.W. and W.S.W., 5 to 3	—
Barometer	—	29·6
Wet bulb	—	56°
Dry „	—	58°

June 11. *Partial Summary.*—This day was occupied in examining the decay of the sound near the edge of the phonic area at both sides of the Foreland. At the pier, and a quarter of a mile from it, no sound heard. Wind, force 2, almost dead against sound. On the South Sand Head side decay also very striking, and more striking at 2 miles than at 3¾ miles: wind here, force 2, in favour of sound. The wind unable to atone for the acoustic opacity of the day. As before, near edge of phonic area the whistles surpassed the horns.

	Afloat.	Ashore.
Wind	S.W. and W.S.W., 2 to 3	—
Barometer	29·7	29·5
Wet bulb	—	55°
Dry „	—	56°

June 25. *Partial Summary.*—Maximum range of horns in axis 6½ miles. Sounds lost at this distance, but again revived to loudness. A second trial caused the sounds to vanish at 6·4 miles; wind in favour of sound. The 18-pounder inferior to horns at 5½ miles on axis.

	Afloat.		Ashore.
Wind	W.N.W., W. by N. to N.W., 4		—
Barometer	—		29·7
Wet bulb	59°		57°
Dry „	62½°		63°

June 26. *Partial Summary.*—Maximum range for whistle and horns 9¼ miles, with paddles quiet; the horns could have been heard further, though the wind, with a force of 4, was against the sound. On the 25th the maximum range, with the wind favourable, was 6·4 miles. Something, therefore, besides the wind must affect the sound-range. Sound heard at 5·4 miles through paddle-noises and slop of the sea.

	Afloat.		Ashore.
Wind	S.W.		W.N.W., then W.S.W.
Barometer	30·3		30
Wet bulb	60°		62°
Dry „	62°		66°

July 1. *Partial Summary.*—Maximum range Varne light-ship, 12¾ miles; maximum observed on board the 'Irené' 10½ miles. Heard through paddle-noises 6¾ miles. Afloat, wind almost dead against sound; on shore across it. Thick haze, but acoustically a very transparent day. Horn rotated, and strengths of its various blasts expressed numerically; direct or axial blast strongest. Proved that subsidence of sound at edge of phonic area is not due to deviation from axis, because it occurs when axis is directed along the edge. The American whistle did not manifest any remarkable power: this day was the first occasion of its trial.

	Afloat.		Ashore.
Wind	S.S.W. to S.W., 2		N.W., 3, to W.S.W., 2
Barometer	30		29·8
Wet bulb	61° to 60°		62°
Dry „	67° to 62°		67°

July 2. *Partial Summary.*—A very different day, acoustically, from yesterday. Maximum range in axis only 4 miles; wind oblique and against sound; sea noisy. Acoustic opacity of atmosphere augmented from the morning onwards. Near end of Dover Pier, to windward, sounds not heard; on the other side of Foreland, to leeward, sounds well heard.

	Afloat.		Ashore.
Wind	W.S.W. and S.W., 5		—
Barometer	30·1		29·8
Wet bulb	58°		60°
Dry „	61°		62°

July 3. *Partial Summary.*—Maximum range at 2 P.M. under 3 miles: cannon scarcely heard at 2 miles. The obscuration of the sun by a cloud diminished the aërial opacity; the sinking of the sun diminished it still more, and caused sound to reach men in an anchored boat, where half an hour previously nothing could be heard. Range subsequently extended beyond the Varne buoy and to the Varne light-vessel. Day singularly calm and optically clear: dead calm, indeed, afloat.

	Afloat.		Ashore.
Wind	Calm		S.S.E., 2
Barometer	30		29·7
Wet bulb	68° to 61°		63° to 59°
Dry „	71° to 61°		69° to 63°

July 4. *Partial Summary.*—Maximum range probably from 4 to 5 miles: horn heard through paddle-noises, not the whistle. Gun-sounds superior to any thing on this day. Sounds unheard at 5½ miles.

	Afloat.	Ashore.
Wind	W.S.W., 7	—
Barometer	29·9	29·5
Wet bulb	61½°	61°
Dry „	64°	64°

October 8. *Partial Summary.*—Maximum range over 9 miles; howitzer loud, syren clear, horn feeble. No wind at the time of maximum, but following closely upon heavy squall of rain and hail. Previously, at 5½ miles the sounds had been much more feeble.

	Afloat.	Ashore.
Wind	Calm to W.S.W., 3, to N. by W., 2	W.S.W., 2, to N.W., 3
Barometer	29·6	29·4
Wet bulb	52° to 48°	50° to 45°
Dry „	55° to 48°	54° to 46°

October 9. *Partial Summary.*—Maximum range 7½ miles; syren faint but distinct. Syren better than gun or horn with wind across; gun better than syren or horn with wind directly favourable to sound. Good and serviceable sound from syren heard at 3 miles distance to windward, 'Galatea' steaming at full speed.

	Afloat.	Ashore.
Wind	W.S.W., 3 to 6	W.S.W., 5 to 6
Barometer	29·9	29·6
Wet bulb	54°	49°
Dry „	58°	55°

October 10. *Partial Summary.*—The syren maintained its superiority during land-observations: deep in the sound-shadow both its direct sound and long-drawn echoes were heard when the horn and its echoes were inaudible. A strong wind was blowing against the sound at this time.

October 11. *Partial Summary.*—In the land-observations of this day the syren proved itself superior to the gun against a strong wind: indeed at 550 yards dead to windward of the station the smoke of the gun was seen, but no sound heard, whilst at the same time the syren was piercingly intense; to leeward the gun-sound travelled a very much greater distance. The shortness of the gun-sound is not favourable for the purposes of signalling. Moderate gale from S.W. all day.

October 13. *Partial Summary.*—2½ miles E. of station, syren superior to horn, Canadian whistle, or gun. Same distance W. of station, gun louder than any thing; then syren. Canadian whistle equal to horns on both sides. In the axis horn better than whistle. All sounds heard during descent of very heavy rain. Locality of station discovered, when hidden in thick mist, by sound alone.

	Afloat.	Ashore.
Wind	N.W. by W., 2 to 4	N.N.W., 3
Barometer	29·7	29·3
Wet bulb	57° to 53°	52° to 49°
Dry „	58° to 54°	55° to 50°

October 14. *Partial Summary.*—Maximum range 10 miles; in the morning at the Varne buoy, the distance being 7¾ miles, all sounds well heard. With a changed atmosphere at the same spot, four hours later in the day, the syren was feeble and the gun and horns not heard. It was not the wind which caused this difference, for it had then the same direction and force as in the morning. At 5 miles in the axis the sounds were all much more feeble than at 7¾ miles at the Varne buoy, which was out of the axis, in the morning. At 4 miles to leeward the syren was strong, the gun loud, horns and Canadian whistle plainly heard.

	Afloat.		Ashore.
Wind.....	N.N.W. and W.N.W., 2 to 3	.....	N.W., 2
Barometer	30·00	.....	29·6
Wet bulb	54° to 57°	.....	55° to 48°
Dry „	57° to 61°	.....	59° to 52°

October 15. *Partial Summary.*—Inspected Dungeness rotating fog-horn. Maximum range, with horn pointed on us, 3·9 miles, then barely audible: lost at 4 miles. Report of gun much louder than direct sound of horn at 3·9 miles. In the other medial line range of horn still less. Aërial echoes of 4 seconds duration returned in every case from the direction in which the horn was pointed.

	Afloat.		Ashore.
Wind	nil	.....	—
Barometer	30·00, falling	.....	—
Wet bulb	55°	.....	—
Dry „	59°	.....	—

October 16. *Partial Summary.*—Maximum range in axis 5 miles: syren faintly heard; gun seen but not heard. With paddles going syren heard at 3½ miles in axis; gun seen but not heard at 2½ miles. Round arc of 2 miles radius from South Foreland the results were as follows:—Deep in sound-shadow on Dover side syren much enfeebled; Canadian whistle at times equal to it; gun yielded only a thud. In sound-shadow on South Sand Head side the syren was again much enfeebled, and at times surpassed by the Canadian whistle: here the gun gave a loud report, there being a component of wind in favour of the sound. On clearing the shadows of the intervening projecting cliffs on either side all the sounds were much louder; and on nearing the axis of the syren its sound rose to an extraordinary intensity, surpassing the loud report of the gun.

The Canadian whistle has shown itself much superior to the other whistles.

	Afloat.		Ashore.
Wind.....	W.S.W., 2 to 3	.....	S.W., 3
Barometer.....	30·2	.....	29·9
Wet bulb	60° to 56°	.....	55° to 48°
Dry „	64° to 60°	.....	60° to 51°

October 17. *Partial Summary.*—Four horns tried against syren: proved inferior to it. A day of great acoustic transparency. Maximum range with vessel stopped, gun 16½ miles, dull report; syren and horns heard occasionally at 15 miles in axis. Early in the day, with paddles going, syren heard at 7 miles in axis, while gun and horns were inaudible at 5½ miles. In sound-shadow, Dover side, syren proved itself superior to gun and horns. The limit of the sound-shadow was to-day very sharply defind. At the base of the South Foreland cliff the aërial echoes of the syren-sound lasted from 14 to 15 seconds: the echoes were observed to strike in upon the direct sound one second after commencement of the actual blast.

	Afloat.		Ashore.
Wind.....	calm and N.E., 1	.....	S.E., 1
Barometer.....	30·2	.....	29·9
Wet bulb	54° to 61°	.....	54° to 50°
Dry „	57° to 66°	.....	60° to 54°

October 18. *Partial Summary.*—Experiments on pitch and pressure of syren. At 2 miles 2400 probably better than 1500 revolutions per minute. The greater pressure yields the harder and firmer sound. Reports heard in the morning proved to come from rifle practice on Kingsdown beach, 5½ miles off, or twice the distance at which cannons were heard on July 3. Dead to leeward, about 4 miles, missed hearing a gun through not attending. Gun requires a prepared attention to hear it.

	Aflost.		Ashore.
Wind	W., 2	.....	W.S.W., 3
Barometer	30·1	.....	29·8
Wet bulb	54° to 56°	.....	54°
Dry „	56° to 59°	.....	59°

October 20. *Partial Summary.*—A day of atmospheric variation and consequent variability of sound-range; strong wind, high sea, and much local noise. At 5·7 miles to leeward syren more distinct than gun; at some shorter distances gun-sound quenched by local noises, but not so with the syren-sound. Across the wind the syren was heard through paddle-, sea-, and wind-noises at 5 miles; horns (two small ones) weak at 3¼ and inaudible at 4 miles. Gun-report not picked up by some ears so well as the hard and sustained sound of the syren; horn a soft musical sound, inferior to explosive blast of the syren.

	Aflost.		Ashore.
Wind.....	W. by S. to N.N.W., 6 to 8	.....	N.N.W., 5 to 7
Barometer	29·9	.....	29·6
Wet bulb	55° to 49°	.....	55° to 42°
Dry „	55° to 51°	.....	57° to 45°

October 21. *Partial Summary.*—Strong wind, high sea. At 6¼ miles in the axis, paddles going, wind across, syren heard clearly; gun seen, but not heard: horns lost after 4 miles. At 4¾ miles in axis vessel stopped in the midst of heavy rain and high wind; gun, horns, and whistles heard, but syren burst out with sudden power, overmastering them all.

	Aflost.		Ashore.
Wind	W.N.W. to W., 5 to 7	.....	W., 5
Barometer	29·8, falling	.....	29·6, falling
Wet bulb	48° to 52°	.....	46°
Dry „	49° to 52°	.....	48°

October 22. *Partial Summary.*—Land-observations only. At the Cornhill Coastguard Station, distant 1 mile, with a very strong wind blowing directly against the sound, the syren and horns (the latter pointed towards us) were heard distinctly, while the smoke of the gun was seen, but the sound was not heard. Sheltered from the wind the gun was heard distinctly. At a greater distance, sheltered from the wind by a furze bush, the syren and horns were heard, but not the gun. Gun-sound liable to be obliterated by a passing puff of wind. To leeward, on the cliff beyond St. Margaret's Bay, the gun-report was very loud, the syren and horn-sounds being also very powerful.

	Aflost.		Ashore.
Wind.....	—	.....	W.S.W., 5
Barometer.....	—	.....	29·1
Wet bulb	—	.....	56° to 49°
Dry „	—	.....	57° to 52°

October 23. *Partial Summary.*—Land-observations only. Strong wind, at times increasing to a gale. Mr. Ayres, 600 paces to windward, heard syren distinct, horn indistinct, gun not all. Syren also heard distinctly through thunderstorm; gun and horns inaudible. At 1¼ mile in same direction syren always heard, generally distinct and strong; gun and horns only heard 3 times. Mr. Douglass, to leeward, carried syren-sound to 7⅛ miles, having lost the gun at 6¾. During thunderstorm and heavy rain, sounds were heard plainly at 2½ miles which had previously been inaudible. In rear of instruments effective range of sounds about a mile, syren predominating.

	Aflost.		Ashore.
Wind	—	.....	W. and W.S.W., 6 to 8
Barometer	—	.....	28·6, rising
Wet bulb	—	.....	50°
Dry „	—	.....	52°

October 24. *Partial Summary.*—Observations by Mr. Douglass in 'Palmerston' steam-tug. Wind light, and sea quiet. Maximum range, syren 7¾ miles, feeble sounds, distinct at 5 miles. Gun unheard at 5 miles, low thud at 3·9, but unheard at 3 miles to windward. All sounds heard through rain and squall 3 miles distant. In sound-shadow, South Sand Head side, distant from station 3 miles, syren only heard.

	Afloat.		Ashore.
Wind	E.S.E., 3	.....	S.E. and E.S.E., 3 to 5
Barometer	—	.....	28·9
Wet bulb	—	.....	45° to 43°
Dry „	—	.....	49° to 43°

October 27. *Partial Summary.*—Wind strong, sea rough. At sea in axis syren heard at 6 miles; nothing else. Heard also with paddles going up to 5 miles. Gun feeble at 4 miles, unheard at 5; horns barely audible at 4 miles. Wind across, force 5. Nearly dead to windward 1½ mile, syren distinct through paddle-noises; horns just audible; gun seen but not heard. To leeward syren good at 5 miles, gun fair, horns feeble. On land to leeward all heard at 6 miles. Sounds very loud in Dover all day.

	Afloat.		Ashore.
Wind.....	E.N.E., 6	.....	N.N.E., 5
Barometer.....	—	.....	30·00
Wet bulb	—	.....	44° to 41°
Dry „	—	.....	47° to 44°

October 28. *Partial Summary.*—Thick haze round horizon, South Foreland invisible; zenith blue; sun shining. Maximum range 7½ miles in axial line; gun-report fair, syren-sound also fair; 2400 revolutions rather better than 1500. At 6 miles, syren with 2400 revolutions distinctly superior to gun; subsequently, vessel having drifted out of axis and the atmosphere having changed, neither gun nor syren heard at 6 miles, though nearly dead to leeward. Nothing heard all day on board South Sand Head light-ship; but, through action of wind, sound loud in Dover, and heard in the middle of Folkestone.

	Afloat.		Ashore.
Wind	N. by E. to N., 3	.....	N.N.W. to N., 3
Barometer	30·5, falling	.....	30·1, falling
Wet bulb	48°	.....	45° to 39°
Dry „	51°	.....	49° to 40°

October 29. *Partial Summary.*—Afloat, in axial line, high and low notes of syren faint at 7 miles; high note rather better than low. Gun barely heard at 5 miles. Subsequently, gun was seen and not heard at various distances down to 2½ miles. Wind across, force 3. At 3½ miles dead to windward the syren was faintly heard; gun not heard at 2½ miles. Echoes observed to come strongest from windward. On land, to windward, Mr. Douglass carried sounds to between 2 and 2½ miles; to leeward Mr. Edwards heard the sounds at 7 miles; and Mr. Ayres, in rear of instruments, carried the sounds inland for 5 miles.

	Afloat.		Ashore.
Wind	E.S.E. to E.N.E., 3	.....	E. to E.N.E., 4
Barometer	30·3	.....	29·9, falling
Wet bulb	48°	.....	42° to 38°
Dry „	52°	.....	48° to 44°

October 30. *Partial Summary.*—No wind. Nearly at right angles to their axes, gun and syren heard at 11½ miles: horns not heard. At 8½ miles syren-sound efficient, with paddles going, strikingly superior to gun. Subsequently in axis, with a rising wind, no sounds heard at 6½ miles; horns lost at a little over 5 miles; at the same time gun very feeble. At South Sand Head, distance 3¾ miles, wind across, force 5, syren very feeble; gun and horns not heard. On land, wind across, syren only heard 3 miles N.E. of South Foreland; but on Folkestone Pier, 8 miles on the other side, syren heard plainly. Guns and horns not heard beyond Folkestone Hill.

	Afloat.		Ashore.
Wind	Calm to N.N.W., 5	.....	N.W., 2 to 4
Barometer	—	.....	29·5
Wet bulb	—	.....	43° to 40°
Dry „	—	.....	45° to 42°

October 31. *Partial Summary.*—Sea rough, wind and rain. Nearly at right angles to its axis, syren exceedingly faint at 3¼ miles; no sound of gun or horns. In axis, syren well heard through paddle- and other noises up to 2¾ miles. In a violent rain-squall, wind force 8, more than 2 miles from station, forcible sound heard from syren; other signals not heard: with less wind, syren heard off end of Admiralty Pier, directly to windward; gun-sounds quenched; horn-sounds very faint. Atmosphere exceedingly opaque to sound.

	Afloat.		Ashore.
Wind	S.W. by S. to W., 4 to 8	.....	W.S.W., 6 to 4
Barometer	29·6	.....	29·6
Wet bulb	52°	.....	47°
Dry „	54°	.....	50°

November 1. *Partial Summary.*—Land-observations by Mr. Douglass. On ramparts of Dover Castle, 2 miles from South Foreland, wind obliquely against sound, syren distinctly more audible than horn, though the latter was pointed towards the castle; horn feeble; gun not heard. On the other side, with wind obliquely favourable to the sound, at 0·9 miles distance, the syren and horn (the latter now pointed eastward) were barely audible; the gun was not heard. Aërial echoes, 11 seconds in duration, at South Foreland.

	Afloat.		Ashore.
Wind	—	.....	W.S.W., 5 to 3
Barometer	—	.....	28·9
Wet bulb	—	.....	49° to 46°
Dry „	—	.....	52° to 50°

November 21. *Partial Summary.*—The gun was first fired along the line between the observers and the Foreland, and then at right angles to this line. The report in the former case was sensibly, though not much, louder than in the latter. The syren was then blown in the same manner, first with its axis directed upon us, and then in the perpendicular direction. The fall of intensity in passing from the one position to the other was far more considerable than in the case of the gun. Striking subsidence of sound near the boundary of acoustic shadow. Guns fired to leeward produced louder and longer echoes than when fired to windward.

	Afloat.		Ashore.
Wind	S.S.W. to W.S.W., 2 to 5	.....	S.W., 3
Barometer	29·9, falling	.....	29·5, falling
Wet bulb.....	41° to 44°	.....	40°
Dry „	44° to 46°	.....	42°

November 22. *Partial Summary.*—Yesterday's experiments were repeated and amplified; result substantially the same.

	Afloat.		Ashore.
Wind	W.N.W. to N.W., 7	.....	N.N.W., 7
Barometer	29·5	.....	29·1, rising
Wet bulb	50°	.....	47°
Dry „	54°	.....	51°

November 24. *Partial Summary.*—Sound in rear of syren, to leeward, stronger than in front; stronger also at 2½ miles than at 1 mile. A vast augmentation of the sound when syren was turned to leeward. Whistles tested: Canadian and 8-inch whistle proved to be the best. The syren, with disks alone, unaided by trumpet, proved to be about equal to the best whistles.

	Afloat.		Ashore.
Wind	W., 2 to 3	.....	W.S.W., 3
Barometer	29·9	.....	29·6
Wet bulb	47° to 52°	.....	46° to 49°
Dry „	48° to 52°	.....	47° to 50°

November 25. *Partial Summary.*—Acoustic opacity nearly equal to that of July 3. At 2·8 miles, with all quiet, sounds exceedingly feeble; no guns heard. Gun yielded only a faint crack at 2½ miles. Day then very calm. Change of wind brought with it augmentation of sound. The Canadian whistle compared with the horn to-day, and found superior to it. Syren for a time not in action. Eight-inch whistle feeble to-day. Syren sounded afterwards both level and depressed; no sensible difference.

	Afloat.		Ashore.
Wind	Calm and E.N.E., 1	.....	S. and S.E., 1
Barometer	30·2	.....	29·8
Wet bulb	56° to 51°	.....	51° to 47°
Dry „	52°	.....	53° to 48°

The barometer and thermometer observations were made afloat by Mr. EDWARDS; ashore by Mr. AYRES.

Table of Ranges of HOLMES'S *Horns.*

May 19th	3½ miles.	
„ 20th	4 to 5½ „	
June 2nd	6 „	
„ 3rd	9 „	
„ 10th	8¾ „	
„ 25th	6½ „	
„ 26th	9¼ „	
July 1st	12¾ „	
„ 2nd	4 „	
„ 3rd	2½ „	and 12¾ miles.
„ 4th	3¾ „	

Syren and Horns.

		Syren.		Horns.
October 8th	.....	10 miles	.....	9 miles.
„ 9th	.....	7½ „	.....	4 „
„ 14th	.....	10 „	.....	10 „ and 7½ miles.
„ 15th	.....	no syren	.....	3·9 „ (Daboll).
„ 16th	.....	5 miles	.....	no horn.
„ 17th	.....	15 „	.....	15 miles (4 horns).
„ 20th	.....	6¼ „	.....	3¾ „
„ 21st	.....	6½ „	.....	4 „
„ 23rd	.....	7½ „	.....	6 „ (on land).
„ 24th	.....	7¾ „	.....	3½ „
„ 27th	.....	6 „	.....	4½ „
„ 28th	.....	7½ „	.....	no horn.
„ 29th	.....	7 „	.....	„
„ 30th	.....	11½ „	.....	8½ miles.
„ 31st	.....	3½ „	.....	2½ „ bearing off Varne.
November 1st	.....	6·9 „	.....	—— (on land).
„ 25th	.....	4 „	.....	——

Range of syren to range of horn as 7 : 5 nearly.

INDEX.

§ 1. Introduction 183
§ 2. Condition of the Question 184
§ 3. Instruments and Observations 186
§ 4. Influence of Sound-Shadow 190
§ 5. Rotation of Horn 191
§ 6. Contradictory Results 194
§ 7. Aërial Reflection and its Causes; solution of contradictions 194
§ 8. Aërial Echoes 197
§ 9. Experimental Demonstration of the stoppage of Sound by Aërial Reflection 202
§ 10. Action of Hail and Rain 205
§ 11. Action of Snow 207
§ 12. Action of Fog. Observations in London 209
§ 13. Experiments on Artificial Fogs 214
§ 14. Observations at the South Foreland 216
§ 15. Atmospheric Selection 220
§ 16. Action of Wind 224
§ 17. Influence of Pitch and Pressure 227
§ 18. Concluding Remarks 228
Appendix:—
On Gun-flashes as Fog-signals 232
Remarkable Instances of Acoustic Opacity 234
Partial Summaries of Observations in Chronological Order 235
Table of Ranges of Holmes's Horns 243
Syren and Horns 243

VIII. *On the Fossil Mammals of Australia.*—Part VIII. *Family* Macropodidæ: *Genera* Macropus, Osphranter, Phascolagus, Sthenurus, *and* Protemnodon.

By Professor Owen, *F.R.S. &c.*

Received November 11, 1872,—Read January 23, 1873.

§ 1. *Introduction.*—Through the adventurous journeys of John Gould, F.R.S., in the wilds of Australia, and by the noble works * in which he has given the results of his zoological observations in that continent and the adjoining island of Tasmania, we mainly know the extent and kinds of variation under which the Kangaroos there exist.

The present communication gives part of the researches into the forms of those saltatory herbivorous Marsupials which have passed away, or, at least, are known to naturalists only by their fossil remains. I shall be happy if I am able to complete a work which may be regarded as worthy to rank as a supplement to that of my old and esteemed friend and fellow labourer.

As the extinct species which I now attempt to define (their restoration awaiting further materials) have chiefly been made known to me by their fossil jaws and teeth, some remarks on the latter organs will be briefly premised.

The dentition of the Kangaroos (bilophodont *Macropodidæ*†) is summarily described and figured in my 'Odontography'‡: in later works § its phases of development and mutation are exemplified in detail in *Macropus major* ‖. The last phase delineated (Anat. of Vert. vol. iii. fig. 296, F) is that which is shown in the subject of figs. 15 & 16, Plate XX., in the mandible of *Macropus major*, in which the anterior of the four retained molars (d_4) is "nodding to its fall." I have seen a specimen of an older Kangaroo of this species in which the series of grinders was reduced to two, viz. m_2 and m_3. Fig. 15, Plate XX., is also introduced to exemplify the largest size of mandible to be derived from any known existing kind of Kangaroo. The other figures in that Plate

* Those which relate to the present Paper are:—Monograph on the Macropodidæ, or Family of the Kangaroos, folio, 1841–42; The Mammals of Australia, folio, 1845–54.

† This is a section distinct from the Kangaroo-rats, Bettongs, &c., with quadrituberculate molars, included in the subfamily *Hypsiprymnidæ*.

‡ 4to, 1840–45, pp. 389–393, pls. 100, 101, 102.

§ Art. Teeth, in 'Cyclopædia of Anatomy,' vol. iv.; also 'Anatomy of Vertebrates,' 8vo, vol. iii. p. 380, fig. 296.

‖ I have always referred to this large and first-discovered species of Kangaroo under Shaw's later name (General Zoology, vol. i. pp. 505, 800). Mr. Waterhouse alludes to it sometimes (as in p. 52 of his excellent 'Natural History of Mammalia') as *Macropus major*, sometimes as *Macropus giganteus* (*ib.* p. 55), the synonym of Zimmerman's *Jerboia gigantea* (1777) and of Schreber's *Didelphis gigantea* (1778).

show modifications in the size, form, structure, and order of succession and shedding of teeth requisite for the description and comprehension of characters of fossil jaws and teeth of the present family of Marsupials.

Thus, in *Macropus* (*Osphranter*) *robustus*, Gd. (Plate XX. figs. 13 & 14), the premolar (p 3), which is not larger than that in *Macropus major* *, is later retained; and the following molar (d 4) in my subject had undergone a much greater degree of wear than in *Macropus major* before p 3 had risen into place. This is plainly shown by the lower level of the much-worn d 4 in fig. 13. It would also seem to have been originally a relatively smaller tooth than its homologue in the "greater Kangaroo." The last molar is in place, and shows the same slight degree of masticatory wear in both species; but with this the molar series is reduced to four teeth in one, and shows five teeth, or four and a half, in the other.

In *Macropus* (*Halmaturus*) *ualabatus*, Less. & Gd. (Plate XX. fig. 11), the premolar (p 3), which is relatively larger than in the two preceding Kangaroos, has risen into place before the crown of the following molar (d 4) was worn down to its base. The hinder thickened end of p 3 is worn nearly level with the more complex grinding-surface of d 4, which nevertheless indicates, by the extent of exposed dentine, that it had been in use when the deciduous predecessors (d 2 and d 3) of the premolar (p 3) were in place. The last molar is here fully developed; its front lobe is abraded, and the series of five teeth are in a condition to continue together the work of mastication for a great part, at least, of the lifetime of this smaller kind of Kangaroo.

In the Red-necked Kangaroo (*Macropus* (*Halmaturus*) *ruficollis*, Dm., Gd.) (Plate XX. figs. 9 & 10) the penultimate molar (m 2) is in place and use before the first two deciduous molars (d 2, d 3) are shed, and when the premolar (p 3) is concealed, with the two roots as yet unformed, in its cell of development. The crown of the last molar (m 3) is also formed, and was about to pierce the gum. The permanent dentition of *Halmaturus ruficollis*, Gd. (*Kangurus ruficollis*, Dm., 1817), is that of *H. ualabatus*.

In *Macropus* (*Halmaturus*) *erubescens*, Scl.†, the premolar (Plate XX. figs. 1–8, p 3) has, in the upper jaw (fig. 1), nearly risen into place, the crown being extricated from the formative socket before the penultimate molar has appeared through the gum. The skull here figured gives an interesting phase of dental development. The premolar has displaced the second deciduous molar (d 3) on both sides of the upper jaw, d 2 continuing on the left side, and its socket being unobliterated on the right side. In the lower jaw (fig. 4) the premolar (p 3) has pushed half its crown above the socket on the left side, from which both d 2 and d 3 are displaced, whilst d 3 remains on the right side, the premolar

* Anat. of Vertebrates, vol. iii. fig. 296, *x*, *p* 3.

† Sclater, 'Proceedings of the Zoological Society,' March 7th, 1871, p. 240 (Cut, figs. 5 & 6). This eminent zoologist remarks:—"The muffle of *M. erubescens* is quite naked; and the species therefore belongs strictly to the section *Halmaturus* of Mr. Waterhouse's arrangement." But the bony palate is entire, as in most large Kangaroos, including *M. antilopinus*, *M. robustus*, and *M. rufus* "of the present convenient but, as it appears to me, arbitrary division" (Waterhouse, *op. cit.* p. 95).

not having come into view; a trace of the socket of the shed d_2 remains. In a younger individual of *Macropus erubescens*, the skull of which, marked "'*Uroo*,' far north," was kindly sent to me from Adelaide, South Australia, by G. F. Waterhouse, Esq., the three deciduous molars are in place and use (Plate XX. figs. 6 & 7, d_2, d_3, d_4); m_1 has nearly risen into place in the upper jaw (fig. 6), but is not so far advanced in the lower jaw (fig. 7). The germ of the premolar (p_3) is exposed by removal of bone in the upper jaw.

In the skull of a nearly full-grown Kangaroo (*Macropus* (*Boriogale*) *magnus*, Ow.), also from the "far north" of the province of South Australia, the premolar is represented by the foremost deciduous tooth. On the left side of the upper jaw it is in contact with d_4, and m_3 is nearly risen into place; on the right side (Plate XX. fig. 12) a vacuity corresponding with the hind half of d_3 remains, and shows the socket of the hind root of that deciduous tooth. Its homotype in the lower jaw (fig. 12 *a*) is shed in both rami, and the very small bilobed crown of d_2, or p_3, is in close contact with d_4. If d_2 had a predecessor, it must then be the tooth (p_3) which I suppose it to represent. In either case the modification is rare, and, so far as I know, unique in the bilophodont section of *Macropodidæ*: assuming the foremost tooth to be d_2, it repeats the condition and formula of the molar series in *Diprotodon* and *Nototherium*.

I shall not here carry further the account of the dental changes in living species of Kangaroos; but there are modifications of the grinding-surface and crown of the molar teeth which are useful in tracing out the affinities of extinct species.

The premolar, like the foremost deciduous molar, has an antero-posteriorly extended crown, with a more or less trenchant margin, supported by two roots. The margin may be slightly thickened and obtuse posteriorly, with a still more feeble swelling anteriorly, and the crown may not show any other modification; such is the very small lower premolar of *Macropus* (*Osphranter*) *robustus*, Gd. (Plate XX. fig. 13, p_3). In the upper jaw of this species the premolar (fig. 14', *a*, *b*), with an increase in antero-posterior and transverse extent, shows none in the vertical direction; but the thickened fore part of the crown is divided by a notch from the rest of the trenchant border, and this by a smaller notch from the hind swellings; moreover the base of the crown is produced inward, and this ridge swells out posteriorly. The fore-and-aft dimension of the upper premolar does not, however, exceed that of the adjoining molar, d_4.

In *Macropus* (*Boriogale*) *magnus* (fig. 12) the upper premolar, or its representative, is not so long from before backward as the adjoining two-ridged molar. The anterior thickening is not marked off by a notch; it is connected by a basal rising with the hinder thickening, and the intermediate rather depressed outer surface shows two faint vertical ridges. An inner basal ridge swells into a small tubercle posteriorly.

In the lower jaw (fig. 12 *a*) the still smaller homotype has the crown transversely cleft to its base, and the hinder, somewhat larger lobe is thickest behind, with a feeble internal tubercle.

The upper premolar of *Macropus erubescens* (ib. figs. 1 & 2, p_3) is similarly cleft,

though not quite to the base; it has an inner basal ridge swelling behind into a tubercle, which abuts upon the hinder and larger division of the cleft crown. The lower premolar, of smaller size (figs. 4 & 5, p_3), is cleft in a minor degree.

In *Macropus ualabatus* the premolar exceeds the adjoining molar (d_4) in antero-posterior extent. In the upper jaw the trenchant border is slightly notched by a few vertical grooves traversing the outer side of the crown; and the inner basal ridge is similarly but more deeply notched; the entire crown is also broader than in the premolars of the previously cited species.

The modifications of the crown in the transversely two-ridged or "bilophodont" molars add characters in the discrimination of fossils, and it is convenient to define and name the parts affording them. The main "lobes" (Plate XX. figs. 29, 30, & Plate XXI. fig. 13, m_3) are "front" (*a*) and "back" (*b*); a ridge along the fore part of the base of the crown is "prebasal" (*f*); if, as is usual, there be one at the back part of the crown it is "postbasal" (*g*, ib. fig. 29, & ib. figs. 12 & 15, m_3). Commonly these several transverse elevations are connected together by ridges which affect a longitudinal course: that which ties the prebasal ridge to the front lobe is the "fore link" (ib. fig. 29, & ib. fig. 13, m_3, *s*), that which ties together the main lobes is the "mid link" (*r*), that which descends to the "postbasal" ridge is the "hind link" (*t*), of which ridge it frequently seems to be the sole representative (Plate XXI. fig. 18, *t*).

The upper molars, as usual, are broader than the lower ones, and the prebasal ridge is usually narrower (antero-posteriorly); but the ridge descending from the hinder and inner angle of the back lobe to the base of the hind surface of that lobe ("hind link" and "postbasal ridge") is usually better marked or more commonly present in the upper than in the lower molars.

The coronal modifications of these teeth are represented in certain existing species in figs. 23 to 28, Plate XX.; to these are added figures of a lower molar in two of the extinct species of Kangaroo (ib. figs. 29, 30), which I next proceed to define.

§ 2. *Macropus Titan*, Ow.—This species was founded on a portion of the right ramus of a lower jaw from the Breccia-cave in Wellington Valley, New South Wales; in which jaw, notwithstanding the superiority of size of the molar and of the portion of molar in place to any of those in *Macropus major*, I was led from certain indications of immaturity to ask permission from the possessor and discoverer of the then (1836) unique fossil to excavate the substance of the bone; this being granted, led to the detection of the nearly complete premolar or successional tooth in its formative alveolus, such as is figured in vol. ii. pl. 29. fig. 3 of Sir THOMAS MITCHELL'S work*.

The discovery of the premolar was a satisfactory addition to the less conspicuous differences in the molars of the present as compared with those in the fossil jaw of a similarly sized extinct Kangaroo, also in Sir THOMAS MITCHELL'S collection, on account of the remarkably large and complex character of the premolar in that fossil, now the

* Three Expeditions into the Interior of Eastern Australia, &c. 8vo. 1838, vol. ii. p. 360 (2nd edit. 1839, p. 366, pl. xlvii.).

type of *Sthenurus* (*Macropus*) *Atlas* (comp. fig. 18, p^3 with fig. 4, p^3, Plate XXII.). But I had not at that time the further satisfaction of determining the characters of the maxillary dentition of *Macropus Titan* by fossils of that species, either at the corresponding immature stage of the animal affording the mandibular fragment or of full-grown individuals. I have subsequently received both desiderata, some of which reached me in time to notice in the under-cited work*, and of which figures are now for the first time given. The maxillary specimen (Plate XXI. figs. 6–9), in its phase of dentition, relates as closely to the mandibular one (Plate XXII. figs. 17, 18) as does the upper jaw of *Sthenurus Atlas* (Plate XXIV. figs. 4 & 5) to the portion of lower jaw (Plate XXII. figs. 3 & 4).

The fossil in question (Plate XXI. figs. 6–9) is not from the Breccia-cave of Wellington Valley, but from a freshwater bed or drift in Queensland, where it was obtained and transmitted to me by my friend George Bennett, M.D., F.L.S. This is interesting as evidence of the range of the large and now extinct species. It shows the usual state of petrifaction of fossils from that formation and locality. It is a portion of the left maxillary bone, with a series of five molars *in situ*. The first (ib. ib. d^2), slightly mutilated externally, has a simple subcompressed unilobate (or subbilobate?) crown, broadest behind, of much smaller size than that of the following two-ridged grinder (d^3); its working-surface had been worn so as to expose a broad field of dentine. The next tooth (d^3) shows a minor degree of abrasion, the third molar (d^4) still less. In the fourth (m^1) the summits of the two transverse ridges have just been touched; those of the hindmost molar (m^2) in place had not come into use, although they attained nearly the level of the ridges of the antecedent tooth. Moreover, behind the fifth molar was the fore part of a smooth subspherical cavity (ib. fig. 9), plainly the formative alveolus of another molar (m^3) still to come into place.

Accordingly the five molar teeth in this maxillary fossil I interpreted as homotypal in the upper jaw with the five molars in the lower jaw of a similarly immature *Macropus major*. Adopting the symbols in fig. 296, D, vol. iii. of my 'Anatomy of Vertebrates,' those of the five teeth in the present fossil would be:—d^2, d^3, d^4, m^1, & m^2. To test this conclusion I proceeded to remove the outer table of the jaw-bone above d^3, and detected the germ of p^3 (ib. fig. 6), in a stage of development like that of p^3 in the lower jaw of the type specimen (Plate XXII. fig. 18), and corresponding with the state of the dentition in the upper jaw of *Macropus erubescens* (Plate XX. fig. 6). The back tooth, when formed in the hindmost closed alveolus, would be m^3, completing the total of seven teeth developed in the molar series of the *Macropodidæ*.

In the upper premolar of *Macropus Titan* the crown consists of two simple, conical, subcompressed lobes, the hindmost being thickest posteriorly; it is supported on two roots, the formation of which had commenced in the specimen described: its movement into place, or into the masticatory series, would have involved the shedding of d^2 and

* Catalogue of the Fossil Mammalia and Birds in the Museum of the Royal College of Surgeons, 4to, 1845, p. 324, Nos. 1500 and 1510.

d_3; its crown would then contrast with that of d_4 by its freshness or freedom from wear. The convexity of the outer surface of the two lobes, and the depth of the dividing indent, accord with the characters of the lower premolar of the type specimen of *Macropus Titan* expressed in fig. 18, Plate XXII.

The bilophodont* upper molars of *Macropus Titan* (Plate XXI. fig. 8) show a well-developed "prebasal ridge" connected by a "link" of enamel with the fore part of the front lobe, near the middle and inclining rather toward the inner angle. In *Sthenurus Atlas* (Plate XXIV. fig. 6) this link is feebly if at all developed.

The mid link connecting the two main lobes in *Macropus Titan* (Plate XXII. fig. 11) is rather sinuous and tumid; it is better developed in this species than in *Sthenurus Atlas* (Plate XXIV. fig. 6). The oblique posterior ridge (Plate XXII. fig. 11, m_3, g, and Plate XXI. fig. 9, m_3) is strongly marked, and defines a depression at the inner and under side. The main lobes have broad convex bases in the side view of the molars, and the entire crown is longer in proportion to its transverse breadth than in *Sthenurus Atlas*.

The front pier of the zygomatic arch (Plate XXI. fig. 6, zy) is in advance of the hindmost molar in place (m_3) in this young specimen. The anterior outlet of the suborbital canal (ib. fig. 6, zl) is 9 lines in advance of the orbit. Behind the outlet (zl) is the small orifice (a) of a (vascular?) canal, descending into the substance of the maxilla. I have not observed this orifice in the large existing Kangaroos. So much of the bony palate as is preserved (ib. fig. 8) is entire and imperforate, as in *Macropus major*. This character, associated with the small size and simple structure of the premolar, and, as will be seen in subsequently described fossils, its comparatively early loss, support a reference of the present large Kangaroo to the genus *Macropus*, as restricted by most zoologists of the present day.

In the specimen from the Breccia-cave, Wellington Valley, of the left upper maxilla and molar series (Plate XXI. fig. 10) the premolar had risen into place; the last molar (m_3) was protruding from the formative cell, but had not come "into line;" the first two deciduous molars had been shed.

The crown of the foremost tooth was broken off, but the fangs remained (ib. p_3). They were two in number (the hindmost the largest), corresponding in relative size, degree of divarication, and extent of jaw occupied by their insertion with those developed in the unprotruded premolar of the younger specimen (ib. fig. 6, p_3). The choice of the tooth belonging to the fangs in front of the series in the subject of fig. 10 lies between p_3 and d_3; but the latter tooth has, in conformity with its broader bilophodont crown, four roots, each pair diverging from a transversely extended base. The evidence of the roots remaining in the socket of the broken molar is therefore decisive of its homology; the loss of the crown of p_3 is nevertheless regrettable. Its working-surface would have contrasted with that of the tooth d_4, which, having been longer in place and use,

* This term signifies not only that the crown is composed of two principal ridges or lobes, but that these are transverse in position.

shows each transverse lobe worn to near its base, exposing corresponding broad tracts of dentine united by a linear strip along the base of the mid link. In m_1 the dentine exposed on the transverse lobes is a linear tract, rather broader on the front lobe; the front (s) and mid (r) links show abrasion, but not carried to the exposure of the dentine. In m_2 the enamelled summits of the ridges are slightly abraded; m_3, as before stated, had not risen into place.

The molar characteristics of the species (*Macropus Titan*) are well exemplified in this cave-specimen. Sufficient of the palate is preserved to show, as in the preceding one, that it had no large vacuities. The relative position of the zygomatic pier (n) seems to have retrograded as compared with fig. 8; but it still strengthens the jaw where the hindmost molar here (m_2) was in use; when m_3 comes into place and takes its share, the jaw, as we shall see again, becomes concomitantly modified.

The specimen described and figured formed part of the collection of duplicate fossils obtained, under the favourable circumstances detailed in the Philosophical Transactions for 1870, p. 569, by Professor Thomson and Mr. Krefft from the Breccia-cavern discovered by Sir Thomas L. Mitchell, C.B.

In a collection of marsupial fossils at Worcester I recognized a portion of the right upper jaw, with the molar series, of a *Macropus Titan* exemplifying the stage of dentition when the last molar as well as the premolar had come into place, but the former so recently that the zygomatic pier had not much receded in position. The first and second deciduous molars (d_2 and d_3) had been shed. The part of the series d_4 to m_2 inclusive occupied a space rather short of that containing the homologous teeth in the younger specimen (Plate XXI. fig. 10); but the structure of the last two teeth and the proportions of the premolar were those of *Macropus Titan*. Unfortunately the crowns of the first three teeth had suffered fracture. A portion of the hinder fold of enamel remained on the broken base of the crown of the premolar, showing that the hind lobe of that tooth, besides being thicker than the fore one, was divided into an outer and inner lobule. Its longitudinal extent agreed with the crown of the germ of p_2 exposed in the subject of fig. 6, Plate XXI.

The same phase of dentition is exemplified in a similar portion of the right maxillary of another and somewhat larger individual of *Macropus Titan* (Plate XXI. fig. 11), in which the crown of the premolar is entire, and shows by its unworn condition that it had but recently risen into place. This tooth instructively contrasts with the next grinder, which is worn down so as to expose a continuous field of dentine, encroached upon by two opposite folds of enamel from the inner and outer sides of the crown meeting at the middle. In the next tooth the dentine is exposed upon each of the transverse lobes and upon part of the anterior "link." In the penultimate molar a thin line of dentine appears on the front lobe, but the enamel is not worn down so far in the hind lobe. The enamel ridge of the front lobe of the last molar is touched by abrasion. The crown of the premolar shows it to have been the last of the series of five teeth now come into place. It is trilobed: externally it shows only the bilobed structure (as in fig. 6);

but there is a smaller third tubercle on the inner side of the hind lobe, increasing the breadth of that part of the tooth, as was indicated by the last-described specimen (ib. fig. 10, p_3). The length of the entire series of five teeth is 2 inches 9 lines; that of the premolar (fore-and-aft diameter of crown) is 5 lines, that of the next tooth (d_4) being the same; that of the penultimate molar is 8 lines. The whole series is bounded on the inner side by an almost straight, very feebly concave line; the outer contour is rather more convex.

The two specimens above described are in the Museum of the Natural-History Society of Worcester, to the Council of which I am indebted for the opportunity of describing and figuring them. They were obtained by the donor, Henry Hughes, Esq., in the freshwater deposits of Darling Downs.

The subject of figs. 15, 16, & 17, Plate XXI., is also from the freshwater deposits of Queensland. It includes a considerable proportion of the right maxillary, with the last four grinders *in situ*, the dentine being exposed along a very narrow strip of the front lobe of the hindmost tooth (m_3). In the foremost (d_4) the channel of dentine along the mid link is not quite exposed, the enamel at the base of the link still remaining. The two anterior deciduous molars and the premolar have been shed and the alveoli obliterated. This, therefore, is from a fully mature individual. The three teeth (d_4, m_1, m_2) homologous with the last three molars of the young specimen (ib. figs. 6–8) occupy the same longitudinal extent, viz. 1 inch 8½ lines: with the fully developed succeeding teeth they exemplify the later stage of the upper molar dentition in the present extinct species. The last molar (ib. figs. 15 & 16, m_3) shows well the characteristic modifications of its working-surface in *Macropus Titan* as compared with that in *Sthenurus Atlas* (Plate XXIV. fig. 6, m_3): the prebasal ridge (f) is broader; its margin rises (the tooth being viewed prone) from the outer end to near the middle of its transverse course, then sinks more rapidly to its inner end, which bends up upon the front lobe. From the low or open angle thus described by the sharp margin of the prebasal ridge, the linking process (s) extends to near the middle of the fore part of the front lobe. In *Sthenurus Atlas* there is no front link; the margin of the narrower and lower prebasal ridge forms no angle as it sinks to terminate at the fore and inner end of the front lobe.

The mid link (Plate XXI. fig. 15, m_3, r) comes off from the front lobe nearer to its inner end in *Macropus Titan*, but not from that end as in *Sthenurus Atlas* (Plate XXIV. fig. 6, m_3, r). It is more developed in *Macropus Titan*, and its course is more longitudinal as it recedes to abut against the middle of the hind lobe; the postbasal ridge (g) extends from the postinternal angle of the hind lobe downward and outward to the postexternal part of the base of that lobe, leaving a well-marked oblique dent or cavity on the posterior surface of that lobe. In *Sthenurus Atlas* a general slight concavity of the hind surface of the hind lobe of m_3, upper jaw, is bounded below by a feeble postbasal ridge. With an equality of breadth, the fore-and-aft extent of the last molar in *Macropus Titan* exceeds that of *Sthenurus Atlas* by 1 line.

The hind border of the front or maxillary pier of the zygomatic arch is on the vertical

parallel of the interval between the fore and hind lobes of m_3 (Plate XXI. fig. 15). The retrogression of this buttress of bone is concomitant with the grinding-function now assumed by the last of the molar series (compare with figs. 10 & 8).

The anterior outlet of the suborbital canal (Plate XXI. fig. 16, 21) is 1 inch in advance of the anterior border of the orbit. Three lines behind the antorbital foramen is the smaller oblique aperture (*a*) leading down to the interior of the maxillary bone. The outer plate of the maxillary, in advance of and below the antorbital foramen, shows a depression; while the maxillary wall of the nasal cavity swells outward in existing Kangaroos. The proportion of the bony palate preserved shows the small narrow fissure where the maxillo-palatine suture bends inward opposite the fore part of m_3; elsewhere the palate is entire, as in *Macropus* proper, in *Boriogale*, and *Osphranter*. The fore part of the palate near d_4 shows a longitudinal channel (ib. fig. 15, *b*), 4 lines broad, bounded anteriorly by a ridge, or hind part of the diastema, extending forward and inward from the fore part of the socket of d_4, where the sockets (here obliterated) of p_3 and d_3 had been. This prepalatal groove is not shown in *Macropus major*, *Macropus rufus*, or *Osphranter robustus*.

The maxillary bone extends for 10 lines behind the last molar, on the level of the alveolar openings, and is there impressed by the shallow groove leading to the foramen and canal between the back part of the maxillary and the pterygoid process of the alisphenoid. The figures (Plate XXI. figs. 15–18) being of the natural size preclude the need of verbally noting admeasurements.

The side view of a corresponding part of the upper jaw of a large male *Macropus rufus*, at the same stage of dentition as in the present fossil, is given in Plate XXIII. fig. 1; it is from one killed by Mr. Gould, and was the largest Kangaroo which he saw in Australia.

In reference to the constancy in size and other characters of *Macropus Titan*, I was fortunate in finding a second specimen from an adult of this fine extinct Kangaroo in the Geological Museum of the University of Oxford, which the learned and estimable Professor of Geology, John Phillips, D.C.L., F.R.S.*, liberally transmitted to me for comparison and delineation. It was accompanied by an almost entire lower jaw of the same species, at the same phase of dentition, and apparently of the same individual. Both had been obtained by Dr. Nicholson, of Sydney, New South Wales (now Sir Charles Nicholson, Bart.), from the freshwater deposits of Queensland.

The subject of figs. 10, 11, & 12, Plate XXII., is part of the left upper jaw with the last four molars (d_4–m_3) in place; d_3 and p_3 have been shed and their sockets obliterated. The crowns of the remaining teeth show different degrees of abrasion, the summits of the last molar being slightly worn, not so as to expose the dentine. This specimen, therefore, bespeaks a fully mature but not aged animal.

The bone includes the base of the anterior pier of the zygomatic arch (from which the dependent process has been broken away), part of the floor of the orbit with the orbital

* His friends and science have to lament an irreparable loss since this was written.

aperture of the antorbital canal, and a considerable extent of the bony palate (showing the same imperforate structure as in the preceding specimens of *Macropus Titan*).

The pier of the zygoma extends obliquely from the under and fore part of the orbit downward and backward, the hind border being on the vertical parallel of the middle of the last molar. The ridge from the outer side of the masseteric process subsides, as it rises toward the orbit, sooner than in *Macropus major* or *Macropus laniger*. As in the last described specimen, the anterior outlet of the suborbital canal is relatively further in advance of the orbit than in *Macropus major*, being an inch from that part and on a vertical parallel with the diastema in advance of the front molar ($d\,4$); in *Macropus major* it is above the interval between $d\,4$ and $m\,1$, and opens only $4\frac{1}{2}$ lines in advance of the orbit. In *Osphranter robustus*, Gd., the antorbital foramen is 10 lines in advance of the nearest part of the orbital margin, and is on the vertical parallel of $d\,4$. It thus more nearly resembles *Macropus Titan* than does *Macropus major*; but the larger extinct Kangaroo differs from both the large existing species in the following structure, which I now have ground for regarding as constant. There is, as observed in former fossils of *Macropus Titan*, a foramen (*a*) 3 lines behind the antorbital one (21), fig. 10; it is not another outlet of the suborbital canal, but leads obliquely downward into the entrance or substance of the maxillary bone. Of this foramen (*a*) I have not seen a trace in any existing Kangaroo, save *Macropus erubescens* (Plate XX. fig. 1, *a*). The degree of attrition of the upper molars in fig. 11, Plate XXII., agrees with that of the lower molars in fig. 14, ib. The exposed tract of dentine in $d\,4$ is continuous, the mid link being worn down to its base; the fore part of the crown is broken off. In $m\,1$ the front lobe is worn down to the level of the prebasal ridge, which is well marked, overlies the back part of $d\,4$, and shows a rudiment of a link or mid rising to the front lobe of its own tooth. The line of abrasion of this lobe is from without inward and a little backward, not transverse to the skull's axis: a mid link is continued from it to the middle of the front surface of the hind lobe; this is worn, but not so as to obliterate the oblique outer cleft dividing it from the postbasal ridge which rises to be lost in the inner end of the hind lobe.

In $m\,2$ the characteristic configuration of the crown of the upper molar of *Macropus Titan* is well shown. The two chief lobes are more nearly transverse in the direction of their summits than in *Macropus major*; the prebasal ridge with its linking process and the mid link are as well marked as in that species, and the oblique postbasal ridge is longer. In the last upper molar of *Macropus Titan* this ridge (*g*), which is almost obsolete in *Macropus major*, is as well marked as in the preceding molar, $m\,2$. The mid link of the last molar is more curved than that of $m\,2$; the concavity of the curve is turned inward.

Compared with the molars of *Sthenurus Atlas* (Plate XXIV. fig. 6) the prebasal ridge is rather more developed, the mid link is thicker, the outer and inner sides of the transverse ridges are thicker and more prominent, and the fore-and-aft extent of the crown is relatively greater.

The crowns of the upper molars are, as usual, broader than those of the lower jaw, and, as in *Macropus major* and *Nototherium*, the last lower molar has a greater longitudinal extent of grinding-surface than the tooth above.

In another specimen of a smaller portion of the left maxillary of *Macropus Titan* in the University Museum of Geology, Oxford, the dentition is shown at the same phase of development as in the preceding fossil, with a rather greater degree of abrasion. A thin line of dentine is exposed upon the summit of the anterior lobe of m 2; the mid link is worn to its base, exposing a linear tract of dentine uniting the broader field upon the anterior and posterior lobes. The size and other characters of the upper molars in figs. 10–14 (Plate XXII.) are satisfactorily repeated in the present evidence of *Macropus Titan*.

Both specimens are from the freshwater beds or drifts of Queensland, and were presented to the Oxford Museum by Sir CHARLES NICHOLSON, Bart., M.D., and formerly Speaker of the Legislative Assembly at Sydney, New South Wales.

The portion of right upper maxillary (Plate XXIII. figs. 2 & 3) in which the adult series of five grinders had been acquired, but with posthumous mutilation of the crowns of the two anterior ones, shows a modification of those of the three following (m 1, m 2, m 3) which I now know to be a variety, although not such as to induce me to refer the fossil to another species. The mid link (fig. 3, *r*) as it passes forward from the hind to the front lobe expands and divides; the more direct or normal continuation, after reaching the front lobe, bends to terminate or be continued into the inner border of that lobe; the other lower and shorter division turns outward to be lost upon the lower part of the outer half of the hind surface of the front lobe.

This character I briefly expressed as "a more complex form of the longitudinal ridge connecting the two principal transverse eminences" than in *Macropus major* or *Macropus laniger*, in my 'Catalogue of the Fossil Mammals* in the Museum of the Royal College of Surgeons'; at the date of which work this specimen was the sole evidence of the upper jaw and teeth which appeared to me to be referrible to *Macropus Titan*.

Besides the structure of the grinding-surface of the molars above defined, those in fig. 3, Plate XXIII., are arranged with a curve rather more marked than in the subject of fig. 11, Plate XXII.; but as the teeth here are less straight than those in the subject of fig. 8, Plate XXI., this seems to be but a ground of variety. The relative position of the zygomatic pier in figs. 2 & 3, Plate XXIII., may relate to the recent movement of m 3 into its working position: the untouched lobes of this tooth are longer and sharper than usual; yet the general concordance with the molar characteristics of *Macropus Titan* lead me still to refer the specimen No. 1510 to that species.

The modification of the mid link seems a small matter, but is not so in the actual phase of zoology. Evolutionally speaking, this variety may be viewed as either a remnant or a dawn of a complex condition of the part which will be described in subjects of a subsequent section.

* 4to, 1845, p. 324, No. 1510.

Of the mandibular dentition of *Macropus Titan* an early stage is exemplified in the fragment of lower jaw from the Breccia-cave of Wellington Valley on which the species was founded*.

I have given an improved figure of the outer side of this specimen, with the premolar exposed in the primitive alveolus, in Plate XXII. fig. 18, and have added a view of the grinding-surface of the two mutilated molars *in situ* (ib. fig. 17). This portion of (right) mandible of *Macropus Titan* includes the hind part of the first molar (d_4) and a larger proportion of the succeeding molar (m_1). The anterior lobe of this tooth is entire; the hind part of the posterior lobe is broken away.

The anterior talon or "prebasal ridge" of m_1 has almost the character of a lobe; it is united to the anterior normal transverse lobe by a well-developed fore link, commencing near the outer angle of the fore lobe, and describing a slight bend in its forward course to expand upon the hind part of the "prebasal ridge" nearer its outer than its inner end. The projecting angle of the "link" is directed inward. The valley between the anterior lobe and the prebasal ridge is thus divided into two hollows, the inner one being the largest. The inner border of the prebasal ridge is sharp, and abuts against nearly the middle of the back part of the antecedent tooth (d_4). The outer border of the prebasal ridge is thicker than the inner one, less inclined inward, and projects freely a little external to the level of the hind lobe of d_4. The back part of this lobe is entire; it shows a submedian posterior vertical indent; there is no perceptible trace of basal ridge.

The mid link (fig. 17, Plate XXII.) repeats the characters of the fore link, save that it sinks lower to connect itself with the anterior lobe, leaving more of the summit of that lobe free than is left to the prebasal ridge. The summit of the anterior ridge of m_1 inclines a little forward as it crosses the tooth from without inward, and is slightly bent with the convexity backward. The mark of wear, which in the young animal owning this tooth had not exposed the dentine, affects the hinder slope of the summit of the transverse ridge.

The characters of the crown in the two lower grinding-teeth of the type specimen of *Macropus Titan* above described are, in the main, those of the largest existing representatives of the true or subgenerically restricted *Macropus*.

In the lower jaw of *Macropus major* (Plate XX. fig. 15) the prebasal ridge (fig. 16, *f*) of m_1 and m_2 has a like size and shape, and is connected with the anterior lobe by a similar link; but this is less bent inwardly in its forward course. In *Macropus rufus* the prebasal ridge is less developed (Plate XXI. fig. 4); there is no postbasal ridge or talon. A feeble vertical notch is shown by the back part of m_1 and m_2; this does not

* Three Expeditions, 8vo, 1838, by Sir Thomas L. Mitchell, C.B., vol. ii. p. 359, pl. xxix. fig. 3. This specimen, with other fossils from the Wellington Valley cavern, submitted by its discoverer to me, were presented to the Geological Society of London, to the President and Council of which I am indebted for the opportunity of reexamining and figuring this collection, which initiated our knowledge of the fossil Mammals of Australia.

appear in m_3. The proportion of length to breadth of the grinding-surface of the true molars is the same in the recent as in the extinct species compared; the difference is mainly in size.

In a portion of the right mandibular ramus of *Macropus Titan*, with the three posterior molars *in situ*, these, like the single entire molar in Plate XXII. fig. 17, m_1, show a proportionally greater antero-posterior extent of the prebasal ridge than in *Macropus major* or *Macropus* (*Osphranter*) *rufus*. Of the latter existing species* Mr. Gould, F.R.S., was so good as to place in my hands, for the purpose of these comparisons, the jaws and teeth of a male which he killed between the rivers Murray and Adelaide, Australia; it measured 8 feet 2 inches from the nose to the end of the tail, and was the largest Kangaroo which that eminent naturalist saw in the continent of which he has so admirably illustrated the rich ornithology as well as its singular mammalogy.

These specimens I presented, in Mr. Gould's name, to the Royal College of Surgeons†, after their application to the requisite comparisons with the fossils from the Wellington Valley caves and freshwater beds of Australia. Figs. 1 & 14 in Plate XXIII. give a side view, and figs. 2 & 4, Plate XXI. the grinding-surface, of the right series of upper and lower molars of this animal, of the natural size.

So much of the mandibular ramus of a *Macropus Titan* (Plate XXII. figs. 13–16) as remains in the specimen in the Oxford Museum closely agrees, save in size, with that of *Macropus major* (Plate XX. fig. 15). As in that recent specimen, the individual affording the present fossilized relic had shed both the premolar and the two anterior milk-teeth; d_4 also shows a wear of crown and exposure of roots indicative of speedy expulsion. The long diastemal border (between d_4 and i) is trenchant to near the outlet of the incisive alveolus. It descends, more rapidly than in the living Kangaroo, from the anterior molar socket, with a concave curve, reducing the vertical extent of the symphysial part of the ramus at the outlet of the dental canal (ib. fig. 13, v) to two thirds of that at the outlet of the anterior molar socket, d_4. In advance of the dental canal the symphysial part of the jaw is reduced to a mere case of the root of the long procumbent incisor, i.

The descent is less sudden, and the concavity of the diastemal border somewhat less, in another specimen of the mandible of *Macropus Titan*, which more closely resembles in this respect the recent Kangaroo.

The symphysial surface in *Macropus Titan* (Plate XXII. fig. 15) begins behind, in advance of the vertical parallel of the fore part of the first molar socket; it expands so as to cover the lower half of the inner surface of the ramus at the part opposite the outlet (v), and then contracts to terminate before attaining the outlet of the incisive alveolus, at least as regards its grooving and other rough markings for ligamentous union. The contrast between this structure of the symphysial joint and that in fig. 6, s (*Sthenurus Atlas*), is considerable, and supports the inference that the junction between

* Then known as the *Macropus laniger*.

† See 'Catalogue of the Fossil Mammals and Birds' &c., 4to, 1845, pp. 324, 325, Nos. 1510, 1511.

the right and left rami was not more close in the large *Macropus Titan* than in *Macropus major*. The direction of the elongated socket of the incisor and the procumbent position of that tooth in the fossil are as in the existing species of *Macropus*. The crown of the incisor, so far as it is preserved, agrees in shape, relative size, disposition of enamel, position and obliquity of the back part of the abraded working-surface, with that of *Macropus major*. The configuration of both outer and inner surfaces of the horizontal ramus, especially the ridge indicating the lower limit of insertion of the crotaphyte muscle and extending a little below the margin of the ectocrotaphyte cavity, as shown in *Macropus major* (Plate XX. fig. 15), are repeated in *Macropus Titan* (Plate XXII. fig. 13, *e*). The last molar stands out more freely, or entirely, in advance of the fore margin (ib. fig. 13, *q*) of the coronoid process in *Macropus Titan* than in *Macropus major* (Plate XX. fig. 15); and it advances further as the animal grows older and the molar series is further reduced.

The inflection of the inner and lower border of the ascending ramus begins anteriorly nearly in the same relative position. The anterior border of the intercommunicating vacuities (Plate XXII. fig. 15, *e*, *d*) between the outer and inner cavities of the ascending ramus appears to be the same in the present fossil as in the largest existing species of Kangaroo. The inner postalveolar border is smoothly rounded, and forms no angle indicative of a postalveolar process.

The molars in the fossil under description are more worn than in the *Macropus major* compared, with a similarly reduced series of teeth. In d_4 (Plate XXII. fig. 14) the exposed tract of dentine is continuous, the mid link crossing the valley being worn through. The prebasal ridge is indicated only by the internal notch; the basal remnant of the crown is supported by fangs, which are partially exposed by absorption of the alveolus, and the crown overhangs the beginning of the diastema, indicative of the impending fall of the tooth (ib. fig. 13, d_4); whence I infer that the molar dentition of *Macropus Titan* would be reduced in advanced age, like that in *Macropus major*, by the loss of d_4, and perhaps ultimately of m_1*.

The pattern of the working-surface of the succeeding molars closely accords with that in *Macropus* proper. The prebasal ridge is considerable, both longitudinally and transversely: the fore link is well marked; it joins the front lobe external to the mid line, leaving a fossa on each side. The contrast with the rudiment of this link in *Sthenurus Atlas* (ib. figs. 8 & 9) is considerable, as is that also in the development of the mid link and the breadth of the anterior margin of the prebasal ridge.

The antero-posterior breadth of the transverse ridges is greater in *Macropus Titan* than in *Sthenurus Atlas*, especially at their outer sides (comp. figs. 13 & 5); the longitudinal extent of the crown is relatively greater as compared with the transverse diameter in *Macropus Titan*.

In the next illustration of the mandibular characters of *Macropus Titan*, so much as is preserved of the two rami shows the angle at which they meet to unite at the sym-

* Since this passage was penned I have received from my friend Dr. Rienets, F.L.S., evidence of the fact.

physis (Plate XXVI. fig. 9). It also shows that the comparatively loose union of the symphysis had permitted the right ramus to glide a little forward from the left one before they were fixed in position by the petrified matrix; but this correspondence with the large living Kangaroos is more decidedly shown in the subject of fig. 11, Plate XXVI. The right ramus of fig. 9 includes the last four molars, d_4, m_1, m_2, m_3, and a part of the premolar, p_3 (this tooth, like the crown of m_1, has suffered more from fracture than from masticatory action). The left ramus includes the last three molars and the hind half of the crown of d_4. The present fossil was obtained by Henry Hughes, Esq., in the freshwater deposits of Queensland, and is now in the Museum of the Natural-History Society of Worcester.

In a fossil with three molar teeth (d_4, m_1, and m_2), and the formative cavity of m_3, these teeth are somewhat inferior in size to their homologues in fig. 13, Plate XXII., and probably indicate that they come from the female of *Macropus Titan*. The subject of figs. 12 & 13, Plate XXIII., is the original specimen in the Museum of the Royal College of Surgeons, No. 1512*, which first afforded the characters of the penultimate and last molars of *Macropus Titan*: this I now believe to have come from a female of that species.

The mandible of *Macropus Titan* (Plate XXVI. figs. 11 & 12), after solution of the soft parts in its original burial-place, shows the effect of the disturbance of the grave by the dislocation of the rami, which had been somewhat loosely attached during life by the partial syndesmosis of the symphysis. So separated and shifted, the right ramus being pushed about 2 inches in advance of the left, the parts have rested without further disturbance long enough to permit the dislocated rami to become connected together by the petrified matrix. The bone, which during the same period had undergone some degree of petrifaction, appears again to have been subject to movements of the matrix, resulting in the amount of fracture of the most prominent parts which is common in the fossils from the freshwater beds of the Australian localities yielding the subjects of the present paper. But the later disturbances have not affected the artificial union of the previously separated and dislocated rami.

The jaw-bone in this specimen exceeds in depth and a little in length that of the *Macropus Titan* in the Oxford Museum (Plate XXII. figs. 13, 15), but the longitudinal extent of the four molars is the same. The present fossil is from an older individual: d_4 is worn down to its base, and the ridges of m_3 (both of the lobes and links) show more abrasion. The vertically oblong pit toward the inner side of the back part of the last molar (ib. fig. 15) is well marked. The symphysial articular surface (ib. fig. 12) is neatly defined behind; its rougher part subsides anteriorly, and ceases about an inch from the outlet of the incisive socket. The vertical diameter of this socket is 8 lines; that of the base of the incisor, where the tooth has been broken off, is 7 lines.

The portion of a left mandibular ramus of a fine old male of *Macropus Titan* (Plate XXVI. fig. 13) shows the largest size of the lower jaw which I have as yet seen in fossils

* Catalogue, *ut supra*, p. 325.

of this species. But though the depth of the mandible at the interval between d_4 and m_1 is nearly half an inch greater than in the subject of fig. 11, or in the Oxford specimen (Plate XXII. figs. 13, 15), the teeth are not much larger. A figure of the working-surface of the last molar in this large *Macropus Titan* is given in fig. 14, and one of the hind surface of the same tooth in fig. 15, to exemplify the characteristic pit there in the fossil.

In the hind part of a mandibular ramus of a fine old *Macropus Titan*, with the last molar well worn, and now much in advance of the coronoid process, the depth of the jaw behind this tooth is 1 inch 6 lines, and the same at the interval between m_2 and the débris of the socket of m_1*.

§ 3. *Macropus affinis*, Ow.—In a small collection of Marsupial fossils made by Sir Thomas Mitchell, C.B., in a survey undertaken after his return to Australia in 1839, and which he was so good as to transmit to the Royal College of Surgeons, there were confirmatory evidences of the two large species represented by the fossils of his first collection in Wellington Valley, described and figured in his work published in 1838†, and also indications of a third species of large Kangaroo, which I described in my Catalogue of the Fossil Mammalia of the Museum of the College, and referred to a *Macropus affinis*‡. This second collection was obtained, according to the notes accompanying it, "from the alluvial or newer tertiary deposits in the bed of the Condamine river, west of Moreton Bay."

The best evidence it contained of the *Macropus affinis* was a portion of the left mandibular ramus, now for the first time figured (Plate XXIII. figs. 10 & 11), including the antepenultimate and penultimate molars, and the sockets and fangs of the premolar (p_3) and of the first (d_4) and last (m_3) two-ridged molars. The two molars (m_1 and m_2) retaining their crowns showed the specimen to have come from an aged individual. The pattern of that of m_1 had been worn away, with mere indications of the two chief divisions and the prebasal ridge. The crown of the penultimate molar agreed in its general proportions more with that of *Macropus Atlas* than with that of *Macropus Titan*, but was narrower in proportion to its antero-posterior diameter than in *Macropus Atlas*, and the mid link was more developed. From its homologue in *Macropus Titan* the tooth differed in having no trace of a postbasal ridge (compare with fig. 13, Plate XXIII.). The depth of the jaw containing the teeth was greater than in *Macropus rufus* (of which a corresponding part of the mandible of a large individual is given in fig. 14, Plate XXIII.). The teeth, however, indicate a species of less size than either of the two extinct ones above cited. I therefore continue to regard this fossil as evidence of an extinct Kangaroo of intermediate proportions between the largest known living species and those defined in my original memoir, and of which additional illustrations are given in the present.

* This is the specimen alluded to as having been received, since the present paper was prepared, in the last collection of fossils from the freshwater deposits of Queensland, transmitted by George Bennett, M.D., F.L.S.

† Three Expeditions &c., 8vo, vol. ii. ‡ *Op. cit.* 4to, 1845, p. 328.

§ 4. *Osphranter Cooperi*, Ow.—The subject of figs. 17 & 18, Plate XXIV., is the fore part of the left mandibular ramus of an aged individual of a Wallaroo, of the size of *Osphranter robustus*. It retains the first three molars (p 3, d 4, m 1), the second of which, as having been longest in place, has the crown worn down to its base, from within obliquely outwards, and in a rather greater degree than in the corresponding tooth of the recent species compared, the mandible of which is the subject of fig. 13, Plate XX. The premolar (p 3) shows three small tubercles on its working-surface, arranged from before backward; the crown is subcompressed, and very slightly thickened behind; the inner surface of the fore part of the crown is gibbous, as in *Osphranter*, and its proportions are as in *Osphranter robustus*. The degree of wear of the next tooth is such as would be incompatible with the retention of the foremost if it were the deciduous tooth, d 3; but, for decisive evidence, I removed the inner wall of the ramus where the germ of p 3 would have been, and there was no trace of such successional tooth. The present fossil, therefore, has come from a fully mature individual. A species of true *Macropus* would not have retained the premolar or the following tooth at this age, or have kept d 4 with a crown so far worn down. Thus the fossil accords with *Osphranter* in the proportions of p 3 and d 4 and their long continuance in line with the following molars. The third tooth (m 1) in the fossil is relatively broader than in *Osphranter robustus*. The outer side of the diastemal and symphysial part of the mandible is less convex vertically than in *Osphranter robustus*. The symphysis begins behind in the same relative position to the premolar. I indicate the present fossil Kangaroo by the name of the donor, Sir Daniel Cooper, Bart.; it was discovered in the freshwater beds of Darling Downs, Queensland.

§ 5. *Osphranter Gouldii*, Ow.—The subject of figs. 15 & 16, Plate XXIII., is a corresponding part of the lower jaw of a young specimen of the same subgenus, but of smaller size. The fossil shows a remnant of the socket of d 2, and the much-worn crown of d 3; that of d 4 is also much worn, but not reduced to the degree shown in fig. 18, Plate XXIV. To this smaller kind of fossil Wallaroo (*Osphranter Gouldii*) I have attached the name of the discoverer and founder of the genus.

§ 6. *Phascolagus altus*, Ow.—Of this species a portion of the upper jaw and teeth was figured in the Palæontological Appendix to Mitchell's 'Three Expeditions into the Interior of Eastern Australia,' &c., vol. ii. plate xxix. (plate xlvii. of 2nd edition) figs. 4 & 5, with the following remark:—"This specimen I believe to belong to *Macropus Titan*. The permanent false molar, which is concealed in the upper jaw, is larger than that of the lower jaw of *Macropus Titan*; but I have observed a similar discrepancy of size in the same teeth of an existing species of *Macropus*" (ib. p. 360). Subsequent and closer comparisons have, however, shown that the pattern of the grinding-surface of the upper molars is more like that in *Halmaturus* and *Osphranter* than in *Macropus major* or *Macropus Titan*; and the discovery of the upper jaw of the latter species at a corresponding phase of dentition (Plate XXI. fig. 6) has shown that, in size and simplicity of form, the upper premolar much more closely accords with the

lower one in the type mandible of *Macropus Titan* (Plate XXII. fig. 18) than does the premolar exposed in the specimen under examination (Plate XXII. fig. 1).

These phases of dentition, illustrative of the characters and affinities of the fossil under review, are shown in the specimens Nos. 1741, 1742, and 1743, in the Osteological Series of the Museum of the Royal College of Surgeons of England*, and are illustrated in Plate XX. figs. 1–12 of the present Paper. In the recent species (*Macropus* (*Phascolagus*) *erubescens*) the upper premolar (ib. fig. 6, p^3), in its form and proportions, still more closely resembles that (Plate XXII. fig. 1, p^3) of the larger extinct Kangaroo (*Phascolagus altus*) of the Wellington Valley Bone-cave.

This species combines with the proportion of the premolar, affording one of the characters of the subgenus *Halmaturus*, the entire or imperforate bony palate, which is found in all the species of *Macropus* in its restricted or subgeneric sense (Plate XXII. fig. 2). In this combination of characters the fossil agrees with the existing *Phascolagus erubescens*.

The degree of development of the concealed premolar, the crown being completed with the basal portions of both roots, coincides, as in *Phascolagus erubescens*, with the incompleted eruption of the molar (m^1) and the still hidden and undeveloped state of m^2 and m^3; whence may be inferred a like precocious appearance of the premolar in the working series, with the concomitant shedding of the two anterior deciduous teeth (d^2, d^3), the premolar preceding the penultimate molar in entering upon the work of mastication. The differences observable between the fossil and the recent Kangaroos combining the above characteristics of the proposed subgenus are, at least, specific.

The premolar, divided in both by a vertical cleft into a smaller anterior and a larger posterior lobe, shows in the fossil a more definite basal ridge along the outer side of the latter lobe than in *Phascolagus erubescens*; there is also a more definite outswelling of the hind part of the hind lobe in *Phascolagus altus*; two feeble grooves divide the outer surface of the fore part of the anterior lobe into three vertical prominences, but these are faintly marked in the present fossil.

In the bilophodont molars the prebasal ridge is narrow and the indication of the fore link is minute. The mid link is narrow, neatly defined, and sinks rapidly from the inner and posterior apex of the front lobe to the lower part of the interlobal valley. The postbasal ridge is represented by a similar outbending and descent of a sharp ridge from the inner angle of the hind lobe; which ridge, curving to subside upon the outer part of the base of the hind lobe, circumscribes, below, the depression or transverse concavity on the hind surface of that lobe.

The position and extent of the origin of the anterior pier of the zygoma is the same in the fossil and the recent species compared. The configuration of the hind margin of the bony palate is the same. But our extinct Kangaroo shows these characters of its subgenus on a larger scale than the largest known existing species of *Phascolagus*. The tooth d^4 (Plate XXII. fig. 2) is as large as its homologue in *Macropus major*; the

* Osteological Catalogue, 4to, 1853, p. 324.

antero-posterior extent of m_1 is a trifle more in the fossil. We may infer from the superior size, both absolute and relative, of the premolar in *Phascolagus altus* that the permanent molar dentition would be represented for a longer period of life by the five teeth, p_3, d_4, m_1, m_2, and m_3, than in the existing Great Kangaroo (*Macropus major*).

The specimen above described, with the rest of Sir Thomas L. Mitchell's first collection of cave-fossils from Wellington Valley, is in the Museum of the Geological Society of London. I am indebted to the President and Council for the opportunity of giving new and better figures of the type of *Phascolagus altus* than the original ones in the 'Appendix' of the above-cited work.

In the collection of fossils from the freshwater deposits of Queensland, lately received from Dr. George Bennett, F.L.S., of Sydney, New South Wales, there are instructive evidences of *Phascolagus altus* adding to our knowledge of its cranial and dental characters. The specimen No. 38752, Register of Fossils, British Museum, is part of a right maxillary of a young animal with the dentition in nearly the same state as the subject of figs. 1 & 2, Plate XXII. The germ of the premolar seems rather less than in that type specimen; but the hind angle was broken off in the work of exposure, which the state of petrifaction of the lacustrine fossils made more difficult than in the Cave specimen. The fore link is a little more marked in m_2 than in the type specimen, but the agreement in other characters is sufficiently close to determine the species and subgenus as above defined.

The next Bennettian specimen is from a somewhat older individual of *Phascolagus altus*; it is a portion of the right maxilla with d_4, m_1, and m_2 in place; these three molars occupy the same extent as that in the skull of *Boriogale magnus*, the upper molars of which are figured in Plate XX. fig. 12—an extent about 1 line short of that in *Macropus rufus* (Plate XXIII. fig. 1), and about 1 line more than that in *Osphranter robustus* (fig. 3, Plate XXI.). We have here, therefore, plainly demonstrated, the representative of a Kangaroo about the size of the largest now living in Australia. Independently of the premolar character shown in the previous specimens, the present fossil could not be referred to *Macropus major*. The antorbital foramen is too remote from the orbit and from the ridged beginning of the masseteric process, which also is more directly continued from the fore border of the orbit than it is in *Macropus major*. The foramen in question is 7 lines in advance of the nearest part of the masseteric ridge in the fossil; it is $3\frac{1}{2}$ lines in advance of that ridge in *Macropus major*.

In the position of the antorbital foramen the fossil more resembles *Osphranter robustus*, in which, however, the foramen is about a line further in advance of the masseteric ridge; this, in its prominence, sharpness, and the depression anterior to it, resembles more than does *Macropus major* the fossil fragment compared. *Boriogale* more closely repeats the above-defined cranial character in the fossil. But *Phascolagus* has the palate entire, where *Boriogale* shows the large vacuity (Plate XX. fig. 12) common to it with the type species of *Halmaturus*, F. Cuv. In the molars of *Phasco-*

lagus the prebasal ridge is larger than in *Boriogale*; the breadth of the outer sides of the two main lobes is greater; the postbasal ridge better defines the hinder depression below.

Both the cranial and dental characters of *Phascolagus* forbid its reference to a *Boriogale*. In the upper molars of *Osphranter*, with a prebasal ridge developed in the same degree as in *Phascolagus*, the fore link is also present, though feeble; yet in a more conspicuous degree than in *Phascolagus*, where it can hardly be said to exist: the fore link is better developed in the upper molars of *Macropus major*, and the valley is wider between the two lobes.

The remains of the alveolar cavities for the two roots of the premolar show that it had come into place in the fossil under review; and the fore-and-aft extent which the two cavities occupy with the width of the intervening tract of bone indicate a premolar about the size, in that dimension, of that of the type specimen (Plate XXII. fig. 1, p^3), and rather longer than the following tooth (d^4), but far short of the proportions which characterize p^3 in the genera *Sthenurus* and *Protemnodon*, next to be defined. The state of the socket of m^3 in the Bennettian specimen, and the rising of its base between the insertions of the fore and hind fangs, clearly bespeak that this tooth had likewise come into place, and that the fossil under comparison is from a nearly mature individual of its kind. Sufficient of the bony palate remains to show (as in the younger type specimen) that it was entire, as in *Macropus* proper and *Osphranter*.

The interorbital aperture of the suborbital canal in *Phascolagus* is single, subcircular, and well defined; its fore and upper border rises upon a ridge or plate of bone, which extends forward and outward to near where the masseteric ridge subsides into, or rises from, the fore border of the orbit. This structure I have not observed in the skull of any existing species of *Macropus*, *Osphranter*, or *Halmaturus*; the nearest approach to it is seen in the skull of *Boriogale magnus*.

A second and larger proportion of the upper jaw of *Phascolagus altus* in the Bennettian series shows, on the left side, the base part of the crown of the premolar in place, and the sharp summits of the lobes of the last molar emerging from their nursery. The antecedent molars show more wear than in the preceding specimen. The mid link in d^4, in the present, is worn down to the dentine; yet the second lobe of m^3 is less abraded, and the fore link is rather more conspicuous.

On the right side the hind molar and its socket have been broken away. More of the premolar is preserved, but the bilobate outer part of the crown is wanting; it had, plainly, the antero-posterior dimensions of the entire crown exposed in the type specimen (Plate XXII. fig. 1). The tract of the suborbital canal is exposed in both halves of this upper jaw; and we see that its anterior outlet must be far in advance of the orbit, and about half an inch above the fore end of the premolar.

The molar series in this fossil equal in extent and in the size of the teeth those of *Macropus rufus* and *Macropus major*; they rather exceed in size those in the younger, perhaps female, individuals represented by the first-described fossil from the fresh-water beds of Queensland and by the type specimen of *Phascolagus altus*.

§ 7. *Sthenurus Atlas*.—Similar considerations to those which influenced judgment and action in regard to the type fossil of *Macropus Titan*, added to plainer indications of the incomplete development of the rear teeth of the molar series in the fragment under scrutiny, led me, in 1837, to perform the same operation on the subject of fig. 4, Plate XXII.*; and great was my surprise at the result.

The hidden germ (p^3) equalled in antero-posterior diameter both the deciduous molars which it would have displaced, and surpassed in that diameter the largest of the molars to the extent of one half that length of their crown. For the great extinct species of Kangaroo so indicated I proposed the name of *Macropus Atlas*†.

The tooth so discovered recalled a dental characteristic of the Potoroos, or Kangaroo-rats (*Hypsyprymnus*, &c); but the molars in the fossil were strictly bilophodont, more so, indeed, than in *Macropus Titan* or the existing *Macropus major*. There was less indication, for example, in the "links" of any subdivision or reduction of the two transverse ridges to a quadrituberculate grinding-surface; they stood out more definitely and more freely. Moreover, the large premolar of the fossil was primarily divided externally into a fore lobe and hind lobe by a vertical fissure continued as a groove almost to the base of the crown, whilst the oblique extension of that fissure inward and backward gave a transversely subbilobed character to the unworn surface of the hinder part of the tooth. As, however, I have since obtained a portion of the upper jaw with the right series of molars of the same species, I will proceed with its description before entering upon further and requisite details of the mandibular evidences originally indicating the present extinct subgenus of Kangaroo.

The instructive illustration of the maxillary dentition of *Sthenurus Atlas* (Plate XXIV. figs. 4, 5, 6) formed part of a collection of fossils sent to me by F. G. Waterhouse, Esq., Cor.M.Z.S., Curator of the Museum of Natural History in Adelaide, South Australia, in the freshwater deposits of which province this fossil was obtained.

The portion of maxilla includes the masseteric process (ib. fig. 4, zy), the hind border of the maxillary pier from which it is continued being parallel to the interval between the penultimate (m^3) and last (m^4) molars. The process extends down a little below the alveolar border of m^4, and appears to be entire with an obtuse end. It is not so long relatively, does not reach so low, as in *Macropus major* or *Macropus* (*Osphranter*) *robustus*, but is more produced than in *Macropus* (*Halmaturus*) *ualabatus*: its proportions are most nearly those of *Macropus* (*Phascolagus*) *erubescens*. The outer surface of the base of the process is less deeply excavated than in any of the above-named recent species.

The convex tract behind the masseteric process and maxillary pier of the zygoma leads into the orbit, and there, about 8 lines in advance of the hind border of the pier, is the orbital aperture of the suborbital canal. It is single, subcircular, well defined, without any appearance of the oblong depression we there see in *Macropus*, *Osphranter*, and *Halmaturus*, where a second large foramen also communicates with the orbit.

* Plate xxix. fig. 1, Mitchell's 'Three Expeditions,' &c., vol. ii. 1838. † Ibid. p. 359.

The floor of the orbit presents an oblong depression (the "entorbital fossa"), with a sharp anterior and superior margin. From the fore part of this depression proceeds the suborbital canal, commencing by a large circular hole ("entorbital foramen"); a little way behind this is a smaller ("spheno-palatine") foramen.

The inner wall of the orbit, formed by the maxillary and palatine, curves outward and upward from the upper border of the depression to unite with that in advance contributed by the lacrymal, leaving the outer circumference of the entorbital foramen free from any direct rise of the interorbital plate. Anterior to the entorbital canal there is a more shallow and imperforate depression affecting the lower part of the lacrymal, at a little distance from the anterior border of the orbit. This structure of the orbital surface agrees with that in *Macropus* and *Osphranter*, with minor differences.

In *Macropus major* the entorbital fossa is deeper, the sharp upper border being extended backward beyond the spheno-palatine foramen; there is a third smaller "pterygo-palatine" foramen at the end of that border; but the fractured state of the fossil prevents the determination of its agreement or otherwise in regard to that third foramen.

In *Osphranter robustus* the second foramen is as large as the first, and is situated to its inner side and very little posterior to it, the intervening bony plate with a sharp concave edge forming the inner border of the entorbital foramen and the antero-external border of the more oblique spheno-palatine foramen.

In *Sthenurus Atlas* the upper border of the entorbital fossa, in its shortness and degree of sharpness, is more like that in *Macropus Titan*. The inner wall of the orbit ascends rather more directly therefrom than in *Macropus major*. The pterygo-palatine foramen in the palatine part of the inner orbital wall is more minute in *Osphranter* than in *Macropus*. In *Phascolagus erubescens* the proximity of the first and second foramina is closer than in *Osphranter*.

In the unique skull of *Boriogale magnus* it appears that part of the inner wall of the orbit completing, above, the circumference of the second foramen is unossified; and such part of the skull in a petrified state would show only one large circular orifice, answering to the first or entorbital one in *Macropus major* and *Macropus Titan*. In the comparison of the orbital part of the skull, *Macropus Titan*, in the relative size and position of the two anterior foramina (entorbital and spheno-palatine), agrees with *Macropus major* more closely than with the above-cited representatives of other sub-genera of living Kangaroos.

From the upper and anterior margin of the entorbital foramen (Plate XXIV. fig. 5, *o*) rises a plate of bone (*n*, figs. 4 & 5), quickly narrowing to form part of the inner wall of the orbit, or partition-wall between that cavity and the nasal one. This structure implies a less relative depth, or diameter, of the orbit from without inward or transversely than in the existing genera above cited*. But a nearer approach to the above-defined orbital

* Whence may be inferred a smaller eyeball, associated perhaps with more diurnal habits, than in the still living Kangaroos.

character in *Phascolagus* and *Sthenurus* appears in *Boriogale*, where the nerves and vessels, passing by the floor of the orbit to the maxilla, leave only one mark of perforation of that floor by a subcircular entry to the canal, the other elements forming the second and contiguous foramen in *Macropus*, &c., here traversing the above-surmised membranous or unossified state of the inner and under wall of the orbit. The reduced ossified part continued from above the bony canal rises somewhat like the lamellate process shown at *n*, figs. 4 & 5, in *Sthenurus*. *Boriogale* also shows the longitudinal depression above and exterior to the entorbital foramen, terminating anteriorly in a blind end, as is seen in *Sthenurus* and in a feebler degree in *Halmaturus*.

The outlet of the suborbital canal in *Sthenurus Atlas* is relatively further from the orbit than in *Macropus major*, in which respect the present fossil resembles *Osphranter* and *Halmaturus*: the distance in the present example of *Sthenurus Atlas* is 1 inch 1 line. The lower part only of the outlet and canal is preserved in the present specimen; and below the outlet is a second small foramen, the canal from which passes backward, not downward as in *Macropus Titan*.

There is not sufficient of the bony palate preserved to determine whether it was as entire as in the larger living Kangaroos (*Macropus major*, *Osphranter robustus*, *Phascolagus erubescens*), or with vacuities, as in most species of *Halmaturus*; but part of the border opposite the interval between d_4 and m_1 (Plate XXIV. fig. 6) is so smoothly rounded off as to suggest that it is a natural, not a broken, tract.

The premolar has the middle two fourths of its outer surface slightly depressed and feebly concave lengthwise (ib. figs. 4 & 6, *a*), with two chief vertical ridges and others faintly indicated. The fore and hind ends of the outer surface are smooth and convex, or bulging; the free margin is subtrenchant, with the ends of the terminal bulges obtuse. The inner surface or division of the crown (ib. fig. 5, *b*) is much lower than the outer one, yet having more of the character of a part of the crown than of a developed "cingulum;" it increases in height as it recedes, the hind part swelling into an inner lobe, continued at the back part of the crown into the postexternal tubercle and abutting against the inner side of that part by a second transverse ridge. The lower and less developed fore part of the inner division of the crown is similarly connected with the antexternal tubercle, viz. by a low ridge forming the fore part of the crown, and by a buttress-like production against the inner surface of that tubercle. The intermediate part of the inner division is connected with the outer division by three transverse ridges (ib. fig. 6, p_3). A premolar of the size shown in the figures, and with the structure above described, would be held, according to its proportions to the molars behind, as indicative of a subgeneric section of *Macropodidæ*, for which I propose the term *Sthenurus*, suggested by the form and proportions of a vertebra of the very powerful tail of this great extinct Kangaroo*. I shall presently be able to show that the modifications of the mandible and mandibular incisor support this distinction.

The bilophodont upper molars of *Sthenurus* are characterized by a narrow prebasal

* Gr. σθένος, strength; οὐρά, tail.

ridge (Plate XXIV. fig. 6, *f*) without a fore link; by a still narrower and shorter post-basal ridge, represented by that, *g* (fig. 4 *a*), which curves from the outer part of the base to the inner angle of the hind lobe, along the back part of the crown, that surface sinking a little above the part of the ridge nearest the base of the tooth. The mid link (ib. fig. 6, m_{2}, *r*) is thin, low, or rudimental, yet still traceable from the back part of the inner angle of the anterior lobe to the middle of the base of the fore part of the hind lobe. The contour of the working-surface of the molars is more subquadrate than in *Macropus Titan*, the fore-and-aft diameter being not so much greater than the transverse. The series describes a feeble curve convex outward, but changes anteriorly to a slight concavity through the modification of the premolar at *a*, as above described. The abraded state of d_{4} contrasts with the almost untouched crown of p_{3}, showing the earlier development of the hinder tooth. The dentine is just exposed on the inner halves of the lobes of m_{1}. The enamel only shows abrasion at the summits of the lobes of m_{2}; the edges of those of m_{3} are slightly polished by wear anteriorly. This fossil has come from an individual that perished in the prime of life.

In existing Kangaroos the upper premolar of *Macropus ualabatus*, Lesson (Plate XXIV. figs. 1–3, p_{3}), bears the nearest resemblance to that of *Sthenurus Atlas*, and is associated with the same general pattern of working-surface of the molars d_{4}, m_{1-3}, except that the fore link and mid link, though feebly developed, are more neatly defined and readily recognizable in the small existing Kangaroo. The figures of both these recent and fossil Kangaroos, or "Wallabies," being of the natural size, preclude the need of stating dimensions.

Macropus (*Sthenurus*) *Atlas* was first indicated by a fragment of the under jaw from the Breccia-cave in Wellington Valley. In the type specimen (Plate XXII. figs. 3 & 4) three molars (d_{2}, d_{4}, and m_{1}) are in place; the penultimate molar (m_{2}) is lost; the crown of the last molar (m_{3}) is just rising from the formative alveolus.

The first true molar (m_{1}) affords an instructive comparison and contrast with that in the type specimen of *Macropus Titan* (ib. figs. 17, 18, m_{1}). The grinding-surface of m_{1} in *Sthenurus Atlas* is broader in proportion to its length, especially behind. The prebasal ridge is narrower and lower; a simple link descending from the fore and outer angle of the front lobe slopes straight to the middle of the summit of the prebasal ridge.

The outer convex borders of the two lobes (ib. fig. 4, m_{1}) are narrower than in *Macropus Titan* (ib. fig. 18, m_{1}), and maintain their breadth, like columns, more uniformly to their summits. The inner borders are rather broader below, but are narrower than in *Macropus Titan*. In *Sthenurus Atlas* the valley between the lobes is both wider transversely and deeper, the link being lower; it slopes from a point rather external to the middle of the front surface of the hind lobe, and runs almost straight down to the middle of the base of the hind surface of the front lobe. The mid link becomes almost obsolete in the last molar (ib. fig. 3, m_{3}). The summits of the lobes bend slightly backward vertically, and from the thickening of the outer and inner angles are feebly concave

across anteriorly. There is a slight swelling of the base of the hind surface of the hind lobe, but not any distinct postbasal ridge.

I have given a new figure of the side view of part of this fossil (Plate XXII. fig. 4)*, and an upper view of the entire fragment (ib. fig. 3), showing the characters of the working-surface of the molars.

In a visit this year to the Geological Museum, Oxford, I was much gratified and interested in finding, in the series of fossils from the freshwater deposits of Darling Downs presented by Sir Charles Nicholson, Bart., M.D., evidence of which I had been long in quest, of the fully, or nearly fully, developed dentition of the lower jaw of *Sthenurus Atlas*. Through Professor Phillips's kind permission, this unique fossil forms the subject of figs. 5–8, Plate XXII. It is a left mandibular ramus, wanting the ascending branch, of a nearly mature individual of *Sthenurus Atlas*.

The last molar (m_3) has risen into place, and the summits of its transverse lobes have been just touched by masticatory abrasion, acting from above obliquely backward, without exposing the dentine; but the large and characteristic premolar (p_3) has not risen beyond the level of the basal third of the crown of the adjoining molar (d_4), and its summit is quite unworn.

This specimen, moreover, gives the mandibular characters of the genus *Sthenurus* as distinguished from those of *Macropus* (ib. figs. 13, 15)—as, *e.g.*, the shorter symphysis (fig. 6, *s*), the larger extent thereon of the articular surface (which reaches to the outlet of the incisor socket), the angle which its lower border makes with that of the horizontal ramus, and the continuation of the upper or diastemal border to the incisor outlet in a direction more nearly parallel with that of the molar alveolar border, not descending so much or so abruptly from that border as it advances forward. The outlet (ib. fig. 5, *v*) of the dental canal is nearer the molar series, and the part of the jaw anterior to the outlet is shorter than in *Macropus Titan*. The depth of the ramus behind the last molar (m_3) is relatively greater. The inner surface of the horizontal ramus (ib. fig. 6) is less convex vertically than in *Macropus Titan*.

The symphysial surface, though free or unanchylosed in the not quite mature individual yielding the specimen, must, from its greater vertical extent and uniform flatness, fit closer to its fellow, and permit less divaricating movements of the two rami than in *Macropus*. Besides the anterior outlet (ib. fig. 5, *v*) there is a vascular foramen below m_1, midway between the alveolar and inferior borders of the ramus; but this may be an individual character.

The broken border of the ascending ramus shows the fore half of the margin of the wide intercommunicating foramen (ib. fig. 6, *e*), and the fore part of the large cavity from the inner half of which the dental canal is continued forward.

The postalveolar platform has a sharper inner border, and forms a more marked angle at that border, than in *Macropus*, indicating the place of the postalveolar process in *Nototherium*, to which, in the form and proportions of the symphysis, its closer and

* It is shown entire from this view in "Mitchell," *op. cit.* 1st ed. vol. ii. pl. xxix. fig. 1.

firmer junction of the rami, as well as in the characters of p_3, the present genus offers a nearer approach than does *Macropus* proper. Moreover, as the socket of the incisor follows the direction of the symphysis, the tooth projects less horizontally than in *Macropus*, and rises at a similar angle with the horizontal lower border of the ramus*. In all the characters of the symphysial end of the mandible *Halmaturus ualabatus* (Plate XXIV. figs. 10, 12) agrees with *Macropus* and differs from *Sthenurus*.

The lower border of the crown of the incisor, with the free end of that tooth, is broken away in the Oxford specimen, but enough of the crown remains to show that it is shorter but vertically broader than in *Macropus* proper. The enamel is confined to the under and outer sides; the radical cement encroaches on the outer enamel in an angular form (Plate XXII. fig. 5, *i*). The upper border of the base of the crown is trenchant; the tooth gradually gains in thickness to the lower border, but even here it is less than half the vertical breadth of the crown; the inner surface, behind the working one, is vertically more concave than in *Macropus Titan*. The hind part of the narrow surface of attrition upon the upper edge of the crown begins half an inch from the hind border of the enamel.

The premolar with a fore-and-aft extent of crown of 8 lines (17 millims.), a vertical extent of 6 lines (12 millims.), and a greatest breadth, near the hind border, of 3 lines ($6\frac{1}{2}$ millims.), is, externally (ib. fig. 5, p_3), divided into two subequal lobes; but the vertical fissure runs obliquely backward and inward, so that the lobe forming the anterior half of the outer surface of the crown forms the whole of the inner surface.

This lobe has a slight prebasal prominence, and is divided above by two vertical transverse fissures, the foremost of which is in view on the outer surface, extending nearly halfway down the crown; this fissure widens to the upper border, where the two divisions of the lobe which it separates are linked by a slender longitudinal bar of enamel. The second transverse fissure is not so widened above, but the rudiment of an enamel link appears behind the second transverse division of this lobe; the third division is less definitely cleft or marked off from the rest of the antero-internal lobe, which is continued with a trenchant border to the back part of the crown to which it descends; vertical depressions, hardly to be called fissures, are indicated on the inner surface of this hinder portion of the lobe (Plate XXII. fig. 6, p_3). The postero-external lobe (ib. fig. 5, p_3) has a simple trenchant edge, describing a slight convexity lengthwise; it is connected with the postinternal lobe by two transverse enamel links, the foremost being the largest (ib. fig. 8, p_3).

The outer surface of this complex tooth (p_3) is shown in fig. 5, the inner surface in fig. 6, and the upper surface in fig. 8. The homologous tooth in *Halmaturus ualabatus* (Plate XXIV. figs. 10, 11, 12, p_3) shows nothing of the complexity answerable to that which renders the upper premolar of that Wallaby so similar to the upper premolar of *Sthenurus*; it has an undivided trenchant crown, slightly thickened behind, with some very feeble indications of vertical grooving on both inner and outer sides. Two teeth

* Compare fig. 6, Plate XXII. with fig. 4, Plate VI. Phil. Trans. 1872.

(d_2, d_3) have been displaced by the rise of p_3 in the Oxford specimen of *Sthenurus Atlas* (Plate XXII. figs. 5–8): one of these, viz. d_2, is retained in the type specimen (ib. figs. 3 & 4, d_3).

The molar following p_3 in Plate XXII. figs. 5, 6, 8, answers to d_4 in the placental diphyodont dentition*; its crown differs from that of the younger (type) *Sthenurus Atlas* only in a slight superiority of size. The prebasal ridge is linked to near the middle of the front transverse lobe, a little nearer the outer side, making the worn surface there somewhat thicker; the mid link rising from the middle of the base of the hind part of the front lobe rises to join the hind lobe at a similar position, and with a similar result to the grinding-surface. The more produced prebasal ridge of the next molar presses upon the back part of the hind transverse lobe of d_4, above the feeble outswelling of the base of that tooth. The fore-and-aft extent of the prebasal ridge, with its linking bar inclining obliquely outward to abut upon the front transverse lobe, characterizes the three true molars, $m_{1, 2, 3}$. The transverse breadth of the hind lobe of m_3 is rather less than that of the front lobe.

The molar series of the present mandible describes a feeble convexity outward. The figures in Plate XXII. being of the natural size precludes the need of recording admeasurements.

The subject of figs. 7 & 8, Plate XXIV., and of fig. 9, Plate XXII., is a portion of the left mandibular ramus (drawn in the Plate without reversing) of *Sthenurus Atlas*, from an older individual than that which afforded the more entire ramus, but retaining the four first molars and part of the socket of the fifth.

The outer and posterior lobe of the premolar (Plate XXII. fig. 9, p_3, *b*) has been worn down below the level of the posterior part of the antero-internal lobe (Plate XXIV. figs. 7 & 8, *c*), which stands up as an angular trenchant ridge; on the broader outer lobe a flat field of dentine is exposed, showing that the back part of this premolar, as in *Nototherium*, took some share in mastication, not merely in division of the food as in *Halmaturus ualabatus*; so much of the grooves, ridges, and other accentuations of the crown of p_3 as remain in the present specimen repeat those characters in the unworn homologue of the two preceding specimens.

The crown of d_4 (Plate XXII. fig. 9) shows a field of dentine enclosed by a border of enamel encroaching by a curved indent in opposite sides, and with a feeble fold at the outer part of what was the prebasal ridge. The dentine of this ridge is worn in m_1 into continuity with that of the front lobe, and a small portion of the same tissue is exposed on the back lobe. In m_2 the enamelled summits of the transverse lobes are worn obliquely backward, the enamel showing there a finely polished tract. The basal swelling at the back part of this molar is better defined than in the type specimen; in neither is the hind surface of the molars impressed as in *Macropus Titan*. The present specimen agrees in size with the comparable or homotypal parts of the upper jaw (Plate XXIV. figs. 4, 5, 6). The slight difference of size as compared with the mandible

* Phil. Trans. 1870, p. 539, fig. 3 (*Sus*).

of the younger individual (Plate XXII. figs. 5, 6, 8) is well within the limits of individual and sexual range of variety.

The outer surface of the portion of mandible of *Sthenurus Atlas* (Plate XXIV. fig. 7) shows a longitudinal sinuous shallow channel, extending from below the fore part of p_3 to m_1, at a distance varying from 2 lines to 5 lines below the alveolar border. Below the channel the ramus swells to greater thickness than in the largest of the mandibular fossils of *Macropus Titan*. The lower border has been broken away; and in the longitudinal extent of mandible here preserved, the fractured surface shows a pretty uniform breadth or thickness of 9 lines.

The increase of the fossil (Plate XXIV. fig. 7) over the younger *Sthenurus* (Plate XXII. fig. 5) is shown by the bone more than by the teeth. But even in the smaller specimen (ib. figs. 5, 6) the mandible is relatively stronger and deeper than in *Macropus Titan* (ib. figs. 13, 15). In this species the last four molars (d_4 and m_3) occupy a longitudinal extent of 2 inches 4 lines, but in *Sthenurus Atlas* of 2 inches. These differential mandibular and dental characters come well out in comparing figs. 5 & 13, and figs. 8 & 14, in Plate XXII.

§ 8. *Sthenurus Brehus*, Ow.—This species is represented by two fossils from the Breccia-cave of Wellington Valley, presented to the British Museum by the Trustees of the Natural-History Museum of Sydney, New South Wales, and forming part of the results of the exploration by Prof. Thomson and Gerard Krefft, Esq., carried out with the aid of the legislative grant*.

The specimens formed part of a series of duplicates, thickly encrusted, like those of *Thylacoleo* and *Phascolomys*†, with the reddish stalagmite of the cave.

The most acceptable and instructive results of the clearance of the fossils from their matrix were the subjects of figs. 5–9 of Plate XXVII. The largest specimen (figs. 5 & 6) consists of a portion of cranium including a great part of both maxillaries, with the intervening palatal plates and both palatine bones; the zygomatic masseteric process came out entire on both sides of the skull. The molar series of the left maxillary (p_3 to m_3) had undergone fracture of the crowns of the two anterior teeth; the portion of the right maxillary included the two posterior molars.

The pattern of the molar crowns closely accords with that of *Sthenurus Atlas*, and the narrow but well-defined prebasal ridge (ib. fig. 6, m_2, f) was without the link; the mid link (ib. ib. r) was represented by a rudiment at the bottom of the valley between the two transverse lobes, a, b; the postbasal ridge (g) was represented by the crescentic border of a depression on the hind surface of the hind lobe; the main ridges were rather narrow antero-posteriorly in proportion to their breadth and vertical extent.

The superiority of size of *Sthenurus Brehus* over *Sthenurus Atlas* may be estimated by comparing figs. 5–9, Plate XXVII., with figs. 4–6, Plate XXIV. The base of the broken premolar (Plate XXVII. fig. 6, p_3) shows similar proportions of that tooth, although, as the crown swells out beyond the part retained, this does not yield the whole

* Phil. Trans. 1870, p. 570. † Ib 1871 and 1872.

fore-and-aft extent. Fortunately another cranial fragment of the species (Plate XXVII. figs. 7, 8, 9) included the premolar and adjoining molar entire, and yielded the required subgeneric character of the anterior tooth. The crown is 10 lines in fore-and-aft length, $5\frac{1}{2}$ lines in vertical extent, $4\frac{1}{2}$ lines across the thickest part of the base, which is near the hind end of the tooth. The fore end (figs. 7 & 9, *d*) is subtrenchant, with a prebasal triangular prominence, one angle subsiding halfway along the trenchant fore border. The middle two thirds of the outer surface (fig. 7, *a*) show the usual concavity lengthwise between the smooth and prominent fore (*d*) and hind (*e*) ends of the crown, and on this depressed surface are three vertical obtuse ridges, dividing four shallow linear grooves. The cutting-edge (*a*) similarly sinks below the angular summits of the terminal prominences (*d*, *e*). On the inner side of the crown may first be noticed a low narrow ridge (fig. 8, *f*), extending a few lines backward from the inner basal angle of the prebasal prominence. Above the ridge (*f*) begins the broader rising, which soon stands out as a low inner basal division of the crown; it bends up posteriorly to abut against the inner side of the hind expansion (*e*), leaving a small triangular depression between the buttress and the hind margin of the tooth. The interval between the inner basal lobe or ridge (*b*) and the outer or main part of the tooth is less depressed than in *Sthenurus Atlas*, and does not show the small transverse connecting bars in the hollow. Masticatory attrition has polished the inner side of the blade or outer main part of the crown of this premolar and the inner basal prominence, indicative of a corresponding transverse extension of the crown of the lower premolar. A speck of dentine has been exposed on the buttress. As compared with the upper premolar of *Sthenurus Atlas*, the generic pattern is closely retained, but with the specific modifications above defined.

The crown of the adjoining molar (d_4, fig. 9) is worn obliquely from without nearer to the base of the inner side of the tooth. The very narrow prebasal ridge is shown. The dentine exposed on each lobe is broadest at the inner half of the grinding-surface, and it extends in both lobes into an angular form behind, that of the fore lobe indicating the rudimental link, that on the hind lobe the rudimental postbasal ridge.

The third tooth (Plate XXVII. figs. 5 & 6, m_1) shows, as in *Sthenurus Atlas* (Plate XXIV. figs. 4–6, m_1), a marked increase of size over d_4. The prebasal ridge is more developed; the exposed dentinal tracts resemble those in Plate XXVII. fig. 9, d_4, but are rather less extensive. The characters of the slightly worn penultimate and last grinders have already been defined, and are sufficiently given in figs. 5 & 6, m_2, m_3.

The hind border of the bony palate is so entire in the present evidence of *Sthenurus Brehus* as to show that it described a moderate unbroken concave curve, as in *Osphranter robustus*. So much of the palate itself as is preserved suffices to exemplify its correspondence with that and other larger existing Kangaroos (*Macropus major*, *Macropus rufus*, *Phascolagus erubescens*) in the degree of its integrity.

The masseteric process descends opposite the hind lobe of the penultimate molar, and the hind margin of the anterior zygomatic pier is opposite the fore part of the fore lobe of the last molar (ib. fig. 6, m_3).

The extent of the alveolar part of the maxillary in advance of the masseteric process is relatively greater than in *Macropus major*, and more resembles that in the Kangaroos, which longer retain the premolar and which have that tooth of larger relative dimensions than in the type of *Macropus* proper. The amount of fracture and variety of distortion which this cave cranial fragment has undergone indicates a persevering exercise and diverse direction of force, such as only accords with the operations of the powerful jaws of a large carnivore.

§ 9. Genus *Protemnodon**, Ow.—The genus *Protemnodon* is allied to *Sthenurus*, but distinguished therefrom chiefly by the more simple trenchant shape of the crown of the premolar.

Having ascertained the characters of that tooth in the upper jaw of *Sthenurus Atlas*, in the specimen from the lacustrine deposits of South Australia (Plate XXIV. figs. 4, 5, 6), I subjected to a reexamination the fossil upper jaw brought to me in 1842 by Count Strzelecki from the Breccia-cave of Wellington Valley, and the specimen transmitted in the following year from the bed of the Condamine by Col. Sir Thomas L. Mitchell, C.B., both of which specimens, from the size of the germ of the premolar (Plate XXIII. figs. 6 & 9), had been referred, in my 'Catalogue of the Fossil Mammalia in the Royal College of Surgeons,' to *Macropus Atlas*†.

I had fortunately begun the quest of this tooth from the inner side of the formative alveolus, and was now able to recognize, in the absence of the inner ridge or lobe characteristic of the upper premolar of *Sthenurus*, and giving the crown of that tooth a breadth corresponding with the lower premolar, that the fossils Nos. 1513, 1519 must belong to another species, and, according to the estimate of the value of premolar modifications, to another subgenus of *Macropodidæ*.

The subsequent acquisition of mandibular fossils, with the premolar simple and trenchant, and with equivalent modifications of the form of the bone, have afforded the requisite ground for proposing the genus, and for referring these maxillary specimens to the species *Macropus Anak*, originally founded on characters of the lower jaw and teeth.

The upper molars of *Protemnodon* are more like those of *Sthenurus Atlas* than of *Macropus Titan*; they have a narrow prebasal ridge without the link. The oblique ridge extending downward and backward from the inner and hinder angle of each chief lobe is more definitely marked, and the two lobes are more alike than in *Sthenurus Atlas*. The breadth of the crown in m_1 and m_2 of *Protemnodon Anak* (Plate XXIII. figs. 5 & 8) is greater in proportion to the fore-and-aft length than in *Sthenurus*; and the inner border of the two lobes (ib. figs. 6 & 9) is narrower and more sharply pointed than the outer border (ib. figs. 4 & 7), in a more marked degree even than in *Sthenurus*.

Proceeding to the characters afforded by the mandible and teeth (Plates XXV. & XXVI.), I have first to remark that the premolar (p_2 in all the figures), in its relative antero-posterior extent to the molars which follow, rather exceeds that tooth in *Sthenurus*

* πρό (before), τέμνω (to cut), ὀδούς (tooth)—in reference to the sectorial form of the anterior molar or premolar.
† 4to, 1845, pp. 325, 327, Nos. 1513, 1519.

Atlas. The proportion of p_3 in *Protemnodon* is much the same as in the Bettongs*; it is not equal to that in *Dendrolagus dorcocephalus*†. As in this Tree-Kangaroo, the lower molars, like the upper ones, retain the Macropodal bilophodont character. But the lower premolar of *Protemnodon* shows no indication of the three-lobed division which is marked on the outer surface of the crown of that premolar of *Dendrolagus*. It is rather more like that in the Potoroos, though the indications of vertical grooves and ridges on the compressed part of the crown between the slight fore and hind thickened ends are feebler.

The greatest height of the crown of the premolar, which is at the fore part (Plate XXV. figs. 3, 5, 7, & 8, p_3), is but half the antero-posterior diameter; the utmost thickness (at the back part of the crown) (ib. fig. 2) is less than the height. The free or trenchant margin is straight, and runs nearly parallel with the base of the crown. The fore border is subtrenchant, the hind one flattened, and closely adpressed against the contiguous molar. The fore part is defined behind by the subsidence of the narrower following part of the crown on the outer side (Plate XXV. fig. 3, p_3), and, less definitely, by the foremost of the shallow vertical grooves on the inner side (ib. fig. 8). The base of the fore part of the crown bulges forward beyond the anterior root. The hind part of the crown slightly expands, but is not defined, like the front expansion, from the rest of the crown. A feeble indication of a "cingulum" runs along the outer side of the base of the crown, and is more dubiously represented by a slight smooth out-swelling along the base of the inner surface. The tooth is implanted by two antero-posterior, slightly divergent, fangs.

§ 10. *Protemnodon Anak*, Ow.—The subject of Plate XXV. figs. 1 & 2, the type specimen on which the species *Macropus* (*Protemnodon*) *Anak* was founded‡, is a portion of a left mandibular ramus, including the molar series. All the teeth of the permanent dentition are in place, and from the degrees of wear of their crowns it may be inferred that the foremost (p_3) was the last to come "into line."

Only the hinder angle of the enamelled trenchant border of the crown of this tooth is touched, whilst the dentine is exposed on the ridges of the last molar (ib. fig. 2, m_3). The crown of d_4 has been worn down nearly to the bases of the two lobes, and the dentine of the mid link connects the two exposed wide tracts of that tissue, forming the bases of the worn-out ridges. The next molar (ib. ib. m_1) shows a greater degree of wear; the dentinal part of the mid link is broader, and the lobes, as seen in the side view (ib. fig. 1, m_1), are worn down lower or nearer to their base than in d_4. The front lobe of m_2 has been abraded to the level of the link, which, being low in this species of *Protemnodon*, is hardly touched. A broad tract of dentine is also exposed on the hind lobe. A narrower bilobed tract appears on the front lobe of m_3; the enamelled summit of the hind lobe is smoothly worn downward and backward. The prebasal ridge (f) is broadest in this tooth, and shows a low link (s) continued

* Philosophical Transactions, 1871, p. 250, fig. 18. † Ibid. fig. 16.
‡ Proceedings of the Geological Society of London, vol. xv. p. 185 (June 23, 1858).

from the fore part of the outer swelling of the front lobe. The inner alveolar border (Plate XXV. fig. 8) runs from the postalveolar ridge (*t*) with a feeble concavity to $m\,1$, and then takes as feeble a convex course to the diastema (*l*).

The subject of figs. 1 & 2, Plate XXV., was obtained by Henry Hughes, Esq., from the freshwater deposits exposed in the beds of creeks in Darling Downs. It is now in the Museum of the Natural-History Society of Worcester, to the Council of which Society I was indebted in 1858 for the permission to take the above description and figures of this instructive and, at that time, unique fossil.

To Sir Daniel Cooper, Bart., I have since been indebted for the opportunity of describing and figuring a larger proportion of the left mandibular ramus of the same species of *Protemnodon*, with the molar series at nearly the same stage of attrition. It was discovered by Ed. S. Hill, Esq., in the freshwater deposits of Eton Vale, Queensland. Of this fossil I give an external view (Plate XXV. fig. 3), in which it will be observed that, as in the foregoing example, the crown of $m\,1$ is more worn (has borne more of the work of mastication) than that of the antecedent molar ($d\,4$). I have noted the same circumstance in a *Macropus major* of similar age. This may not relate to an earlier period of $m\,1$ coming into the line of work than the molar which precedes it, but more probably is due to the greater degree of pressure upon a tooth nearer the centre of motion of the mandible. The last molar (ib. figs. 3 & 4, $m\,3$) shows the narrower hind lobe (*b*): the seemingly broader prebasal ridge (*f*) than in $m\,2$ may relate to the less amount of attrition in $m\,2$. The links are low and ill-defined in this, as in the type specimen. There is a slight bulge behind, but no indent marking a postbasal ridge in the hindmost molar. The inner vertical plate of the horizontal ramus is continued further back than in existing Kangaroos and Wallabies, forming an inner wall (ib. fig. 8, *t*), with a definite and sharp margin, beneath the base of the coronoid process; and from the point where this hind margin of the inner mandibular plate is continued upward into the coronoid, a low ridge extends on the side of the plate next the large cavity of the ascending ramus forward and downward to the entry of the dental canal. This ridge (Plate XXV. fig. 14, *g*) divides the cavity into an upper (*f*) and lower (*a*) compartment. The structure is repeated, as will be seen, in the specimen next to be described.

The curve and direction of so much of the diastemal ridge (ib. fig. 3, *l*) as is here preserved resemble rather that of *Macropus* and *Halmaturus* than of *Sthenurus*; but the less mutilated specimen (ib. figs. 7 & 8) shows the toothless tract (*l*, *s'*) to be relatively shorter as compared with the molar series than in either of those genera of existing Kangaroos.

This specimen likewise forms part of the series of fossils from the river-beds at Eton Vale, Darling Downs, presented by Sir Daniel Cooper, Bart., to the British Museum.

The molar series (Plate XXV. figs. 7, 8, 9, $p\,3$–$m\,3$) agrees in extent and in the proportions of the five teeth with the type specimen, but the fossil is from a less aged individual. The hind angle of the sectorial crown of $p\,3$ (fig. 9) is made obtuse and polished by wear.

The dentine exposed on both lobes of m_2 is transversely linear, with a slight forward production in both links (*r* and *s*). In m_3 a speck of dentine appears on the inner angle of the front lobe (*a*): the enamelled ridges of this and the hind lobe show the oblique polished tract at their hinder surface. The characteristic proportions of both fore and mid links are well shown in m_3, and contrast with the better developed ones in the somewhat larger *Protemnodon* represented by the subject of figs. 11, 12, & 13, Plate XXV.

The last molar in *Protemnodon Anak* rises, in the outer side view of the mandible, clear of the front root of the coronoid process (ib. fig. 7, *q*). The fore part of the outer crotaphyte excavation (fig. 7, *f*) sinks as abruptly from the prominent anterior border as in *Macropus* and *Halmaturus*; but the cavity appears to be divided in *Protemnodon* by a curved ridge into an upper (*f*) and lower (*a*) channel, the latter being that which leads to the large hinder orifice of the dental canal. A similar fracture of the ascending ramus of the mandible in existing Kangaroos would not produce this appearance, but it may be due to the minor development and closer approximation to the coronoid plate of the base of the inflected mandibular angle in *Protemnodon*.

In the depth or vertical breadth of the ramus beneath the last molar and the minor degree of vertical convexity of that part, *Protemnodon* contrasts with the narrower and more bulging character of the same part of the jaw in *Macropus*. It is rather less convex, though narrower, in *Halmaturus*. There is a trace at *a*, fig. 8, of the beginning of the excavation, or lower channel, leading to the intercommunicating aperture and to the entry of the dental canal; but the extent of the inner plate of the mandible, from *t* to *a*, is not matched by any existing Kangaroo the lower jaw of which I have compared with the fossil.

In the extent of the edentulous and symphysial part of the mandible (ib. fig. 8, *l*, *s'*) *Protemnodon* agrees with *Halmaturus* rather than with *Macropus*; but the syndesmotic surface extends nearer to the alveolar outlet of the incisor (*i*), although it does not indicate so firm a union as in *Sthenurus* (Plate XXII. fig. 6). It extends more in the axis of the ramus than in *Sthenurus*.

The breadth of the incisor and that of the surface (Plate XXV. fig. 10, *i*) which was opposed to the upper incisors point significantly to a Nototherian tendency.

The configuration of the crown of the unworn molars (ib. figs. 11–13, m_2, m_3) in a portion of a mandibular ramus of a large *Protemnodon Anak* supplements the illustrations of the mandibular dentition of the species. The fossil was part of an individual in which the hindmost molar had recently risen "into place." The links (fig. 13, *r*, *s*) are more neatly defined in this unworn tooth, which also had not moved forward so clear of the coronoid process (fig. 11) as in the older example (fig. 7).

§ 11. *Protemnodon Og*, Ow.—The subject of figs. 5 & 6, Plate XXV., with a certain increase of size of both mandible and molar teeth, repeats the form and size of the premolar (p_3) in *Protemnodon Anak*, but shows a distinct linear indication of the post-basal ridge *g*, and a more definite development of the links *r* and *s* in the last molar, m_3.

These characters may be subsequently found in other individuals, and sufficiently evince an established variety; but they are so strongly marked in the still larger mandibular fossils next to be noticed as to justify their ascription to another (zoological) species, and the imposition of the name which heads the present section.

§ 12. *Protemnodon Mimas*, Ow.—In this species a greater depth and thickness of mandible and a concomitant larger size of molars are associated with a relatively smaller size of the trenchant premolar, which does not exceed that in *Protemnodon Anak*. Such character of the variable tooth might be expected, having regard to those which it exhibits in different species of existing Wallabies (*Halmaturus*, Cuv.).

In the present large extinct species of *Protemnodon* a marked modification of the molar teeth accompanies their relative proportions to the premolar, and confirms the taxonomic deductions as to specific status, but does not give ground for assigning thereto subgeneric value.

The postbasal ridge (Plate XXVI. fig 3, *g*) though narrow is definite; the prebasal ridge (ib. *f*) is proportionately as well developed as in *Protemnodon Anak*; its "link" (ib. *s*) also, and that (ib. *r*) of the two chief lobes, are more distinct than in the type species.

The smaller mandible and teeth (Plate XXV. figs. 7 & 8) cannot have come from a younger specimen of the present species; both molars and premolars are more worn, and prove that fossil to have been derived from an older Kangaroo than the animal which owned the subject of the present description.

The premolar of *Protemnodon Mimas* (Plate XXVI. fig. 1, $p\,3$) shows on the outer side of the intermediate part of its crown five vertical grooves and four ridges, more strongly developed than in *Protemnodon Anak*; these are, in part, worn smooth on that side of the tooth of the subject of fig. 7, Plate XXV. For the rest, the characters of the premolar of the larger species are those of *Protemnodon Anak*.

The crown of $d\,4$, figs. 1–3, has suffered more from fracture than abrasion. A linear tract of dentine is exposed in each transverse lobe of $m\,1$, slightly expanding at the origin of the "link" from near the outer end of their anterior surface. Only the enamel shows abrasion in $m\,2$. The crown of $m\,3$ is entire, has but recently risen into place, and, contrasted with that tooth in the subject of fig. 13, Plate XXV., exemplifies the coronal character of the molars of the present well-marked species. It is partially concealed in a direct outer side view by the coronoid process, *q*, fig. 1, Plate XXVI.

For this fine evidence of *Protemnodon Mimas* I am indebted to my friend Dr. George Bennett, F.L.S., of Sydney, New South Wales, who obtained it from the freshwater deposits forming the bed of "Gowrie Creek," Darling Downs.

From the same fertile district, but in another locality (Eton Vale), Sir Daniel Cooper, Bart., received and presented to the British Museum the portions of mandible (Plate XXIV. figs. 13 & 14, and Plate XXVI. figs. 4 & 5), little, if at all, exceeding in size the corresponding part of that of *Macropus major* or *Macropus rufus*. The best-preserved

molar in each of these fossils indicated, however, a larger species. This molar, moreover, presented good differential characters in the presence of the well-defined, though small, postbasal ridge (ib. fig. 7, *g*), the large prebasal ridge (ib. *f*), and the well-developed and almost equal-sized fore link (*s*) and mid link (*r*); the proportions of the two principal transverse lobes in the minor breadth of their outer and inner convex borders as compared with their height were rather those of *Sthenurus* than of *Macropus*. But *Sthenurus Atlas* shows no postbasal ridge (comp. Plate XXIV. fig. 15, *Protemnodon Mimas*, with the same view, Plate XXII. fig. 8, of the homologous tooth in *Sthenurus*).

On the hypothesis that the specimens Plate XXIV. figs. 13–16 and Plate XXVI. figs. 4 & 5 belonged to the same species as the specimen Plate XXVI. figs. 1 & 2, the last, largest and best-preserved unworn molar in the smaller jaws would be homologous with the antepenultimate and worn molar in the larger jaw. The test-scrutiny was accordingly applied, and the germ of the large premolar characteristic of the genus *Protemnodon* was brought to light in both the smaller fossils (Plate XXIV. fig. 14, p_3, Plate XXVI. fig. 5, p_3). The Kangaroos leaving these remains had each perished at the same phase of dentition as that shown in the type specimen of *Sthenurus Atlas* (Plate XXII. figs. 3 & 4); the subgeneric characters afforded by the premolar are well exemplified thereby. The comparatively flat undivided outer surface, with the continuous straight trenchant margin of the crown of p_3 in *Protemnodon*, contrasts with the two convex lobes defined by the median fissure notching the trenchant margin and deeply grooving the outer surface of the crown of p_3 in *Sthenurus*; and these differences are better marked in the originals than in the figures above cited, although these give the details with quite sufficient accuracy.

The mandibular fossils of the young *Protemnodon* supply acceptable additional evidence of the dental characters of the species. Thus the crown of d_4, which is mutilated in the type mandible (Plate XXVI. fig. 3), is entire in figs. 4 & 6, save as regards the degree of masticatory abrasion to which it has been subject, exposing a linear tract of dentine on each main lobe expanding where the link joins such lobe. The postbasal ridge (Plate XXIV. fig. 13, d_4) is as conspicuous in this as in the succeeding tooth, m_1; the prebasal ridge shows also a proportionate development, with the fore link distinct (Plate XXVI. fig. 6, d_4, *s*). The first and second deciduous molars (ib. d_2, d_3) occupied an alveolar extent of 9 lines; they were displaced, as usual, by the rise of the premolar with a crown of corresponding antero-posterior extent. The subject of figs. 13–15, Plate XXIV., was from a younger animal than that of figs. 4–6, Plate XXVI.; in the former the molar (m_1) had very recently risen into place; in Plate XXVI. figs. 4–6 the enamelled summits of the transverse ridges of m_1 are a little worn, as usual, from above downward and backward.

The socket of the incisor in the subject of fig. 4 is broken across about an inch from its closed end; the fracture (ib. fig. 8, *i*) gives, therefore, the breadth and thick-

ness of the front tooth at that part, which would be, at least, the same as that of the exserted crown of the large procumbent incisor in *Protemnodon Mimas*.

Of the upper jaw and teeth of this species (*Protemnodon Mimas*) my present evidence consists of photographs of the natural size of a specimen obtained by Professor Thomson and Mr. Krefft in the Breccia-cave of Wellington Valley, and deposited in the Museum of the Natural-History Society of Sydney, New South Wales.

The photographs, liberally transmitted to me by the Trustees of that Museum, and prepared under the superintendence of their able Curator, Mr. Krefft, give an outer side view (Plate XXVII. fig. 1), an inner side view of part of the left premaxillary and teeth (ib. fig. 2), an inner side view of the premolar (ib. fig. 3), and a view of the grinding-surface of the two best-preserved molars (d^4, m^1, left side, ib. fig. 4). These teeth, the premolar of the left side, and perhaps the front and second incisor are tolerably perfect; the remaining teeth have suffered more or less fracture; but the remains of the molar series *in situ* on the left side enable the requisite admeasurements and comparisons as to size to be made with the mandibular teeth of *Protemnodon* previously described. From their close accordance in this character with the mandibular teeth of *Protemnodon Mimas* (Plate XXVI. figs. 1–3) I refer the subject of the photographs to that species.

The upper incisors, as in existing *Macropodidæ*, are three in number in each premaxillary. The foremost (Plate XXVII. fig. 1, i^1) is curved lengthwise, with the convexity forward, and has a thick enamelled crown, with the fore part convex transversely; its convex cutting-edge projects slightly beyond that of the second incisor. The crown of this tooth (i^2) is smaller, less convex, and less prominent than that of the foremost one. The indications of the socket of the third incisor support the inference that, as in the large existing Kangaroos (*Macropus major*, *Macropus* (*Osphranter*) *robustus*, *Macropus* (*Osphranter*) *rufus*), the antero-posterior dimension of the crown of that tooth exceeded that of the second and first incisors; but of the precise proportions of these teeth exemplifying specimens are still *desiderata*.

The antero-posterior extent of the incisive alveoli of the left premaxillary is 1 inch 5 lines, that of the toothless interval between the third incisor and the premolar is 1 inch 9 lines; the extent of the molar series is 3 inches 2 lines. The diastema is relatively shorter than in the above-cited existing Kangaroos, and indicates a corresponding condition of the lower jaw, whereby, as regards length, *Protemnodon* resembles *Sthenurus*.

The premolar (p^3), however, retains in the upper jaw the more simple trenchant form which afforded the subgeneric distinction in the homotypal tooth below. There is a slight expansion of the fore and hind parts of the crown, the intermediate part of the blade having an entire and nearly straight trenchant edge, with the indication of a low ridge or cingulum along the base. The corresponding part on the inner side of the crown (ib. fig. 3), though much less developed than in the upper premolar of *Sthenurus*, adds another character differentiating *Protemnodon Mimas* from *Protemnodon Anak*.

The bilophodont molars have both pre- and postbasal ridges; the former, as usual in upper molars, less produced than in the lower molars. The indication of the fore link is recognizable, and that between the main lobes is more plainly shown (Plate XXVII. fig. 4); the mid link is worn down to the base, exposing a broader tract of dentine in the foremost (d_4) and a linear tract in the next tooth (m_1). A broad field of dentine had been brought to the grinding-surface in both molars. Mr. Krefft has noted on one of the photographs of a fossil upper jaw, which I refer to *Protemnodon Mimas*, "Molars worn down, premolar in good condition"—an appearance which is the consequence of the later development of the front tooth of the series. The crowns of the other molar teeth seem to have suffered mutilation from fracture in the original of the photograph. The maxillo-premaxillary suture (between $_{21}$ and $_{22}$ in Plate XXVII. fig. 1) is unmistakable in the photograph; anterior to it, in a line with the hind part of the last incisive socket, the premaxillary has suffered fracture. The extent of the diastema contributed by the maxillary ($_{21}$) is 1 inch 1 line. The course of the suture resembles that in *Halmaturus*; it does not describe an angle or curve forward before ascending obliquely backward to the nasal, as in *Macropus major*.

If photographs alone, such as those in Plate XXVII. figs. 1–4, of which I have given the foregoing interpretation, should be thought insufficient evidence of an extinct species, I may remark that the characters of *Protemnodon Mimas*, and the determination of that species of extinct Kangaroo, are independent of them, and are sufficiently exemplified in the fossil remains of the mandible and mandibular teeth of this gigantic Wallaby.

§ 13. *Protemnodon Ræchus*, Ow.—The subject of figs. 10–13, Plate XXVII., from King's Creek, Clifton Station, presented by the proprietor, George King, Esq., is a part of a left mandibular ramus, with the permanent dentition, save the last molar, in place and use; and, from the degree of attrition of the crown of m_2, it is plain that m_3 had risen into place, and been lost with the supporting part of the jaw by mutilation of the fossil. The retained molars have characters of those of *Protemnodon Anak*, in wanting the postbasal ridge (fig. 13), and having the links less sharply defined (fig. 12) than in *Protemnodon Mimas*. But the increase of size is more than can be granted to difference of sex. The protemnodont pattern of premolar is closely adhered to; the hind swelling of the crown (ib. fig. 10, *b*) is relatively somewhat greater than in *Protemnodon Anak*, and a smooth triturating surface has been worn upon its summit; the trenchant border is abraded, as usual, upon its outer side. The anterior lesser expansion is defined externally by an oblique, not vertical groove. The lower border as well as both ends of the mandible have been broken or worn away.

The preserved teeth describe a slight curve, convex inward—a character (if it be specific) which is not shown by any of the other and smaller kinds of extinct Kangaroos forming the subject of the present communication. In this I have continued the practice, began in my Appendix to Mitchell's work (1838), of attaching the names of giants, familiar to the students of biblical and mediæval histories, to the several extinct species which towered of old above the tallest of the living Kangaroos.

Description of the Plates.

PLATE XX.

Fig. 1. Side view of the cranium and teeth of a nearly full-grown Uroo (*Macropus (Phascolagus) erubescens*).

Fig. 2. Working-surface of the molars, left side, upper jaw, of a nearly full-grown Uroo.

Fig. 3. Inner surface of the molars, left side, upper jaw, of a nearly full-grown Uroo.

Fig. 4. Outer side view of mandible and teeth of the same skull.

Fig. 5. Working-surface of the lower deciduous molars of the same skull.

Fig. 6. Side view of molar series, left side, upper jaw, of a younger *Phascolagus erubescens*.

Fig. 7. Working-surface of the molars in place of a younger *Phascolagus erubescens*.

Fig. 8. Side view of portion of mandible, with the molar series, of the same skull; the germ of the premolar is exposed in its formative cell.

Fig. 9. Side view of molar series, left side, upper jaw, of a young Red-necked Kangaroo, *Macropus (Halmaturus) ruficollis*.

Fig. 10. Side view of molar series, left side, lower jaw, of the same skull.

Fig. 11. Side view of the left mandibular ramus and teeth of the yellow-foot Kangaroo, *Macropus (Petrogale) xanthopus*.

Fig. 12. Working-surface of upper molars, right side, of *Macropus (Boriogale) magnus*.

Fig. 12 *a*. Side view of three anterior lower molars of *Boriogale magnus*.

Fig. 13. Side view of the left mandibular ramus and teeth of *Macropus (Osphranter) robustus*.

Fig. 14. Working-surface of the molar series of ditto. 14′. *a*, side view of upper premolar; *b*, working-surface of ditto.

Fig. 15. Side view of the left mandibular ramus and teeth of the Giant Kangaroo, *Macropus major*.

Fig. 16. Working-surface of the molar series of the Great Kangaroo.

Fig. 17. Side view of the left upper incisors, *Macropus major*.

Fig. 18. Idem, *Osphranter robustus*.

Fig. 19. Idem, *Boriogale magnus*.

Fig. 20. Idem, *Halmaturus ualabatus*.

Fig. 21. Idem, *Halmaturus ruficollis*.

Fig. 22. Idem, *Petrogale xanthopus*.

Fig. 23. Working-surface of left upper molar (m^2), *Macropus major*.

Fig. 24. Idem, *Macropus rufus*.

Fig. 25. Idem, *Osphranter antilopinus*.

Fig. 26. Working-surface of left lower molar (m_3), *Boriogale magnus*.

Fig. 27. Working-surface of left upper molar (m^2), *Halmaturus ualabatus*.

Fig. 28. Idem, *Petrogale xanthopus*.

Fig. 29. Working-surface of left lower molar, *Macropus Titan*.
Fig. 30. Idem, *Sthenurus Atlas*.

PLATE XXI.

Fig. 1. Working-surface of right upper molars, *Macropus major*.
Fig. 2. Idem, *Macropus rufus*.
Fig. 3. Idem, *Osphranter robustus*.
Fig. 4. Working-surface of right lower molars, *Macropus rufus*.
Fig. 5. Idem, *Boriogale magnus*.
Fig. 6. Outer side view of left maxillary bone and molar series of a young *Macropus Titan*.
Fig. 7. Inner side view of left maxillary bone and molar series of a young *Macropus Titan*.
Fig. 8. Working-surface of molars of left maxillary bone of a young *Macropus Titan*.
Fig. 9. Back view of the same fossil, showing formative cell of the last molar, m 3.
Fig. 10. Palatal or under view of a left maxillary, with working-surface of the molar teeth of an older *Macropus Titan*.
Fig. 11. Working-surface of right upper molar series of a mature *Macropus Titan*.
Fig. 12. Working-surface of the last two lower molars, right side, *Macropus Titan*.
Fig. 13. Outer side view of the same teeth, with outline of part of mandible.
Fig. 14. Inner side view of the same teeth, with outline of part of mandible.
Fig. 15. Under or palatal view of right maxillary bone and teeth of an old *Macropus Titan*.
Fig. 16. Outer side view of right maxillary bone and teeth of an old *Macropus Titan*.
Fig. 17. Inner side view of teeth of right maxillary bone of an old *Macropus Titan*.
Fig. 18. Hind surface of last upper molar (m 3) of an old *Macropus Titan*.

PLATE XXII.

Fig. 1. Outer side view of part of right maxillary and teeth of a young *Phascolagus altus*; the premolar (p 3) and penultimate molar (m 2) are exposed in their formative alveoli.
Fig. 2. Under view of the same specimen.
Fig. 3. Upper view of portion of right mandibular ramus, showing the working-surface of the molars in place, and of the crown of the last molar (m 3) in its formative alveolus, of a young *Sthenurus Atlas*.
Fig. 4. Outer side view of part of the same specimen, with the crown of the premolar exposed in its formative alveolus. (These figures are from the type specimen of *Macropus Atlas*, Ow., figured in Mitchell's 'Three Expeditions,' &c., 8vo, vol. ii. 1838, p. 359, pl. xxix. fig. 1.)

Fig. 5. Outside view of a larger portion of the left mandibular ramus of an older individual of *Sthenurus Atlas*; the premolar (p_3) has not quite risen into place.
Fig. 6. Inside view of the same specimen.
Fig. 7. Upper view of symphysis and incisor of the same specimen.
Fig. 8. Working-surface of the molars of the same specimen.
Fig. 9. Working-surface of the lower molars of a full-grown or mature individual of a large male of *Sthenurus Atlas* (or of a larger species of *Sthenurus*).
Fig. 10. Outside view of part of left maxilla and teeth of a mature individual of *Macropus Titan*.
Fig. 11. Under view, with working-surface of molars, of the same specimen.
Fig. 12. Inner side view of the same specimen.
Fig. 13. Outside view of major part of the left mandibular ramus and teeth of a mature individual of *Macropus Titan*.
Fig. 14. Working-surface of the molars of the same specimen.
Fig. 15. Inside view of the same specimen.
Fig. 16. Upper view of part of the symphysis and incisor of the same specimen.
Fig. 17. Upper view of a small portion of the right mandibular ramus of a young *Macropus Titan*.
Fig. 18. Outer side view of the same specimen, with the germ of the premolar (p_3) exposed in its formative alveolus. (This is the type specimen of *Macropus Titan* figured in MITCHELL'S 'Three Expeditions,' &c., p. 359, pl. xxix. fig. 3.)

PLATE XXIII.

Fig. 1. Outer side view of right maxillary, with the molar teeth and their roots exposed, of a large old male *Macropus rufus*.
Fig. 2. Outer side view of right maxillary and teeth of *Macropus Titan*. (From the specimen No. 1519, 'Fossil Mammalia,' in the Museum of the Royal College of Surgeons: Catalogue of Fossils, 4to, 1845, p. 324.)
Fig. 3. Under view, with working-surface of the teeth, of the same specimen.
Fig. 4. Outer side view of portion of right maxillary and teeth of a young *Protemnodon Anak*.
Fig. 5. Working-surface of the teeth of the same specimen.
Fig. 6. Inner side view of the same specimen, with the premolar (p_3) exposed in its formative cavity. (From the specimen No. 1519, 'Fossil Mammalia,' in the Museum of the Royal College of Surgeons; wrongly ascribed to *Macropus Atlas* in my 'Catalogue,' 4to, 1845, p. 327.)
Fig. 7. Outer side view of portion of the left maxillary of a young *Protemnodon Anak*.
Fig. 8. Working-surface of the teeth of the same specimen.
Fig. 9. Inner side view of the same specimen, with the premolar (p_3) exposed in its formative cavity. (From the specimen No. 1513, 'Fossil Mammalia,' in the

Museum of the Royal College of Surgeons; wrongly ascribed to *Macropus Atlas* in my 'Catalogue,' 4to, 1845, p. 325.)

Fig. 10. Inner side view of part of the left mandibular ramus and teeth of *Macropus affinis*, Ow.

Fig. 11. Working-surface of the best-preserved molars (m_1, m_2) of the same fossil. (This is the type specimen of the species, No. 1524, 'Fossil Mammalia,' in the Museum of the Royal College of Surgeons: 'Catalogue,' 4to, 1845, p. 328.)

Fig. 12. Outside view of portion of right mandible, with last three molars, of an old (female?) *Macropus Titan*.

Fig. 13. Working-surface of the teeth of the same specimen.

Fig. 14. Corresponding portion of lower jaw of *Macropus rufus*. (From the largest existing Kangaroo, obtained by JOHN GOULD, Esq., F.R.S., during his sojourn in Australia.)

Fig. 15. Outside view of portion of left mandibular ramus and four molars of an immature *Osphranter Gouldii*, Ow.

Fig. 16. Working-surface of the teeth of the same specimen.

PLATE XXIV.

Fig. 1. Outer side view of skull and upper teeth of a nearly mature *Halmaturus ualabatus*.

Fig. 2. Inner side view of molar series of ditto.

Fig. 3. Working-surface of molar series, and portion of bony palate, of ditto.

Fig. 4. Outer side view of right maxillary bone and teeth of a full-grown *Sthenurus Atlas*.

Fig. 4 *a*. Hind view of penultimate molar (m_2) of ditto.

Fig. 5. Inner side view of the same fossil.

Fig. 6. Under view of ditto, with working-surface of teeth.

Fig. 7. Outer side view of portion of mandibular ramus and teeth of a mature *Sthenurus Atlas*.

Fig. 8. Inner side view of the same fossil.

Fig. 9. Hinder surface of the crown of penultimate molar (m_2) of ditto.

Fig. 10. Outer side view of mandibular ramus and teeth of a nearly mature *Halmaturus ualabatus*.

Fig. 11. Working-surface of molar series of ditto.

Fig. 12. Inner side view of the same mandibular ramus and teeth.

Fig. 13. Outer side view of a portion of the right mandibular ramus and tooth of a young *Protemnodon Mimas*.

Fig. 14. Inner side view of the same fossil, with the premolar (p_3) exposed in its formative cavity.

Fig. 15. Working-surface of the molar (m_1) of the same fossil.

Fig. 16. Back view of the molar (m_1), with part of the formative cavity of m_2, of the same fossil.

Fig. 17. Inner side view of fore part of the mandibular ramus of an *Osphranter Cooperi*.

Fig. 18. Outer side view of the three molars, p_2, d_4, m_1, of ditto.

PLATE XXV.

Fig. 1. Outside view of portion of mandibular ramus, with the molar series, of a full-grown *Protemnodon Anak*.

Fig. 2. Working-surface of the teeth of the same fossil.

Fig. 3. Outside view of a larger portion of a mandibular ramus, with the molar series, of *Protemnodon Anak*.

Fig. 4. Working-surface of the two hindmost molars of the same fossil.

Fig. 5. Outside view of a portion of a mandibular ramus and molar series of a *Protemnodon Og*.

Fig. 6. Working-surface of the teeth of the same fossil.

Fig. 7. Outside view of a left mandibular ramus, nearly entire, with incisor and molar teeth, of *Protemnodon Anak*.

Fig. 8. Inside view of a left mandibular ramus, nearly entire, with incisor and molar teeth, of *Protemnodon Anak*.

Fig. 9. Working-surface of the molar series of the same fossil.

Fig. 10. Upper view of symphysial part, with the incisor, of the same fossil.

Fig. 11. Outside view of a portion of mandible, with last two molars, of *Protemnodon Og*.

Fig. 12. Inside view of a portion of mandible, with last two molars, of *Protemnodon Og*.

Fig. 13. Working-surface of the two molars, m_2, m_3.

Fig. 14. Hinder fractured surface of the mandibular ramus of *Protemnodon Anak* (fig. 3).

PLATE XXVI.

Fig. 1. Outside view of part of left mandibular ramus, with the molar series, of a mature *Protemnodon Mimas*.

Fig. 2. Inside view of part of left mandibular ramus, with the molar series, of a mature *Protemnodon Mimas*.

Fig. 3. Working-surface of the teeth of the same fossil.

Fig. 4. Inside view of part of the left mandibular ramus of a young *Protemnodon Mimas*.

Fig. 5. Outside view of the same fossil, with the premolar (p_3) exposed in its formative cavity.

Fig. 6. Working-surface of the molars preserved in the same fossil.

Fig. 7. Working-surface of the first lower molar (m_1) of the specimen, fig. 14, Plate XXIV., of a young *Protemnodon Mimas*.

Fig. 8. Fractured anterior end of specimen, fig. 4, with section of alveolus of incisor (*i*).

Fig. 9. Upper view of a large portion of the mandible and molar series of a full-grown *Macropus Titan*.

Fig. 10. Working-surface of the lower molars (m 2, m 3), with remains of m 1 and d 4, of an older (female?) *Macropus Titan*.

Fig. 11. Outer side view of a nearly entire right mandibular ramus, with the fore part of the left ramus of the same jaw attached by matrix, of a (male?) *Macropus Titan*.

Fig. 12. Inner side view of symphysial part of right ramus of the same fossil.

Fig. 13. Inner side view of part of right mandibular ramus of a larger individual, or variety, of *Macropus Titan*.

Fig. 14. Working-surface of last molar (m 3) of the same fossil.

Fig. 15. Hinder surface of the same molar.

PLATE XXVII.

Fig. 1. Left side view of fore part of cranium and teeth of *Protemnodon Mimas*.

Fig. 2. Right side view of part of premaxillary and broken incisors of the same fossil.

Fig. 3. Inner side view of left premolar of the same fossil.

Fig. 4. Working-surface of the right upper molars (d 4, m 1) of the same fossil. (The above four figures are from photographs of the specimens now in the Museum of Natural History, Sydney, New South Wales.)

Fig. 5. Left side view of part of cranium and teeth of *Sthenurus Brehus*.

Fig. 6. Under or palatal view of the same fossil.

Fig. 7. Outer side view of left upper premolar (p 3) and following molar (d 4) of *Sthenurus Brehus*; with fore end view of p 3.

Fig. 8. Inner side view of the same teeth.

Fig. 9. Working-surface of the same teeth.

Fig. 10. Outer side view of part of left mandibular ramus and teeth of a mature *Protemnodon Rœchus*.

Fig. 11. Inner side view of the same fossil.

Fig. 12. Working-surface of the teeth of the same fossil.

Fig. 13. Hind surface of the molar (m 3) of the same fossil.

Fig. 14. Portion of palate and of right molar series of *Protemnodon Mimas*. (From a photograph of the specimen in the Museum of Natural History, Sydney, New South Wales.)

All the figures are of the natural size.

IX. *On the Structure and Development of the Skull in the Pig* (Sus scrofa).

By W. K. Parker, *F.R.S.*

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My intention for some time past has been to follow up the Morphology of the Fish's skull by that of the Mammal; and as amongst the "Placentalia" the Guineapig (*Cavia aperea*) takes a very *low* place, it was chosen as the type to work out. I have been led to change my plan, however, and to take a *medium* type by an unexpected supply of materials kindly put into my hands, in November 1871, by my friend Mr. Charles Stewart; these were about seventy embryos of the Common Pig, a considerable number of which were barely two thirds of an inch in length, whilst others measured 6 inches in a straight line from the snout to the tuberosity of the ischium*.

As the tissues in the earlier stages were only in a nascent condition, the greatest care has been taken to harden them for slicing into sections and for dissection from without inwards; and no labour has been spared in this matter the sections being made after the hardened embryos had been imbedded in solid paraffin. These extremely thin objects were coloured with an ammoniacal solution of carmine, and then transferred to slides, on which they were mounted in acid glycerine. The coarser sections of the larger embryos, to be used as opaque objects, were made without imbedding, after the specimens had been immersed in a dilute solution of nitric or muriatic acid, to which had been added some chromic acid†; in the former way I have been able to obtain views of the tissues of the earliest stage under a magnifying-power of as much as 600 diameters, although about 50 diameters has been found to be the most useful, showing, as such a lesser enlargement does, the various parts in relation to each other, and enabling the eye to follow the granular thickenings which are becoming differentiated into special tissues.

The study of this particular type of Mammalian skull has been facilitated by preparatory work in many other types of this Class, extending over a period of thirty-three years; but I have determined not to bring any thing forward relating to special modifications until this more exhaustive piece of work has seen the light.

The first impulse in this direction was given me by an invaluable work which appeared long ago; I refer to W. Cheselden's 'Anatomy of the Human Body' (London, 1722, 8vo). But my newer stand-point is from the 'Elements of Comparative Anatomy' (1864), by Professor Huxley (Lecture 7th to the end).

* The actual length of these embryos, measured along the curved line of the spine to the end of the tail, is about one half more than is given by my practical and easier method of admeasurement.

† All the finer sections and preparations were made by my son, Mr. T. J. Parker.

Since the older writer, no native anatomist has arisen more fitted to hold and to handle this difficult subject than the author of those "Lectures."

I shall follow up this matter from point to point in the same manner as that pursued in the papers already offered to the Society; and my endeavour is to link on paper to paper so that they may form an organic whole, the idea and purpose being the same in each, and the special mode of treatment the same.

In the present communication more *relative* anatomy has been given than in the former papers; I have to steer between the confusion arising from the display of too many parts, and the baldness of a mere account of skeletal structures.

If the nasal and auditory sense-capsules were as easy of elimination as the eyeball, the skull and face would present a much less complex problem; but they soon become part and parcel of a most intricate cranio-facial *unity*, and everywhere intrude themselves upon the observer.

A certain convenient subdivision of this especial piece of morphological work can be made; thus we have—

1st. The notochordal region of the skull.

2nd. The pronotochordal region of the same.

3rd. The facial arches.

4th. The sense-capsules.

The metamorphosis of the original and, as it were, *larval* parts here obtains its highest degree; the distance which has to be travelled by the morphologist between the starting-point and the goal may be conceived of if the primary form (Plate XXVIII. fig. 5) be compared with the finished condition of the skull (Plate XXXVI. fig. 4).

In observing the growth-changes that bring about this result, a large amount of *histological* labour is involved; in the present piece of work that part of the research has been taken pains with as much as if it had been intended to write upon the tissues, and not upon their massing and arrangement. The determination of *homologous* parts in this type, as compared with the elements that build up the skull of the Fish, the Frog, and the Bird, has not been by any means the most difficult part of my toil; they arrange themselves, and assume their own titles, in a very ready manner; for the difficulties of terminology will all melt away as soon as a sufficient number of types have been traced down to their embryonic "roots." Many parts will have to be re-named; but this will be easy work when the true reason for the change is made plain.

As the skeletal parts are all composed of the various kinds of "connective tissue," and as these kinds are intimately related to, and often pass insensibly into, one another, it is not easy to keep to a consistent terminology in describing them.

This class of tissues becomes hardened by bone-salts at very different *ages*; and in any homologous territory, if ossification is *late*, the tissue becomes hyaline cartilage first; in other types the like tract may become bony, whilst, as yet, the tissue is extremely soft and young: in intermediate conditions bone is formed in a tissue which is *indifferent*; it

looks like hyaline cartilage, but the cells are crowded, and it is formed into bone before the intercellular substance has time to appear*.

In the Mammal, more than in any other type, the original parts are all the more completely transformed, in that the bony substance formed in the primordial cartilage becomes very large in relation to its first model; and, moreover, the "investing bones" formed in the subcutaneous web become very large indeed, as compared with the small granular territory, the soft model in which they first appeared.

Also in no other type do the primary facial rods become segmented, arrested, and metamorphosed to the same degree as in this the highest vertebrate Class.

First Stage.—Embryo Pig, 7½ to 8 lines long.

The primordial skeleton of the most highly specialized Mammal is as simple as that of the lowest brain-bearing Fish; the form of the fœtal head (Plate XXVIII. fig. 1) may be aptly compared with that of the Fish and Frog (see my former papers on those types). In embryos 7½ lines in length the three brain-vesicles (C 1ª, C 2, C 3) are hollow, the film of soft brain-substance merely lining the enclosing membranous cranium. The foremost of the vesicles has budded into the two rudimentary hemispheres above the primary sac, the "thalamencephalon" (Plate XXVIII. figs. 3 & 6, C 1, C 1ª); yet at this stage the cutis does not cover the whole of the third vesicle (C 3) nor the whole of the auditory sac (*au.*). The head is bent over upon the thin-walled thorax, and the cervical region of the spinal chord is very outbent and swollen (fig. 1).

The Visceral Clefts.—After noticing the brain-vesicles and the three pairs of sense-capsules (*ol.*, *e.*, *au.*), the foundations of which are already well laid, the eye detects that peculiar dehiscence of the facial wall, the continuous face being cloven by the formation of a series of slits or cuts, which pass quite through the substance of the cheek and neck. By the older embryologists these are counted from behind the mouth; but in my last paper, especially, I have shown that the mouth itself is a great, double, completed cleft, and that there is a *secondary* cleft in front of it, the "palato-trabecular" or preoral cleft (*cl.* 1). But the "first postoral" is in reality the second cleft; this is the largest in the embryo pig, with the exception of the mouth. Behind this there are three others; and the first of these, the "second postoral," is the counterpart of the most anterior of those through which the *water-currents* pass in the osseous fish. Below and behind the clefts the fore limb is seen in rudiment. Here it will be seen that there is a deficiency in the *number* of clefts behind, as compared with the gill-bearing vertebrates (see papers on Frog and Salmon). Only the first "postoral" cleft is persistent and functional, the three behind soon closing in. I shall describe, anon, what becomes of the persistent clefts, that in front of and that behind the great mouth-cleft. Between the clefts are formed the arches; these facial bars have some resemblance to ribs, but are formed independently of axial parts, whereas the ribs are evident downgrowths from the vertebral portions of the "Somatomes."

* See on this subject, "On the Connective Tissues," by A. Rollett, in Stricker's 'Human and Comparative Histology,' translated by H. Power for the New Sydenham Society. London, 1870, pp. 47–146.

Where the facial arches most closely imitate ribs, as in the first and second postoral, the "capitulum" and the "tuberculum" are applied to a part to which they have no proper morphological relation—namely, to a sense-capsule. This is one of the many modifications the morphological elements are subjected to in the cephalic region.

The original pattern of the facial system of a vertebrate is simple in the extreme; the paired rods are accurately like each other, but their development is not quite synchronous; the *secondary* preoral pterygo-palatine (*p.pg.*) is overshadowed and slow in growth (see Plate XXVIII. fig. 2, where the arches are drawn as though the object were transparent)*. The facial thickenings between the clefts which contain the arches may be seen with considerable clearness, especially in front (fig. 3); here in front of (above) the mouth towards the mid line we see the clubbed ends of the trabeculæ (*tr.*) roofed over by the nasal sacs. Below and somewhat behind these are the pterygo-palatine arches (*p.pg.*), in the thick outer wall of which the maxillaries and malars will be developed, and the pith of which will become, by early ossification, the palatine and pterygoid bones. Below the *inferior*, transverse, large mouth, the thickenings which contain the first and second postorals are seen—Meckelian and hyoid. The cleft which is formed between the trabecular and the pterygo-palatine bars is best seen in the side view (figs. 1 & 2, *cl.*1); it opens in the inner canthus of the eye. The two pairs of preoral rods will be best understood by reference to a palatal view of the skull with the postorals cut away (fig. 4), and to the diagrammatic view of the skull and face as seen from below (fig. 5). It is easy to see, by a reference to the palatal view (fig. 4), that we are now standing on the same level as the "Dipnoi" amongst the Fishes; the external nostril (*e.n.*) and the internal (*i.n.*) lie on the same plane; a free intervening growth of cartilage, binding the arches together, with no further metamorphosis of the parts, would produce a true parallel to the skull of those remarkable Fish. The sinuosities of the upturned palate (fig. 4), its plaits and its crevices, are easily understood by reference to the diagram (fig. 5).

First Preoral Arch.—The trabecular rods form together an elegantly lyriform structure; they already have begun their extensive "commissure," being parallel now in their fore half. Behind, they are like callipers, and the blades are at some distance from each other; their apices, sharpened off, seem to approach the fore end of the investing mass (*i.v.*); but a sectional view (fig. 6, *tr.cm.*, *i.v.*) corrects this error, and shows that these diverse parts lie on a totally distinct plane and far from each other, a fact I pointed out long ago in my paper on the Frog (Phil. Trans. 1871, Plate III. p. 143). These trabecular blades embrace the pituitary body (*py.*); but their curve does not conform to its shape, and is altogether independent of it, being the proper "habit" or morphological *fashion* of the arch. After forming the elegant, pyriform, primordial pituitary space, the trabeculæ become thicker, narrower, and lie closely side by side; this is soon followed by fusion of their edges—the formation of the trabecular commissure (see Plate XXIX. fig. 4, *tr.cm.*). These two rods do not end as a straight bar, but in front

* In my paper on the Frog (Phil. Trans. 1871, p. 148) the pterygo-palatine arcade is described as a secondary structure; in that on the Salmon (*ibid.* 1873, p. 109) it is spoken of as independent. *It is a secondary arch.*

are bent upon themselves, as the fingers in clutching; hence the transverse crevice seen in the palate between the inner nares (fig. 4, *i.n.c.*, *tr.*). This retral growth of the trabecular "cornua" is not so pronounced in the Frog (Phil. Tráns. 1871, Plate v.), but is equal to the Mammal in the Bird ("Fowl's Skull," Phil. Trans. 1869, Plate LXXXI. figs. 1 & 2, *tr.*). The median part of the upper lip, which is transverse and quite rudimentary in the youngest embryo (fig. 3, *u.l.*), has developed in a somewhat older specimen (fig. 4, *pn.*) into a pointed retral flap. This flap hides an azygous projection of the trabecular commissure, the "prenasal cartilage;" this axis of the premaxillaries is a part largely developed in Birds (see "Fowl's Skull," Plates LXXXI.–III. *pn.*), where it is first retral, then vertical, and then foreturned, so that it is the principal factor in the exaggerated prognathism of that Class. Outside this process the trabecular cornua are at present clubbed and bulbous (Plate XXVIII. figs. 3, 4, & 5, *c.tr.*); afterwards they each send backwards a recurrent rod*. The general appearance of the trabeculæ, as seen from above, is shown in Plate XXIX. fig. 4; their varying thickness is displayed in sections (Plate XXIX. figs. 1, 2, 3, & 5, *tr.*).

Second Preoral Arch.—Even in the Osseous Fish I found the pterygo-palatine arch both *late* and feeble in its development; in the Frog it is a long time before it appears, and grows very slowly, and is never more than a long conjugational band between the trabecular and mandibular rod. In the Mammal, as in the Bird, this primarily feeble rod is ossified hurriedly, as it were, before the cells can acquire any intermediate substance (see "Fowl's Skull," Plate LXXXI. figs. 1, 6, & 11); yet in the present instance the bony plates that arise in and around these small sigmoid granular rods are some of the most complicated and the most massive in the whole head and face. Even through the palatal skin the hooked tops of the preoral arches can be seen (fig. 4); but whilst those of the trabeculæ grow *inwards*, those of the pterygo-palatine bars grow upwards and outwards, persistent in the "hamular process." The direction of the whole bar (Plate XXVIII. figs. 4 & 5, *p.pg.*) is downwards and forwards, and their extremities or "cornua" approach each other below the trabeculæ: they are at present far apart in this originally cleft palate (figs. 4 & 5); the fold of mucous membrane covering each on its inner side gradually grows towards its fellow, and they eventually meet and coalesce. The thick cushion outside each bar is the nidus in which the maxillary and malar are developed; and the whole maxillo-palatine mass is a mere process or outgrowth of the first (postoral) arch, and is not an independent morphological region. At present the arch is *subocular*; but it does not correspond to the subocular bar of the Tadpole ("Frog's Skull," Plate v.), which is formed by the extremely long pier of the mandibular arch, the arrested conjugational pterygo-palatine lying quite in front of the eyeball.

* The distinctness of these rods from the surrounding tissues is purposely exaggerated in the accompanying illustrations, for they are imbedded in a gelatinous tissue rich with enclosed granules or young cells, whose protoplasmic substance takes up the carmine very freely; and the differentiation of these rods is at present a matter of degree, that part of the blastema which will become hyaline cartilage being the most compact and crowded with young cells; next to this the nascent perichondrium; and the most gelatinous part outside is the rudimentary condition of the loose stroma or areolar connective tissue.

In this stage a section of the head (affected as it is by the "mesocephalic flexure") which passes through the first cerebral vesicle and its upgrowth takes the pterygo-palatine rods almost from end to end, whilst the trabeculæ are cut directly through (Plate XXIX. fig. 5, *tr. p.pg.*). Thus, as compared with the trabecular apices, the pterygo-palatines *descend* a little before they send upwards the apical hook.

Third Arch, or First Postoral.—This rod, like its immediate successor, is stout, sigmoid, and strongly inhooked above; it does not at present meet its fellow at the mid line. This is the primordial mandible, *mn.*; but it remains as the lower jaw for a very short time, and is not segmented into an upper and lower piece. There is a stage in all the oviparous Vertebrata in which this rod is free from segmentation; but, above the Lamprey, a pier and free arch are formed by subdivision of the bar. The tissue over it is thick, and in this overlying part the persistent mandible is formed (see Plate XXVIII. figs. 1, 4, & 6, and Plate XXIX. figs. 5 & 6, *mk.*).

The morphological changes that take place in the hooked and inbent apex are of the greatest interest; for now we arrive at the point where not only the hyoid arch is arrested and modified in relation to the outworks of the organ of hearing, but the mandible of the embryo is also suddenly given up to these secondary correlations. Considered in relation to their new function, the parts of the mandible of the mammal might, like those of the upper part of the hyoid arch, be included in the *stapedian* terminology*.

The Meckelian rod itself is shown in the vertical section near its extremity (Plate XXVIII. fig. 6, *mk.*), and in the palatal view (fig. 4, *mk.*) near its apex; near its apex it is seen on the outside in the lateral view of the sliced head (Plate XXIX. fig. 6, *mk.*). But horizontal sections of the head are necessary to show the relation of the apex of this bar to the first postoral cleft, the rudimentary ear-drum cavity (see Plate XXIX. figs. 7, 8, 9, *mk.*). These sections show that this very expanded cleft is being divided into two spaces, one of which (the inner) becomes the tympanic cavity, and the other the "meatus auditorius externus." The septum or diaphragm is formed by the lining skin of the cleft growing outwards from the side of the ear-sac, and inwards from the outer face; this latter growth is the most intense, being pushed in by the ingrowth of the apex of the embryonic mandible, which, growing inwards and backwards, carries the lining skin of the cleft before it; thus the "membrana tympani" is formed. Looking at these figures, we see at once that the "manubrium mallei" is the hooked apex of the primordial mandibular arch, and that therefore it must correspond with the large bifaceted backwardly placed head of the Bird's quadrate bone"†.

The shoulder or tuberculum of this rib-like bar becomes the thick head of the

* See "On the Representatives of the Malleus and the Incus of the Mammalia in the other Vertebrata," by Professor Huxley, Proc. Zool. Soc., May 27, 1869, pp. 391–407.

† I was under the impression that the "internal angular process" of the Bird's mandible ("Fowl's Skull," Plate lxxxi. fig. 13, *i.a.p.*) was the homologue of the manubrium mallei of the mammal; it is not; both it and the posterior process (*p.a.p.*) are outgrowths formed lower down, and correspond in nature to the "opercular knob" of the next or hyoid arch.

hammer; the solid rod itself develops for a while, but by the time of birth has shrunk into the feeble, pointed "processus gracilis."

Fourth Arch, or Second Postoral.—At present this arch is extremely like the one in front of it (Plate XXVIII. fig. 5, and Plate XXIX. figs. 5 & 6, *hy.*); but it is flatter, and the right and left bars meet more closely and at an obtuse angle; its shoulder, also, is more upturned. This arch has been cut through in the palatal vertical views (Plate XXVIII. figs. 4 & 6, *hy.*); but the form of its tuberculum and capitulum are best seen in the horizontal section (Plate XXIX. figs. 7–10, *hy.*), and its shoulder or tuberculum is exposed in the sliced head (Plate XXIX. fig. 6, *hy.*). In this latter figure it is seen that the shoulder stands out like that of a Bird's rib, the head or capitular portion thrusting itself as far inwards as it can on to the periotic wall. The *landmarks* exposed in this figure are the portio dura and the top of the jugular vein (Plate XXIX. fig. 6, *hy.*, 7^a, *j.v.*). In figs. 7, 8, & 10 of the same, Plate, the horizontal sections show that the head of the hyoid growing towards the auditory mass is exactly like the head of the mandibular rod. The portio dura nerve is seen both at its entrance into and its exit from the facial wall in this figure, and it is of the utmost consequence to the morphologist as being a most safe landmark. In the outer lateral view it is seen escaping behind that part of the hyoid rod which becomes the "stylohyal" (Plate XXIX. fig. 6, *hy.*, 7^a).

In one horizontal view (Plate XXIX. fig. 7, *hy.*, 7^a) its whole *auditory* course is seen, on one side its entrance into the wall in front of the first postoral cleft, and its exit behind the hyoid in the other; the same thing is shown in Plate XXVIII. fig. 8, 7^a (see also Plate XXIX. figs. 8–10, 7^a). In most of these figures the head and neck of the hyoid are shown from *above* (Plate XXIX. figs. 7, 8) and from *below* (fig. 10, *hy.*); but in another seen from above (Plate XXIX. fig. 9) the section is through the rods a little lower down; and here we get a most instructive view, the shoulder evidently becoming dislocated from the neck, a process which will go on to complete separation of the parts.

Fifth Arch, or Third Postoral.—In this arch the Mammal has developed merely the counterpart of the "hypobranchial" segment of the first branchial arch; it is shown in a subhorizontal section *in situ* attached to the larynx (Plate XXIX. fig. 5, *th.h.*, *lx.*), and in the diagrammatic figure (Plate XXVIII. fig. 5, *th.h.*) is seen beneath the auditory sacs. In my paper on the Frog (Plate VII. p. 171) I showed how that the thyrohyals were the hypobranchial remnants of the first and second branchial arches developed backwards; those of the Mammal are therefore strictly homologous with those of the Frog, the latter being formed by retention of a part, which part is alone developed in the former.

Looking again at the five pairs of facial arches as a whole, we see that the only arch, at present, which has developed a conjugational keystone piece is the first or trabecular: this is the "prenasal rostrum" which figures so largely in my former paper on the Bird's Skull. No other keystone appears afterwards in the Pig, save in the last pair;

this becomes the "basihyal" of anthropotomy, but answers to the first "basibranchial of the Fish. These and the other "conjugations" will be shown in the more advanced stages.

The Notochordal Region and Membranous Cranium.—With the arrest of the somatomic divisions in the cephalic region of the embryo, and the great modification of the nerves of common sensation and of motion, we have no certain guide as to how much or how little of the *spine* the notochordal region corresponds to. The notochord retreating, relatively, from the fore end of the investing mass and becoming in time the temporary axis of a single basal bone, the basioccipital, although it gives a *vertebral* character to its own territory, is yet placed by its altered conditions in a new category.

In my first stage I take the skull when it has been fully bent upon itself—the "mesocephalic flexure;" and at this time the large notochord (Plate XXVIII. fig. 6, *nc.*) bends suddenly upwards, and ends in a free blunted point, exactly opposite that infolding of the membranous cranium which partially severs the second from the third cerebral vesicle (C 2, C 3). The investing mass stops short of the apex of the notochord and lies *beneath* its plane. The relation of the two, as seen from above, is given in the horizontal view (Plate XXVIII. fig. 8), and as seen from below, diagrammatically, in fig. 5. The vertical section (Plate XXVIII. fig. 6) shows the notochord covered above with the membranous cranium (dura mater and cells of the *cutis*), and bearing in its hollow the medulla oblongata (*m.ob.*) and the vesicular cerebellum (C 3). The three structures here seen *behind* the pituitary body (*py.*) form the primordial "posterior clinoid wall;" and the rounded mass of delicate gelatinous stroma which lies above these three parts, in the *hilus* of the kidney-shaped third cerebral vesicle, is the "third or median trabecula" of Rathke—a structure quite temporary, as that excellent author averred, and of no morphological import*.

Only in the basal region is there at present any developed hyaline cartilage (Plate XXVIII. figs. 5, 6, 8, and Plate XXIX. figs. 4 & 7, *i.v.*), although it does appear in large tracts afterwards infero-laterally, and even *above* also in the occipital region. At present all but the notochordal region of the cranium is a very soft and delicate membrane, inclosing the large blebs into which the great neural axis has developed. Afterwards this membrane will in certain parts split up into three strata—the dura mater within, the granular territories in which the "investing bones" develop will lie on the outside, and in the middle the hyaline cartilage of the occipital and sphenoidal regions. At present the skin is represented, but not thickened into distinguishable dermis, over the third vesicle (Plate XXVIII. figs. 1, 2, & 6, C 3); afterwards this vesicle will be entirely enringed behind, in the manner of a vertebra, the middle layer of the membrane chondrifying directly upwards from the investing mass. But in the basisphenoidal region only as much cartilage as was primarily related to the free end of the notochord (namely, the "postclinoid wall") has any remnant in it comparable to a

* Rathke erred in supposing the "paired rafters," or symmetrical trabeculæ, to be outgrowths of the investing mass of the notochord.

vertebral structure; and the whole basisphenoidal territory, small as it is in the Pig, is very *compound*, having its origin in the two apices of the investing mass, in the apices and posterior end of the commissure of the trabecular rods, in a chondrified part of the cranial wall (related to the investing mass merely by coalescence at its postero-inferior angle), and, lastly, in a *secondary* growth of cartilage beneath the pituitary body. This last growth of cartilage is found in Sharks and Batrachians, but not in Teleostean Fishes nor in Lizards and Birds.

The basisphenoidal territory may be understood by drawing one imaginary line across the end of the trabecular commissure (Plate XXIX. fig. 4, *tr.cm.*), and a second across the apices of the investing mass, leaving a short tract in front of the posterior line; the figure itself ends at this second line.

The cranial wall in the anterior sphenoidal region is altogether soft and gelatinous at present (Plate XXIX. fig. 5, a section through the primary cerebral vesicle, thalamencephalon, and its two outgrowths, the *hemispheres*); beneath this part the trabecular bands are now fairly differentiated (*tr.*), and these form the lower half of the *compound* presphenoid, as we shall soon see. These are the three proper cranial regions, corresponding to the cerebral vesicles, but not in any proper morphological sense answering to the divisions of the body, the somatomes; no other segment can be found, for the immense outgrowths of the first cerebral vesicle lie upon the nasal roofs.

Even the posterior sphenoid loses what little relation it had to the notochord, which is absorbed by the basioccipital, and is largely formed by borrowing substance from the first facial arch; but the anterior sphenoid is a mere chondrification of the middle layer of the membranous cranium, the two wings mutually sending downwards an azygous plate which coalesces with the common crest of the trabecular bars.

The Sense-capsules.—The extent of the olfactory region is at present very small; afterwards the whole labyrinth takes up three fourths of the cranio-facial length. The squarish septum (Plate XXVIII. fig. 6, *s.n.*) looks almost directly downwards beneath the first cerebral vesicle, and the double roof of the labyrinth has the relation of an eave to the cerebral roof. The most projecting kidney-shaped part of the nasal roof is the "ala nasi;" and both lateral and front views of the embryonic head show how this is, as it were, grafted on to the upper surface of the down-bent trabecular bars. A great difficulty is got rid of here, which has cost me much trouble; for the alæ nasi are not formed out of the substance of the trabeculæ, nor can they be considered, in the adult, merely the front and portico of the roof of the nasal labyrinth; they completely coalesce with the trabecular knuckles; and the rooting, ossified snout of the adult Pig is of a compound nature, principally, however, formed of the genuflection of the first visceral arch.

The passage from the outer to the inner nostril (Plate XXVIII. fig. 4, *e.n.*, *i.n.*) is already tortuous (Plate XXVIII. fig. 7, a vertical section *beyond* the septum nasi); for already the mucous membrane has been thrown into baggy folds, into which soft outgrowths of the roof are entering, afterwards to become a labyrinth of cartilaginous

coils. The bilobate mass directly above the trabecular horn (*al.tb.*, *i.tb.*, *c.tr.*) has its anterior lobe developed into the curious "alinasal turbinals" immediately within the snout, whilst its hinder lobe becomes the long "inferior turbinal." The swelling below the down-turned roof is the rudiment of the "nasal turbinal," scarcely developed in the adult of this type; and the mass which lies beneath the rudiment of the olfactory crus (1) becomes the upper and middle turbinal (one mass in the Pig) and the true olfactory region. Vertical sections* show, most instructively, what could in nowise have been guessed at—namely, that the nasal labyrinth has its skeletal parts formed by the approximation and coalescence of two imperfect cylinders open freely below, and by these receiving at their junction below the ascending crest formed by the conjugated trabeculæ (Plate XXIX. figs. 1–3, *al.s.*, *al.e.*, *tr.*). When these trabecular bands continue flat (as in the embryo, Plate XXIX. figs. 1 & 2, *tr.*), then we have, as in the Frog and the Crow, a cartilaginous floor to the nasal passages (see memoir on "Frog's Skull," Plate VII. fig. 6, and Plate X. fig. 3, *s.n.l.*; and also Proc. Roy. Micr. Soc., Oct. 2, 1872, p. 224, pl. 38. fig. 1, *s.n.*). In most Mammals, and in Birds not belonging to the Passerine group, the trabeculæ narrow in to form the rounded thickened base of the whole "ethmo-presphenoidal bar;" this process is seen to be beginning in the section through the posterior part of the nasal region (Plate XXIX. fig. 3, *tr.*). The section through the inner nares (Plate XXIX. fig. 3, *i.n.*) also shows the back wall of each nasal passage (*p.n.w.*). These large rounded spaces are seen to have the rudiments of the last of the middle turbinal coils already continuous with the end wall. These posterior walls correspond to the end of each "sphenoidal sinus;" this is therefore the presphenoidal region, and *behind* the mesoethmoid; and the pyriform openings above were made through the front of the cranium and through the fore end of the cerebral hemispheres, where they bud-off the olfactory crus (see also Plate XXVIII. fig. 6, C 1*, *ol.*). This same section has cut through the first cleft (first preoral or lacrymal passage) on its way to the nasal passage. The anterior extremity of the, as yet, soft palato-pterygoid rod (*p.pg.*) is here cut through, where it passes below the inner nostril (*i.n.*). The space between the rudimentary olfactory crus and the budding upper turbinal (Plate XXVIII. fig. 6, *ol.*, *s.n.*) is composed, at present, of an almost structureless gelatinous stroma; it is slow to form those cartilaginous bands afterwards, which, creeping between the olfactory filaments, form the cribriform plate—a *secondary* morphological structure almost entirely peculiar to the Mammalian skull.

The eyeball only affects the skull from without, by modifying the facial and cranial structures to form its safe recess or orbit; but the *eyeball* is constructed after the fashion of the old cottage *oven*, being built into the side walls of the skull, bulging out on the outside, and having its *nerve-mouth* projecting also within.

In my last paper, on the Salmon's skull, I was able to show the infolding of the

* These *vertical* sections of the nasal region were made by the same slicing as the *horizontal* sections of the head further back: this depended upon the hooked shape of the head at this stage; the razor passed at right angles to the nasal roof, but parallel to the notochord.

dermal layer to form the ear-sac, the cavity of which was wide open on the outer surface. In this piece of work "it was my hap to light upon" embryos the youngest of which were filming over this primordial "aqueduct;" the skin (*cutis*) is incomplete over the top (Plate XXVIII. figs. 1 & 2, *au.*), but the passage itself, leading into the rudimentary labyrinth, is closed by a gelatinous plug. The periotic walls come as near to cartilage, in their commenced solidification, as the investing mass and facial arches, and the outline of the sacs can be fairly made out. Their general form can be seen by referring to the horizontal views (Plate XXVIII. figs. 5 & 8, and Plate XXIX. fig. 7, *au.*); but they are well outlined on the external surface (Plate XXVIII. figs. 1 & 2, *au.*), and they are seen to be tuberose bodies, having a straightish inner margin, a sublobular outer margin, and their broadest end behind. They are separated by the breadth of the investing mass with its enclosed notochord, and this tract is narrowest in front. When the upper face is slightly pared off (Plate XXIX. fig. 6, *au.*), the opening of the aqueductus vestibuli is shown; but this is best seen in a horizontal section, viewed from below (Plate XXIX. fig. 10), where it is seen imbedded in the periotic wall inside the "tegmen tympani" (*t.ty.*); a little behind it is seen the portio dura, which forms by its boring the "aqueductus Fallopii." In the same figure the opposite side of the section was made lower down, so that the roof of the tympanum (tegmen) is cut away, and the tympanic cavity cut through, exposing the head of the second postoral arch and the "aqueductus" just above its entrance into the cavity of the ear-sac*.

The reader will observe that this passage has the appearance of being double; I could not, however, find more than one perforation. This opening into the auditory sac, which is large in my first and second stages in the Frog, has closed in the third stage ("Frog's Skull," Plates III. & IV. *au.*); in the Salmon ("Salmon's Skull," Plate V.) it has not closed in "fry" of the first summer.

As for the cavity of the ear-sac, it is at present very rudimentary; the canals are but beginning to bud out from the postero-superior region, and the cochlea is perfectly ornithic (compare Plate XXVIII. fig. 8, *cl.*, and Plate XXIX. fig. 7, *cl.*, with "Fowl's Skull," Plate LXXXII. fig. 1, *cl.*).

The sections show the larger nerves and vessels, which serve as excellent landmarks, especially the trigeminal, the portio dura, the glosso-pharyngeal, the vagus, and the hypoglossal nerves (5, 7*, 8, 8*, 9). The three last nerves all pass through soft stroma in the angle between the auditory sac and the investing mass; the large vessels also ("basilar artery" and "internal carotid") all lie, as dense ensheathed masses of young blood-corpuscles, in the most diffluent stroma, the fluidness and instability of which makes it an admirable "soil" for these fast-growing countless "roots." Before passing to the next stage I must again refer the reader to the diagrammatic figure (Plate XXVIII. fig. 5), that he may compare it with what I have already described in the embryos of the Fowl, Frog, and Osseous Fish at a similar stage. With the vantage-ground of this

* I have not been able to determine what relation this primary opening bears to the "aqueductus cochleæ," or whether it is related to it at all.

simple platform it will be easy to follow the metamorphosis of the primordial parts, even in the Mammal, where such changes are most of all displayed, and to compare and harmonize them with the lesser degrees of transformation to be seen in Fowl, Frog, and Fish.

Second Stage.—Embryo Pig, 1 *inch long.*

Most of my illustrations of the complete embryonic skull will be made from a somewhat more advanced stage than this; but this second stage is of great importance in illustrating the morphology of the facial arches and auditory sacs*.

In this embryo *chondrification* has fairly set in, although the cells of the hyaline cartilage are still close together, quite as close as in the nidus of the vomer in the next stage, or the tissue in which the rostrum of the parasphenoid is developed in the embryo bird ("Fowl's Skull," Second Stage, Plate LXXXI. fig. 7, *r.st.*). *Ossification* has commenced also, and can just be seen in the nidus of the vomer, maxillary, and dentary; this last is the forwardest of the bony plates (Plate XXX. fig. 2, *d.*). Beginning at the snout we see that the alæ nasi have chondrified, and that the retral trabecular horns (Plate XXX. fig. 1, *al.n.*, *c.tr.*) have coalesced with them: the little papular prenasal cartilage (*p.n.*) is well seen in this *front* view; beneath this, a little further back, the stroma becomes dense on each side and forms the premaxillary territories, and is ready to ossify. In the deepest and widest part of the ethmoidal region, a vertical section of which (Plate XXX. fig. 2) shows the commenced ingrowing of the proper turbinal folds, we see now that the descending nasal roofs and the ascending trabecular crests have all coalesced together to form the large mesoethmoid (*m.eth.*). A long scoop-shaped territory lies immediately under the trabecular base of this septum, and this granular tract is undergoing endostosis; it is forming the vomer (*v.*). Far on each side, above a rudimentary tooth-pulp, is a faint trace of the maxillary (*mx.*). In this "schizognathous" stage the root of the tongue is seen at no great distance from the freely exposed vomerine region, and the oral cavity (*m.*) has here steep sides, in the walls of which the primary palatal bars (*p.pg.*) are seen as compressed granular plates. On each side, below an inferior tooth-rudiment (*t.*), a large mass of nascent cartilage is seen, having a kidney-shaped section; and inside this a round rod of cartilage is seen, converging towards its fellow of the other side as it passes forwards. If my observations had ended here, the thick slab of granular tissue, with its incurved edges, would have merely been noticed as the proper dentary territory or nidus of the mammalian mandible; it is more than this, as the next two stages will show: the "rod" is MECKEL'S cartilage (*mk.*), the shaft of the first postoral arch. The dentary bone itself appears in this section, and is of a rich rose-colour in the preparation, one stained with carmine; the tissue around the osseous deposit is becoming *colourless*, like MECKEL'S rod, for the carmine scarcely tints the cartilage. The other postoral bars are shown in this section; the "cornu minor"

* From a large number of exquisite sections of this stage I have only made the six illustrations here given; for what the rest show is better seen in a somewhat more advanced stage, the morphological level being essentially the same.

(*c.hy.*) is cut through near its junction with the long stylohyal, and the "cornu majus" (*br.* 1) is shown in its whole extent on each side; whilst between we have the basal piece (*b.br.*), not, truly, a basihyal, but answering to the first basibranchial of the Fish. The larynx (*lx.*) is below and behind them, and behind it is the œsophagus (*œ.*).

Another section (Plate XXX. fig. 3), taken further back than the last, shows the sphenoidal, auditory, and occipital regions as seen from above. The differentiation of parts has gone on very rapidly, whilst the embryo has merely become longer by one half, and the difficulties in the way of interpretation are largely removed.

In front the "anterior clinoid wall" (*a.cl.*) is seen to be symmetrically divided at the mid line; this is the junction of the trabeculæ at the end of their long commissure, in front of the outbent blades which pass around the pituitary region. The pituitary cup is deep, wide, narrow above, and has a crescentic form, the concavity of which looks forwards. Between this actual cavity and the free ascending ends of the notochord and investing mass there is a large amount of gelatinous tissue, through which the wavy internal carotids (*i.c.*) pass, converging and again diverging. The gelatinous tract, the base of the so-called "middle trabecula" (see Plate XXVIII. fig. 6, *m.tr.*), is widest close to the ear-sacs, and narrows to the edge of the pituitary pit; on each side of it is seen a bulbous mass, the Gasserian ganglion (5). On each side of the extremity of the notochord and investing mass are seen the well-defined ear-sacs, which are here cut through in their cochlear region; the *coils* are now well developed. On the left side the section is *behind* and *below* that shown on the right, where the "malleus" or head of the first postoral is cut through, the shaft of the second arch, and the "meatus externus" and outer ear. Part of a similar section, taken lower down (fig. 4), displays the orbito-sphenoids in section (with their upper part cut away); and where they have coalesced with the trabecular commissure, there the optic nerves (2) are crossing. The alisphenoids (*al.s.*) are sections seen in their whole extent, but not their connexion with the basisphenoid, the notochordal region of which is displayed, backwards to where the basioccipital territory begins.

At a lower level closely packed cells are developing into cartilage, which will form a secondary floor to the pituitary body, the *seat* of the "sella turcica;" then the posterior sphenoid will be morphologically complete. The connexion of the two great postoral bars with the auditory capsule will be better understood by two more sectional views similar to the large figure (Plate XXX. fig. 3), but of more limited extent and more highly magnified: all these figures are made from the *antero-inferior* aspect of the up-tilted basis cranii, the sections, which in the nasal region were *vertical* (figs. 1 & 2), being *horizontal* behind. Such a section (Plate XXX. fig. 5) through the outer ear or concha (*co.*) and head of the first postoral bar shows how curiously incurved this capitular portion is, and how that its apex is developed into an orbicular part, like that on the apex of the next bar. The shoulder, which articulates with the upper part of the next bar, is very bulbous, and at the root of the neck a conical boss is sent outward; the shoulder is the head of the malleus, the boss is the process for attachment of the

tensor tympani muscle, and the rest of the neck with the rounded head is the "manubrium mallei" (*mb.*). The dark jagged space is the tympanic cavity, a development of the first postoral cleft, and which runs forwards into the mouth-cleft as the Eustachian tube. Now it is easy to see how the membrana tympani is formed; for the inhooked apex of the mandibular rod, creeping like a tendril toward the auditory sac, necessarily carries with it the lining membrane of the cleft wrapped over its head. The shaft is not shown here, because it has been severed with the fore part of the shoulder or "tubercular" portion.

On the left side of the larger figure (Plate XXX. fig. 3) the fellow of this is seen, but cut away further backward. Below the "manubrium" is seen the shaft of the next arch (now to be called stylohyal); its direction is downwards and forwards to the root of the tongue; a good distance must be supposed between this and the section through the ceratohyal already described (Plate XXX. fig. 2, *c.hy.*).

The outer ear or concha (*ca.*) is fast passing into cartilage; it is curiously folded upon itself, and runs round the external orifice of the cleft; it is much modified already from its Batrachian and Plagiostomous prototypes, the "annulus tympanicus" of the former, the "principal opercular" of the latter. The interest attached to the vegetative gemmation of the membrana tympani is more than rivalled in the metamorphic changes that take place in the succeeding arch and in the neighbouring territory of the ear-sac. In the first stage we saw that the simply oval primordial ear-pouch was developing into a lobular form, and that there were three bulgings on the outer side of the sac (Plate XXVIII. figs. 5 & 8, *au.*). The middle of these, by a process of gemmation, has freed itself to a great extent from the wall of the ear-capsule, thus forming a "fenestra" in that wall, which, however, is closed by the separated nodule of cartilage. This twin bud (Plate XXX. fig. 6, *st.*) (it has two papular elevations which look forward and outward from its free surface) is covered externally by delicate indifferent tissue, ready to become cartilage. Being in the posterior wall of the first postoral cleft, the second arch (hyoid), whilst sending its orbicular head inwards, does not become infolded in the mucous membrane lining the cleft, but is free to creep, tendrilwise, to the surface of the ear-sac; this it does, and conjugation takes place between its orbicular "capitulum" and the freed auditory bud. But in the first stage we saw that a curious kind of segmentation was taking place through the shoulder of the second postoral bar (Plate XXIX. fig. 9, *hy.*); now that process is much more complete, and the simple bar has undergone a process the exact counterpart of that by which the blade of the *orange*-leaf articulates with its petiole: whilst this has been going on, a rounded "tuberculum," distinct as that in the rib of a bird, has been developed on the detached upper segment (Plate XXX. fig. 6, *s.c.i*); this is the "short crus of the incus;" the *neck* growing towards the ear-sac is the "long crus" (*l.c.i.*); its expanded, conjugating end the nidus of the "os orbiculare;" the *half-shoulder* above is the body of the incus, which articulates with the shoulder of the arch in front (Plate XXX. fig. 3, *i., m.*); and the bigeminal segment of the auditory sac is the young stapes (*st.*). The other half of the shoulder, or tubercular

part of the rod, is continuous with the long descending part of the arch (see Plate XXX. fig. 9, *st.h.*); it is the head of the stylohyal. Here we see that the second postoral arch grafts its capitular portion on to the auditory segment, and splits its tubercular portion into two new condyles, one of which, covered by the squamosal on the outside, articulates with the tegmen tympani; whilst the other, retreating very sensibly, *coalesces* with the ear-sac further backwards and downwards, close in front of the exit of the portio dura nerve. In the figure these parts are continuous; but the continuity is kept up at present by *new cells*, and these younger cells are all soft indifferent tissue as yet; their morphological differentiation will be explained in the next stage*.

Behindthe stylohyal and some distance outside the promontory (*pr.*), the portio dura nerve (7^a) is seen in section, an excellent land-mark for the stylohyal; further backward the compound 8th nerve (8^a, 8^b) is seen in the "foramen lacerum posterius" (*f.l.p.*); the hypoglossal (9) is enclosed in the upgrowing exoccipital cartilage (Plate XXX. figs. 3, 9, *e.o.*).

The relation of the auditory sac to the exoccipital (*e.o*) is shown in fig. 7; the whole arch of the horizontal canal is seen shining through the cartilage, and its ampulla is obscured by the fibres of the portio mollis nerve (7^b); the Gasseran ganglion, and the compound 8th nerve are also severed (5, 8^a, 8^b).

Third Stage.—Embryo Pig, 1⅓ inch in length.

Those metamorphic processes which were rapidly proceeding in the last stage have become very complete in this, where the embryo is one third longer: this stage must be copiously illustrated and described at length, as it is the best stepping-stone between the early simple and the later transformed conditions. The sections now to be described are a series from the end of the snout to the occipital region. Parallel to each other, yet they do not keep the same *vertical* relation to the embryonic head, but become almost horizontal sections of the occipital region: the whole head at this stage is about equal in size to a horse-bean.

The first of these slices is through the end of the snout (Plate XXXI. fig. 1), and shows the coalescence of the alinasal cartilages with the backwardly bent trabecular cornua (*al.n.*, *a.tr.*). The next (fig. 2) is through the foremost part of the septum nasi (*s.n.*) and valvular fold of the nostril—rudiment of alinasal turbinal (*al. tb.*): a more enlarged view of the lower half of the septum (fig. 2^a) shows the large and massive trabecular cornua, and the prenasal part of the trabecular commissure between them. In the next (figs. 3 & 3^a) the cornua are now seen to be *retral*, for here they are becoming separate from the "prenasal;" still the base of the septum nasi as well as their

* I have studied the development of this *interhyal* tract in the Batrachia Anura and Ophidia, where it never even chondrifies; in the Eel (*Anguilla*), where it is very small and indistinct as cartilage, and fades into a mere ligament; in the Osseous Fish (*Salmo salar*) and the Ganoid (*Accipenser sturio*), where it becomes an ossified rod of cartilage; and in the embryo of *Linota cannabina*, where it chondrifies after a time and *fuses together again* the incus and stylo-hyal.

own direction is backwards. Further back (fig. 4) the septum is increasing in height, and the retral processes of the trabecular horns are now much smaller; these slender laminæ I propose to call the "recurrent cartilages"*.

In the *fifth* and *sixth* sections (figs. 5, 6, 6ª) there is no longer any distinction between the rudimentary prenasal cartilage and the completely fused portion of the trabecular bar, the lower part of the septum nasi; but as this bud-like wedge of cartilage (see Second Stage, Plate XXX. fig. 1, *p.n.*) never becomes vertical, having its apex downwards as in the Chelonians, still less protruded forwards as in its large counterpart in the Bird, the bony plates that appear as its splints are not *superior* as in the Bird, nor *anterior* as in the Chelonian, but *inferior*. These plates are the premaxillaries, which appear in the Mammal below the snout (see also CALLENDER, Philosophical Transactions, 1869, p. 166, Plate XIV. fig. 6, *c*, for its inferior position in Man). Yet in the Mammal the premaxillaries are related, as splints, more to the retral trabecular cornua themselves than to the arrested azygous cartilage impacted between them. So much of the alinasal cartilages as are depicted in the enlarged figure (fig. 6ª, *al.n.*) is a separate segment—the "appendix alæ nasi." The next section (fig. 7) is behind the first third of the septum nasi, where the rudiments of the "hard palate" begin in front, the lips of which appear now on each of the padded bases of the septum, but are here far apart. The long cushion-like valvular mass in which the aliseptal folds (*al.s.*) end and dilate is the early form of the inferior turbinal, which is not so sharply separate from the alinasal turbinal (*al.tb.*) as in the Bird. A sharp process of mucous membrane is seen above this on each side, where the aliseptal cartilages bend inward; this is the rudiment of the "nasal turbinal" (see HUXLEY, 'Elem. Comp. Anat.' p. 248), which is but feebly developed in the Pig. In the thick mass which envelops the base of the septum two flat straps of cartilage are seen in section; these are the "recurrent laminæ" (*r.c.c*), and they are continued in this stage back to that part of the septum which, ossifying earlier than that in front, gets the name of "perpendicular ethmoid" (see fig. 11). On each side of the lower palatal lip there is a rudimentary tooth-pulp shown in section; and above this, up to the nasal roof, the tissue is marked off from the skin and subcutaneous tissue; this is the granular nidus for the posterior margin of the premaxillary and the anterior margin of the maxillary. The detached piece of this section (fig. 7ª) is the fore end of the lower jaw with tooth-pulps appearing, and with a curious result of the great prognathism of the type, namely, complete fusion of the ends of the primordial mandibular rods—"MECKEL'S cartilages (*mk.c.*)." A section made near the middle of the septum (fig. 8), although answering on the whole to the 7th, shows the tip of the vomer (*v.*) and a very near approach of the lips of the hard palate, and, below (fig. 8ª), the convergence of the mandibular rods and the fore end of a bony tract outside them; this is the dentary (*d.*); the tongue (*tg.*) is here cut across.

The *ninth* section (fig. 9) takes in part of the frontal wall, with the foremost part of

* The "recurrent cartilages" are of great morphological importance; in future communications I hope to show their form and meaning in the Ophidians and in Birds, "Passerines," "Hemipods," the *Rhea*, &c.

the membranous cranium; here we shall see the "recurrent laminæ" (*rc.c.*) on each side of the vomer (*v.*), and a still nearer approach of the edges of the under palatal floor. The *tenth* section (fig. 10) is through the most projecting part of the hemispheres, but in front of the olfactory crus; here, above the inferior turbinal, folds of cartilage are appearing in the roof, the foremost part of the upper turbinal (*u.tb.*). Here the septum, now to be called the perpendicular ethmoid (*p.e.*), is at its highest, and we are now behind the recurrent processes of cartilage. At this part the palatal bands meet each other, and in them a new bony centre has appeared, the maxillary (*mx.p.*); like the premaxillary, it begins below. The next section (fig. 11) is through the widening ethmoidal region and the partly separated olfactory crura (1), as well as through the hemispheres (C 1ª). The *twelfth* section (fig. 12) is through the cavity of the hemisphere, which is being cut off from that of the olfactory crus (1): it is immediately in front of the eye, the anterior (inner) canthus being cut through. This section is of great interest; here we are behind the aliethmoidal cartilages above, and the olfactory bulbs rest upon the soft mat through which their fibres root down into the nasal cavity. The walls, both outer and middle, reach further backward than the roof of the nasal sac; and in the middle wall we see the ribbed condition caused by thickening at the junction of the trabecular crests with the double keel sent down by the nasal alæ. Here the middle turbinal arises above the inturned nasal wall which ends the inferior turbinal (*m.tb.*, *i.tb.*). This section is behind the new maxillary centre, but shows a large tooth-rudiment on each side of the conjugating palatal flaps; the antagonist teeth appear in rudiment below on each side of the severed tongue (*tg.*), above the growing dentary (*d.*), whilst inside the dentary is the Meckelian rod (*mk.*).

The next section (fig. 13) is through the anterior third of the eyeballs (*e.*) and the middle of the hemispheres (C 1ª), and *behind* the perpendicular ethmoid; here the rest of the septum is almost, if not entirely, of trabecular origin; the section is in front of the junction of the orbito-sphenoidal cartilages (*o.s.*) with the basal median part. But if the mesoethmoid is dying out here, the nasal wall (*n.w.*) continues much further backward, bordering the greater part of the so-called presphenoid (*p.s.*). We are now behind the upper and lower turbinal regions, and here the "middle turbinal" (*m.tb.*) is near its extremity; one interspace is cut through. This narrow, subcranial, presphenoidal part of the nasal labyrinth, running so far backwards parallel with the trabecular plate, is the "sphenoidal sinus;" and if any osseous centres were to form in the wall of this narrow region, they would be, as in Man, the "bones of Bertin," the hindermost of the ossifications of the olfactory sacs. Bound strongly beneath the basi-facial wall is the granular nidus of the vomer (*v.*), kidney-shaped in section; and beneath it the posterior nares are imperfectly floored in this the region of the palatal bone (*pa.*), which is a centre just commencing in granular indifferent tissue less solid and clear than that of the vomer. Below the mouth this section differs from the last in that the dentary (*d.*) is thicker and lies closer to the mandibular rod (*mk.*).

The *fourteenth* section (figs. 14 & 14ª) is through the middle of the eyeball, and

gives us the first intimation of the frontal (*f.*) outside the orbito-sphenoid (*o.s.*). The end of the middle turbinal is seen from behind in the sphenoidal sinus (*sph.s.*); at this part the nidus of the vomer is most solid, and comes nearest to hyaline cartilage (fig. 14ᵃ, v.), a state of things first pointed out to me by my friend Mr. Chas. Stewart.

The proper territories of each investing bone in the Pig evidently only want *time* that they might all become true cartilage; ossification sets in too soon for the formation of the intercellular substance, but each tract, before ossification, is a true morphological element or organ, as much so as the cartilaginous "operculars" and "branchiostegals" of a Shark or the "labial cartilages" of a Myxinoid. In illustration of these remarks I have now to mention a fact new both to Professor Huxley and myself, namely, that the substance which ossifies to become the dentary (figs. 14 & 14ᵃ, *d.*) becomes for the most part very typical solid colourless cartilage, as much so as Meckel's cartilage, which it invests: I shall show this more fully in the next stage.

The *fifteenth* section (Plate XXXII. fig. 1) is through the fore edge of the optic foramen; and here we see the nasal wall (*n.w.*) closing in upon the presphenoid (*p.s.*), and joining the end of the sphenoidal sinuses. Part of each optic nerve (2) is seen in this section, and for that reason the orbito-sphenoid appears distant from the median cartilage below; its continuity is seen in the vertical section (Plate XXXIII. fig. 4). Here the frontal (*f.*) is growing down towards the orbit, to which it will form a ceiling.

The *sixteenth* section (Plate XXXII. fig. 2) is through the largest part of the hemispheres and the underlying thalamencephalon; the eye is cut through near its posterior canthus, and the optic chiasma is severed (2). This section is through the lowest part of the presphenoid, which is still invested below by the vomer (*v.*); and opposite the section of the hindermost part of the ascending palatine plate we see the fore part of the *cartilaginous* "external pterygoid plates" (*e.pg*). The orbito-sphenoids (*o.s.*) are here at their greatest size, creeping far up the cranial wall and protecting the swelling hemispheres; the section through the dentary (*d.*) is close in front of the "coronoid process."

The *seventeenth* section (Plate XXXII. fig. 3) is through the large orbito-sphenoidal leaves, where they join the presphenoid behind the optic foramen (see also Plate XXXIII. fig. 4, *o.s.*, *ps.*, 2). This is the last section which shows the vomer (*v.*), and here the razor passed through the soft, faintly ossified "internal pterygoid plate"—pterygoid proper. The fore-turned external pterygoid plates (*e.pg.*) are here thick massive cartilages; and here, also, both the primary (*mk.*) and secondary (*ar.*) elements of the mandible are composed entirely of hyaline cartilage; this part of the permanent lower jaw is the coronoid and fore part of the articular regions. This section through the posterior part of the palato-pterygoid bar is of great interest, as it gives the direction taken by the apex of the second facial bar, namely upwards and outwards, although the *upward* bend is less in the Pig than in many Mammals; it has its fullest development in that small Ruminant, *Tragulus javanicus*.

In the *eighteenth* section the basisphenoid and its alæ (Plate XXXII. fig. 4, *b.s.*,

al.s.) are cut through obliquely, so as to show only the *floor* part of the latter; and the cartilage beneath the pituitary body is made to appear thicker than it is in reality (see Plate XXXIII. figs. 2 & 3, *py.*). On each side of the pituitary body the internal carotids are seen passing to the "circle of WILLIS," and outside these the Gasserian ganglia. Overlapping the whole are the orbito-sphenoids (*o.s.*); and on each side of the flaps of the soft palate (*s.pa.*) MECKEL'S cartilages are severed; and lower down the ceratohyals (*c.hy.*), thyrohyals (*t.hy.*), and larynx (*lx.*) are shown.

The *nineteenth* section (Plate XXXII. fig. 5) is drawn a little more than half across, and on a larger scale. The curve of the cranio-facial axis makes this and the succeeding section very oblique; and in this figure the basilar artery (*b.a.*) is tilted to show the bulbous end of the notochord (*nc.*): this thickish section is viewed from behind. The notochord ends now in the region of the future "spheno-occipital synchondrosis;" this narrow part of the basilar plate or investing mass is subcarinate. The razor has passed through the cochlea (*cl.*) parallel to the plane of its coils; over this part of the auditory sac, which is scooped above at the side, lies the great Gasserian ganglion (5); and inside and above this the fore edge of the large superoccipital lamina (*s.o.*) is seen, severed close to the edge, so that there is here a discontinuity between it and the ear-sac, the reason of which is shown in the *inner* lateral view (Plate XXXIII. fig. 3, *au.*, *s.o.*), where a large rounded notch is seen in front of the occipital cartilage, bulged out at this part by the lateral sinus. At some little distance from the cochlea the first postoral (now a veritable "malleus") is severed through its head, neck, and shoulders; the *head* is now flattened: this section shows the thin edge of this manubrium, the thicker part being cut through in the next section (fig. 6, *mb.*). This view (fig. 5, *m.t.*) well shows the imbedding of the manubrium in the membrana tympani, and the inner and outer regions of the first postoral cleft, thus divided by the head of the bar.

The *twentieth* section (Plate XXXII. fig. 6) is a little oblique from side to side; it is thus practically double; the left hand shows parts in front of those displayed on the right. Here, on the left, the manubrium mallei (*mb.*) is cut through at its thickest, posterior part, and its solid shoulder (*ml.*) is seen, the part which articulates with the incus. The incus (*i.*) is seen on the right side, hiding the manubrium partly, and having its "short crus" cut away: the figure shows the *back* of the thickish section. On the right side the section is behind the notch on the base of the fore edge of the superoccipital (*s.o.*). Below and outside the tympanic cavity (*t.c.*) the stylohyal (*st.h.*) is seen in oblique section as it passes downwards and forwards, and mesiad of this there is the large jugular vein (*j.v.*) with a coiled radicle. The notochord (*nc.*) is very clearly seen in the substance of the basioccipital cartilage. On the right side the stapedial bud is seen projecting from the auditory capsule just above the section of the promontory (*st.*, *pr.*), and to this the orbicular "capitulum" of the second postoral is applying itself limpet-like. The *twenty-first* section (fig. 7) is from behind the left side of the last (reversed), and shows on the whole what is displayed on the right; in both (figs. 6 & 7) we see the opening of the "aqueduct of the vestibule," and in this the "aqueduct of Fallopius" containing the

portio dura (7^a), and mesiad of the jugular vein the glossopharyngeal nerve (8^a) is cut through; outside the nerve, and below the stylohyal (*st.h.*), a part of the exoccipital (*e.o.*) appears. A similar section to the other side is also shown as a front view (fig. 8); and here the relation of the portio dura (7^a) to the hyoid arch is well seen. This figure does not show the incus, which is removed to display a new segment (*i.hy.*) that has arisen, the counterpart of the Sauropsidian "infrastapedial" (H.) and of the so-called "stylohyal" of the Fish (Cuv.); here it will be called the "interhyal" (P.), as it really wants a proper name. In the second stage (Plate XXX. fig. 6) I have displayed the relation of the segmenting second arch to the very Batrachian stapes (see "Frog's Skull," Plate VII. figs. 12–16, *st.*); and on the same Plate (fig. 8) I have put for comparison the state of things in the third stage; both are *front* views, and the apex of the stapes is towards the eye. The bigeminal papulæ on the younger stapes are now connected by a bridge of cartilage, and the lateral dimples are the *foot-hole* of the little stirrup.

The round head of the short crus of the incus is seen articulating with the tegmen tympani (Plate XXX. fig. 8, *t.ty.*, *i.*); and *below* and *behind* it is the clavate head of the stylohyal, which is ready to coalesce with the periotic cartilage at the junction of the epiotic and opisthotic regions (see Plate XXXVI. fig. 2, *i.*, *st.h.*, *op.*, *ep.*). The descent of the dislocated hinder half of the shoulder of this second arch is not so great as in the Osseous Fish ("Salmon's Skull," Plate VI. fig. 2), but it is considerable; and this is a true third stage, as may be seen by comparing fig. 9, *hy.*, in Plate XXIX. with figs. 6 & 8, Plate XXX. The binding, intervening band of new indifferent tissue which has grown in the gap of these divided parts has acquired a hardening nucleus of new cartilage, exactly as we see it in Ganoid and Osseous Fishes, e. g. *Accipenser*, *Anguilla*, *Salmo*. The portio dura (7^a) is seen passing down its aqueduct behind these segments, and the upturned, inbent, long crus or neck of the rib-like bar ends in an elegant sucker-shaped disk, its capitulum or apex*.

On the same Plate the postoral arches of the third stage are shown in a side view (Plate XXX. fig. 9); and a comparison of the undivided mandibular bar with the displaced fragments of the hyoid will make things plain to the mind. The apices of the two bars come very near together; but whilst the first hooks backwards, downwards, and inwards, it does not graft itself upon the auditory sac; nor does its shoulder send backwards a secondary pedicle so distinct as the "short crus of the incus." Moreover the shaft of the bar on the first arch keeps on its way normally, as in the early embryo; but in the second arch this part has been segmented off, and displaced backwards and downwards, catching in its descent at the *neck* and *head* of the arch, but travelling still further in more advanced stages, until it rests and combines with the postero-inferior angle of the auditory mass. In the new web which grows between the two segments comes the secondary "interhyal" segment; this, however, loses all its first relations, and finally

* In both these figures, put together for comparison, the parts of the second arch are coloured, and those of the auditory capsule are plain, for the sake of distinction, that the *eye* may learn to separate the "stapes" from the segments of the hyoid arch.

coalesces with the neck of the projecting stapes (Plate XXXVI. figs. 2 & 3, *i.hy.*, *st.*), and is half lost, at last, in the tendon of the "stapedius" muscle. The "cornu minor" (*c.hy.*) of Man is here seen (Plate XXX. fig. 9, *c.hy.*) below, articulating with the keystone of the arrested third postoral. This lesser horn answers to the "hypohyal" (P.) of the Osseous Fish. The *twenty-second* section, a front view (Plate XXXIII. fig. 1), is through the three semicircular canals; the anterior canal (its arch) is above, near the superoccipital cartilage (*a.sc.*, *s.o.*), whilst the horizontal canal (*h.s.c.*) is laid bare at its crown and where its non-ampullar end enters the vestibule (*vb.*). The posterior canal has the base of its ampulla laid open, and half the arch is seen shining through the cartilage; the lateral cerebellar recess (*l.c.r.*) is seen above the aqueductus vestibuli (*aq.v.*). The space between the auditory mass and the basi- and exoccipitals is the posterior foramen lacerum (*f.l.p.*; see also fig. 3); the basilar artery lies on the basioccipital, and the notochord is within it (*b.o.*, *b.a.*, *nc.*).

From a series of sections taken horizontally in the nasal region, but which were cut through the investing mass at a right angle, or nearly so, I have selected two as of most importance in this demonstration. The first of these (Plate XXXII. fig. 9) is a front view, and shows the left mandibular rod coming forwards and downwards, its manubrium being buried in the tissue behind. The portio mollis (7^a) is seen lying at the entrance of the meatus internus. The anterior (superior) canal (*a.s.c.*) is cut through near its ampulla, and the cochlea (*cl.*) is divided at right angles to the plane of its coils.

On the other side (left of the figure, right side of the head) the portio mollis (7^a) is seen streaming into the labyrinth, and the portio dura (7^e) is cut through close to the tegmen tympani (*t.ty.*). The crown of the anterior canal is here cut through, and the horizontal canal near its ampulla; here the "tegmen" is seen at its high anterior part, and the "short crus" of the incus lies in front of its descending portion, where the overlying horizontal canal dips before it turns inwards to enter the vestibule. The "long crus" of the incus (*i.*) is shown hooking upwards, and expanding into its orbicular portion on the stapes (*st.*), the apex of which has been cut away, exposing the hole. The head of the stylohyal (*st.h.*) is cut through, and in the angle between it and the long crus there is a large pisiform "interhyal" (*i.h.y.*). The basioccipital has an irregularly pentagonal section, shows the notochord in its centre, and is very distinct from the auditory mass: this distinction is very clear and persistent in the Mammalia.

The plane from which the last section was taken being sliced again, yielded what I have depicted in fig. 10: this is a *back* view, and the left side of the figure corresponds to the left side of the head. On the right side the occipital arch (*s.o.*) appears almost to its crown; on the left its fore edge is just missed.

The left side of this figure corresponds very nearly to the left of the last, but the razor has passed close behind the auditory nerve and through the promontory, where its cartilage passes as a narrow band between the fenestræ (*f. ovalis* and *f. rotunda*), a tract which receives osseous matter from the "opisthotic" centre. Here the base of the stapes is towards the eye, and half of it is seen through the cochlear wall (promontory): the rest is as in the last figure.

But the section of the right side, just a degree further back, is most instructive. The large superoccipital cartilage is seen embracing and walling in the great sinus (*s.o.*, *Ls.*), and the periotic mass is severed at the junction of the anterior and posterior canals, so that the tube opened here is the "common canal" (*c.c.*). The posterior non-ampullar half of the horizontal canal (*h.s.c.*) is opened over the hinder part of the tegmen tympani. The promontory is cut through at the fenestra rotunda (*pr.*, *f.r.*), and outside this is seen the head of the stylohyal; all this long sinuous rod (*st.h.*) is exposed, and also the short ceratohyal (*c.hy.*) at its base, where it is seen articulating with the three rudiments of the next arch (base and larger cornua of hyoid of Man).

Reconstruction of the skull at this third stage from the foregoing materials will be rendered easier by light obtained from longitudinally vertical sections (Plate XXXIII. figs. 2 & 3). In these the first is made, in the facial region, a little to the left of the mid line, so as to give the left face of the septum of the nose; in the next (fig. 3) the septum is cut away, and the left face of the right turbinals exposed. In the first (fig. 2), the brain is sketched in outline *in situ*; in fig. 3 it has been removed to display the inner wall of the cranium. The notochord (*nc.*) has retired from the posterior clinoid wall, and has been buried in cartilage; it still lies, however, nearest the upper surface of the investing mass. This may be compared with the like view in the first stage (Plate XXVIII. fig. 6). There the basifacial axis scarcely made a right angle with the basicranial; here these parts meet at an angle larger by one half: there the notochord mounted above the investing mass; here it has retired, and lies below the clinoid wall. The gelatinous space called by RATHKE the "middle trabecula" is gone, and the reduplicated lining membrane of the cranium has formed the "tentorum cerebelli" (fig. 2, *t.cb.*). The huge expansion of the hemispheres (C 1[a]) has hidden the middle vesicle (C 2), as seen from the outside. Behind the large cerebellum (C 3) the occipital cartilage (*s.o.*) is seen in section; and below the rounded margin of the basioccipital (*b.o.*) is shown, the tract becoming thinner forwards, and then much thicker close to the pituitary body (*py.*); it ends above in the overlapping "posterior clinoid wall" (*p.cl.*). The pituitary depression is not saddle-like, but is a deep cup, floored by a good plate of cartilage. The anterior clinoid wall (*a.cl.*) is rounded, and belongs to the presphenoid; the depression in front of it is for the optic chiasma. The median plate rises gently in front of the optic depression, and this higher part for a short distance belongs to the anterior sphenoidal territory: it is formed principally by the trabecular commissure and crests. The rest of the plate belongs to the perpendicular ethmoid and septum nasi; the latter is the longest region and the former the highest. The lateral ethmoid (*al.s.*) is scarcely seen in this view (fig. 2). Below the pituitary body the Eustachian opening (*eu.*) is seen, in the root of the tongue the ceratohyal (*tg.*, *c.hy.*), and in the substance of the lower jaw the commissure of MECKEL'S cartilages (*mk.*).

These things and some others are seen in the next figure (fig. 3). Outside the foramen magnum (*f.m.*) is seen the occipital condyle (*o.c.*); in front of this the "anterior

condyloid foramen" (9), and then the "foramen lacerum posterius" (8). Crest-like, above the foramen magnum and auditory mass, is the superoccipital cartilage (*s.o.*), ending in front in a sinuous manner, being notched and bulged out by the lateral sinus (*l.c.*). The ear-sac (*au.*) is an ovoidal flattened body, lying obliquely outwards and backwards, with its bored and scooped face on the inner side. The blind recess under the arch of the anterior canal is for the cerebellar process; the antero-inferior spaces are for the compound seventh nerve; the meatus internus has a small cartilaginous bridge in front of it, which passes upwards inside the canal for the portio dura. Between the ear-sac and the small thick alisphenoid (*al.s.*), there is a large shallow fossa for the Gasserian ganglion (5), and the space for the main part of the fifth nerve is merely the great chink between these two parts—the alisphenoid and the ear-sac. Hence in the Pig we see no "foramen ovale," and the "f. rotundum" has no distinctness from the chink between the orbito- and alisphenoids. Most of the alisphenoid is spent in forming the large "external pterygoid process," and its cranial part is small; here it is not the "ala major," as in Man. On the other hand, the "alæ minores" of Man are represented in the Pig by huge wings of cartilage, that spread themselves from the nasal to the auditory regions. This reversal in size of the anterior and posterior wings is like what we see in the Lizard, &c.,—unlike the Bird's skull in this respect, where the orbito-sphenoids are aborted, the alisphenoids huge. As in the Lizard, the Mammalian orbito-sphenoid has a postneural band, which encloses the optic nerve (2) in a complete foramen: this is well developed in the Pig (Plate XXX. fig. 3, *o.s.* 2). In front of the optic nerve the base of this orbital wing is continuous with the trabecular commissure for some extent; the greater part of the so-called presphenoid is, however, trabecular in nature. The olfactory roof and wall extends backwards behind the septum, which graduates into the presphenoid; thus a large rounded notch exists on each side, and the roof of the true olfactory region and floor of the rhinencephalon is soft; through this delicate tissue the olfactory filaments root downwards to the rudimentary upper and middle turbinal septa (*u.tb.*, *m.tb.*). Between the nerve-fibrils cartilage is beginning to appear, and thus a cribriform plate will be formed of secondary cartilage (fig. 3, *cr.p.*). In front of the upper turbinal a rudimentary "nasal turbinal" (*n.tb.*) is formed by bending inwards of the aliseptal cartilage. Lower down this cartilage turns inwards, and develops into the inferior or *anterior* turbinal (*i.tb.*), attached to which in front is the small alinasal turbinal (*al.tb.*).

Fourth Stage.—Embryo of the Pig, from 2 *inches* 4 *lines to* 2 *inches* 6 *lines in length.*

From dissections and sections of embryos not larger than the grub of the honey-bee in the first, we come in this stage to specimens as large as a mouse.

This is an excellent stage for morphological comparison, as the skull may well be placed side by side with that of the adult Osseous and Ganoid Fish, Amphibian and Reptile, and with that of the ripe chick of the Common Fowl. It also corresponds very closely in development with an early stage of the skull of *Balæna japonica*, Lac., excel-

lently illustrated by the late Dr. ESCHRICHT ("Ni Tavler til Oplysning af Hvaldyrenes Bygning, udferte til utrykte Foredrag af afdede Etatsraad Dr. D. F. ESCHRICHT:" Copenhagen, 1869. Edited by Professor REINHARDT, plate ii. figs. 1–3).

The well-marked granular territories that at first invested the primordial skull and face are now largely ossified, and these ossifications are massive in relation to so small a skull. As in the strong-legged "Herbivora" generally, and in the "Aves præcoces," the development before birth and before hatching is very rapid, so that they are strong and in good liking at their first appearance. Moreover, the primordial parts are undergoing endostosis at many points, and from this time the bony metamorphosis takes place very rapidly. If this skull (Plate XXXIV. figs. 1–7) be compared with that of the Bird ("Fowl's Skull," Plate LXXXIV.–LXXXVII.), it will be seen that the premaxillaries (*px.*) do not reach to the end of the snout, instead of projecting beyond it, and they do not send a nasal process between the nasal bones up to the frontal. Here, in the Mammal, the maxillary is by far the largest bone, and, with the linking malar and zygomatic spur of the squamosal, forms a strong subocular arch, one pier of which is formed by the maxillary and reaches near to the nostril, whilst the other pier is formed by the supratemporal and stretches over the auditory capsule to the occiput (Plate XXXIV. fig. 1, *mx.*, *j.*, *z.sq.*). This sigmoid, trilobate temporal (squamosal) bone, besides creeping over the infero-lateral wall of the cranium by its squamous part, clamping the outer wall of the ear-capsule by a long falcate process, and perfecting the great facial yoke (zygoma), also takes in a new relation; it articulates with a well-differentiated *secondary* mandible (*d*). This is distinctively Mammalian; for in the highest Sauropsida (the Bird) the primordial and secondary mandibles have an equal development, and are permanently combined as the free arch of the mandible, the large "pier" of which is merely the hugely developed head, neck, and shoulder of the first mandibular rod. In this stage of the Mammalian skull we catch the equivalence of these primary and investing parts; but the new hinge is formed already, and the primary bar, now at its highest relative development, shows no sign of segmentation into a pier (quadrate) and a free arch (articulo-Meckelian). By the time of birth, the whole of the large succulent rod of cartilage which runs along the inside of the lower jaw (fig. 7, *d.*, *mk.*, *m.*) and coalesces largely with its fellow in front will have shrunk up into a delicate fibrous band, leaving a small bony style (processus gracilis) to the arrested upper part of the rod*. A bony ring is growing round both the preoral and the first postoral clefts; these are the lacrymal (*l.*) and the tympanic (*ty.*); the first of these has an outer facial development, and is not hidden in the orbit as in Man. The nasal, frontal, and parietal bones (*n.*, *f.*, *p.*) form a regular double series; they are only equivalent to the *inner layer* of the scutes seen in the same region in Ganoid Fishes; yet they are very thick, the thickness depending upon the free development of connective (indifferent) tissue between the cutis and the primordial skull. The fontanelles are still wide open; but

* In the figure (Plate XXXIV. fig. 7, *d.mk.*) the primary rod is cut through, and the mandible detached from its new hinge: these parts will be described more in detail from the figures of sections.

the lower edge of the frontal has sent inwards from its eave a plate which reaches the orbito-sphenoid—the orbital plate. The agreements and the differences seen by comparing the Ornithic and Mammalian skulls are made very evident if this palatal view (Plate XXXIV. fig. 2) be put side by side with the figure of this region in the Ostrich's embryo ("Ostrich's Skull," Phil. Trans. 1866, Plate VII. fig. 4); at this stage the conformity is more remarkable than the difference.

The dental (*d.px.*) part of the pig's premaxillary is broad and filled with tooth-sacs, which deeply groove it; the palatal processes (*pp.x.*) are slender. The approximating maxillaries (*mx.*) do not hide the vomer (*v.*); they are grooved by vessels down the middle of their palatine plate, whilst their dentary portion is hollow and shell-like, containing as it does large growing tooth-germs. The palatines (*pa.*) are *ornithic*, scarcely showing so much of the "hard palate" as a Green Turtle (*Chelone mydas*). The pterygoids (*pg.*) are thin in their ascending part, and are clubbed hooks below; they and the palatines both articulate with the great conjugational "basipterygoid," which here, as in the Ophidia, mainly arises from the alisphenoid; it is, however, formed of true cartilage, as in all the Sauropsida in which it occurs. This part, the "external pterygoid plate" (*e.pg.*), is a pronotochordal secondary structure; it arises at its root from the side of the apex of the trabecula. These apices of this first pair of bars do not project outwards and backwards in the Pig as in the Kitten, nor does the "basitemporal" appear here in rudiment as the "lingula sphenoidalis;" both these, the process and the bone, are exquisitely and most instructively displayed in the Guinea-pig (*Cavia aperea*). The ring on which the tympanic diaphragm is stretched (*ty.*) is at present U-shaped, with its crura pointing backwards, and the larger on the outer and *upper* side; this crus has a flat flange which looks upwards. The vomer has the same relative size as in the embryos of the Ostrich and the Whale ("Ostrich's Skull," Plate VII. fig. 4, *v*, and ESCHRICHT "On the Cetacea," plate ii. fig. 2, V.).

In the endoskeletal parts we have to deal with two tissues at once, hyaline cartilage and bone, principally endosteal at present, although rapidly gaining the surface and beginning to affect the perichondrium; I shall describe it first in the dissections and then in the sections. In the side view (Plate XXXIV. fig. 1) the tracts that are hardening in the arch of the occiput are shown; and of these there are five, namely the superoccipital and *two pairs* of exoccipitals (see also fig. 3). Moreover the superoccipital is double, as may be seen in a younger specimen (fig. 4, *s.o.*), but the two patches run into each other in a day or so. The ossification of the exoccipital is remarkable; for within the substance of the massive condyle an epiphysial centre appears, quite distinct at first from the large rambling growth above (figs. 1 & 3, *e.o.*); these two points soon coalesce. The basioccipital (*b.o.*) is best studied in a sectional view (fig. 5), but its form is seen from above and below (figs. 6 & 2); it is spearhead-shaped in outline and thick as to substance; it is fast obliterating the notochord. The newer cartilage which underfloors the pituitary body is rapidly ossifying as basisphenoid (fig. 5, *b.s.*); the form of this centre is seen from below in fig. 2: this is the only bone at present in the posterior

sphenoid. The real harmony between the outstanding bars on each side this bone and the basipterygoid spurs of the Lizard and Ostrich is here clearly shown; whilst the "external pterygoid plate" was only studied in Man (where it is said to be merely a periosteal outgrowth of the "ala major"), its homology with the "Sauropsidan" bar could not be determined; here it is a direct cartilaginous outgrowth of both *base* and *wing*, and its basal origin is from the side of the trabecular apex. Here it articulates with both palatine and pterygoid, being so huge and developed to so great an extent laterally; there (see "Ostrich's Skull," Plate VII. fig. 4, *pg.*, *a.p.*) the pterygoid is wedged in bodily between the basipterygoid spur and the palatine; it is in that memoir called the "anterior pterygoid process" (*a.p.*), and by Professor HUXLEY "basipterygoid" ("On the Classification of Birds," Proc. Zool. Soc. April 11th, 1867, p. 418). There is no special ossification in the confluent trabecular base beneath the orbito-sphenoids (figs. 2 & 5, *p.s.*), and these large wings have no centre over and behind the optic foramen, as in Man (figs. 1, 5, 6, *o.s.*, 2); from these only the whole mass will be leavened. These wings have now coalesced with the lateral ethmoid (*al.e.*) in front, and overlap the auditory capsule (*au.*) behind, exactly as in ESCHRICHT'S figure of the embryo of *Balæna japonica* (*op. cit.* plate ii. figs. 1 & 2, *k*, *mG*). The orbito-sphenoids are now at their highest degree of development (see in third stage, Plate XXXIII. fig. 3, *o.s.*, and in the sixth, Plate XXXV. figs. 1, 3, 4, *o.s.*). In front of these orbito-sphenoidal nuclei there is no endosteal deposit, nor is there any for some time to come in the ear-sacs (*au.*). The bird's-eye view (fig. 6) shows how far, as in the Bird, the great septum of the nasal sacs ("mesoethmoid," continuous perpendicular plate, and septum nasi) continues backward beyond the primary roof (here compare fig. 6 with primordial struthious skull, *op. cit.* Plate VII. fig. 1, *al.e.*, *cr.g.*, *o.s.*). The cribriform plate is now sufficiently advanced on each side of the retral septum of the sacs to be fairly understood; it is a delicate comb-shaped lamina of *secondary* cartilage, with four long "teeth" growing inwards and forwards from its margin or "back;" the long interspaces admit the olfactory filaments. The common outer band does not fill in all the space which forms the floor to these huge rhinencephalic fossæ, but, as in the embryo of the Ostrich and Fowl ("Fowl's Skull," Plate LXXXI. fig. 4, *eth.*), the septum is continued backwards to the verge of the anterior sphenoid, and here, in the Pig, ends in a club-shaped manner.

Between the anterior edge of the orbito-sphenoid (*o.s.*) and the back of the comb-like lamina (figs. 5 & 6, *cr.l.*) there is a considerable membranous space. The bulgings in the olfactory roof (*al.e.*, *al.s.*) are caused, behind, by the upper and middle turbinals, now increasing in complexity, and further forwards by the inferior turbinals. Behind the postneural commissure of the orbito-sphenoid (figs. 5 & 6, *o.s.*) the alisphenoids are obscurely seen (*al.s.*), overshadowed and obscured by the so-called "alæ minores." They have no foramina in their substance, but the cranial nerves root down in front of and behind them; on the upper view the whole of the alæ and the floor of the "sella turcica" are far from the eye, the posterior clinoid wall (*p.cl.*), the end of the investing mass, cropping up high into the cranial cavity. The ear-sacs are seen from without,

within, above, and below (figs. 1, 2, 5, 6, *au.*), but as a mass they are not much changed from the last stage.

In the sectional view (fig. 5) the Eustachian tube (*eu.*) is seen below the basisphenoid (*b.s.*); below the soft palate the ceratohyal (*c.hy.*) is cut through, and along the inside of the lower jaw the primary mandible is seen. This is better shown when dissected out (fig. 7); and now its malleal end is ossified (this part is cut through in fig. 1), whilst, below, the Meckelian commissure is severed, the bars uniting along their anterior fourth. I can only find one osseous centre here in the mandible, the dentary (*d.*); this is found in the rapidly chondrifying nidus, which, like a huge "inferior labial," obliquely overlaps the primary mandible; in Man, according to CALLENDER, there is an osseous centre at the *chin* in MECKEL'S cartilage, and a second splint (splenial) on the inner face of the dentary (see Phil. Trans. 1869, Plate XIII. figs. 6 & 7, p. 170).

A few of the many sections prepared of this stage will now be described, and they will thoroughly explain the structure of the parts which have above been described mainly from *dissections*.

The *first* is through the snout (Plate XXXIV. fig. 8), and shows the arched cartilages, united together in front, which were formed by fusion of the alæ nasi with the overbent trabecular horns.

The *second* (Plate XXXIV. fig. 9) is through the alæ nasi (*al.n.*), fore part of septum (*s.n.*), alinasal turbinal (*al.tb.*); and the masses with trilobate outline below are the recurrent trabecular horns (*rc.c.*).

The *third* section (Plate XXXIV. fig. 10) shows a curious triradiate cartilage separated from the "alæ nasi;" this is the "appendix" (*a.al.n.*); here the trabecular cornua are becoming slenderer.

The *fourth* (Plate XXXIV. fig. 11) shows the same parts further back; here the recurrent process has become a smallish band lying flat on each side of the base of the septum, which is now becoming high, but has not commenced the inferior turbinal fold. The four-winged section, on each side, below the septum and recurrent cartilages is the severed premaxillary (*px.*, see also fig 2).

The *fifth* (Plate XXXIV. fig. 12) is through the middle of the inferior turbinal (*i.tb.*); the pedate section here shows the upper limb coiled on itself, but not the lower at present. The recurrent laminæ of the trabecular horns are running even past this point backwards; they are here vertical, and in close relation with the nidus of the scoop-shaped vomer (*v.*)*. In this section the outer stratum of granular tissue overlying the nasal canals is now ossified as the nasal bones (*n.*), and the mass of tissue overlying the pterygo-palatine bar has become the maxillary (*mx.*), with its deep dental groove and pulps and its palatine plate.

The *sixth* (Plate XXXIII. fig. 4) section is through the solid anterior third of the

* The relation of these recurrent developments of the trabecular horns to the splints that belong to the first facial arch is of intense interest; I am working out this subject in various groups; it is most complicated in Passerine Birds, "Ægithognathæ" (H.). They are evidently formed by the fusion of a "labial" with each trabecular horn. If we add to this the "appendix alæ nasi" and the secondary mandible, we get *three pairs* of suctorial cartilages in an ordinary Mammal.

frontals (*f.*), the true olfactory region, and the zygomatic process of the maxillary (*z.mx.*); above this is the overlapping malar or jugal (*j.*) The thick frontal slabs send inwards and downwards an "orbital plate," which clamps the ethmoidal wall; this wall is seen to be separated by a very large space from the fore top of the septum or perpendicular ethmoid (*p.e.*), on each side of which lie the olfactory crura (1), and above these the fore part of the hemispheres (C 1ª). Here we see that the comb-like floor of the olfactory crura (Plate XXXIV. figs. 5, 6, *cr.p.*) is connected with, and is the top of, a system of cartilaginous ingrowths, the upper and middle turbinals (*u.tb.*, *m.tb.*), which, by repeated splitting, as it were, or rather by a process of foliation, is becoming more complex day by day. This section is through the most solid part of the vomer (*v.*), where it is squared below to rest upon the "hard palate" over its median suture; here the palatines are cut through their fore part, where they are thin bony scoops, protecting the outside of the posterior nostril passage. Large tooth-pulps (*t.p.*) are seen above and below, and the lower are in relation to a large dentary groove, the outer wall of which is very massive and the inner a smaller rod: both of these sections are parts of a continuous dentary (*d.*); below the inner bony bar is the Meckelian rod (*mk.*), on each side of the base of the tongue (*tg.*).

The *seventh* section (Plate XXXIII. fig. 5) is through the fore part of the eyeballs (*e.*) and the sphenoidal sinuses (*sp.s.*), the hinder part of the backwardly projected nasal labyrinth. At this point the septum, "perpendicular ethmoid," ends; and the pyriform section seen here, at the posterior end of the large "rhinencephalic fossæ" (see Plate XXXIV. fig. 6, *ol.*, *cr.p.*), is no longer indebted to the inturned nasal roofs for its height, which is due to the upgrowth of the trabecular crests*. This section is through the most solid part of the palatines (*pa.*), and their interior edge is thickening and growing inwards ready to form their part of the hard palate. Only the malar (*j.*) is seen on the side and below, and mesiad of it is the coronoid process of the lower jaw (*cr.*). MECKEL'S cartilage (*mk.*) is now high up the inside of the jaw, which is here mainly composed of solid hyaline cartilage, the inner cells of which are rapidly proliferating as "osteoblasts." In the root of the tongue (*tg.*) the ceratohyals are seen articulating with the basi- and thyrohyals (*c.hy.*, *b.hy.*, *th.h.*), now one piece of cartilage.

The *eighth* section (Plate XXXIII. fig. 6) is one of the most instructive; it severs the orbito-sphenoids (*o.s.*) where they pass into the presphenoidal trabecular wall close at the back of the sphenoidal sinuses (see also in third stage, Plate XXXII. fig. 1). This is the high part immediately in front of the optic foramina (Plate XXXIV. figs. 1, 5, 6, ₃). The orbito-sphenoids are overlapped above by the frontals (*f.*), and the presphenoid has the end of the vomer (*v.*) beneath it. Here the thin ascending plate of the pterygoid, and its thick "hamular" part, is cut through; the osseous matter is scarcely continuous in the ascending part, and every now and then a separate "mesopterygoid" is developed.

* If the reader wishes to see an exact counterpart of this structure displayed in the second facial arch or "palato-pterygoid," it is ready at hand in the skull of the Pelican, where both the preoral arches form a long and solid "commissure," from which a high crest ascends.

The fore part of the curious thick leaf of cartilage which grows out of the ali- and basi-sphenoid is here cut through; it is the conjugating process between the first and second preoral arches. Here the zygomatic process of the squamosal (*z.sq.*) overlaps the jugal (*j.*), and here the cartilaginous part of the lower jaw is nearly at its thickest; in the root of the tongue the stylohyals are severed at their junction with the ceratohyals (*c.hy.*).

The *ninth* section (Plate XXXIII. fig. 7) has been made through the orbito-sphenoid (*o.s.*) close in front of the osseous nucleus (see Plate XXXIV. fig. 6, *o.s.*); it has passed down through the low part of the presphenoid, where it is crossed by the optic nerve, and where its territory ends and that of the basisphenoid (*b.s.*) begins.

Hence in this figure we have the alisphenoids (*al.s.*) cut through beneath the orbital wings. At this point the hooked coronoid process is severed at its apex (*cr.*); and outside this is the "squamosal" (*sq.*), with its articular cartilage and "meniscus." Below these the articular and angular part of the lower jaw is shown; it is one mass of hyaline cartilage. MECKEL'S cartilage and the stylohyal are also cut through.

Part of this section is shown from the *right* side (Plate XXXIII. fig. 8), magnified twice as much. Here the three cartilages that form the mandibular hinge are all secondary, and, like the outer ear, suggest a reversion to the labial and opercular cartilages of the Shark.

The *tenth* section (Plate XXXIII. fig. 9) is through the malleal portion of the first preoral arch (*ml.*), the front of the tegmen tympani (*t.ty.*), the long, overlapping, posterior angle of the orbito-sphenoid (*o.s.*), the cochlea (*cl.*), the stylohyal (*st.h*), and the "occipito-sphenoidal synchondrosis" with its enclosed notochord (*b.o.*, *nc.*).

The *eleventh* section (Plate XXXIII. fig. 10) displays the ampulla of the anterior canal (*a.sc.*), the general cavity of the labyrinth (*vb.*, *cl.*), and the tegmen tympani (*t.ty.*), with a bony eave formed by the squamosal (*sq.*) and roofing over the body of the incus (*i*). The incus is seen turning up its "long crus" and spreading its *orbicular* apex over the top of the stapes (*st.*), which has been cut through from top to bottom, exposing the *foot-hole*. Below the stapes is the cartilaginous wall of the promontory (*pr.*), and outside this is a continuation of the tympanic cavity, in the outer wall of which is the stylohyal (*ty.*, *st.h.*).

Fifth Stage.—Embryo Pig, 3 inches long.

This is merely introduced to show the ossification of the "alisphenoids" (Plate XXXIII. fig. 11, *al.s.*), which had not begun in the last, whilst in the next they are one solid mass of bone with the basisphenoid (*b.s.*). Here it will be seen that the posterior sphenoid is much simpler in the Pig than in Man; in the Sheep at this stage I find it simpler still, not being able to discover any median centre; but the two alisphenoids are to be seen becoming pointed below, and growing towards each other; here, then, there appears to be no median piece, and thus the postsphenoid is like the anterior region, in which the orbito-sphenoids themselves fill in the mid region with bone. The contrary takes place in the Rodents; and especially in the Guineapig (*Cavia aperea*)

are the sphenoidal structures complex. As in other Rodents, there is a large presphenoid as well as a basisphenoid; the alisphenoids are ossified from two centres on each side; long "lingular" pedicles are formed by the apices of the trabeculæ, and to these are articulated a pair of long, outstretched falcate bones, the evident counterparts of the "basitemporals" of the Bird. In this animal also the "external pterygoid processes" are *basipterygoids*, and the small pterygoid bone is attached to their apex. Even in the Ruminants these spurs are *basal* in their origin. (For the development of the human sphenoid, see Huxley, 'Elem. Comp. Anat.' p. 144.)

Sixth Stage.—Embryo Pigs, 6 inches long, measured from snout to ischium.

The head in this stage equals in size that of a Squirrel, but its bones are much more dense. The roof-bones (Plate XXXV. figs. 1–3) are now applied to each other edge to edge by sutures, and in certain places overlapping as squamæ. The "anterior fontanelle" (*fo.*) is still open, but is much lessened; the parietal and occipital bones now form a good "lambdoidal suture." The nasals, frontals, parietals, squamosals, lacrymals, premaxillaries, maxillaries, and malars (*n.*, *f.*, *p.*, *sq.*, *l.*, *px.*, *mx.*, *j.*) are all so far formed as to require but little change of size to fit them for their adult relationships. The palatal region (Plate XXXV. fig. 2) shows a great development of the secondary floor or "hard palate," the palatines themselves being now tied together at the mid line below. The ossification of the cartilaginous skull has advanced greatly; the superoccipital is a large, strong shell of bone, and it is at present separated by a larger tract of cartilage from the exoccipitals than in the last stage (*s.o.*, *e.o.*). The latter have now only one centre, for the epiphysis formed primarily in the substance of the condyle has coalesced with the outer deposit; this is now creeping far down into the substance of the long twisted paroccipital process (*p.oc.*). The basioccipital (*b.o.*) now reaches from the foramen magnum to the spheno-occipital synchondrosis; and in front of it (fig. 2) the basisphenoid is now a thick mass of bone (see also fig. 3, *b.o.*, *b.s.*). The presphenoidal region is hidden below by the vomer (fig. 2, *v.*), but in the section (fig. 3, *p.s.*) it is seen to be largely unossified. The alisphenoids (figs. 3 & 4, *al.s.*) are solidly anchylosed to the median piece; they are larger relatively to the orbito-sphenoids, but are still inferior in size and in place; they largely owe their size, laterally, to the external pterygoid processes (*e.pg.*), for their cranial region is small. The whole orbital wing is much more contracted, relatively (figs. 1, 3, 4, *o.s.*); it has become detached from the ethmoid, and is some distance from the auditory mass. The two centres are completely anchylosed at the mid line, and quite enring the optic passages (2); below (fig. 3), they are forming the presphenoid.

The remainder of the facial axis and nasal septum is one sheet of solid cartilage; and so also is the complex nasal labyrinth, now much more complex in its turbinal growths and cribriform plate (*u.tb.*, *cr.p.*).

From the intimate impaction of the auditory mass into the sides of the cranium, its osseous centres have caused much confusion; this has, however, been greatest in the

Oviparous Vertebrata, where the fusion of the periotic capsule with the skull proper is the greatest. Here, in the Pig, the bony deposits are formed much as in Man (see Huxley, 'Elem. Comp. Anat.' pp. 147–156). A description such as that quoted above serves almost equally well for both types. Looking at the inner face of the capsule (figs. 3 & 4), we see a creeping endosteal patch, which surrounds the "meatus internus," runs under the fore part or apex of the cochlea, and has found its way supero-posteriorly to the junction of the anterior and posterior canals (*a.sc.*, *p.sc.*). Seen from the outside (fig. 5, *pro.*) the same bony tract is seen above and in front of the "fenestra ovalis" (*f.ov.*), below which it forms a sudden hook-like bend, which turns forwards, passing into the tract seen on the inside of the cochlea: this is the "prootic" ossification.

On the inside (fig. 3) a small hook of bone is seen in front of the "foramen lacerum posterius" (8); this is a spur sent round the back of the capsule *from the outside*, and the plate of which it is a process is seen from that aspect (fig. 5, *op.*) covering the most bulbous part of the cochlea, the "promontory" (*pr.*). This is the "opisthotic bone;" it sends forwards and downwards a long hook, which binds behind the hook of the prootic, beneath the apex of the cochlea. Another process of the opisthotic runs between the fenestra ovalis and fenestra rotunda (*f.ov.*, *f.r.*) close in front of the head of the styloid cartilage (*st.h.*). *Above* the head of the styloid, and below the hinder end of the tegmen tympani, a smaller spatulate scute has appeared; this is the mastoid proper, or "epiotic." Behind this little bone the auditory mass is much contracted in the Pig, this part of the capsule being strongly clamped by the squamosal and impinged upon by the exoccipital (*e.o.*), where it gives off its "paroccipital process" (*p.oc.*). The "epiotic" centre will, moreover, ossify the true mastoid region; although it arises in a more forward position, it is less than that of Man (see Huxley, *op. cit.* p. 154, fig. 61, *ep.o.*). In the vertical section (fig. 11) the prootic and opisthotic centres are cut through, each at two places, the first (*pro.*) above the stapes (*st.*) and inside the cochlea (*cl.*), and the opisthotic appears below the stapes and in the substance of the promontory (*pr.*). On the outside the semicircular canals are seen (*a.sc.*, *h.sc.*, *p.sc.*) imbedded in solid cartilage. The structures of the "middle ear" have now acquired their almost full metamorphosis; these are enclosed in a large imperfect ring of bone, the tympanic (figs. 1 & 2, *ty.*). This bone is now becoming very thick, and its breadth has greatly increased; but as yet there is no bony meatus stretching outwards beyond the membrana tympani (*m.t.*). It will be seen in the lower view that there is an additional bone clinging to the inner edge of the tympanic; this wedge and two smaller ossicles which I shall describe in the next stage are the feeble counterparts of the auditory "bulla" of the "Felidæ" and their congeners*.

* On the subject of the auditory "bulla" of the Carnivora, see Professor W. H. Flower's very valuable paper "On the Value of the Characters of the Base of the Cranium in the Classification of the Carnivora" &c., Proc. Zool. Soc., Jan. 14, 1869, p. 4. Whilst Professor Huxley's 'Elements' was going through the press, I showed him the bulla of the new-born Lion's whelp, a thin spoon-like lamina of true hyaline cartilage growing outwards from the inferior edge of the opisthotic region. Soon after this, Professor Rolleston presented me

In the Pig the bullar ossifications are found in a very soft stroma of connective tissue, and not in thin cartilage; yet that stroma is connected with the outer and lower edge of the opisthotic and prootic regions; it is the membranous floor of this huge air-cell.

The "os bullæ" already developed is seen in section, in its fore part (fig. 11, *o.b.*), a little in front of the auditory capsule, at the edge of the lower limb of the tympanic (*ty.*). The parts of the facial system of cartilages entering into the structure of the middle ear are shown in figs. 1 & 12; the "manubrium mallei" (fig. 1, *mb.*) is now ossified, and so also is the incus (figs. 12, *i.*). The lingual part of the hyoid arch is continued upwards to the epiotic region (figs. 1 & 5, *st.h.*), and behind this flattened rod the portio dura nerve is seen escaping.

The little ceratohyal (*c.hy.*) turns backward to articulate with the basal piece, to which also is attached the thyrohyals (fig. 1, *b.h.*, *t.hy.*); the latter three parts belong to the "third postoral arch."

The ossification of the lower jaw (fig. 1, *d.*) is almost complete, the unossified cartilage being principally condylar.

A series of sections selected from a large number now remain to be described; they will more fully illustrate this stage.

The *first* of these (Plate XXXV. fig. 6) is through the anterior third of the "inferior turbinal;" the pedate lower edge of the "aliseptal" cartilage here is seen to be coiled inwards above and below, the common back of the two coils lying towards the septum (*i.tb.*, *s.n.*). The sudden inbend of the aliseptal lamina higher up is the rudimentary "nasal turbinal" (*n.tb.*); below the septum is the vomer (*v.*), and below this the palatine processes of the premaxillaries are seen (*p.px.*). The nasals (*n.*), the side of the premaxillaries, and maxillaries (*px.*, *mx.*) show very thick in the section. On each side of the vomer the "recurrent apices of the trabecular horns" are still present (*rc.c.*).

The *second* section (fig. 7) is through the complex upper and middle turbinal regions (*u.tb.*, *m.tb.*) and the high part of the perpendicular ethmoid (*p.e.*). The olfactory crura (1) are also cut through as they lie on the cribriform plate (*cr.p.*). This widest part of the true olfactory region is roofed in by very massive frontals (*f.*); the thin lower edge of these bones is the orbital plate. The vomer (*v.*) is here very deep; on each side of it the posterior nares are cut through, and these are protected by the long scoop-shaped processes of the palatines (*pa.*). A part of the maxillary is seen on each side of a large molar tooth with its pulp; and above, the outer piece of bone is the jugal (*j.*).

The *third* section (fig. 8) is through the low part of the perpendicular ethmoid and the end of the cribriform plate, where it overlies the middle turbinal (*m.tb.*) only. The palatines (*pa.*) are here at their fullest development, their scooped portion underlying the end of the nasal wall (*n.w.*), and their subvertical plate sending inwards the palatal part of the hard palate.

with the head of a new-born Hyrax; and in this I found a large bulla, ossified separately from the true tympanic "annulus," and evidently formed in a shell of true cartilage.

The *fourth* section (fig. 9) instructs us how the olfactory labyrinth ends, as the sphenoidal sinus, on each side of the presphenoid; and it is seen that the fore edge of the orbito-sphenoids wall in this region, and underprop the thin descending orbital plate of the thick arching frontals (*o.s.*, *f.*). On each side is the eye-socket, and below the presphenoid is the thin part of the vomer (*v.*); the palatines (*pa.*) are here cut through behind the hard palate, and opposite them a section of the malar bone and lower jaw is shown (*j.*, *d.*).

The *fifth* section (fig. 10) brings a number of bones into view; the presphenoidal (trabecular) cartilage is rapidly ossifying from the coalesced orbito-sphenoidal centres (*o.s.*, *p.s.*); beneath this is the vomer (*v.*); and on each side, protecting the hinder nostrils, we see the thin part of the palatine and pterygoid (*pa.*, *pg.*); outside these is the fore-bent wing of bone known as the "external pterygoid plate" (*e.pg.*).

The *sixth* section (fig. 11) is through the postero-external part of the external pterygoid (*e.pg.*); it binds strongly against the inner face of the articular ("glenoid") part of the many-spurred squamosal (*sq.*). Here the basisphenoid is cut through behind the "wings," and the internal carotids mount up here to reach the "circle of WILLIS;" here the two limbs of the tympanic, the "os bullæ," and the stylohyal (*ty.*, *o.b.*, *st.h.*) are cut through. The "glenoid hinge" is here with its meniscus, and the articular region of the lower jaw is still largely cartilaginous beyond the head of the articular surface.

The *seventh* section (fig. 12) is through the lower edge of the parietal (*p.*), and also through the upper and lower edges of the squamosal (*sq.*) above, where it binds upon the mastoid region, and below, where it flanks the long paroccipital process (*p.oc.*). The extremity of the sinuous, creeping tympanic cavity is here seen (*ty.*), and outside it the stylohyal and tympanic (*st.h.*, *ty.*) are severed; below these is a section of the paroccipital spur. Over the incus (*i.*) the horizontal canal is seen, and below the stylohyal the portio dura (7^a). Inside the upper and outer portion of the rambling prootic centre the portio mollis (7^b) is seen streaming in; the inner face of the cochlea has the prootic in its edge, and the outer and lower the opisthotic (*pro.*, *op.*). The basi-occipital (*b.o.*) is seen to have a subcrescentic form in section, and the cartilage at the edges of this elegant "basilar plate" will have its outer edge ossified by the exoccipitals.

Seventh Stage.—New-born Pigs.

Since the last stage the skull has almost doubled its length, and the process of ossification has gone on very rapidly; moreover the form of the entire skull has become much more specialized.

Wishing to limit the illustrations to this paper, I have given but few figures of the preparations made for my own research; but the head at this stage is most easily obtained by those interested in these studies. Moreover the semi-adult condition of the skull, prior to any extensive anchylosis, will be fully illustrated. Besides the finish given to the general form of the skull by the now complete investing bones, the endoskeletal parts are well ossified. The whole of the occipital arch and its paroccipital processes,

and the anterior and posterior sphenoids, are now thoroughly hardened. The "sphenoccipital synchondrosis" is of small extent, very thin, and a scarcely thicker tract of cartilage remains between the posterior and anterior sphenoids; yet these are each a single bone at this stage. The perpendicular ethmoid and septum nasi are still unossified; but the inferior turbinals are almost completely, and the "lateral masses" partially, converted into endosteal bone. The cribriform plate is soft, and so is the snout (Plate XXXVI. fig. 1); but this latter is everywhere burrowed with vessels prior to hardening.

Beneath the skull we see a most compact building together of the palatal, pterygoid, external pterygoid, and tympanic bony pieces; the thick, clubbed "hamular process" of the internal pterygoid is fixed as an undersetter to the solid nut-like tympanic, and dints it as an inturned horn dints the frontal in certain varieties of the Cow; this, however, is only a temporary state of things, and is quite recovered from in the lengthening head and face.

No "interparietals" have been found, adding two bones to the growing superoccipital, as seen in Man; this part, the superoccipital, is now a nearly vertical wall, the parietals finishing the roof above. The lower jaw is well ossified, and is now entirely free from the arrested primordial bar, MECKEL'S cartilage. The three periotic centres have completely ossified the auditory mass, "petrosal" and "mastoid" (Plate XXXVI. fig. 2); and a small bilobate ossicle has appeared in the attached (confluent) head of the stylohyal (*st.h.*). Further down another centre has appeared in the middle of the long rib-like bar, taking up nearly the middle third. The upper piece of bone (formed from two nuclei in the Lamb, and apparently also in the Pig) is called by Professor FLOWER* the "tympano-hyal," a term it may be well to retain. The rudimentary stylohyal of Man is ossified from the upper centre; for "a centre of ossification appears in the styloid cartilage, and extending upwards and downwards, gives rise to the pyramid and styloid process" (HUXLEY, 'Elem.' p. 160). Hence it will be seen that the tympano-hyal and the *upper* styloid bone are identical; both these bones are largely developed in the Osseous Fish, the so-called "epi- and "ceratohyals;" they occupy the great flat "cornu," at the base of which the short ceratohyal proper, with its two bony centres, is articulated. The unciform ceratohyal ("cornu minor") of the Pig is strongly attached by fibrous tissue to the transverse basal piece (Plate XXXVI. fig. 2, *c.h.*), now coalesced with its own rudimentary arch, the "cornua majora" of Man; these pieces are ossified proximally (fig. 2, *th.h.*), and these centres correspond with the first pair of hypobranchials of the Osseous Fish, the median part answering to the first basibranchial. The "stylo-mastoid foramen" (fig. 2, *s.m.f.*, 7^{a}) is seen transmitting the portio dura nerve; and this sends upwards and forwards the "chorda tympani" ($7^{a'}$), to which is attached the smallest of the three "ossa bullæ" (*o.b'.*); the middle-sized piece is seen in front of the stylohyal (*o.b''.*). The rest of the drum-walls being removed and the squamosal, the outer face of the periotic mass shows the three semicircular canals

* In his valuable little work 'On the Osteology of the Mammalia,' 1870, p. 173.

above (*a.sc.*, *h.sc.*, *p.sc.*) and the cochlea below (*ol.*). The hollowed tegmen tympani (*t.ty.*) has in its hinder recess the head of the incus (*i.*); the recess ends in a round cup-like facet for the short crus of the incus, with its down-turned rounded head; the "acetabulum" for this head is finished, externally, by the squamosal, part of which, having become adherent, is shown in the figure. Below this wall-chamber for the incus is the fenestra ovalis with the enclosed stapes (*f.ov.*, *st.*); the long axis of the oval space and oval base of the stapes is upwards and forwards. The inturned hook of the long crus of the incus is now coupled to the neat head of the stapes by means of an intermediate bone, the "os orbiculare" (*o.ob.*), a special centre developed in the *primary head* of the second postoral bar, which, limpet-like, applied itself to that periotic "bud" which became the stapes, by a process similar to that which detaches the axillary buds in *Lilium tigrinum*. In this figure the malleus is not given; it is shown in fig. 3 (*ml.*). The processus gracilis (*p.gr.*) is reduced to a style, ending in fibrous tissue; the manubrium (*mb.*) is flat and slightly arcuate; the "head," articulated with the incus, is elegantly notched for this purpose, and fits on to the incus by a synovial joint, the miniature of that by which the tibia fits on to the astragalus in this same animal. Between the head and the manubrium the bone is thin, and is scooped externally; the head sends inward a rounded process (*i.p.m.*), and the manubrium sends backwards an angular snag; this latter is for the attachment of the "tensor tympani" muscle. The little *secondary* nucleus of cartilage which we saw developed between the dislocated incus and stylohyal (Plate XXX. figs. 8 & 9, *i.hy.*) is now attached to the neck of the stapes by its broad outer end, whilst its bluntly pointed distal end is buried in the fibres of the tendon of the "stapedius" muscle (*st.m.*). This is the *last* effect of the high degree of metamorphosis exhibited by the second postoral bar of the Mammal. The fore edge of the exoccipital, with its paroccipital spur (fig. 2, *e.o.*, *p.oc.*), is strongly clamped upon the auditory capsule; this is also made still stronger by the large posterior flange of the overgrowing squamosal, not shown in this figure.

The under surface of the snout is also given at this stage, to show the complete coalescence of the alæ nasi with the recurved trabecular horns, and their continuation backwards as the "recurrent laminæ" (*rc.c.*), also the alinasal external segment or "appendix." The solid fore end of the snout, already full of small blood-vessels, is ready to become the snout-bone for *rooting*; this bone is formed in the ossified knees of the trabeculæ: the stunted, recurved prenasal cartilage is now undistinguishable from the base of the septum nasi, formed by the complete coalescence of a large tract of the trabecular bars, the long commissure of the foremost facial arch.

Eighth Stage.—The Skull of a Pig 6 months old.

This makes a more convenient *last* stage than the adult, as here are still in existence the greater number of the sutural landmarks, so soon to be largely obliterated, whilst the change in general form is rather of interest to the zoologist than to the morphologist. The long angular skull (Plate XXXVI. fig. 4, and Plate XXXVII. figs. 1 & 2)

is an irregular pyramid, with two equal and two unequal sides and an oblique base. A complete contrast in outward form to the human skull, that of the Pig is straightest of all the types; it is very angular and strongly built, but its bone-tissue is inferior in density to that of the Sheep, being intermediate in this respect between the bone of a Ruminant and that of a Cetacean. The flat top of the skull, with its orbits flush with the top, indicate the semiaquatic habits of its owner; and the immense depth and squareness of the base of the pyramid is correlative to the high neck and strong shoulders of this type: *leverage* is suggested by every ridge and every snag. Coming back to the morphology of the matter, I may remark that the long straight nasals (Plate XXXVII. fig. 1, *n.*) overlap the snout in front, and only show their edge in the side view. They are articulated by suture along their outer edge with the upper edge of the long premaxillary wedge (*px.*), and for a less extent with the maxillary (*mx.*); they terminate in a transverse line half an inch behind their *maxillary* suture. The frontals (*f.*) together form a somewhat pentagonal plate, divided along the mid line by the sagittal suture. The anterior third is deeply grooved, the grooves issuing from the "infraorbital foramen;" the posterior half of their outer margin is thick, and somewhat raised as the superorbital ridge. There is a large orbital plate (Plate XXXVII. fig. 1) which is bounded behind above by the short arrested postorbital process, and lower down and within by the orbito-sphenoid (*o.s.*). The upper surface of the parietals (*p.*) is of short extent, and divided by the continuation of the sagittal suture; they are greatly pinched in to form the large temporal fossa (*t.f.*); and behind they are somewhat impinged upon by the large superoccipital wall (*s.o.*).

The premaxillaries (Plate XXXVI. fig. 4, and Plate XXXVII. fig. 1, *px.*) have a large facial and a lesser palatine region, the palatine spurs (*p.px.*) being slender and compressed. The huge maxillary (*mx.*), besides forming most of the fore face and the anterior root of the zygoma, by its last tooth-socket (a large pupiform cavity), binds behind upon the external pterygoid plate and descending extremity of the palatine (Plate XXXVII. fig. 1, *mx.*,*pa.*,*e.pg.*). Below (see Plate XXXVI. fig. 4, *mx.*) it forms three fourths of the elegant grooved and ribbed hard palate; the double "posterior palatine foramen" (*p.p.f.*) is partly in this bone and partly in the palatine. The median suture of the hard palate (Plate XXXVI. fig. 4) is two thirds the length of the skull and face. The palatine bones (*pa.*) by their primary ascending plate articulate with the vomer, and send forwards a long scoop-like process beneath the lateral ethmoids. After forming the elegant end of the hard palate they grow downwards as a thick boss, which articulates with the maxillary and external pterygoid process on the outside, and with the internal pterygoid plate on the inside. This latter bone, the "pterygoid" proper (*pg.*), is very thin in its ascending part; and on the right side the uppermost squamous part is a separate piece, the mesopterygoid (Plate XXXVI. fig. 4, *ms.pg.*), a bone not commonly distinct in the Mammalia; yet in my collection it occurs in the Fox (*Canis vulpes*), and is subdistinct in the Hedgehog (*Erinaceus europæus*); here, in the Pig, it is a small triangular scale. The inferior part of the pterygoid is thicker and is subfalcate, the

hooked portion ("hamular process") being the *apex* of the pterygo-palatine arch. The top of the pterygoid, behind the mesopterygoid, has already coalesced with the posterior sphenoid at the root of the "ala," and externally also it is almost completely confluent with the "external plate" (*e.pg.*).

The thin dentate end of the vomer (Plate XXXVI. fig. 4, *v.*) ends at the same transverse line with the mesopterygoids, whilst in front it reaches the fore margin of the premaxillaries within a barleycorn's length. The vomerine plate broadens a little to sit on and articulate with the upturned and dovetailed edge of the palatine plate of the palatine bones, and also with the longer "harmony suture" of the maxillaries; its groove for the trabecular subseptal beam is rather deep. The complex nasal labyrinth is well ossified, but the septum between the inferior turbinals, supplied by the fifth nerve, is still soft. Beneath the large lacrymal, the "lamina papyracea" of the ectoethmoid is just seen beside the pupiform socket of the developing last molar tooth.

The lacrymal (Plate XXXVII. fig. 1, *l.*) is a notable bone on the outer cheek, with an upper and a lower canal; it is deeply scooped where it articulates antorbitally with the orbital plate of the frontal (Plate XXXVII. fig. 1, *l.*, *f.*); it forms part of the anterior root of the zygoma, and is nearly equally developed both within and without the orbit.

The malar or jugal (*j.*) is a massive bar of bone, strongly set in at the front of the orbit between the lacrymal and maxillary (Plate XXXVII. fig. 1, *j.*, *l.*, *m.*). It is saddle-backed in its front thicker part, and its hinder half suddenly becomes only as thick as in front, so that the squamosal may yoke on to it; its lower surface is arcuate, its inner surface scooped, and its outer surface convex: almost an inch of space intervenes between the highest part of the malar and the descending postorbital spur of the frontal. The proper temporal bone, fitting scale-like to the temporal region of the skull, over the hinder edge of the frontal and the lower part of the parietal, is properly called the squamosal (*sq.*). This temporal part turns suddenly outwards to catch the pyriform condyle of the lower jaw, and then runs forwards and rides on the upper edge of the malar, stopping behind the concave portion of that bone. This zygomatic process of the squamosal rises sharp above the transverse "glenoid" bridge, which is scooped above and below, has a convex transverse part in front, and an angular scooping behind: this part is clothed with articular cartilage, and the mandibular condyle rides freely *under* and *up* it, a meniscus intervening.

There is an acute ridge which runs obliquely up to the upper third of the superoccipital (Plate XXXVII. fig. 1, *sq.*, *s.o.*); this rising wall closes in the deep temporal fossa. This ridge of bone, running downwards also, binds strongly upon and coalesces with the rough rounded uplooking mouth of the "meatus auditorius externus" (*m.e.*). Below this part the squamosal splits into two narrow rough leaves; the hinder of these is the "posttympanic process," and the front leaf is the "postglenoidal." The latter binds upon the side of the tympanic (*ty.*), the former runs down the fore edge of the paroccipital (paramastoid) process (*p.oc.*), and scoops its upper third. In front of the

glenoid facet the squamosal is strongly sutured to the alisphenoid (*al.s.*) and to its great expanded wing (*e.pg.*). Together, the zygomatic elements make a strong, deep, and convex arch on each side, which starts rather sharply upwards.

The tympanic (*ty.*), now like a large *filbert* in form and size, although snaggy and ridged below, is principally a mass of square-chambered diploë. It has a small cavity, to the produced edge of which the membrana tympani is attached; and its former "crura" have met, run upwards, and formed the curious, ascending, coral-like meatus.

The flange of the anterior crus is now a squamous process beneath the squamosal, and close to the inner edge of the glenoidal cartilage. As there are no proper "foramina ovalia" in the posterior sphenoid, so there is a continuous "foramen lacerum" round the tympanic, and between it and the basis cranii (Plate XXXVI. fig. 4, *f.l.p.*). Looking through these large chinks we can see a small part of the periotic mass, which is very separate from the surrounding parts. The great occipital plane (Plate XXXVII. fig. 2, *s.o.*, *e.o.*, *b.o.*) is scooped above (*s.o.*), and then the bone bends forwards, wedging itself in between the parietals: the upper element is alate above, and then narrows in and rests obliquely upon the exoccipitals (*e.o.*), forming the keystone of the archway for the medulla spinalis, the foramen magnum (*f.m.*). The arch again expands its sides, the exoccipitals spreading out behind and over the mastoids, which further outwards are plastered over by the "posttympanic processes" of the squamosal. These lateral pieces run downwards as the long paramastoid, or, more correctly, *paroccipital* spurs; whilst their middle region juts out, and forms the diverging, semioval, subpedunculate articular condyles (*oc.c.*). Inside the paroccipital process there is a considerable foramen for the hypoglossal nerve (Plate XXXVI. fig. 4, *s*). In front of the paroccipital process there is a canal, bounded on the outside by the posttympanic spur of the squamosal, and on the inside by the unciform tympanic. Looking up this canal we see that its inner half is occupied by a rod of bone, thickest below, where it is flat, the continuing cartilage being macerated off. This rod is the apex of the stylohyal—"tympano-hyal" (Flower), and the open tube is the canal for the portio dura; its mouth is the "stylo-mastoid foramen."

The basioccipital (Plate XXXVI. fig. 4, *b.o.*) is a pentagonal lozenge of bone, joining the sides of its own arch by suture, and separated from the next basal piece (*b.s.*) by a narrow synchondrosis. This basioccipital plate is mammillate at the sides below, and subcarinate mesially: it is the *notochordal* bone. Next in front is the basisphenoid (Plate XXXVI. fig. 4, *b.s.*), now merely the basal part of an inverted arch of bone, the "posterior sphenoid." The narrow cartilaginous tract between this and the bar in front is hidden below by the end of the vomer, and by a sharp ridge which grows mesiad from each alisphenoid (Plate XXXVI. fig. 4, *al.s.*, *b.s.*, *v.*). The orbito-sphenoids have met below, ossifying the underlying trabecular bar, which part is ensheathed by the vomer. These large wings can be seen in the posterior part of the orbit around and above the optic foramen (Plate XXXVII. fig. 1, *o.s.*, 2), and also from behind through the foramen magnum (Plate XXXVII. fig. 2, *o.s.*, 2). The change which has taken

place in the little auditory ossicles is too small to require special notice. The mandibular rami (Plate XXXVII. fig. 3) are quite distinct from each other at present: they are large deep bones, with a small notch between the coronoid and articular regions; the latter is somewhat pyriform, with the narrow end inwards; the angular region is flat, with a round outline and a thick rugose edge.

Ninth Stage.—The Skull in Adult Pigs.

The further changes that take place in the Pig's skull are mainly increase of size and extensive ankylosis. Besides referring the reader to the actual object, I may mention that a short and useful account of this type of cranium will be found in Professor FLOWER'S work, 'An Introduction to the Osteology of the Mammalia,' 1870, p. 172, and also in Professor HUXLEY'S 'Anatomy of the Vertebrated Animals,' 1871, p. 368.

SUMMARY.

The most important results of the present investigation may be stated as follows:—

1. In a pig embryo, in which the length of the body did not exceed two thirds of an inch, and four postoral clefts were present, the cranio-facial skeleton was found to consist of:—

(*a*) the notochord, terminating by a rounded end immediately behind the pituitary body.

(*b*) On each side of the notochord, but below it, a cartilaginous plate, which in front ends by a rounded extremity on a level with the notochord, whilst behind it widens out and ends at the free lower margin of the occipital foramen. These two plates, taken together, constitute the "investing mass" of RATHKE. In this stage they send up no prolongations around the occipital foramen; in other words, the rudiment of the basi-occipital exists, but not that of the exoccipital nor superoccipital.

(*c*) The large oval auditory capsules lie on each side of the anterior half of the investing mass, with which they are but imperfectly united: there is no indication of the stapes at this stage.

(*d*) The *trabecular* or first pair of preoral visceral arches enclose a lyre-shaped pituitary space; they are closely applied together in front of this space, and, coalescing, give rise to an azygous prænasal rostrum: they are distinct from one another and the investing mass.

(*e*) The *pterygo-palatine* or second pair of visceral arches lie in the maxillo-palatine processes, and are therefore subocular in position. Each is a sigmoid bar of nascent cartilage, the incurved anterior end of which lies behind the interior nasal aperture, while the posterior extremity is curved outwards at about the level of the angle of the mouth. The pterygo-palatine cartilages are perfectly free and distinct from the first preoral and the first *postoral* arch, although developed in a process of the latter, and are therefore *secondary* arches.

(*f*) The *mandibular* or first pair of postoral visceral arches are stout cartilaginous rods of cartilage, which lie in the first visceral arch behind the mouth. The ventral or

distal ends of these arches are not yet in contact; the dorsal or proximal end of each is somewhat pointed and sharply incurved, pushing inwards the membrane which closes the first visceral cleft and is the rudiment of the membrana tympani.

(*g*) The *hyoid* or second pair of postoral arches are in this stage extremely similar to the first pair, with which they are parallel. They are stout sigmoid rods of cartilage, which are separated at their distal ends, present an incurved process at their opposite extremities, and are not segmented.

(*h*) The *thyrohyal* or third postoral arches, which correspond with the first branchial of branchiate Vertebrata, are represented by two short cartilaginous rods which lie on each side of the larynx.

(*i*) The olfactory sacs are surrounded by a cartilaginous capsule, which has coalesced below with the trabecula of its side; while, within, the mucous membrane lining the capsule presents elevations which indicate the position of the future turbinal outgrowths of the capsule.

In this stage the posterior nares are situated at the anterior part of the oral cavity, as in the Amphibia; and the roof of the mouth is formed by the floor of the skull, the palatal plates of the maxillæ and palatine bones being foreshadowed by mere folds. The outer end of the cleft between the trabecula and the secondary preoral arch appears to be the rudiment of the lacrymal duct, while its inner end is the hinder nasal aperture. The gape of the mouth is the cleft between the second preoral and the first postoral arch. The auditory passage, representing the Eustachian tube, tympanum, and external auditory meatus, is the cleft between the first and second postoral arches. The proximal end of the mandibular arch, therefore, lies in the front wall of the auditory passage, and the hyoid in its hinder wall.

2. In an embryo pig, an inch in length, (*a*) the notochord is still visible; (*b*) the investing mass, the halves of which are completely confluent, has become thoroughly chondrified, and is continued upwards at each side of the occipital foramen to form an arch over it.

(*c*) The auditory capsules are still distinct from the investing mass, and a plug on the outer cartilaginous wall of each has become marked off as the stapes.

(*d*) The hinder ends of the trabecular arches have coalesced in front of the pituitary body, but they are not yet confluent with the investing mass.

(*e*) The pterygo-palatine rods have increased in size; they have not become hyaline cartilage, but are beginning to ossify in their centre.

(*f*) In the mandibular arch the proximal end has become somewhat bulbous, and is recognizable as the head of the malleus, whilst the incurved process, still more prominent than before, is the *manubrium mallei*. The rest of the arch is Meckel's cartilage; outside this a mass of tissue appears, which is converted into cartilage, rapidly ossifies, and eventually becomes the ramus of the mandible.

(*g*) The proximal end of the hyoidean arch, similarly enlarging and articulating with the corresponding part of the mandibular arch, becomes the incus, the incurved process

attaching itself to the outer surface of the stapes and becoming the long process of the incus. The incus, thus formed out of the proximal end of the hyoidean arch, becomes separated from the rest of the arch by conversion of part of the arch into fibrous tissue, and the moving downwards and backwards of the proper hyoid portion of the arch. A nodule of cartilage left in the fibrous connecting band becomes a styliform "interhyal" cartilage, while the proximal end of the detached arch becomes the stylohyal.

(*h*) The thyrohyals have merely increased in size and density; they closely embrace the larynx by their upper ends.

(*i*) The olfactory capsules are well chondrified, and their descending inner edges have coalesced with each other and with the trabeculæ below to form the great median septum: the turbinal outgrowths are apparent.

In this stage the alisphenoids and orbito-sphenoids appear as chondrifications of the walls of the skull, quite independent of the investing mass and the trabeculæ.

The floor of the pituitary space chondrifies independently of the trabeculæ and the moieties of the investing mass, but serves to unite these four cartilaginous tracts.

3. In an embryo pig $1\frac{1}{8}$ inch in length, (*a*, *b*, *c*) the primordial cranium is completely constituted as a cartilaginous whole, formed by the coalescence of the investing mass and its exoccipital and superoccipital prolongations, the modified trabeculæ, the subpituitary cartilage, the auditory capsules, the alisphenoidal and orbito-sphenoidal cartilages, and the olfactory capsules. The notochord is still to be seen extending in the middle line from the hinder wall of the pituitary fossa (now the clinoids of the sella turcica) to the posterior edge of the occipital region.

(*d*) The trabecular arches form the sides of the sella turcica, the presphenoid, and the base of the septum between the olfactory capsules; in front, where they form the azygous prenasal or "basitrabecular" element, they are developed backwards as "recurrent bands," elongations of the free recurved cornua.

(*e*) The pterygo-palatine arches, still increasing in size, but not chondrifying, now rapidly ossify; they are half-coiled laminæ bounding the posterior nasal passages.

(*f*) The mandibular arches and the rudimental ramus have become solid cartilage, and the latter is ossifying as the dentary; the distal part of each mandibular rod unites with its fellow for some distance.

(*g*) The hyoid arches are now more fully segmented as incus, with its orbicular head, interhyal, stylohyal, and ceratohyal.

(*h*) The thyrohyals are merely larger and denser.

(*i*) The olfactory capsules have now the turbinal outgrowths all marked out as alinasal, nasal, upper, middle, and lower turbinals.

4. In pigs of larger size the form and proportions of all the parts of the cranium become greatly altered, and ossification takes place on an extensive scale, but no new structure is added.

5. It follows from these facts that the mammalian skull, in an early embryonic condition, is strictly conformable with that of an Osseous Fish, a Frog, or a Bird at a like period of development, consisting as it does of:—

(*a*) A cartilaginous basicranial plate embracing the notochord, and stopping, like it, behind the pituitary body.

(*b*) Paired cartilaginous arches, of which two are preoral, while the rest are postoral.

(*c*) A pair of cartilaginous auditory capsules.

(*d*) A pair of cartilaginous nasal capsules.

Further, that in the Mammalia, as in the other Vertebrata the development of the skull of which has been examined, the basicranial plate grows up as an arch over the occipital region of the skull, and coalesces with the auditory capsules, laterally, to give rise to the primordial skeleton of the occipital, periotic, and basisphenoidal regions of the skull. The trabeculæ become fused together, and, uniting with the olfactory capsules, give rise to the presphenoidal and ethmoidal parts of the cranium; and the moieties of the skull thus resulting from the metamorphosis of totally different morphological elements become united to give rise to the primordial cranium.

As in the Salmon and Fowl, the second pair of preoral arches give rise to the pterygopalatine apparatus; in the Frog this arch is late in appearance, and is never distinct from the trabecular and mandibular bars, serving as a conjugational band between them. The mandibular arch, which in the Salmon becomes converted into MECKEL'S cartilage, the os articulare, the os quadratum, and the os metapterygoideum, in the Frog into MECKEL'S cartilage and the quadrate cartilage (which early becomes confluent with the periotic capsule), in the Bird into MECKEL'S cartilage, the os articulare, and the os quadratum (which articulates movably with the periotic capsule), in the Pig is metamorphosed into the malleus, which is loosely connected with the tegmen tympani, an outgrowth of the periotic capsule.

MECKEL'S cartilage persists in the Fish and the Amphibia, but disappears early in the Bird, and still earlier in the Mammal. The permanent ossifications formed outside the primary mandible are all membrane-bones in Fish, Frog, and Fowl, but in the Mammal (exceptionally) the ramus has a cartilaginous foundation. In the Fish the hyoidean arch becomes closely united with the mandibular, and then segmented into the hyomandibular, the stylohyal, ceratohyal, and hypohyal—the hyomandibular or proximal segment articulating with the outer wall of the periotic, and many of the segments becoming dislocated.

In the Frog the hyoid also becomes segmented into three pieces. The middle segment becomes the suprastapedial (hyomandibular) with its extrastapedial process, and, extending inwards as mediostapedial, articulates with the stapes, developed by segmentation from the outer wall of the auditory capsule, the proximal part, or interstapedial, intervening. The stylohyal is dislocated and becomes connected with the auditory capsule below the stapes (opisthotic region).

In the Bird the hyoidean arch remains distinct from the mandibular; whilst in its primordial condition it coalesces by its incurved apex with the auditory capsule in front of the promontory, before the stapedial plug is segmented. It then chondrifies as three distinct cartilages—an incudal, a stylohyal, and, distally, a ceratohyal. The stapes becomes free from the auditory capsule, but remains united with the cartilaginous part

of the incus (mediostapedial); the ascending part islargely fibrous (suprastapedial), and the part loosely attached to the mandibular arch is the elongated extrastapedial. The short stylohyal afterwards coalesces with the body of the upper or incudal segment by an aftergrowth of cartilage (the "interhyal" tract); a long membranous space intervenes between it and the glossal piece (ceratohyal). Thus the "columella" of the Bird is formed of three hyoidean and one periotic segment.

In the Pig the hyoidean arch is distinct, but articulates closely with the mandibular; its upper segment (hyomandibular) is converted into the incus, and becomes connected with the stapes, its disciform apex being ossified as the "os orbiculare." The stylohyal is dislocated and coalesces with the opisthotic region of the auditory capsule.

The views which have hitherto been entertained respecting the mode of development of the *ossicula auditus* of the Mammalia fall under four heads:—

1. According to Reichert*, the malleus and incus both result from the metamorphosis of the cartilaginous skeleton of the mandibular arch, while the stapes proceeds from an after segment of the hyoidean arch, which becomes separated and imbedded in the outer wall of the auditory capsule.

The latest writer on the subject, Semmer†, supports Reichert's views in the main, but is not quite sure about the origin of the stapes.

2. Günther‡ holds that not only the malleus and the incus, but the stapes as well, are the product of the metamorphosis of the skeleton of the mandibular arch.

3. Magitot and Robin§, on the other hand, maintain that the malleus only takes its origin from the skeleton of the mandibular arch. They consider the incus and stapes to arise independently, but do not expressly refer them to the skeleton of the second postoral visceral arch.

4. Professor Huxley‖, arguing from the anatomy of the mandibular and hyoidean arches in the lower Vertebrata, has put forward the view that the malleus of the Mammalia is the product of the metamorphosis of the proximal end of the cartilaginous skeleton of the mandibular arch, while the incus proceeds from the proximal end of the hyoidean arch, and is the homologue of the "suprastapedial" of the *Sauropsida*. He expresses no opinion respecting the origin of the stapes.

* "Ueber die Visceralbogen der Wirbelthiere," Müller's Archiv, 1837.

† Untersuchungen über die Entwickelung der Meckel'schen Knorpels und seiner Nachbargebilde. Dorpat, 1872.

‡ Beobachtungen über die Entwickelung des Gehörorganes bei Menschen und höheren Säugethieren. Leipzig, 1842.

§ "Mémoire sur un organe transitoire de la oie fœtale désigné dans le nom cartilage de Meckel," Annales des Sciences Naturelles, sér. 4, i., xviii. 1862.

‖ "On the Representatives of the Malleus and the Incus of the Mammalia in the other Vertebrata," Proceedings of the Zoological Society, 1869.

Explanation of Abbreviations.

a.a.n. appendix alæ nasi.
a.cl. anterior clinoid wall.
al.e. aliethmoid.
al.n. alinasal.
al.s. alisphenoid.
al.tb. alinasal turbinal.
aq.v. aqueduct of the vestibule.
a.sc. anterior semicircular canal.
a.ty. annulus tympanicus.
au. auditory sac.
b.a. basilar artery.
b.br. basibranchial.
b.h., *b.hy.* basihyal.
b.o. basioccipital.
b.s. basisphenoid.
b.tr. basitrabecula.
c.a. concha auris.
c.c. common (auditory) canal.
c.h., *c.hy.* ceratohyal.
cl. cochlea.
cl. 1, 2, &c. 1st, 2nd, and following visceral clefts*.
cr. coronoid process of mandible.
ct. cutis.
c.tr. cornua trabeculæ.
C 1. thalamencephalon.
C 1ª. hemisphere.
C 2. 2nd cerebral vesicle.
C 3. 3rd cerebral vesicle.
d. dentary.
d.px. dentary part of premaxillary.
e. eyeball.
ep. epiotic.
e.n. external nostril.
e.o. exoccipital.
e.pg. external pterygoid plate.
eu. Eustachian tube.
f. frontal.
f.l.p. foramen lacerum posterius.
f.m. foramen magnum.
f.ov. fenestra ovalis.
f.r. fenestra rotunda.
h.br. hypobranchial.
h.sc. horizontal semicircular canal.
hy. hyoid.
i.c. internal carotid artery.
i.hy. interhyal.
i.n. inner nostril.
i.p.m. internal process of malleus.
i.tb. inferior turbinal.
iv. investing mass.
j. jugal.
j.v. jugular vein.
l. lacrymal.
l.c.i. long crus of incus.
l.c.r. lateral cerebellar recess.
l.l. lower lip.
lp. lip.
l.s. lateral sinus.
lx. larynx.
m. mouth.
mb. manubrium mallei.
mc. meniscus.
m.e. meatus externus.
m.eth. mesethmoid.
mk. Meckel's cartilage.
mk.c. Meckelian commissure.
ml. malleus.
mn. mandible.
m.n. middle nostril.
m.ob. medulla oblongata.
ms.pg. mesopterygoid.
m.tb. middle turbinal.
m.tr. middle trabecula.
m.ty. membrana tympani.
mx. maxillary.
mx.p. maxillo-palatine.
n. nasal.
n.w. nasal wall.
o.b. os bullæ.
oc.c. occipital condyle.
œ. œsophagus.
ol. olfactory sac.
o.ob. os orbiculare.
os. orbito-sphenoid.
o.v. ophthalmic vein.
p. parietal.
pa. palatine.
p.e. perpendicular ethmoid.

* These may be counted from the lacrymal or first preoral, or from the tympanic or first postoral.

pg. pterygoid.
p.gr. processus gracilis.
p.n. prenasal cartilage.
p.n.w. posterior nasal wall.
p.oc. paroccipital.
p.p.f. posterior palatine foramen.
p.pg. palato-pterygoid.
p.p. palatine floor of palatine.
p.px. palatal part of premaxillary.
pr. promontory.
pro. prootic.
ps. presphenoid.
p.sc. posterior semicircular canal.
py. pituitary body.
rc.c. recurrent cartilages.
s.c.i. short crus of incus.
s.m.f. stylo-mastoid foramen.
s.n. septum nasi.
so. supero-occipital.
s.pa. soft palate.
sp.s. sphenoidal sinus.
sq. squamosal.
s.t. sella turcica.
st. stapes.
st.h. stylohyal.
st.m. stapedius muscle.
t.c. tympanic cavity.
t.cb. tentorium cerebelli.
t.f. temporal fossa.
tg. tongue.
th.h. thyrohyal.
t.p. tooth-pulp.
tr. trabecula.
tr.cm. trabecular commissure.
t.ty. tegmen tympani.
ty. tympanic.
u.l. upper lip.
u.tb. upper turbinal.
v. vomer.
vb. vestibule.
z.m.x. zygomatic process of maxillary.
z.sq. zygomatic process of squamosal.
1. olfactory nerve.
2. optic nerve.
5^a. ophthalmic nerve.
5^b. main part of 5th nerve.
7^a. portio dura.
7^{cr}. chorda tympani.
7^b. portio mollis.
8^a. glossopharyngeal.
8^b. vagus.
9. hypoglossal.

Description of the Plates.

PLATE XXVIII.

First Stage.—Embryo Pig, $\frac{3}{4}$ inch in length.

Fig. 1. Side view of upper part of embryo. × 7 diameters.
Fig. 2. A plan of the same, with facial arches. × 7 diameters.
Fig. 3. A front view of the same. × 7 diameters.
Fig. 4. A palatal view of the same, with the mandible and lower face removed. × 15 diameters.
Fig. 5. A plan of the skull and face, seen from below. × 10 diameters.
Fig. 6. A vertical section of the head. × 10 diameters.
Fig. 7. Part of the same, with median part of nasal region removed. × 20 diameters.
Fig. 8. Upper view of a horizontal section of the head. × 10 diameters.

PLATE XXIX.

Fig. 1. Transversely vertical section of the nose, in front. × 12 diameters.
Fig. 2. A similar section through the middle of the nasal region. × 12 diameters.
Fig. 3. Another section through the posterior nasal region. × 12 diameters.

Fig. 4. Horizontal section below the cranial cavity, exposing first arch, notochord, and investing mass. × 10 diameters.
Fig. 5. A subhorizontal section through the eyes and root of tongue. × 10 diameters.
Fig. 6. Part of the head, with outer part of cheek pared away. × 12 diameters.
Fig. 7. A section made through the plane of the hinder part of cranium. × 12 diameters.
Fig. 8. Part of same section. × 20 diameters.
Fig. 9. A similar section. × 26 diameters.
Fig 10. Another subhorizontal section through the top of the first postoral cleft. × 20 diameters.

PLATE XXX.

Second Stage.—Embryo Pig, 1 *inch long.*

Fig. 1. Section across the end of the snout. × 10 diameters.
Fig. 2. Section through ethmoid region, root of tongue, and larynx. × 12 diameters.
Fig. 3. Section nearly in plane of the notochordal region, front view. × 10 diameters.
Fig. 4. A similar section, lower down. × 10 diameters.
Fig. 5. Part of a similar section through apex of mandibular arch, front view. × 15 diameters.
Fig. 6. A similar section through apex of next arch, front view. × 15 diameters.
Fig. 7. Another similar section through periotic capsule in plane of horizontal canal. × 10 diameters.

Third Stage.—Embryo Pig, 1½ *inch long.*

Fig. 8. Part of a section through top of second postoral arch, corresponding with fig. 6 of second stage, front view. × 15 diameters.
Fig. 9. Side view of the three postoral arches. × 15 diameters.

PLATE XXXI.

Third Stage (continued).

Fig. 1. 1st section through nasal region. × 7½ diameters.
Fig. 2. 2nd section of same. × 7½ diameters.
Fig. 2*. Part of fig. 2. × 22½ diameters.
Fig. 3. 3rd section of same. × 7½ diameters.
Fig. 3*. Part of same section. × 22½ diameters.
Fig. 4. 4th section of same. × 7½ diameters.
Fig. 5. 5th section of same. × 7½ diameters.
Fig. 6. 6th section of same. × 7½ diameters.
Fig. 6*. Part of fig. 6. × 20 diameters.
Fig. 7. 7th section of nasal region. × 7½ diameters.
Fig. 7*. Mandibular portion of same section. × 7½ diameters.

Fig. 8. 8th section of same. × 7½ diameters.
Fig. 8ª. Mandibular portion of same section. × 7½ diameters.
Fig. 9. 9th section of nasal region. × 7½ diameters.
Fig. 10. 10th section of same. × 7½ diameters.
Fig. 11. 11th section of same. × 7½ diameters.
Fig. 12. 12th section of same. × 7½ diameters.
Fig. 13. 13th section of same, head. × 7½ diameters.
Fig. 14. 14th section of same, head. × 7½ diameters.
Fig. 14ª. Part of fig. 14. × 14 diameters.

PLATE XXXII.

Third Stage (continued).

Fig. 1. 15th section of same, head. × 7 diameters.
Fig. 2. 16th section of same, head. × 7 diameters.
Fig. 3. 17th section of same, head. × 7 diameters.
Fig. 4. 18th section of same, head. × 7 diameters.
Fig. 5. 19th section of same (part back view). × 14 diameters.
Fig. 6. 20th section of same, head (back view). × 10 diameters.
Fig. 7. 21st section of same (part back view). × 14 diameters.
Fig. 8. 21st part of section (front view). × 14 diameters.
Fig. 9. Hinder part of a section taken *horizontally* through the nasal region (front view). × 10 diameters.
Fig. 10. A similar section of same taken further backwards and lower down (back view). × 10 diameters.

PLATE XXXIII.

Third Stage (continued).

Fig. 1. 22nd section of the same head (part seen from the front). × 14 diameters.
Fig. 2. Vertical section of head. × 5 diameters.
Fig. 3. The same, with brain and septum nasi removed. × 5 diameters.

Fourth Stage.—Embryo Pig, 2½ inches long.

Fig. 4. Vertically transverse section through fore part of brain. × 5 diameters.
Fig. 5. A similar section through fore part of orbit (part). × 5 diameters.
Fig. 6. A like section through hind part of orbit (part). × 5 diameters.
Fig. 7. A similar section through sphenoid (part). × 5 diameters.
Fig. 8. Part of same section (right side). × 14 diameters.
Fig. 9. A similar section through the fore part of auditory sac. × 7 diameters.
Fig. 10. Another section through hinder part of same. × 14 diameters.

Fifth Stage.—Embryo Pig, 3 inches long.

Fig. 11. Section through posterior sphenoid. × 3½ diameters.

PLATE XXXIV.

Fourth Stage (continued).

Fig. 1. Side view of skull. × 3½ diameters.
Fig. 2. Lower view of same. × 3½ diameters.
Fig. 3. End view of same. × 3½ diameters.
Fig. 4. Upper occipital region of a somewhat younger embryo. × 3½ diameters.
Fig. 5. Section of skull (vertical). × 3½ diameters.
Fig. 6. Bird's-eye view of primordial skull. × 3½ diameters.
Fig. 7. Inner view of compound mandible. × 3½ diameters.
Fig. 8. 1st section through nasal region. × 7 diameters.
Fig. 9. 2nd section of same part. × 7 diameters.
Fig. 10. 3rd section through nose. × 7 diameters.
Fig. 11. 4th section of same. × 7 diameters.
Fig. 12. 5th section through inferior turbinal. × 7 diameters.

PLATE XXXV.

Sixth Stage.—Embryo Pig, 6 inches long.

Fig. 1. Side view of skull. × 1½ diameter.
Fig. 2. Lower view of same. × 1½ diameter.
Fig. 3. Vertical section of same. × 1½ diameter.
Fig. 4. Bird's-eye view of primordial skull. × 1½ diameter.
Fig. 5. Outer view of occipital and auditory regions. × 2 diameters.
Fig. 6. Transversely vertical section through inferior turbinals. × 3 diameters.
Fig. 7. Another through ethmoids. × 3 diameters.
Fig. 8. Part of section through hinder part of nasal labyrinth. × 3 diameters.
Fig. 9. Section through orbits. × 2 diameters.
Fig. 10. Part of section through anterior sphenoid. × 3 diameters.
Fig. 11. Section through hinge of lower jaw. × 2 diameters.
Fig. 12. Section through periotic mass and basioccipital. × 3 diameters.

PLATE XXXVI.

Seventh Stage.—Pig at birth.

Fig. 1. Under view of snout-cartilages. × 5 diameters.
Fig. 2. Auditory capsule, hyoid, and occiput. × 3½ diameters.
Fig. 3. Auditory chain of bones. × 5 diameters.

Eighth Stage.—Pig, 6 months old.

Fig. 4. Lower view of skull. Natural size.

PLATE XXXVII.

Fig. 1. Side view of same. Natural size.
Fig. 2. End view of same. Natural size.
Fig. 3. Side view of lower jaw. Natural size.

X. *Contributions to the History of Explosive Agents.*—Second Memoir.

By F. A. Abel, *F.R.S., Treas. Chem. Soc.*

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Since the publication of the experiments on the explosion of gun-cotton and other compounds and mixtures by detonation, which are detailed in a memoir submitted by me to the Royal Society in March 1869 *, this matter has received considerable practical development, and has also been made the subject of further scientific investigation, both in England and on the Continent. In continuing my researches into the conditions to be fulfilled for accomplishing the detonation of explosive substances, I have arrived at further results confirmatory of, and additional to, those described in my former memoir. I have also been led to pursue experiments bearing upon this subject in somewhat new directions; and I venture to believe that an account of the results arrived at may possess some value as tending to throw further light upon the behaviour and properties of explosive agents.

The exceptional behaviour which I have described as being exhibited by certain explosive compounds when applied to the development of detonation, in comparison with other substances, to which they were not inferior as regards the force and heat developed by their explosion, has been confirmed by further experiment. It was stated by me that 0·32 grm. (5 grains) of mercuric fulminate, if applied under favourable conditions, suffice to develop the detonation of compressed gun-cotton, that 3·25 grms. (50 grains) of chloride of nitrogen appeared to be the minimum amount by which detonation of gun-cotton could be developed, that 6·5 grms. (100 grains) of iodide of nitrogen failed to produce this result, and that repeated trials with different quantities of nitroglycerine ranging up to 31·2 grms. (1 ounce) did not in any one instance result in the detonation of gun-cotton by that substance, although the mechanical force and heat developed by its explosion were at least fully equal to those brought into operation by corresponding quantities of the most violent of the above explosive agents.

The experiments with nitroglycerine have since been considerably extended by me. Vessels containing 62·4 grms. (2 ounces) and 124·8 grms. (4 ounces) of nitroglycerine have been placed upon disks (weighing 8 ounces) of compressed gun-cotton, and the liquid has been exploded by means of a fulminate-fuse. The only effects of the violent explosion produced were, in each case, the pulverization and dispersion of the mass of gun-cotton, some of the particles being occasionally ignited; but no

* Phil. Trans. vol. clix. p. 489.

symptom of detonation was developed *. Two ounces of nitroglycerine, in the form of dynamite (*i.e.* converted into a plastic mass by admixture with one fourth its weight of Kieselguhr), were also exploded in close contact with a cylinder of compressed gun-cotton; but the same negative result furnished by the preceding experiments was obtained. While, therefore, gun-cotton might be detonated by 0·32 grm. (5 grains) of mercuric fulminate, or by 3·25 grms. (50 grains) of chloride of nitrogen (of which substances only 0·07 grm.,=1 grain, and 0·1 grm.,=1·5 grain, are respectively required to detonate nitroglycerine), it was found not to be detonated even by the explosion in contact with it of 124·8 grms. (4 ounces) of nitroglycerine.

The obvious incompatibility of such results as these with the general conclusion (founded upon extensive and varied experiments with different explosive substances), that the facility with which an explosion accomplishes the detonation of such substances is proportionate to the mechanical force and heat developed by that explosion, led me to suggest that a similarity in character, or synchronism, of the vibrations developed by the explosion of particular substances might operate in favouring the detonation of one of such substances by the initial detonation of a small quantity of another, while, in the absence of such synchronism, a much more powerful initiative detonation, or the application of much greater force, would be needed to effect the detonation of the material operated upon. This view has been favourably regarded by many, as affording a reasonable explanation of the apparently anomalous results above referred to, and appears to have received some support from the results of certain interesting experiments recently instituted by MM. Champion and Pellet† with iodide of nitrogen and one or two other explosive substances. They observed, in the first instance, that the detonation of about 0·03 grm. of iodide of nitrogen at one extremity of a tube 13 mm. diameter and 2·4 metres (and even 7 metres) in length immediately determined the explosion of a similar quantity of the iodide placed at the other extremity. By inserting a pith-ball pendulum into the tube they demonstrated that the concussion transmitted through it was very slight. They also found that the detonation of small quantities of nitroglycerine, mercuric fulminate, or nitroerythrite at one end of the tube exploded the iodide placed at the other extremity; and that if the tube were divided, so as to introduce an interval of 5 or 6 mm. between the two parts, a much more powerful explosion was required to determine the detonation of iodide at the furthest extremity. Some experiments which they made, by attaching small quantities of iodide of nitrogen to the strings of a double-bass and causing these to vibrate, appeared to indicate that only a particular pitch or rapidity of vibration determined the explosion of the iodide, similar results being also obtained with vibrating plates of metal. These, and some other results obtained with the aid of parabolic mirrors, led them to conclude that the

* In *one* instance, among a large number of experiments, the detonation of two ounces of nitroglycerine in contact with a disk of compressed gun-cotton (supported by a wrought-iron plate) furnished a result which appeared to indicate that the gun-cotton had detonated.

† Comptes Rendus, tome lxxv. p. 210.

explosion of a detonating substance must be attributed to a peculiar vibratory motion, differing in character with the constitution and properties of the substance, and acting independently of the concussion and heat developed by an explosion. In a subsequent paper* MM. Champion and Pellet described some experiments in which they compared the effects produced upon a series, or scale, of sensitive flames by explosions of mercuric fulminate and iodide of nitrogen, the quantities of explosive substances and their distance from the flames being varied. The explosion of the fulminate was found to affect certain flames in the scale, leaving others unaffected, at a distance at which a corresponding quantity of iodide of nitrogen produced no effect upon any of the flames. When the distance of the explosion from the flames was reduced, the iodide affected those which represent the highest notes in the scale, while a corresponding experiment with the fulminate acted upon the entire series. By increasing the quantity of iodide used and diminishing its distance from the flames, the whole series was eventually affected by the explosion of that substance. Want of success in attempts to establish a difference between the effects of the explosion of mercuric fulminate and nitroglycerine upon the series of flames was ascribed by the experimenters to the limited range of the series (or of the analyser of vibrations); but they regard their results as having demonstrated that a marked difference exists between the character of vibrations developed by the explosion of iodide of nitrogen and mercuric fulminate, and that the kinds of vibrations developed by a particular explosive substance are modified by an augmentation of the quantity of material exploded, so that a definite relation should be observed between the susceptibility to explosion of the substance (such as gun-cotton or nitroglycerine) operated upon and the quantity of explosive substance required to produce the initiative detonation.

The results described by MM. Champion and Pellet have certainly demonstrated that different explosive agents, detonated under the same conditions, may differ importantly in regard to the character of vibrations which they develop; and their experiments with iodide of nitrogen afford some support to the hypothesis that a particular explosive agent is peculiarly susceptible to the disturbing influences of the class of vibrations which its explosion develops most readily, if not altogether to the exclusion of others. The observation made by them, that iodide of nitrogen, if employed in increased quantities, will develop eventually those vibrations obtained with small quantities of mercuric fulminate, which are not obtained with it when it is employed under the same conditions as the fulminate, is in complete accordance with the facts pointed out by me, that the chloride and iodide of nitrogen will only detonate gun-cotton when employed in very much larger quantities than the requisite amount of fulminate. It still remains, however, to be demonstrated why nitroglycerine, which is so readily detonated by a much smaller quantity of fulminate than has to be used with gun-cotton, and is also susceptible of detonation by small quantities of the latter material, is incapable of detonating gun-cotton, even when employed in comparatively overwhelming quantities.

* Comptes Rendus, t. lxxv. p. 712.

The employment of the iodide of nitrogen in experiments of the kind carried out by MM. CHAMPION and PELLET appears to me open to objection: first, because it is of uncertain composition, and consequently varies very considerably in sensitiveness and stability; secondly, because the quantities employed in experiments can only be approximately estimated, and hence experiments instituted with small quantities cannot possess great accuracy; thirdly, because it is so very readily exploded by vibrations imparted to the air by concussions of all kinds, that its use can scarcely afford much scope as a means of investigating the character and effects of different vibrations.

Hence, in carrying out a series of experiments on the transmission of the concussion or vibration developed by explosions, I have preferred to operate with explosive agents of thoroughly constant composition, and more readily and accurately applicable because less highly susceptible of explosion.

I.—ON THE TRANSMISSION OF DETONATION.

In describing the experiments instituted by them on the detonation of iodide of nitrogen at considerable distances through the medium of tubes, MM. CHAMPION and PELLET allude to an analogous experiment made by M. BARBE with dynamite. I have been unable to find a description of this particular experiment; but Captain TRAUZL, of the Austrian Engineers, has made numerous experiments on the transmission of detonation of a cartridge of dynamite by means of tubes to charges separated from it by considerable distances. A cartridge of dynamite, between 2 and 3 ounces in weight, was inserted into each extremity of an iron tube (a gas-pipe) 6 feet (1·82 m.) long and 1·25 inch (·031 m.) diameter; by the explosion of one of these charges with a detonating fuse, the detonation of the charge at the opposite extremity of the tube was accomplished, the two explosions being apparently simultaneous. A similar result was obtained by employing a much wider tube; and the practically simultaneous detonation of several charges, connected with a long tube by short branch-pipes at a distance of 2 feet from each other, was accomplished by inserting dynamite-cartridges into this long tube over the opening of each branch-tube, and detonating a charge of the material at one extremity of the tube. These interesting results induced me to institute a series of experiments, with the view of more closely examining into the transmission of detonation to widely separated masses of different explosive agents through the medium of tubes. The following is an account of the principal results obtained.

a. *Experiments with compressed gun-cotton.*

The first experiments were made with wrought-iron tubes (gas-pipe), 1·25 inch (·0317 m.) in diameter. One ounce (31·2 grms.) of gun-cotton, in the form of a 1-inch disk, was placed *against* each opening of the tube; the disk at one extremity was exploded by means of an electric detonating fuse. The length of tubes experimented with was gradually reduced from 1·92 metre (6 feet 3 inches) to ·91 metre (3 feet) before any transmission of detonation was obtained; the mass of gun-cotton at the

further end was shattered and dispersed, portions of it being sometimes inflamed as the length of the tube used approached 3 feet. With such a tube much of the gun-cotton at the far extremity was scattered in a burning state; but a *partial* detonation of the mass was obtained, as indicated by the shattering of the extremity of the iron tube against which it rested. The experiment was repeated with a tube of the same length (3 feet), the disks of gun-cotton being *just inserted* into each extremity: a partial detonation of the second disk again occurred, portions being dispersed and inflamed. Similar results were obtained with tubes ·760 metre (2 feet 6 inches) long, except that the portion detonated was more considerable, 4 inches of the iron tube being broken up at that extremity. In order to ascertain that the shattering of the tube at the opposite end to that in which the initiative detonation was produced was really due to a partial detonation of the gun-cotton, and not to the check experienced by the rush of gas through the tube upon its encountering the obstacle presented by the disk or plug of gun-cotton, the following experiment was made. A cylindrical plug of wood, fitting *loosely* into the tube, was inserted at one extremity, and a disk of gun-cotton (1 ounce =31·2 grms.) was introduced into the other end and detonated: the wooden plug was entirely broken up into small splinters, which were scattered about, but the end of the tube containing it was not injured. A similar experiment was made with a tube 6 feet 6 inches (2 metres) long, a wooden plug being inserted at one end, and a cartridge of dynamite weighing 2 ounces = 62·4 grms. exploded in the other extremity*: in this case also the wood was reduced to small splinters, but the tube at that end was not injured. The shattering of the tube at the far end in the preceding experiments was therefore conclusive evidence of a detonation of the gun-cotton, or of some

* A remarkable illustration upon a large scale, but analogous to these experiments with tubes, of the work which a wave of gas set in motion by an explosion will accomplish when resistance is opposed to it was furnished by an experiment made a short time back at Portsmouth in reference to the application of compressed gun-cotton to the rapid demolition of fortifications.

Among the old works required to be demolished was an arched "Counterscarp Gallery" of curved form, 250 feet in length, 7 feet wide, and of a total height of 7 feet 4 inches. The arch was of 120° and 18 inches thick; the back wall and part of the arched roof abutted upon solid earth; the front wall was from 5 feet to 5 feet 9 inches thick, and was pierced with nineteen oblong conical openings or "loopholes," the inner dimensions of which were 3 feet 2 inches by 5 inches. Each end of the gallery had a doorway 6 feet high and 2 feet 9 inches wide, provided with a wooden door, outside which was a grating composed of iron bars 1 inch thick, framed together so as to be 4 inches apart.

Three charges of compressed gun-cotton, weighing 20 pounds each, were arranged for simultaneous explosion in this gallery by means of electric detonating fuses; they were quite unconfined, being simply suspended side by side against the outer wall, between two of the loopholes, at a short distance from one extremity of the gallery. By the detonation of these charges the gallery at this end was destroyed to a distance of about 60 feet; and the brickwork was fissured to a much greater distance, the front wall being partly pushed forward up to a length of about 140 feet. At the other end of the gallery, where the wave of gas was checked or brought up, the destruction accompanied by the explosion was nearly equal in extent to that effected at the seat of the charges; the arch of the gallery was destroyed to a distance of about 70 feet, and the front wall was pushed forward to a length of about 80 feet. The bars of the iron grating which closed the entrance into the gallery at this end were twisted up into fantastic shapes, and projected to considerable distances.

portions, at the extremity most distant from the initiative explosion. With tubes 2 feet (·608 m.) in length, the gun-cotton disks being just inserted into each end, perfect detonation by transmission was attained. A variation of the mode of arrangement furnished the same result. One pound of compressed gun-cotton was placed in a pit dug in the ground; one extremity of a tube (1·25 inch in diameter and 2 feet long) was allowed to rest upon it, and was buried in the ground in this vertical position, the earth being firmly rammed round the charge and tube. A perforated disk of gun-cotton, weighing 1 ounce (31·2 grms.), and containing a detonating electric fuse, was just inserted into the open end of the tube, and exploded; the charge at the opposite end of the tube was detonated, a large crater being produced in the ground, and the iron being violently projected to a great height. In a corresponding experiment made with a tube 3 feet (·91 metre) in length, the buried charge was only inflamed by the detonation of 1 ounce of gun-cotton at the upper extremity.

With the employment of 2 *ounces* (62·4 grms.) of compressed gun-cotton as the detonating agent, experiments were made with tubes gradually reduced in length from 6 feet (1·82 metre) to 5 feet (1·53 metre); with such a tube complete detonation of the gun-cotton at the distant extremity was obtained *.

These experiments were subsequently repeated with the employment of stouter wrought-iron tubes, but of the same lengths and diameter (1·25 inch, ·031 m.) as those before used. In these the shattering and opening-up of the tube at the seat of the initiative detonation were less considerable, but increased effects as regards the transmission of detonation were obtained. The detonation of only 0·5 ounce (15·6 grms.) of gun-cotton, inserted into one extremity of a tube 2 feet long, detonated a charge in the opposite extremity—a result which it required 1 ounce (31·2 grms.) to accomplish in the previous experiments, while the latter quantity, employed in the stouter tubes, induced detonation through a tube 3 feet 3 inches (1 metre) long.

Trials were made of wrought-iron tubes only 1 inch (·0254 m.) in diameter, and of similar thickness to those first used in the experiments with the wider tubes; no detonation was transmitted, when the disk of gun-cotton was simply placed *against* the opening of one extremity of the tube, on the detonation of 1 ounce just at or within the other extremity; but when *both* charges were inserted into the extremities, the detonation was transmitted to a distance of 3 feet with the employment of 1 ounce of gun-cotton, while with the wider tube of similar thickness complete detonation was only obtained with certainty at a distance of 2 feet (·608 m.).

In other experiments wrought-iron tubes of larger diameter were used. A stout wrought-iron tube, 1·75 inch (·45 m.) in diameter and 3 feet (·91 m.) in length, had a charge of 3 ounces of gun-cotton (1 inch, ·025 m. diameter) just inserted

* In these experiments, when detonation of the *distant* charge was obtained, the destructive action upon the iron tube was always greatest at that extremity; the sudden obstruction of the wave of gas by the induced detonation was evidently productive of greatly increased destructive effects from the point at which the opposing columns of gas met.

into one extremity; its detonation did not induce that of a disk of the same diameter inserted into the other extremity of the tube; whereas the detonation *was* effected by the explosion of only 1 ounce (31·2 grms.) in the further extremity of a tube the same length and substance, but only 1·25 inch (·031 m.) in diameter. With a tube of the larger diameter (1·75 inch) and 2 feet 3 inches (·684 m.) long, the detonation of 2 ounces of gun-cotton of the above diameter in one extremity induced *partial* detonation in the other extremity. In these experiments the diameter of the tubes was 0·75 inch (·019 m.) larger than that of the disks of gun-cotton inserted in them; had the diameter of the latter corresponded more nearly to that of the tube, detonation would have been effected by transmission through a greater length, as was demonstrated by several experiments; thus 2 ounces of gun-cotton, 1·75 inch (·044 m.) in diameter, detonated in one extremity of a tube 3 feet (·91 m.) long and 2 inches (·05 m.) in diameter, accomplished the detonation of a disk of the same diameter inserted in the other extremity.

Attempts to explode gun-cotton through the medium of tubes of considerably greater width were not successful. Cast-iron tubes, 3 inches (·076 m.) in diameter and of different lengths, from 5 feet (1·52 m.) to 2 feet (·608 m.), were employed, and 2 ounces (62·4 grms.) of gun-cotton were detonated in one extremity; disks 1 inch (·025 m.) in diameter, inserted into the other extremity of these tubes, were not detonated. It appeared probable that the complete shattering and dispersion, on the instant of the explosion, of those portions of the cast-iron tube which were at the seat of the detonation might operate against the effective transmission of the concussion through the tube (and such was demonstrated to be the case in corresponding experiments with dynamite): the experiments were therefore repeated with the employment of *wrought*-iron tubes 2·75 inches (0·069 m.) in diameter, ranging in length from 6 feet 6 inches (2 m.) to 2 feet 10 inches (·858 m.); but only negative results were obtained; the detonation of the charge in one extremity of the tubes shattered the gun-cotton disk in the opposite extremity, inflaming portions, but the latter was in no instance detonated.

A few experiments were made with tubes of different materials, with the view of examining into the influence exerted by such variation upon the transmission of detonation. A tube 1·25 inch (·031 m.) in diameter and 2 feet (·608 m.) long was constructed of several superposed layers of strong brown paper. The detonation of 1 ounce of gun-cotton inserted in one extremity did not accomplish the explosion of gun-cotton in the other end; the disk was dispersed in fragments, some of which were inflamed, and the tube was torn to pieces. Detonation would have been induced with certainty by that quantity of gun-cotton in a thin wrought-iron tube of the above dimensions, and in a stout tube of that material with half the quantity, as already shown. With employment of a lead tube of the given dimensions, but slightly stouter in substance than the thickest wrought-iron tube used, detonation was induced by means of 1 ounce of gun-cotton: the metal composing the tube was opened up and

distended into thin sheets at each end by the detonations. The experiments with tubes made of different materials were extended during the progress of this investigation, when silver fulminate was used; the above results, in addition to those obtained with iron tubes differing in strength, indicated, however, that the transmission of detonation is regulated to an important extent, in experiments of some magnitude, by the strength of material composing the tube and the consequent resistance which it opposes locally to the force developed by the initiative detonation.

The almost instantaneous manner in which the detonation appeared to be transmitted from the point of first explosion to the distant mass of gun-cotton, through the medium of tubes, led me to believe that the mechanical condition of the gun-cotton might, under these circumstances, be without effect upon the results obtained. If gun-cotton yarn or wool be struck with a hammer upon an anvil, unless the layer interposed between the support and striking body be very thin, repeated blows are required to accomplish the detonation of any part of it, as the force is mainly expended, in the first instance, in imparting motion to the particles of the mass, which becomes compressed, and thus reduced to a condition in which the particles offer great resistance to mechanical motion before the force applied can develop chemical metamorphosis. For the same reason*, if a fuse charged with mercuric fulminate, to an extent greatly in excess of that required to detonate a mass of compressed gun-cotton, be exploded in the centre of a mass of gun-cotton yarn or wool, freely exposed, or even if a small disk of compressed gun-cotton be detonated in contact with the loose material, the latter will not be detonated, but be simply dispersed in small fragments, with or without inflammation, because the particles of the wool or yarn are not in a condition to oppose resistance to the force applied, which therefore is expended in imparting motion to them. It can be conceived, however, that the blow, or gas-wave, to which a mass composed even of quite loose fibres is opposed may be so sudden in its operation that the particles which it first encounters undergo chemical disintegration before time can operate in causing the force to expend itself in imparting mechanical motion to the mass. In some experiments to be presently described, this action of a sudden blow upon those particles of a compressed mass of gun-cotton (placed so as to be perfectly free to move) which it first encounters will be found conclusively demonstrated (p. 361); but the same kind of action of the gas-wave upon a mass of uncompressed gun-cotton fibre was strikingly illustrated by one or two experiments with wrought-iron tubes of the kind described.

In the first instance, a loosely twisted thread or yarn of gun-cotton was wound pretty firmly into a ball of such size as to fit rather tightly into one extremity of a tube 1·25 inch in diameter and 2 feet (·608 m.) long. The detonation of a 1-ounce (31·2 grms.) disk of gun-cotton in the opposite extremity of the tube induced the detonation of the ball of yarn. Had the latter been in close contact with the detonated disk, freely exposed, it would simply have been scattered, some particles being probably

* Philosophical Transactions, 1869, vol. clix. pp. 497, 498, 501.

inflamed. The experiment was repeated with this difference, that the gun-cotton yarn was inserted into the end of the tube simply in the form of a loose and light plug; in this instance, also, the loose gun-cotton was detonated. The resistance to mechanical motion offered by the mass, supported by the sides of the tube, was sufficient, as opposed to the sudden rush of gas, to ensure the detonation of the loose material. In one experiment a piece of yarn, which formed part of the gun-cotton plug, was allowed to protrude outside the tube; this portion was projected in the air in a burning state, while the plug contained in the extremity of the tube was detonated.

The successive though practically simultaneous detonation of several distinct and somewhat widely separated masses of gun-cotton, through the agency of tubes, affords further demonstration of the great rapidity with which force is transmitted by these means. One or two instances will suffice as illustrations. A tube 1·25 inch (·031 m.) in diameter, 6 feet 6 inches (2 metres) long, had a disk weighing 1 ounce (31·2 grms.) inserted to a distance of 2 feet (·608 m.) from each extremity; similar disks were also just inserted into the openings; thus the tube contained four charges, each one separated from the nearest by a distance of 2 feet. On the detonation of the disk at one extremity only one explosion was heard; but all the charges were detonated, the tube being rent into many pieces.

A tube of the same diameter as the foregoing, but 10 feet in length, had four openings bored into it at intervals of 2 feet, into which were fitted short branch-pipes (1 inch in diameter) at right angles to the main tube. One ounce of gun-cotton was inserted into the latter, opposite each opening, and also into its open ends; half an ounce of gun-cotton was also inserted into the extremity of each branch-pipe. The arrangement therefore included ten distinct charges; those in the main pipe were separated from the nearest by spaces of 2 feet, and those in the branch-pipes were 1 foot distant from the corresponding charge in the long tube. Just as in the preceding experiment, only one sharp explosion was heard when the disk at one extremity of the main tube was detonated; all the disks were exploded, the action being apparently instantaneous. The tubes were rent into many and curiously contorted fragments, which were projected to considerable distances; and small craters were formed, in the ground on which the tube-arrangement rested, at the seat of each charge.

These results rendered it a matter of considerable interest to endeavour to determine the velocity with which detonation is transmitted from mass to mass through the medium of tubes, and to compare it with that at which detonation is transmitted, in open air, by contiguous masses of gun-cotton. The results obtained will be given hereafter*.

* Numerous practical exemplifications of this mode of transmitting detonation have been obtained both with gun-cotton and dynamite; and some decided advantages appear likely to accrue from the application of this mode of exploding charges to certain blasting and mining operations. Thus it is obviously possible, by suitable arrangement of tubes, to fire a number of charges with the practical effect of simultaneous explosions. The Austrian engineers have already availed themselves usefully of this method of firing charges in the application of dynamite to purposes of demolition. Again, it has been demonstrated by the author, in actual

b. *Experiments with dynamite.*

The cartridges used consisted of 2·5 ounces (78 grms.) of dynamite, made up into plastic rolls 4 inches (·1 m.) long and 1 inch (0·025 m.) in diameter, and wrapped in waterproof paper.

The experiments were commenced with wrought-iron tubes 1·25 inch (·031 m.) internal diameter, and 6 feet (1·82 m.) long, a cartridge being just inserted into each end. Detonation was induced, as was the case with a nearly corresponding weight of gun-cotton, when the length of tube intervening between the initiative explosion and the other cartridge was reduced to 5 feet.

Other experiments with dynamite, corresponding to those carried out with gun-cotton in tubes of 1 inch and 1·25 inch diameter, as already described, furnished analogous results. Attempts were made to transmit the detonation developed from a charge of 2·5 ounces (78 grms.) of dynamite through cast-iron tubes 4 inches (·1 m.) in diameter, 5 feet 5 inches (1·64 m.), 3 feet 3 inches (1 m.), and 2 feet 2 inches (·666 m.) in length, the dynamite charges being 2 inches (·025 m.) in diameter. Only negative results were obtained; but on repeating the experiments with *wrought*-iron tubes ·069 m. (2·75 inches) in diameter, the detonation of the above quantity of dynamite produced detonation of a charge of that material in the opposite extremity of this tube, through a length of 5 feet 3 inches (1·59 m.). This was the only indication, but a very decided one, furnished by the tube-experiments, that detonation produced by dynamite is more readily susceptible of transmission to a distant mass of the same material, under severe conditions, than is the case with gun-cotton. The great difference between the results furnished by the cast-iron and wrought-iron tubes with dynamite was most probably due to the circumstance that the former, which were not strong, presented insufficient resistance at the seat of detonation to prevent a great escape of force, the concussion being therefore much less completely transmitted through the tube.

Results quite similar to those described as furnished by gun-cotton, in which detonation was transmitted to a number of distinct and widely separated charges, enclosed in tubes, have been obtained in experiments instituted some time since with dynamite by Captain Trauzl of the Austrian Engineers.

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mining operations, that modifications in the method of charging and firing blast-holes in rock &c. may be introduced with advantage in point of safety and expedition in working. Thus it is unnecessary to insert the fuse, with detonator attached, to the bottom of the blast-hole. After entering the principal part of the charge of gun-cotton or dynamite to the bottom, the remainder, with the fuse attached, may be just inserted a short distance into the hole, and the charge may then be exploded with full effect by firing the fuse; or, in holes of considerable depth in hard rock, the charge of explosive agent may with decided advantage be subdivided, a portion only being inserted to the bottom of the hole, and the remainder at intervals of 1 foot or more, the fuse, with priming charge, being just inserted into the opening as above pointed out.

c. *Experiments with mercuric fulminate.*

The fulminate employed in these experiments to furnish the initiative detonation was enclosed in strong tubes or cases of tinned iron, the openings of which were firmly closed by means of the electric fuse used for exploding the charge. The quantity of fulminate inserted into the opposite extremity of the tubes was, in all instances, 100 grains (6·48 grms.), and was loosely confined by being screwed up in moderately stout paper.

1. *Employment of wrought-iron tubes of* 1·25 *inch* (·031 *m.*) *diameter.*—The explosion of 5 grains (·324 grm.) of the fulminate, in one extremity of a tube 1 foot (·304 m.) in length, detonated the charge which was just inserted in the other extremity; but the detonation produced varied in violence, and it appeared as if 1 foot were about the limit of distance through which detonation was susceptible of transmission under the above conditions, as regards dimensions and force of initiative explosion. The explosion of 10 grammes of the fulminate did not induce detonation through a tube of double the length (2 feet = ·608 m.), and with 15·4 grains (1 grm.) the same negative results were obtained. On inserting the fulminate-fuse, containing 15·4 grains (1 grm.) of the substance, to distances of 6 inches, 3 inches, and 2 inches (·15 m., ·075 m., ·050 m.) into the tube 2 feet long, explosion of the fulminate inserted into the other extremity was always induced; but the effects were those of a violent detonation only when the fuse was inserted to the maximum distance (6 inches), leaving a length of 18 inches (·45 m.) through which detonation was transmitted. When tubes of the same diameter, only 18 inches in length, were employed, the fuse charged with 15·4 grains (1 grm.) of fulminate being just inserted into one extremity, the explosion induced in the other was not so violent as in the preceding experiments when the charges were separated by the same distance, the initiative explosion being, however, produced at some distance (6 inches) inside the tube employed. In these cases the transmission of the concussion was obviously favoured by the circumstance that the tube projected some distance beyond the seat of the initiative detonation, the loss of force by dispersion in other than the desired directions being thereby much reduced. With 23 grains (1·49 grm.) of the fulminate just inserted into one opening of a tube 2 feet (·608 m.) long, detonation was induced at the other extremity; and the same result was obtained with the above quantity of fulminate when tubes 3 feet (·91 m.) long were employed.

2. *With tubes* 1 *inch* (·025 *m.*) *in diameter and* 2 *feet long*, the detonation of 10 grains (·65 grm.) of the fulminate inserted into one extremity produced explosion, though not of a very violent character, at the other end. In one instance, among several experiments with these tubes and the above quantity of fulminate, violent detonation was induced, the conditions of the experiment being evidently just bordering on the limits of those essential to the development of detonation by transmission of the concussion.

Tubes of the same diameter and thickness, 5 feet (1·52 m.) in length, were next employed; the detonation of 20 grains (1·2 grm.) and 23 grains (1·3 grm.) inserted in one extremity did not inflame the fulminate enclosed in paper and placed in the other end; and the detonation of 25 grains (1·62 grm.) also failed in one instance even to produce inflammation; but in other experiments, under apparently the same conditions, the fulminate at the opposite extremity was either inflamed or exploded, though not with the violent action which a perfect *detonation* of the same quantity of fulminate would exert.

It would appear from these and similar results obtained with mercuric fulminate that the amount of the latter required to induce detonation of the same substance, under the conditions described, is generally in direct proportion to the length and diameter of the tube through which detonation is transmitted, except when its length is so reduced as probably to bring the fulminate operated upon within the range of the flash of fire, as well as of the blow given by the initiative explosion.

d. *Experiments with mercuric fulminate and gun-cotton.*

It was shown by me, in my former memoir on Explosive Agents*, that 0·32 grm. (5 grains) of fulminate, enclosed in a thin metal case, was required to develop the detonation of compressed gun-cotton, care being taken to secure *close contact* between it and the detonator. In accordance with the fact demonstrated in that memoir, that the sharpness of a detonation and its consequent power of *developing* detonation in other masses was dependent upon the degree of confinement or the strength of the envelope enclosing the explosive substance, I have since found that only 2 grains (0·13 grm.) of the fulminate are required to detonate compressed gun-cotton *with certainty*, provided the case in which it is enclosed be constructed of stout metal (sheet iron), the detonator being so applied as to be closely surrounded by the mass to be detonated (*i. e.* inserted into a perforation in the piece of compressed gun-cotton). If there is not close contact between the two, a considerable larger proportion of the fulminate, confined as above described, is needed to ensure detonation; and in actual practice, when it may frequently be difficult to ensure close contact of the detonator with even some small portion of the charge to be exploded, it is found advisable to use about 1 gramme (15·45 grains) of the fulminate in the detonating fuse.

In attempts to transmit the detonative force from a confined fulminate-charge (or "detonator") to gun-cotton through the agency of tubes, as in the experiments described, somewhat remarkable results were obtained. Contrary to expectation, it was found impossible to accomplish this result through the medium of a tube 1 inch (·025 m.) in diameter and only 1 foot (·304 m.) in length, by the detonation of even so large a charge as 108 grains (7 grms.) of fulminate inserted into one extremity, the gun-cotton being introduced into the other end. With a charge of 154 grains (10 grms.) only *partial* explosion of the gun-cotton was effected through a tube of

* Philosophical Transactions, 1869, vol. clix. p. 498.

that length and diameter: a perfect detonation was in one instance produced when this charge was used in a tube only 9 inches long; but in others, with tubes of this length and diameter, the detonation was also only partial.

Through a tube only 6 inches long detonation was accomplished by means of 108 grains (7 grms.) of fulminate; but the result was doubtful with 100 grains (6·5 grms.). It therefore appears that in order to accomplish the detonation of gun-cotton through the medium of transmission afforded by a narrow tube, at a distance of not more than 6 inches, it is necessary to use at least fifty times the quantity of fulminate, strongly confined, which is required to ensure detonation when the "detonator" is in *close contact* with the charge. This result presents a marked contrast to the fact demonstrated by the experiments described in the transmission of the detonation of gun-cotton from one mass to another, through the agency of tubes,—that the detonation of 0·5 ounce (14·2 grms.) of compressed gun-cotton will induce that of another mass of gun-cotton if separated from it by a tube, of a particular diameter and thickness, 2 feet in length, the distance which thus separates the two masses being about ninety times greater than that through which the detonation of 0·5 ounce of gun-cotton, exposed in all directions, could accomplish the explosion of another mass of the compressed material.

If, however, the quantity of confined mercuric fulminate employed as the initiative charge be increased not very considerably beyond that (154 grains=10 grms.) which is only just sufficient to detonate gun-cotton through a tube 1 inch (·025 m.) in diameter and 9 inches long, very different results are obtained. 219 grains (0·5 ounce or 14·2 grms.) of confined fulminate were employed, in the first instance, in conjunction with tubes of 1·25 inch (·031 m.) diameter and 2 feet long, and the detonation of gun-cotton inserted in the opposite extremity of the tube was accomplished; on gradually increasing the length of the tubes (of the same diameter) employed, it was found, moreover, that the above quantity of fulminate would induce the detonation, with certainty, of gun-cotton through a tube *five feet* long, in which the same quantity of *compressed gun-cotton* would only induce detonation of the same substance, under the same conditions, through a distance of *two feet* (·608 m.). The power of the fulminate to develop detonation by transmission through considerable distances was still more strikingly demonstrated. Two tubes of the same diameter (1·25 inch) and substance, one of them 5 feet (1·52 m.) and the other 1 foot (·304 m.) long, were placed on the ground end to end, so as to form a channel 6 feet (1·82 m.) long. The two ends of the tubes were brought together as closely as possible, and they were then covered with a piece of paper, over which was heaped a little sand. The confined charge of 0·5 ounce (14 grms.) of fulminate was inserted into the open end of the 1-foot tube, and a disk of gun-cotton into the far extremity of the 5-foot tube. The latter was not detonated, but shattered and partly inflamed by the detonation of the fulminate; but on employing 1 ounce (28 grms.) of fulminate and substituting a tube 2 feet (·608 m.) long for the 1-foot tube, so that the total length of the channel was 7 feet (2·1 m.), the detonation of the gun-cotton was

accomplished, although the break in the channel, at the part where the two tubes were placed end to end, must undoubtedly have presented a great outlet of force transmitted, or a serious interruption of the gas-wave.

The observations made by MM. CHAMPION and PELLET, in their experiments on the effects of explosions of different quantities of iodide of nitrogen and mercuric fulminate upon series of sensitive flames, indicated that the vibrations developed by a particular explosive substance varied in character with the quantities exploded; and this appears to receive strong confirmation from the remarkable increase in the power of inducing detonation exhibited by mercuric fulminate, when the quantities detonated exceed certain limits.

On comparing with the foregoing results those obtained by reversing the relative positions of mercuric fulminate and gun-cotton, it was found that the fulminate is susceptible of detonation through considerable distances by comparatively small quantities of gun-cotton. A disk weighing 110 grains (0·25 ounce or 7·1 grms.) inserted in the extremity of a tube 5 feet (1·52 m.) long and 1·25 inch (·031 m.) in diameter, and exploded by means of a small detonating fuse, induced the detonation of the fulminate at the other extremity. The same result was obtained through a tube 7 feet (2·1 m.) long, and even when a channel of this length was constructed by placing two tubes, the one 5 feet and the other 2 feet long, end to end, in the manner already described. It was not practicable to determine the minimum quantity of gun-cotton required to induce the detonation of mercuric fulminate, because the mechanical conditions essential to detonation of the gun-cotton itself cannot be fulfilled with any degree of certainty when smaller quantities than about 100 grains (6·5 grms.) of the material are employed.

e. *Experiments with silver fulminate.*

It was considered desirable to examine into some points connected with the transmission of detonation through the agency of tubes more accurately than was possible with the employment of large metal tubes and considerable quantities of explosive materials; with this view the silver fulminate was selected for purposes of experiment, as being one of the most definite and most manageable explosive compounds of highly sensitive character. In all the experiments, the carefully prepared and dried fulminate was placed in small paper boats, which were inserted into the extremities of the tubes used; the initiative charges were exploded by means of a platinum wire, which was imbedded in the material, and was suddenly raised to a red heat by the current from a sufficiently powerful voltaic battery. It was established, in the first instance, that the explosion of 0·5 grain (·033 grm.) of the fulminate in the manner described, when freely exposed to air, induced the detonation of a corresponding quantity of exposed fulminate with certainty at a distance of 3 inches (·076 m.), but that the attainment of this result was very uncertain when the distance was increased to 4 inches (·1 m.). It was next

ascertained, by repeated experiments, that the explosion of 0·6 grain (·038 grm.) of this particular batch of fulminate, inserted into one extremity of a stout (Bohemian) glass tube 3 feet 3 inches (1 metre) long and 0·44 to 0·48 inch (0·011 to 0·012 m.) in diameter, induced, with certainty, the detonation of fulminate inserted into the other extremity of the tube. The result was repeatedly, but not always, obtained by the explosion of 0·5 grain (0·33 grm.) in metre-tubes of the smaller (0·011 m.) diameter. No detonation was induced by employing 0·4 grain (·026 grm.) of this fulminate. As in the case of the larger experiments with gun-cotton &c., an increase in the diameter of the tube was found to reduce the distance at which detonation could be induced; thus 0·6 grain of fulminate exploded in one extremity of a tube 3 feet 3 inches (1 metre) long and 0·7 inch (·018 m.) in diameter did not detonate fulminate in the other extremity, while, as stated, the result was certain when the narrow tubes above described were employed under the same conditions and with portions of the same batch of fulminate. Only a slight increase of effect was obtained by confining the silver fulminate employed as the initiative agent. 0·4 grain (·026 grm.) enclosed in a copper capsule induced detonation through a metre-tube ·0126 m. (0·5 inch) in diameter, though not invariably; that quantity was therefore about on an equality with 0·5 grain (·033 grm.) of the same fulminate exploded without close confinement *.

The explosion of mercuric fulminate enclosed in a stout copper cap was found to be somewhat less effective in inducing detonation of this silver fulminate through tubes than a corresponding amount of the latter substance freely exposed. In one instance 0·5 grain (·033 grm.) of the confined mercury compound induced detonation through a metre-tube of ·0126 m. (0·5 inch) diameter, but in others detonation was only obtained when the tube was reduced to (31·5 inches) 0·8 metre; and in one experiment, although the silver fulminate was exploded, the glass tube at the seat of this explosion was only broken once across, instead of being shattered into small fragments, as in all other instances. It would therefore appear that in this experiment only a partial *detonation* of the silver compound had been developed, a result frequently obtained in the experiments on a larger scale with other explosive agents already described.

Comparative experiments were made on the power of transmitting detonation possessed by tubes of equal diameter and of as nearly the same thickness as could be obtained, but consisting of different materials. The following is a tabulated statement of the results obtained :—

* The silver fulminate, although always prepared with care by precisely the same process, was not always obtained of the same degree of sensitiveness. A particular series of experiments was therefore always made with one and the same batch of material.

TABLE I.

Nature of tube.	Dimensions.			Initiative charge exploded in one extremity (*silver fulminate*).	Result.	Remarks.
	Diameter.	Thickness.	Length.			
	" mm.	" mm.	" m.	grm. grm.		
Glass (Bohemian)	0·48=12	0·049=1·24 0·058=1·47	39 =1	0·5=0·032	Detonation induced.	The glass tubes were always shattered to small fragments to about 0·2 m. (7·9 inches) beyond the seat of the initiative detonation.
" "	" "	" "	" "	" "	No detonation.	
" "	0·5 =14	0·049=1·24	39 =1	0·6=0·039	Detonation induced.	Several times repeated with the same result.
" "	0·5 =14	0·049=1·24 0·058=1·47	33 =	0·4=0·027	No detonation.	" " " "
Pewter	0·5 =14	0·042=1·07 0·049=1·24	39 =1	1 =0·065	" "	The pewter tubes were always deeply indented, but not opened up, at the seat of the initiative detonation. When detonation was induced at the opposite end, the latter was always torn open, and the metal much distended.
"	" "	" "	35·5=0·9	" "	" "	
"	" "	" "	" "	" "	Detonation.	
"	" "	" "	31·5=0·8	" "	"	Several times repeated with the same result.
Pewter, bent in the centre pretty sharply at right angles.	" "	" "	31·5=0·8 (total length).	1 =0·065	"	Tubes of the same length, bent into different curves, were previously tried with the same result.
Brass	0·5 =14	0·042=1·24	39 =1	1 = "	No detonation.	The brass tubes were not even indented at the seat of the explosion.
"	" "	" "	29·6=0·75	" "	" "	
"	" "	" "	23·7=0·6	" "	Detonation.	
Paper	0·5 =14	0·058=1·47	39 =1	1 =0·065	No detonation.	The length of the tube was gradually reduced to 0·5 m.
"	" "	" "	19·7=0·5	" "	" "	The tube was torn at the seat of detonation to the length of about 1 inch.
"	" "	" "	11·8=0·3	" "	Partial detonation.	There was an explosion, but with comparatively little destructive effect.
Vulcanized india-rubber.	" "	0·058=1·47	39 =1	1 =0·065	No detonation.	
" "	" "	" "	19·7=0·5	" "	" "	The india-rubber was not torn in any one instance at the seat of the initiative explosion.
" "	" "	" "	15·8=0·4	" "	Detonation.	

The following points are indicated by the foregoing tabulated results:—

(1) Detonation was transmitted through glass tubes with very much greater facility than through tubes, of corresponding diameter and thickness, of any of the other materials tried. Employing nearly double the quantity of silver fulminate required to induce detonation with certainty through the glass tubes, it was only possible to obtain a similar result through a pewter tube 0·8 m. (31·5 inches) long, a brass tube 0·6 m. (23·7 inches) long, an india-rubber tube 0·4 m. (15·8 inches) long, and a paper tube 0·3 m. (11·8 inches) long.

(2) The difference in the results obtained was not ascribable to a difference in the escape of force on the instant of explosion at the seat of the initiative detonation, in consequence of the fracture of the tube, nor to the expenditure of force in work done upon the tube at that point, since the glass tubes were always destroyed by the first explosion to a very much greater extent than any of the others; and the brass tube, which was in no way injured at the seat of the explosion, did not transmit detonation to so great a distance as the pewter tubes, which were always deeply indented.

(3) The transmission of detonation would not appear to have been favoured by the sonorosity, or the pitch, of the tube employed, as the sonorous brass tube was not found to favour transmission of the detonation to the same extent as the pewter tube. This was corroborated by some special experiments with glass tubes, of the same dimensions as those described in the Table. A coating consisting of two layers of bibulous paper was firmly attached throughout by means of gum to the exterior of one of the metre-tubes; but the same result was obtained with it by the explosion of silver fulminate as with the uncoated tube. Another tube had pieces of tightly fitting india-rubber tubing placed upon the outside, until its sonorosity was reduced to a very low pitch; but, when used in this condition, it transmitted the detonation, developed by 0·6 grain of fulminate, as readily as the naked tubes.

The principal if not the only cause of the great difference exhibited, in power of transmitting detonation, by these tubes composed of different materials appears to have been satisfactorily demonstrated by some experiments which will be presently described.

II.—INTERFERENCE WITH THE TRANSMISSION OF DETONATION BY TUBES.

Attempts have been made by me, and with some success, to demonstrate by experiment that when the limit of distance has been reached to which a tube of a particular diameter will transmit the force developed by a detonation, of a particular kind or magnitude, with sufficient completeness to induce detonation at the most distant part of the channel, the interposition of impediments in the path of the gas-wave, so slight as to be apparently incapable of opposing the transmission of force to any important extent, will effectually interfere with the development of detonation. Some examples of the experiments instituted in this direction, in which mercuric fulminate and silver fulminate were employed to produce the initiative detonation, are given in Table II. (p. 354).

The quoted experiments with iron tubes demonstrated that the interposition of a small loose plug of tow, or, better still, of finely carded cotton-wool, between the initiative detonation and the charge of explosive substance inserted into the opposite end of the tube will protect the latter from detonation under circumstances which, in the absence of the plug of loose material, just fulfil the conditions essential to detonation. The violence of the concussion or blow sustained by the substance while exposed to the action of the detonation, before motion is imparted to the entire mass by the rush of gas, is strikingly demonstrated by the following circumstance. The crystals of mercuric fulminate, which were *quite loosely* confined in thin paper, were found to be more or less completely crushed or pulverized on recovering the small packets (which were not exploded, but only projected to a considerable distance), when the tuft of wool had been interposed between them and the detonation produced at the other extremity of the tube.

In witnessing these experiments it was difficult to realize that the slight resistance

Table II.

No. of experiment.	Description of tube used.		Initiative detonator inserted into one extremity.	Explosive material inserted into the other extremity.	Obstacle interposed in the path of the gas-wave.	Result.
	Wrought-iron gas-pipe.					
	Length.	Diameter.				
1.	1 foot = ·3 m.	1″·25 = ·026 m.	5 grns. (·32 grm.) of mercuric fulminate confined in case of sheet-tin.	Mercuric fulminate, loosely wrapped in thin paper.	None	Detonation *induced*; tube shattered.
2.	,, ,,	,, ,,	,, ,, ,,	,, ,, ,,	None	Explosion of the loose fulminate, but no destructive effect upon the tube.
3.	,, ,,	,, ,,	,, ,, ,,	,, ,, ,,	A small plug of tow, sufficient to completely prevent the passage of light through the tube.	The loose fulminate was simply inflamed.
4.	,, ,,	,, ,,	,, ,, ,,	,, ,, ,,	,, ,, ,,	The same as in preceding experiment.
5.	,, ,,	,, ,,	,, ,, ,,	,, ,, ,,	A thinner plug of tow admitting the passage of light in several places.	Detonation induced.
6.	5 feet = 1·5 m.	1″ = ·025 m.	25 grns. (1·6 grm.) ,,	,, ,, ,,	None	Detonation, but apparently only partial; destructive effect not great.
7.	,, ,,	,, ,,	,, ,, ,,	,, ,, ,,	Plug of finely carded cotton-wool sufficient to prevent passage of light.	No inflammation. The small package of fulminate was projected to a considerable distance, and the *loosely* confined crystals were pulverized by the force of the initiative detonation.
8.	,, ,,	,, ,,	,, ,, ,, (the detonator inserted to a distance of 6 inches into the end of the tube, which had been cracked in the previous experiment).	,, ,, ,,	Small plug of carded cotton-wool ...	Same as above.
9.	(3 ft. 6 in. = ·77 m.	1″·25 = ·026 m.	15·5 grns. (1 grm.) mercuric fulminate, confined as usual, inserted into one extremity of tube.	,, ,, ,,	None	Detonation, but apparently only partial; very lit tle destructive effect. Experiment repeated, with the same result.
10.	,, ,,	,, ,,	,, ,, ,,	,, ,, ,,	Small plug of carded cotton-wool ...	No inflammation. The crystals of fulminate were crushed to powder. Experiment repeated, with the same result; in a third similar experiment the same result was produced as without the plug of cotton.
11.	6 in. = ·15 m.	1″ = ·025 m.	108 grns. (7 grms.) ,,	Disk of compressed gun-cotton.	None	Detonation induced.
12.	,, ,,	,, ,,	,, ,, ,,	,, ,, ,,	Plug of carded cotton-wool inserted so as to obtruct the passage of light.	The gun-cotton was dispersed.
	Bohemian glass.					
13.	39 in. = 1 m.	0″·44 = 0·011 m. 0″·48 = 0·012 m.	0·5 grn. (0·032 grm.) of silver fulminate placed in an open paper boat, exploded by heated platinum wire.	0·5 grn. (0·032 grm.) of the same silver fulminate in open paper boat.	None	Detonation induced, but not invariably, in repetitions of the experiments with corresponding quantities of the same fulminate.
14.	,, ,,	,, ,,	0·6 grn. (0·039 grm.) of the same fulminate applied in the same way.	,, ,, ,,	None	Detonation induced, and the experiment several times repeated with the same result.
15.	39″ = 1 m.	0″·44 = 0·011 m.	0·6 grn. (0·039 grm.) ,,	,, ,, ,,	A diaphragm of thin bibulous paper inserted into the tube halfway between the two charges of fulminate.	No detonation induced. The fulminate in the far extremity of the tube was blown away, and the paper diaphragm was torn in several pieces, which were projected to a distance of 0·2 m. along the tube.
16.	,, ,,	,, ,,	1 grn. (0·065 grm.) ,,	,, ,, ,,	,, ,, ,,	No detonation. The fragments of the diaphragm were projected further towards the distant opening of the tube.
17.	,, ,,	,, ,,	1·5 grn. (0·097 grm.) ,,	,, ,, ,,	,, ,, ,,	No detonation. The diaphragm was very much broken up.
18.	,, ,,	,, ,,	2 grns. (0·13 grm.) ,,	,, ,, ,,	,, ,, ,,	No detonation. A very considerable length of the tube was shattered by the initiative charge.
19.	,, ,,	,, ,,	3 grns. (0·195 grm.) ,,	,, ,, ,,	,, ,, ,,	In this instance the tube was shattered by the initiative detonation to a length of about 0·6 m., and therefore beyond the seat of the paper diaphragm.
20.	,, ,,	,, ,,	0·6 grn. (0·039 grm.) of a different batch of silver fulminate applied in the same way.	,, ,, ,,	None	No detonation. This fulminate was decidedly less sensitive than that used in the preceding experiments.
21.	,, ,,	,, ,,	0·7 grn. (0·045 grm.) of this fulminate ,,	,, ,, ,,	None	Detonation.
22.	,, ,,	,, ,,	0·6 grn. (0·039 grm.) ,,	,, ,, ,,	No diaphragm, but the interior surface of the tube was coated throughout with a thin lining of French chalk, which was made to adhere to the glass by means of thin gum-mucilage.	No detonation; the looser portions of the French chalk lining were blown out of the tube.
23.	31″·2 = 0·8 m. (same tube as in experiment 22).	,, ,,	1 grn. (0·065 grm.) ,,	,, ,, ,,		No detonation; further portions of French chalk were blown out of the tube.
24.	19″·5 = 0·5 m. (same tube as in experiment 23).	,, ,,	0·6 grn. (0·039 grm.) ,,	,, ,, ,,		No detonation.
25.	14″ = 0·36 m. (same tube as in experiment 24).	,, ,,	0·6 grn. (0·039 grm.) ,,	,, ,, ,,		No detonation.

which the very light tuft of wool opposes to the transmission of force through the tube could modify its action to the extent described; the experiments with silver fulminate detailed in Table II. appear, however, to demonstrate conclusively that a slight retardation of the velocity of the gas-wave, or the expenditure of force in overcoming what appears to be only minute obstacles to the unimpeded transmission of the wave, suffices to interfere most materially with the transmission of detonation.

The interposition of a loosely fitting diaphragm of thin unsized paper between the "detonator" of silver fulminate and a charge of that substance inserted into the other extremity of a glass tube of a particular diameter and 1 metre long, prevented the transmission of detonation through the tube of that length until the quantity of fulminate employed as the initiative "detonator" was about five times that required, of the same fulminate, to transmit detonation under the same conditions, but with omission of the paper diaphragm. But it is not only by such an expenditure of force as is involved in overcoming the resistance which the latter opposes to the free passage of the gas-wave that the transmission of detonation is greatly impeded; the retardation in velocity which the gas-column may sustain in consequence of the friction established between it and particles of a fine powder attached to the sides of the glass tube will also greatly reduce the distance through which detonation of any given description and magnitude will be transmitted. In experiment 22 (in the foregoing Table) much of the French chalk, which was loosely attached to the interior of the tube, was carried away by the rush of gas; and a similar result, though to a diminished extent, was observed in experiments 23 and 24: yet the roughness of the interior surface of the tube was still sufficient, in experiment 25, to prevent the detonation of 0·6 grain (0·039 grm.) of silver fulminate from detonating the fulminate in the opposite extremity of the tube only 14 inches (0·36 metre) long; while a corresponding amount of the same batch of fulminate induced detonation through a glass tube of the same diameter, 35 inches (0·9 metre) long, presenting the usual smooth inner surface.

Discrepancies which were not unfrequently observed in results obtained with wrought-iron tubes in the experiments upon a larger scale with mercuric fulminate and with gun-cotton, were now clearly traceable to differences in the degree of roughness of the inner surface of the tubes, and to the consequent variation in the resistance opposed by those surfaces to the passage of the gas-wave. Moreover, the above results obtained with the glass tube coated inside with powder suggested some experiments which clearly demonstrated that the great difference observed (as shown in Table I.) in the transmission of detonation by tubes consisting of different materials was, at any rate chiefly, ascribable to the resistance which the inner surfaces of those tubes opposed to the free rush of gas through them. A paper tube was constructed of the same dimensions and thickness as those employed in the preceding experiments, but the inner surface consisted of glazed paper instead of ordinary brown paper. The interior of this tube was not uniformly smooth throughout like the ordinary glass tubes used; but still it presented a marked difference to the paper tubes with rough interior used in the first

instance, as regards the readiness with which it transmitted detonation. One grain (·065 grm.) of the fulminate, when detonated in one extremity of the tube 0·9 metre (35 inches) long, induced the detonation of fulminate in the other extremity; whereas a similar result was only obtained with paper tubes 0·3 metre (12 inches) long, the interior surfaces being composed of the comparatively rough brown paper. Again, a piece of brass tube, of the same description as that employed in the preceding experiments, the inner surface being dull and somewhat rough, was brightly polished inside. In this condition the tube transmitted detonation, developed by 1 grain (·065 grm.) of silver fulminate, through a length of 36 inches (0·93 metre); whereas in the original condition, with the somewhat rough interior, it only transmitted the detonation developed by 1 grain (·065 grm.) of the same batch of fulminate, with certainty, through a length of 19 inches (0·5 metre).

The tubes of paper and of brass may therefore be considered to have been placed nearly on an equality with glass tubes of the same dimensions, as regards their power of transmitting detonation, by employing them with smooth interior surfaces. It may not be impossible that the slight superiority of the glass tubes in their power of transmitting detonation may still have been entirely due to the establishment of less friction between their inner surfaces and the gas-wave.

The surfaces of the pewter tubes approached in smoothness those of the glass tubes; and it will be seen (by reference to Table I.) that these tubes did not differ from each other considerably as regards the facility with which detonation was transmitted through them. Several attempts were made to impart a smooth interior to the india-rubber tubes by coating them with varnishes of considerable body; no very satisfactory result was obtained; but the smoothest of the varnished tubes afforded decided indications that the transmission of detonation was favoured, though only to a comparatively slight extent, by the diminution in roughness of the interior. In some experiments one extremity of the tubes was lined with sheet copper just at the seat of the initiative detonation, but the results were not at all affected thereby. The conclusion appears warranted, that with india-rubber tubes the effect of the detonation upon the *material* composing the tube operates in a manner decidedly antagonistic to the transmission of detonation: one cause of this is, no doubt, the comparative readiness with which the *sides* of the tubes yield to the pressure of the gas-wave, and the consequent considerable lateral expenditure of force. The particles of the tube appeared themselves to be set in violent and irregular motion by the explosion; the tubes were always thrown into contortions, and violently jerked out of position, by the initiative detonation*; while with tubes of other materials only those portions were projected which were actually destroyed by the explosions, the tube itself remaining undisturbed.

* In some of the experiments these tubes were strongly attached to boards by means of strappings of wire placed at short intervals; but the results were not affected thereby.

III.—CONCLUSIONS REGARDING THE TRANSMISSION OF DETONATION BY TUBES.

The results which have been described on the transmission of detonation by tubes appear to have established the following points:—

1. The distance to which detonation may be transmitted through the agency of a tube to a distinct mass of explosive substance is regulated by the following conditions:—

(*a*) By the nature and the quantity of the substance employed as the initiative detonator, and by the nature of the substance to be detonated, but not by the quantity of the latter, nor by the mechanical condition in which it is exposed to the action of the detonation.

(*b*) By the relation which the *diameter* of the "detonator," and of the charge to be detonated bear to the diameter of the tube employed.

(*c*) By the strength of the material composing the tube, and the consequent resistance which it offers to the lateral transmission of the force developed at the instant that detonation is produced. This condition does not appear to affect appreciably the results produced by detonation on a small scale, but its influence becomes apparent in larger operations.

(*d*) By the degree of roughness of the tube employed for transmitting detonation, or, in other words, by the amount of resistance opposed to the gas-wave and the amount of force consequently expended in overcoming the friction between the gas and the sides of the tube, or other impediments introduced into the latter.

(*e*) By the degree of completeness of the channel, and by the positions assigned to the "detonator" and the charge to be detonated. It need scarcely be pointed out that if the tube be fissured, or much enlarged either at the seat of detonation or at any other part (*e. g.* if injured by the effects of a previous detonation), or if even a very slight break in continuity exists in the tube, the extent to which force is transmitted must be proportionately diminished. It is also obvious that if the detonator, or the charge to be detonated, be placed against the opening of the tube instead of being inserted into the extremity, the conditions are comparatively unfavourable to the development of detonation by transmission; on the other hand, if the detonator be introduced some distance into the tube instead of being simply inserted into one opening, the loss of force by lateral dispersion is considerably reduced, if not altogether obviated, and the gas-wave consequently retains detonative power at an increased distance from the starting-point.

2. The nature (apart from strength or power to resist opening up or disintegration) of the material composing the tube through which detonation is transmitted generally appears to exert no important influence upon the result obtained, so far as could be determined by the experiments described. At any rate the effects produced by differences with respect to smoothness of the interior of the tubes far outweigh those which may prove to be traceable to differences in the nature of material of which the tubes consist.

3. The results furnished by mercuric fulminate, when applied to the detonation of gun-cotton through tubes, showed that the effects produced by the detonation of a particular substance upon other explosive bodies may vary very importantly with the quantity of material detonated. Thus, when mercuric fulminate was employed in as large a quantity as 10 grms., it did not detonate gun-cotton with certainty through a tube ·225 m. (9 inches) long and ·025 m. (1 inch) in diameter, and 7 grms. were required to develop detonation through a tube ·15 m. (6 inches) long; but double that quantity (14 grms. = 0·5 ounce) transmitted detonation to gun-cotton through a tube 1·52 m. (60 inches) long. The latter result (*i. e.* when an initiative detonation of sufficient magnitude was developed) demonstrated that gun-cotton is much more susceptible of detonation at a distance, through the medium of a tube, by mercuric fulminate than by gun-cotton itself, as 14 grms. of the latter would only have developed the detonation of gun-cotton through about one fifth the distance under the same conditions; but when the detonation of *smaller* quantities of fulminate was applied, the result was reversed, the sensitiveness of gun-cotton to detonation by the fulminate being diminished to a remarkable extent. Thus the amount of the latter required to transmit detonation through a tube ·31 m. (12 inches) long was more than two thirds the quantity required for transmission of detonation to gun-cotton five times that distance, and about eighty times the quantity required to develop detonation when applied in close contact with the gun-cotton.

4. The experiments with gun-cotton and mercuric fulminate furnished, moreover, another instance, analogous to that of nitroglycerine and gun-cotton, of a want of reciprocity in the sensitiveness of two explosive agents to the concussions or vibrations which they develop. 10 grms. of the fulminate would not always develop detonation of gun-cotton through a tube ·31 m. (12 inches) long and ·025 m. (1 inch) in diameter, and 14 grms. were required to induce detonation through a distance of 5 feet; while 7 grms. of gun-cotton amply sufficed to develop the detonation of fulminate through a tube of the same diameter but 2·1 m. (7 feet) long, and having moreover a complete break in its continuity. Both nitroglycerine and mercuric fulminate are therefore far more susceptible of detonation by gun-cotton than the latter is prone to detonation by the vibrations which they develop.

5. When the conditions essential to the transmission of detonation are only on the verge of being fulfilled, or when some slight accidental impediment has arisen, a result is frequently produced which is intermediate between the sudden explosion distinguished as detonation, and simple mechanical disintegration and dispersion of the material (accompanied occasionally by inflammation of the particles).

IV.—DEVELOPMENT OF DETONATION, AS DISTINGUISHED FROM EXPLOSION.

In the preceding experiments many instances occurred in which the mass of gun-cotton operated upon was exploded with comparatively little destructive effect, portions being at the same time dispersed and occasionally inflamed. Similarly, the mercuric

fulminate, which was exposed to the concussion of a distant detonation transmitted through a tube, was frequently exploded in a manner quite distinct from the violent detonation developed in other instances. It has even occurred that the silver fulminate, which under all ordinary conditions detonates violently even when only one particle of a mass is subjected to a sufficient disturbing influence, has been exploded without the usual development of force by the transmitted effect of a detonation of mercuric fulminate; that is, the extremity of the glass tube into which a particular quantity of the silver compound has been inserted was simply broken once across, the paper boat in which the fulminate rested being only partly destroyed, instead of their being reduced to small fragments, as was usually the case with the same quantity of the substance.

In all these instances the violence of the concussion transmitted was obviously only just bordering upon that required for the development of detonation; and it appears most probable that only some small proportion of the mass of fulminate or gun-cotton was in a condition or position favourable to the operation of the explosive force transmitted through the tube. The remainder of the mass would then be dispersed by the gases developed from the detonated portion; in some instances the particles would be inflamed at the moment of their dispersion, in others they would even escape ignition. The latter appears to be always the case when gun-cotton is exploded by a blow from a hammer or falling weight. However carefully the arrangements are adjusted with a view to distribute such a blow uniformly over the entire mass struck, the concentration of a preponderance of the force applied upon some portion or portions of the entire mass appears almost inevitable; hence only a small proportion is actually detonated, the remainder being instantaneously dispersed by the gases suddenly generated while the weight is resting upon the support. This was well illustrated by the results of some very carefully conducted experiments, in which cylindrical masses of compressed gun-cotton, all of the same dimensions and density (1 inch diameter and 1 inch thick), were placed on and between two smooth brass plates upon a flat anvil, adjusted in a level position, and were submitted in that position to the blow of a falling weight (of 50 lb.), the striking surface of which was properly levelled and maintained in its adjusted position during the descent of the weight by means of guides. The small cylinder of gun-cotton (which had been originally produced by submitting the pulped material to a pressure of four tons on the square inch) was reduced to one third its original length by the fall of the weight from a height of 3 feet, but no detonation was produced; with a four-foot fall of the weight on another cylinder, a slight detonation was produced, but the principal portion of the gun-cotton was scattered; the results were quite similar in further experiments, in which the height of fall of the weight was raised by increments of 1 foot to 10 feet: the detonations were somewhat sharper when the weight fell from 12 feet and upwards; but in every instance, even when the weight was allowed to fall from the maximum available height (39 feet), only a small proportion of the gun-cotton was detonated, the remainder being violently dispersed in a finely divided

condition. Similar results were obtained in operating upon small slabs of gun-cotton ·0025 metre (0·1 inch) thick and ·025 m. (1 inch) square, which were placed between smooth and level bronze plates, and subjected to blows from a falling weight.

In some experiments made with the object of investigating, from a new direction, the action of a blow in producing explosion, some slabs and disks of compressed gun-cotton of different weights and thicknesses were fired at from a Martini-Henry rifle at distances ranging from 120 to 300 feet. In these experiments the impact of the bullet determined, in a few instances, the complete explosion of the mass; but in others, when circumstances combined to diminish the detonative power of the blow, a comparatively slight explosion was produced, and the greater portion of the mass fired at was violently scattered in small particles, which sometimes were inflamed. The complete or partial explosion of the mass of compressed gun-cotton was effected either when the thickness of the mass which was freely suspended in air was sufficient to cause it to oppose more or less effectual resistance to the penetrative power of the rifle-bullet, or when the slab of gun-cotton fired at rested closely against an iron plate. In the one case, the particles of gun-cotton actually struck by the projectile were effectually prevented from yielding to motion or mechanical dispersion at the *moment* of impact, by the support which the considerable surrounding mass of gun-cotton afforded them; in the other instance, the rigid iron support, or backing, of the thinner masses of gun-cotton operated in a similar way in causing the effects of the blow to be concentrated upon, or confined to, the portions of gun-cotton actually struck by the bullet. Hence the effect of its impact was in both instances quite similar to that of a blow from a hammer applied to some portion of a piece of compressed gun-cotton placed upon an anvil; the particles struck by the hammer are prevented from taking up the motion of the striking body by the rigid support or anvil; chemical disintegration or explosion consequently takes place instead of mechanical dispersion, which would occur if less resistance were afforded to the motion of the hammer or projectile. At the instant of explosion the particles of matter which are undergoing transformation are confined between the striking body and the support (whether the latter be of metal or of gun-cotton); the resulting gas suddenly generated therefore escapes under great pressure, and scatters the contiguous particles of gun-cotton, sometimes even before these, or any large proportion of them, can become inflamed. In rare instances, as above stated, the explosion of the entire mass of gun-cotton followed upon the impact of the bullet; such an explosion did not, however, give rise to the sudden development of force (and consequent destructive effects) obtained by the *detonation* of similar or even much smaller quantities of gun-cotton, as was demonstrated by placing the masses fired at upon iron plates, which remained uninjured, but would have been indented or fractured had the gun-cotton been detonated in the usual way when in contact with them. These comparatively feeble but complete explosions of the gun-cotton masses were most probably due to some very exceptional peculiarities in the physical condition of the disks or slabs, which

favoured the transmission of ignition from the point where explosion was produced by the blow to the whole of the surrounding portions of gun-cotton at the instant when they were undergoing dispersion. The resulting transformation of the solid into gas, though sufficiently rapid to produce the oral effect of an explosion, must be very gradual as compared to that which attends a detonation, and the destructive effects exerted must in consequence be comparatively unimportant*.

The manner in which resistance to mechanical motion favours the chemical disintegration or explosion of the portions of a compact mass of gun-cotton which is subjected to a blow, as from the impact of a bullet, was conclusively demonstrated by a series of experiments with gun-cotton disks of the same density and diameter, but differing in thickness, and therefore in weight. These disks had strings fastened round their circumference by which they were freely suspended; they were fired at from a Martini-Henry rifle with hardened lead bullets, the marksman being stationed at a distance of 100 yards from the gun-cotton which served as a target. Disks which weighed 4 ounces and 8 ounces were perforated by the bullet, not a particle of the gun-cotton being even ignited, and these results were repeatedly obtained. Disks weighing 12 ounces were inflamed when struck by the bullet, but not exploded; whilst disks weighing 16 ounces, fired at under the same conditions, were exploded, portions being, in some instances, dispersed in a burning state. The resistance opposed to the flight of the bullet, even by the 8-ounce disks, was insufficient to cause such retardation of the projectile during its penetration of the mass as to develop sufficient heat to inflame the gun-cotton; with the 12-ounce disk the resistance to motion offered by the particles in the bullet sufficed, during the penetration of the mass, to develop the heat necessary for its ignition; while the penetration of the bullet was opposed by the mass of the 16-ounce disk to a sufficient extent to cause the operation of the force conveyed by the projectile to be concentrated, at the instant of impact, upon the particles immediately in front of it, which therefore were suddenly transformed into gas or exploded.

By attaching a piece of soft wood, 0·4 inch thick, to one of the faces of a disk weighing 8 ounces, which was suspended as in the former experiments, the surface being fired at from a distance of 100 yards, the flight of the bullet was so far retarded by the resistance which the wood opposed in the first instance, that its subsequent penetration of the gun-cotton was effected comparatively slowly, and the heat developed by the further retardation of the bullet inflamed the gun-cotton, while in former experiments with unprotected disks of the same size these were perforated without any instance of ignition.

Wooden boxes containing compressed gun-cotton, both loosely and closely packed, have been repeatedly fired at from rifles; generally the contents of the box were inflamed, but in no instance was an explosion produced. Similar packages containing

* An interesting confirmation of this difference between explosion and detonation was obtained in subsequent experiments made with the view of determining the rate at which detonation is transmitted through tubes, which are described in the concluding portion of this Memoir.

dynamite or other nitroglycerine preparations were always violently exploded by being fired at, nitroglycerine being much more readily susceptible of detonation by a blow.

V.—INFLUENCE OF DILUTION, BY SOLIDS AND BY LIQUIDS, ON THE SUSCEPTIBILITY OF EXPLOSIVE COMPOUNDS TO DETONATION.

It has been pointed out in my former memoir (p. 513) that solid explosive mixtures, consisting of one or more readily oxidizable substances intimately incorporated with an oxidizing agent, are less readily susceptible of detonation than explosive compounds, and that the readiness with which their violent explosion can be developed through the agency of an initiative detonation is in direct proportion to their sensitiveness to explosion by percussion. Mixtures consisting of a powerful oxidizing agent (*e. g.* potassium chlorate) and a substance which is *per se* already endowed with explosive properties approach (if they are not quite equal) to gun-cotton and even to nitroglycerine, in the readiness with which their detonation may be effected; thus an intimate mixture of potassium picrate and potassium chlorate, in which the latter salt exists in the proportion required for the perfect oxidation of the former, may under favourable conditions be detonated by means of almost as small an amount of mercuric fulminate as the minimum required to detonate compressed gun-cotton*.

a. *Dilution with inert solids and with solid oxidizing agents.*

The extent to which the susceptibility to detonation of the more violent explosive compounds is affected by their intimate mixture with non-explosive substances is regulated partly by the physical or mechanical condition of the substance, and partly by the nature of the material with which it is mixed. Nitroglycerine may be very largely diluted by admixture with perfectly inert solid materials without diminution of its sensitiveness to detonation. Thus the preparation to which its inventor, A. Nobel, gave the name of *dynamite*, and which consists of nitroglycerine diluted with about one third its weight of the very bulky infusorial silica known as "Kieselguhr," whereby it is converted into a plastic material, requires no more powerful initiative explosion to ensure its detonation than the undiluted liquid: other preparations of similar nature, based upon Nobel's original idea of employing solid materials as media for the convenient application of nitroglycerine (and of which some contain not more than 20 per cent. of the explosive liquid), are not less sensitive to detonation. This is obviously due to the liquid nature of nitroglycerine, which permits of its being highly diluted with solid material, without isolation of different portions of the explosive by inert or much less explosive material. In the most diluted mixtures of this kind, provided they are not very carelessly prepared, each particle of the diluent is coated with a film of nitroglycerine, so that there is no break in continuity of the explosive agent in the

* Abel "On Explosive Agents," Phil. Trans. 1869, vol. clix. p. 513. Very interesting results have also been attained in a similar direction by Dr. Sprengel (*vide* Journal of Chemical Society, 1873).

mixture; hence when the initiative detonator is surrounded by such a mass it is in contact at all points with some portion of the nitroglycerine, and the latter is in continuous connexion throughout; detonation is consequently as readily established and transmitted through the mass as though it consisted entirely of nitroglycerine.

The case is different when a *solid* explosive compound is diluted with solid inert material. In an intimate mixture of the finely divided substances there must obviously be complete separation of the particles of the explosive at a number of points proportionate to the extent of dilution and to the state of division; a condition of things may therefore arise, within comparatively narrow limits, when the establishment or transmission of detonation is impeded, either by a diminution of the extent of contact between the exploding substance itself and the initiative detonator, or by the barrier which the interposed non-explosive particles oppose to the transmission of detonation, or by both causes. In some experiments made with intimate mixtures of mercuric fulminate and French chalk (selected for the purpose as being a bulky material), it was found impossible to detonate mixtures containing more than one fifth part by weight of the diluent by means of one grain (·065 grm.) of mercuric fulminate confined in a copper capsule and exploded in close contact with the mixtures; that quantity, similarly confined, sufficed to detonate undiluted fulminate through a tube 8 inches (·2 m.) long and 0·5 inch (·013 m.) in diameter. A mixture of four parts of fulminate and one of French chalk was exploded, without destructive effect, by the grain of confined fulminate fired in contact with it; when the diluent was reduced to one eleventh, the mixture detonated under these conditions, and it was also found to be detonated through a pewter tube 8 inches (·2 m.) long, like the undiluted fulminate. In this mixture the fulminate particles were no longer sufficiently separated to effect their ready detonation.

If an intimate mixture of finely divided particles of sensitive solid explosive compound and an inert solid diluent be compressed into compact masses, the mixture is considerably more susceptible of detonation than if it were in the loose condition; in this respect it resembles the undiluted material; indeed, so far as has been ascertained by experiments with gun-cotton, dilution may under these conditions be carried to a considerable extent, with little reduction in the sensitiveness of the material to detonation, provided the diluent consists partly or entirely of a soluble salt, as will be presently shown.

Intimate mixtures of finely divided gun-cotton with *solid oxidizing agents*, converted into compact and very homogeneous masses by compression while wet and subsequent desiccation, furnished interesting results when compared with pure compressed gun-cotton in regard to their susceptibility to detonation. Pure trinitrocellulose requires for the complete oxidation of its carbon the provision of 24·24 of oxygen for every hundred parts, in addition to that which its composition includes. This proportion of oxygen would be furnished by the admixture with that compound of 61·2 parts of potassium nitrate, 51·5 parts of sodium nitrate, or 61·8 parts of potassium chlorate. Gun-cotton, when prepared with the greatest care upon a manufacturing scale according

to the system now practised, contains not less than seven per cent. of lower nitro-compounds, so that even somewhat larger proportions than those specified of the above named salts would be required to completely burn the oxidizable elements. In order, therefore, to obtain the maximum amount of work from a given quantity of gun-cotton, that substance should be supplied with the additional oxygen capable of being furnished by even somewhat higher proportions than those named of the well-known oxidizing salts. Gun-cotton thus largely diluted with saline matter presents the form of very hard masses of uniform structure, with no tendency to lamination, if the salt is intimately incorporated with the finely divided or pulped fibre, and converted by powerful compression, with the aid of a saturated solution of the particular salt used, into masses of cylindrical or other forms, which are afterwards dried.

A careful comparison of the susceptibility to detonation of these compressed mixtures of gun-cotton and oxidizing salts, with that of simple compressed gun-cotton, has shown them to be on an equality with the latter in this respect. Detonators containing only 1 grain (·065 grm.) of mercuric fulminate, when inserted so as to fit tightly into perforations in cylinders consisting of ordinary gun-cotton and of the mixtures above described, never developed detonation by their explosion; the compressed masses were scattered; in the case of the simple gun-cotton ignition sometimes occurred, and always when the potassium-chlorate mixtures were tried; no instance of ignition occurred with cylinders of the "nitrate" mixtures. When detonators containing 2 grains (·13 grm.) of fulminate were employed in the same way, not only the pure gun-cotton cylinders, but also those consisting of the "nitrate" and "chlorate" mixtures were invariably detonated.

To compare with the foregoing results, some compressed cylinders, quite similar in density and hardness to those of the "nitrate" and "chlorate" mixtures, were prepared by substituting an inert salt (potassium *chloride*) for the other salts in corresponding proportions. The detonation of these could not be accomplished by means of detonators containing 2 grains of mercuric fulminate, which sufficed in the preceding experiments; but they were exploded with certainty by means of 3 grains (·192 grm.) of the fulminate.

The conclusions deduced from these results are as follows:—

1. Gun-cotton may be largely diluted with a non-explosive and perfectly inert solid substance with but little diminution of its sensitiveness, provided the mixture is in the mechanical condition most favourable to its detonation. If the explosive compound is thoroughly incorporated with a soluble salt, the mixture being then compressed into compact masses with the aid of the solvent (water) and dried, the material is obtained in a condition of greater rigidity, and therefore in a form more readily susceptible to the detonating effect of a small charge of fulminate, than can be attained by submitting the undiluted gun-cotton to considerable greater compression, because the crystallization of the salt, upon evaporation of the solvent, cements the particles composing the mass most intimately together. Hence the reduction in sensitiveness, due to the dilution of the explosive compound, is nearly counterbalanced by the greater rigidity imparted to the mass.

2. When the solid substance with which gun-cotton is diluted consists of an oxidizing agent, the predisposition to chemical reaction between the two substances so far increases the susceptibility to detonation that, operating in conjunction with the effect of the soluble salt in imparting rigidity to the mixture, it renders the latter quite as sensitive to the detonating action of the minimum fulminate-charge as undiluted gun-cotton is when in a highly compressed condition. Some indication that the most powerful oxidizing agent furnishes, under these conditions, the material most susceptible of explosion was afforded by the circumstance that the fuses containing only one grain of fulminate invariably inflamed fragments of the "chlorate" cylinders by their explosion, while the "nitrate" cylinders were always scattered without ignition*.

* Soon after the discovery of gun-cotton, attempts were made to increase the explosive force of that substance by impregnating it with solid oxidizing agents, such as potassium nitrate; but the quantity of the salt which could be introduced into preparations of gun-cotton by the only practical mode of treatment (namely, by impregnating these with a saturated solution and evaporating the water) was too small to render such treatment of any decisive practical value. The system of reducing gun-cotton to a fine state of division has afforded the means of readily incorporating this substance with the somewhat large proportion of saltpetre or analogous source of oxygen required for fully oxidizing the whole of the carbon in trinitrocellulose, and I have been successful in obtaining results of considerable practical importance in this direction. The general mode of producing "nitrate" or "chlorate" preparations of gun-cotton is as follows:—The requisite proportion of oxidizing agent, reduced to a very fine powder, is intimately mixed with the finely divided gun-cotton, with the aid of a saturated solution of the particular salt employed, and the mixture is granulated or compressed into any desired form by the usual pressure. Care is taken to make due allowance for the fluctuations in the amount of salt held dissolved by the water, consequent upon any change of temperature in the latter during the manufacturing operation, as well as for the extra amount of salt which will be deposited in the product by the evaporation of the solution left in it after the pressing or granulating. The products obtained in this way, especially when compressed, form very hard masses, which are much less liable to break up or dust when roughly handled than ordinary compressed gun-cotton. The gradual evaporation of the water from them during the drying process causes part of the salt to crystallize throughout the mass, and thus the particles composing it become so firmly cemented together, that the application of considerably less pressure than is required to produce very compact cakes of gun-cotton suffices to furnish masses decidedly superior in hardness and compactness. The cakes or granules, when dry, are found to have become coated with a hard film of the salt, which acts as an additional protective against mechanical injury, and also renders them less readily inflammable than simple compressed gun-cotton. It has been, moreover, conclusively demonstrated, by several experiments continued for considerable periods, that these preparations sustain continuous exposure to elevated temperatures without appreciable development of chemical change for much longer periods than the undiluted gun-cotton; the distribution of saline matter throughout the mass operates protectively by impeding the transmission and consequent further development of any minute change established by protracted exposure to heat in some particle of the mass of gun-cotton, which, however carefully prepared, cannot be absolutely uniform throughout in point of purity.

Although the attainment of the maximum work from a *given weight* of gun-cotton demands the supply of oxygen sufficient for the complete oxidation of the carbon, and although, therefore, the products obtained by incorporating gun-cotton with the full theoretical requirement of a chlorate or nitrate will develop considerably more explosive force than an equal weight of the simple gun-cotton, the most advantageous results are obtained, in actual practice, by employing somewhat less than the full theoretical proportions of the oxidizing agent.

Comparing the explosive action of *equal weights* of compressed gun-cotton and of the "nitrate" mixture prepared with the full proportion of the oxidizing agent (in which, therefore, about 38 per cent. of gun-cotton is

The identity in behaviour of the "nitrated" and "chlorated" preparations of gun-cotton, and of the ordinary material, when subjected to the detonative effect of mercuric fulminate, rendered it interesting to compare the behaviour of these materials when in contact with exploding nitroglycerine. Equal quantities of the several preparations (4 oz.) were employed in all these experiments. The disks (3 inches in diameter) were placed upon wrought-iron plates, all of equal dimensions and resting upon a firm anvil hollow in the centre. The nitroglycerine was contained in glass beakers which were placed upon the disks, and the detonating fuse was immersed in the centre of the nitroglycerine. The development of detonation was recorded by the indentation and cracking of the plate. When the explosion of the nitroglycerine simply dispersed the disk upon which it was placed, or only *exploded* the latter (as was several times the case in the course of these trials), no destructive action was recorded upon the plates which served as supports. The interposition of the disk of gun-cotton between the nitroglycerine charge and the plate served to protect the latter from injury, the force of the exploding nitroglycerine being to a great extent expended in pulverizing and dispersing the disk. Only in one instance, among several experiments, did 2 ounces of nitroglycerine develop the detonation of compressed gun-cotton; that quantity of the liquid detonated both "nitrated" and "chlorated" gun-cotton with certainty. One ounce of

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replaced by the salt), the increased work performed by the 62 parts of gun-cotton, with the aid of the oxidizing agent, will be found not quite equal to that obtained from the one hundred parts of pure gun-cotton; in other words, the loss of force due to the replacement of about one third of the gun-cotton by the salt used is not fully compensated for by the extra work obtained from the remaining two thirds of gun-cotton resulting from its complete oxidation. If, however, about three fourths of the theoretical amount of the salt be employed (referring specially to the potassium or sodium nitrate), the resulting products will perform fully the amount of work obtained from a corresponding weight of the undiluted gun-cotton; and as nearly one third of gun-cotton has been replaced in them by material of about one sixth its cost, a considerable advantage is gained in point of economy.

When equal volumes of highly compressed gun-cotton and of the "nitrate" or "chlorate" mixture, similarly compressed, are compared, the explosive force of the latter is much the most considerable. "Chlorated" gun-cotton is decidedly more violent in its action than the "nitrated" preparations; but it is more costly, on account of the comparatively high price of the salt, and because a larger proportion of the chlorate is required to furnish the requisite proportion of oxygen. It is, moreover, very susceptible of ignition by friction or percussion, and is therefore comparatively dangerous. For these reasons it does not compare favourably with the "nitrated" preparations. Of these, the mixtures with potassium nitrate are somewhat the most readily prepared; they, moreover, have but little if any more tendency to absorb moisture than pure compressed gun-cotton. The considerable advantage which the "nitrated" gun-cotton possesses in point both of cost and of power (especially when compared with equal volumes of compressed gun-cotton), added to the fact that it is as readily susceptible of ignition by detonation and possesses other valuable properties above pointed out, render it highly probable that this preparation of gun-cotton will be largely substituted for the ordinary compressed material in many of its applications. The circumstance that carbonic oxide, produced in considerable amount upon the explosion of trinitrocellulose, is present in the products of explosion of nitrated material in scarcely higher proportion than it exists in those of gunpowder, appears likely to remove the objection against the employment of gun-cotton in military mines, which arose from the large quantity of carbonic oxide developed when heavy charges of gun-cotton were exploded.

nitroglycerine, which in no instance detonated compressed gun-cotton, produced detonation of those materials in three out of four instances; in the fourth, with nitrated gun-cotton, the latter was exploded, but there was no destructive effect exerted upon the iron plate. Results similar to the latter were always obtained when 0·75 ounce and 0·5 ounce of nitroglycerine were employed; with these quantities ordinary compressed gun-cotton was never exploded; the disks were simply dispersed in minute fragments. It appears conclusively established by these experiments that the compressed mixtures of gun-cotton with potassium chlorate and potassium nitrate, prepared in the manner described, are decidedly more sensitive to detonation by nitroglycerine than gun-cotton itself in a highly compressed condition. In order to ascertain whether this difference was ascribable to difference in structure, *i. e.* to the greater hardness and rigidity of the gun-cotton preparations containing a large proportion of saline matter, some disks were prepared of an intimate mixture of finely divided gun-cotton and of the inert salt potassium chloride; the proportion of ingredients used corresponded to those existing in the "chlorated" and "nitrated" gun-cotton, and the same method of manufacture and extent of compression were adopted. Two ounces of nitroglycerine produced partial detonation of this material; a few finely divided fragments were recovered, but the iron plate sustained some, though comparatively little, injury. One ounce of nitroglycerine exploded the chloride mixture only partially, some portions escaping ignition; no indication whatever of the development of detonation was obtained with the employment of this quantity, while in the majority of instances the "chlorate" and "nitrate" mixtures were detonated by a corresponding quantity of the liquid.

It appears, therefore, that the ignition or explosion of the gun-cotton by the detonation of nitroglycerine is to some extent promoted or facilitated by the greater resistance which the material opposes to disintegration by the blow, consequent upon the increased rigidity which its incorporation with the salt imparts to it, but that the undoubtedly greater susceptibility to detonation of the "nitrate" and "chlorate" mixtures by nitroglycerine is chiefly due to some predisposing influence exerted by the oxidizing agent, arising perhaps out of the tendency to violent chemical reaction between it and the gun-cotton, when the conditions favourable to chemical activity are suddenly fulfilled.

b. *Dilution by inert liquids.*

If gun-cotton is diluted by impregnation with a *liquid* substance, or with a body solid at ordinary temperatures which is introduced into the mass in a liquid state, its sensitiveness to detonation is reduced to a far greater extent than by a corresponding weight of a solid substance incorporated *as such* with the gun-cotton. The obvious cause of this is just the converse of that which operates in preventing the reduction of sensitiveness to detonation of nitroglycerine by its considerable dilution with an inert solid. In the latter case, the explosive material envelopes the diluent, and occupies the spaces intervening between its particles; the continuity of the explosive material is consequently preserved throughout the mixture, and detonation is therefore readily esta-

blished and transmitted: while in the other case the *diluent*, which is liquid, or is at any rate first applied in the liquid state, envelopes each particle of the solid explosive agent, so that a complete casing (or film) of inert material surrounds each, isolating it from its neighbours; even therefore if the amount of diluent applied is only small, it must oppose comparatively great resistance to the transmission of detonation throughout the mass, either if it remain in the liquid condition or if it afterwards solidifies*.

If the dilution or impregnation with a liquid be carried to the full extent, the diluent will, in the case of gun-cotton, penetrate, at any rate to some extent, to the interior of the small particles of hollow fibre composing the compressed mass, and the deadening effect will thus be greatly increased.

Experiments have been made with the view of ascertaining how small a proportion of water, distributed through compressed gun-cotton, interferes with its detonation by the ordinary means employed in practice; *i. e.* by exploding a fuse containing about 15 grains (1 grm.) of mercuric fulminate, confined in a sheet-tin tube in contact with it. Cakes of gun-cotton of known weight were kept upon supports in a small chamber, the bottom of which was covered with a layer of tow, thoroughly wetted with water; their increase of weight, due to absorption of moisture, and their susceptibility to detonation were periodically ascertained. When 3 per cent. over and above the normal proportion (2 per cent.) of water had been absorbed, the detonation of the cakes was doubtful. Other specimens which were impregnated with oil, or soaked in melted fat until the latter had penetrated to the centre of the mass, could not be detonated by the explosion of the usual "detonator" (containing about 1 grm. of fulminate) firmly imbedded in them. The explosion of the freely exposed damp gun-cotton was, however, accomplished by greatly increasing the strength of the detonator (*i. e.* by employing a large amount of confined mercuric fulminate); and it occurred to my assistant, Mr. E. O. Brown, to apply the detonation of dry gun-cotton itself to the development of the explosive force of the compressed material when in a moist or wet state.

Gun-cotton, immediately on removal from the press, in which the wet pulp has been submitted to a pressure of not less than four tons on the square inch, retains about 15 per cent. of water. In this condition it cannot be burned by the simple application of fire to its surface, nor can it be detonated except by the employment of a fulminate "detonator" of very considerable power; but if a piece of air-dry compressed gun-cotton, weighing about 0·5 ounce (14 grms.), be placed *in contact* with it and detonated by means of two or three grains of confined mercuric fulminate, the moist gun-cotton is

* In the attempts made by me and by Mr. E. C. Prentice, a few years ago, to moderate the rapidity of explosion of gun-cotton sufficiently for its safe application to propulsive purposes, the most successful results were obtained by uniformly impregnating the compressed explosive agent with solid substances (*e. g.* india-rubber, collodion, paraffine, or stearine) applied with the aid of solvents which, on evaporation, deposit the diluent in the form of a film, continuous throughout the mass and completely enveloping each particle of gun-cotton. The rapidity with which explosion extends through a mass may be regulated to a considerable nicety by varying the strength of the solution of diluent employed or the thickness of the layer deposited between the particles.

violently exploded with absolute certainty. If the compressed material be soaked in water until thoroughly saturated with the liquid (in which case it will absorb about 35 per cent.), or if it be similarly saturated with oil or any other liquid, it may still be detonated, provided a sufficient quantity of dry gun-cotton be exploded in contact with it. In a series of experiments instituted with gun-cotton containing precise quantities of water, it was found that a compact cylinder of the air-dry material weighing 100 grains (6·5 grms.) sufficed to develop the detonation of compressed gun-cotton containing as much as 17 per cent. of water, provided it was detonated in close contact with some portion of the moist mass. Occasional failures occurred, however; and when the proportion of moisture was increased to 20 per cent., double the quantity of dry gun-cotton (200 grains=13 grms.) did not suffice to develop detonation, the result being certain only when about one ounce (28 grms.) of dry material was employed as the initiative detonator. When the gun-cotton had absorbed the maximum amount of water (30–35 per cent.) its detonation could not be absolutely relied upon with the employment of less than about four ounces (112 grms.) of air-dry gun-cotton applied in close contact.

On comparing the susceptibility of moist or wet compressed gun-cotton to detonation by confined mercuric fulminate and by dry compressed gun-cotton, freely exposed and exploded by means of the usual "detonator," the results were decidedly in favour of gun-cotton as the initiative agent. Thus with 12 per cent. of water the moist substance was only detonated once in seven experiments by a "detonator" containing 100 grains (6·5 grms.) of fulminate, whereas no failure ever occurred in detonating gun-cotton, containing that proportion of water, with 100 grains of the air-dry material; moveover the latter sufficed to produce detonation when the proportion of water amounted to 17 per cent., whereas this result could not be obtained with 150 grains (9·75 grms.) of fulminate, and appeared to be certain only when the detonating charge was about 200 grains (13 grms.).

The transmission of detonation from dry gun-cotton to a moist disk, through the agency of a tube, appears to take place, so far as two or three experiments showed, with the same facility as though the mass to be detonated were dry. One ounce of gun-cotton, exploded in one extremity of wrought-iron tubes 1·25 inch (·031 m.) diameter and 2 feet (·608 m.) long, induced the detonation of moist disks, containing 15 per cent. of water, inserted in the other extremity; and with a stronger tube of the same dimensions the detonation was, in one instance, transmitted to a distance of 3 feet. These results are quite parallel to those obtained with dry gun-cotton.

The propagation of the detonation of moist gun-cotton from one mass to another in open air, the pieces being ranged in a row, *in contact* with each other, takes place apparently with as much facility as with the dry material, provided the piece *first* detonated contain not less water than the others to which detonation is to be transmitted*; but this is not the case if even very small spaces be allowed to intervene between the individual masses, as is shown by the following experiments.

* For an interesting result bearing on this point see page 382 (footnote).

Four 9-ounce (280·8 grms.) disks of gun-cotton, containing about 30 per cent. of water, were placed on either side of a similar disk of air-dry gun-cotton, with spaces of 1 inch (·025 m.) intervening between it and them. The detonation established by the dry disk was only transmitted to the nearest on each side, the others were broken up and scattered by the explosion. With a row of pieces of air-dry compressed gun-cotton, 28 feet long, the disks used being 3 inches (0·75 m.) in diameter and ranging in weight from 2 to 2·5 ounces (56 to 70 grms.), and the *contact being continuous* throughout, detonation, when established at one extremity, continued to a distance of 42 feet (12·76 m.), the entire length of the train. In another experiment, similar in all respects, except that the disks employed ranged in weight from 2·5 to 2·7 ounces and were saturated with water, detonation extended likewise to the entire length of the train; the destructive effect, and the velocity with which the detonation travelled, being the same at the end as at the commencement. When disks of the same diameter, but of even higher density and of greater weight (consisting of 4·5 ounces of air-dry gun-cotton), were employed, saturated with water (containing about 30 per cent.) and arranged with intervening spaces of 0·5 inch (·013 m.), the detonation being established by the explosion of a 9-ounce (280·8 grms.) disk in contact with the first of the train, only two of the wet disks were detonated. In a second precisely similar experiment, except that the initiative detonation was produced by the explosion of 18 ounces (561·6 grms.) of dry gun-cotton, only the first five spaced disks were detonated; and in a third experiment, with disks of the same weight and containing the same amount of moisture, but separated only by intervals of 0·25 inch (·0062 m.), ten disks were detonated, 9 ounces of dry gun-cotton being used as the initiative detonator. It appears, therefore, that contact of the distinct masses is essential to the transmission of detonation through any considerable number, by wet gun-cotton, in open air. It has also been established by experiment that the same condition is essential, even when the wet gun-cotton is confined, unless the strength of confinement is sufficient to resist the destructive action of the initiative detonation.

It has been amply demonstrated by careful experiments that the gun-cotton preparations which have been described just now as "nitrated" and "chlorated" may be as readily detonated in the moist state as ordinary compressed gun-cotton, and under the same conditions. These preparations absorb less water, as might have been anticipated, than the comparatively porous masses of gun-cotton itself, even in its most highly compressed condition. The cakes of nitrated gun-cotton, when removed from the press, contain not more than about 8 per cent. of water, and the maximum amount which they will absorb does not exceed 28 per cent.

Numerous comparative experiments on a small and large scale have been instituted with moist and wet gun-cotton, both in the ordinary and "nitrated" forms, with the object of ascertaining whether the mechanical effects of its detonation differ from those obtained with these materials in the air-dry condition; and no evidence whatever has been obtained of a falling off in the work done by gun-cotton (or by the "nitrated"

preparations) when employed wet. It appears, therefore, that the diminution in the expansion of the gases generated, consequent upon the expenditure of heat in the conversion of water into vapour, must be counterbalanced by the additional volume of vapour which the water furnishes. Indications were, however, obtained, in the comparative experiments with large charges, that when gun-cotton (or the nitrated material) is employed in the wet state its detonation is somewhat sharper or more sudden than when used dry; and this is in accordance with the previous observation, that the less susceptible a mass of given explosive material is of compression, when submitted to the action of a sufficient initiative detonation, the more readily will detonation be transmitted, and the more sudden will the transformation take place from solid to gas and vapour. Hence, when the air is replaced by water, in a mass of compressed or nitrated gun-cotton, the transmission of detonation, when once established, is favoured by the increased resistance of the particles to mechanical motion*. The conclusions drawn from the behaviour of wet gun-cotton in practical trials on a large scale were subsequently confirmed in an interesting manner by the results of experiments on the rate with which detonation is transmitted by dry and wet gun-cotton.

The change in structure which a mass of wet compressed gun-cotton sustains when its temperature is sufficiently reduced to freeze the water, affects its susceptibility to detonation in a remarkable, though readily explicable, manner. When the water is crystallized, the particles of gun-cotton are no longer uniformly and completely enclosed in the diluent; but the dilution becomes similar in character to that of the mixtures of gun-cotton with soluble (crystallized) salts which have been described, and the wet gun-cotton (or rather gun-cotton diluted with solid water) becomes readily detonated by the ordinary means employed for exploding the air-dry substance. Thus gun-cotton cylinders in the moist condition in which they were removed from the hydraulic press, and others which had afterwards been soaked in water when frozen and reduced in temperature to $-9°{\cdot}4$ C. (17° F.), were found to be exploded with certainty by the fulminate-"detonators" ordinarily used for exploding dry gun-cotton.

Mercuric fulminate, and also the mixture of that substance with potassium chlorate and antimony tersulphide, which is employed in percussion-caps, were found to be readily detonated through the agency of dry confined fulminate when they were mixed with water in sufficiently large proportion to convert them into pasty masses. In the first instance, 5 grains (·325 grm.) of dry fulminate, enclosed in a capsule of sheet-tin, were found to detonate the wet fulminate and "cap-composition" when surrounded by these.

* In some recent submarine experiments, in which means were employed for accurately measuring the comparative force exerted by different explosions, produced under corresponding conditions (as to depth of submersion, confinement of charge, &c.), some gun-cotton of lower compactness than usual was found to furnish a somewhat low result when detonated in the dry state, but gave a decidedly high result when thoroughly saturated with water. In this case the comparatively porous gun-cotton, when its pores were filled up by water, was in a condition most favourable for sudden detonation.

It was afterwards found that a "detonator" containing 3 grains (·195 grm.) of dry fulminate would not explode the wet fulminate or "cap-composition" when fired at a distance of 2 inches (·05 m.) from them, but did so with certainty at a distance of 1·5 inch (·031 m.). When, instead of exploding the dry fulminate confined in a capsule or tube of sheet-tin, it was detonated by a sharp blow from a hammer (a naval "gun-hammer" being used for the purpose), the wet substance was detonated when just in contact with 5 grains (·32 grm.) of dry fulminate thus exploded; but it failed to detonate when removed to a distance of 0·25 inch (·0063 m.). The detonation produced by means of the hammer was therefore much less violent than when the same or a smaller quantity of *closely confined* fulminate was exploded*, no doubt because particles of the latter escape detonation by the falling hammer, being dispelled by the rush of gas from the portions first exploded.

Finely divided gun-cotton, made up into a paste or pulp with water, is not susceptible of explosion under conditions far more favourable than those just pointed out, which determine the detonation of wet mercuric fulminate. Wet gun-cotton pulp was placed in a cylinder of sheet-zinc open at the upper end; a disk of compressed gun-cotton, coated with waterproofing material, and fitted with an ordinary "detonator" (10 grains of mercuric fulminate enclosed in a tube of stout sheet-tin), was inserted into the pulp, so as to be completely surrounded by it. The explosion of this disk only dispersed the pulp, scattering the fragments of the zinc cylinder. This experiment was repeated with the same result; and no other effect was obtained when disks weighing 2 ounces (62·4 grms.) were detonated in the centre of the pulp, which was employed with various proportions of water. Very different effects were, however, produced by a simple modification of the manner of carrying out these experiments, as will be presently seen.

VI.—EMPLOYMENT OF WATER AS A MEANS OF TRANSMITTING DETONATION, AND OF APPLYING THE FORCE DEVELOPED BY EXPLOSION.

In experiments made upon the employment of wet compressed gun-cotton in submarine mines, the charges being enclosed in stout cases of wrought iron, it was demonstrated not only that gun-cotton containing 30 per cent. of water might be thus as efficiently applied as the dry substance, but also that it is possible to detonate masses of wet compressed gun-cotton, surrounding the dry initiative charge, when the interstices between the individual masses are filled up with water. The slight compressibility of the liquid does obviously not present an impediment to the sufficiently sudden transmission of the force, developed by detonation, to closely contiguous masses of gun-cotton and to others surrounding these, if the individual masses are separated from each other only partly and by small quantities of water. Provided the escape of force by transmission through the water be retarded, at the instant of the first detonation,

* The wet fulminate in close proximity to the dry material, which was detonated by a hammer or in a capsule, was always dispersed by the detonation; but it was collected by a receptacle surrounding the charge, and was thus proved not to have been exploded.

either by the resistance which the material of the case offers in which the gun-cotton disks are confined, or by the pressure of a considerable column of water, the detonation of wet compressed gun-cotton, immersed in water and separated from the initiative detonator and surrounding masses by thin layers of the liquid, can be accomplished with certainty. Results fully equal to those obtained with wet charges enclosed in stout metal cases have been furnished by charges closely packed together and confined round the initiative detonator by means of a case of thin sheet-tin, or a bag, or even by a simple fishing-net (being thus freely exposed to the surrounding water), provided they are immersed in water to a considerable depth. Charges of wet gun-cotton arranged similar to these, but in which the individual masses were not firmly enough held together, thus allowing of greater water-spaces between them here and there, and greater liability to movement, or which were submerged to comparatively small depths, failed to be detonated, even when comparatively large initiative charges of dry gun-cotton were employed, the wet disks being simply dispersed by the explosion.

The suddenness and completeness with which detonation was transmitted through small water-spaces, in the experiments with iron cases, suggested to my mind the possibility of applying water as a vehicle for the efficient employment of small detonating charges for bursting or breaking up cast-iron shells into numerous, and comparatively uniform, fragments, thus applying a shell or hollow projectile, of the most simple construction, to fulfil the functions of the comparatively complicated shrapnel or segment shells. The results furnished by experiments in this direction are interesting, and may prove of practical importance; they are fairly represented by the examples given in Table III. (p. 374). The shells experimented with were exploded by electric agency, being placed in a capacious iron chamber, or cell, lined with oak, specially constructed to admit of the collection of fragments of shells after their explosion. The charge of explosive material, fitted with the fulminate-fuse by which it was detonated, was either enclosed in a cylinder of thin sheet-iron or (in the case of gun-cotton charges) simply coated with waterproofing material; it was attached to the screw-plug of the shell, so that, when this was inserted and screwed into its place, the bursting-charge was fixed in the centre of the shell, being surrounded in the latter on all sides either by air or water. The fine wires by which the detonating fuse was fired were passed through a small opening in the screw-plug, which was then filled up with cement, so that the shell, when fitted, was closed as securely as possible. In subsequent experiments, in which the shells were fired from guns, the bursting-charges were fitted to the base of the concussion-fuse used in the service, so that the shells were absolutely closed when the fuse and charge were screwed into position.

These results afford interesting demonstration of the power possessed by water to transmit, uniformly in all directions, the force developed by an explosion, the destructive effect being proportionate not merely to the amount of explosive agent used, but also to the suddenness of the concussion imparted to the water by the explosion. They showed, besides, that a very slight flaw in the continuity of the resistance opposed in

TABLE III.

No. of expt.	Nature and weight of shell.	Nature of bursting-charge.	Shell, how filled.	Number of fragments furnished by the explosion.	Portion of the shell not recovered, broken up into very minute fragments.	Remarks.
	diam. lb. oz.	oz. grms.				
1.	Spherical shell $5\frac{1}{4}$" 16 3	Gunpowder..... 13 =368·54	The shell entirely filled with the explosive agent.	19 (2 fragments under 1 oz.)...	None.	
2.	" " " 16 1	" ..... 13 =368·54	" " "	18 (1 fragment " ")...	None.	
3.	" " " 16 $\frac{1}{2}$	" ..... 1·12= 31·75	Charge contained in the cylinder surrounded by water.	8 (2 fragment " ")...	None.	
4.	" " " "	" ..... 1·12= 31·75	Shell filled with water, charge in tin cylinder surrounded by the water.	None. The water and gases escaped through the small perforation containing the fuse-wires.	.....	A repetition of this experiment furnished the same result.
5.	" " " 16 $1\frac{1}{2}$	Picric powder..... 1 = 28·24	Charge in central tin cylinder, shell filled with water.	89 (37 fragments under 1 oz.)	1·5 oz. = 42·51 grms.	
6.	" " 5 12 1	Gun-cotton cylinder. 0·5 = 14·17	Cylinder fixed in centre of empty shell.	28 (3 " " ")	None.	
7.	" " " "	" " 0·5 = 14·17	Shell filled with water; charge surrounded by it.	254 (181 " " ")	2·75 = 77·94 grms.	Repetitions of these experiments furnished corroborative results.
8.	" " " "	" " 0·25= 7·08	" " "	141 (68 " " ")	None	
9.	16-pounder rifle-shell (cylindro-conoidal), weight 14 lb. 15 oz.	Gunpowder..... 16 =453·59	Entirely filled with the bursting-charge.	29 (2 " " ")	None.	
10.	" " "	" ..... 16 =453·59	" " "	32 (7 " " ")	None.	
11.	" " "	Gun-cotton cylinder. 1 = 28·34	Cylinder in the centre of the empty shell.	53 (16 " " ")	2·75 oz. = 77·94 grms.	
12.	" " "	" " 1 = 28·34	Shell filled with water, cylinder surrounded by it.	300 (236 " " ")	2 lb. 1 oz. = 953·52 grms.....	A repetition furnished a similar result.
13.	" " "	" " 0·5 = 14·17	Cylinder in the centre of empty shell.	25 (9 " " ")	2·5 oz. = 70·85 grms.	
14.	" " "	" " 0·5 = 14·17	Shell filled with water, the charge surrounded by it.	165 (87 " " ")	None	A repetition furnished a similar result.
15.	" " "	" " 0·25= 7·08	Charge placed in the centre of empty shell.	None. The gas escaped through the small perforation containing the fuse-wires.		
16.	" " "	" " 0·25= 7·08	The shell filled with water enclosing the charge.	121 (37 fragments under 1 oz.)	None	Repetitions of these experiments furnished corroborative results.
17.	" " "	Picric powder..... 15 =425·24	The shell entirely filled with the explosive agent.	52 (8 " " ")	2 lb. 14 oz. = 1304·07 grms.	
18.	" " "	" " ..... 2 = 56·68	The shell filled with water, which surrounded the charge enclosed in a tin cylinder.	138 (71 " " ")	1 lb. = 453·59 grms.	

all directions to the water (*i. e.* the existence of a very small vent which permits of an escape of water at the instant when explosion was established) sufficed to protect the shell against rupture, if the explosion in the latter was not of very sudden nature. Thus, on several occasions, 1·12 ounce of fine-grain powder, when exploded by means of a fulminate-fuse in a small shell filled with water, failed to burst the latter, the water and gases finding their escape through the very small opening in the fuse-plug which received the firing wires, and the luting of which did not offer any effectual resistance; but even when less than one fourth that quantity (0·25 ounce=7·8 grms.) of gun-cotton was detonated under the same conditions in shells of the same size, no instance occurred in which the shell escaped being broken into a large number of pieces, the suddenness with which the force was developed and transmitted by the water leaving no time for the small vent to exert any decisive influence upon the results.

It should be stated that the disintegration of the shells by 1 ounce, and even ½ ounce of gun-cotton, through the agency of water (as shown in Table III.), is far too complete to be of practical value, as the large number of very small fragments produced would be of no use as missiles. The fragments furnished by employing only 0·25 ounce of gun-cotton in the particular sizes of shells used were of much more serviceable nature. Even with this very small charge the shells were not merely burst into very numerous fragments, but these were also projected with considerable force.

The powerful effects obtained in those shell-experiments in which gun-cotton was employed, and the fact that detonation was transmitted from one mass of compressed gun-cotton to another through small water-spaces under the conditions described at page 372, led me to attempt the transmission of detonation from one mass of gun-cotton or dynamite to another in a tube, with the intervention of water. The tubes (of wrought iron, 1·25 and 1 inch in diameter) were fixed in a vertical position; the lower extremities were closed with plugs of wood and of clay, and in some instances with strong metal screw-caps; the gun-cotton or dynamite was placed at this extremity, and the initiative charge, consisting of 2 ounces of compressed gun-cotton or of dynamite, enclosed in waterproof material, was just immersed in the water at the upper extremity and was detonated in that position. The distance between the two charges of explosive material was reduced to 2 feet without detonation being transmitted; and it was evident that, under the conditions fulfilled by these experiments, the intervening column of water, even if much reduced in length, must act as a protective to the submerged explosive charge, the force being mainly expended in the opposite direction, where comparatively little resistance was opposed to it.

Other experiments, to which I was led by the remarkable effects produced in detonating small charges of gun-cotton in shells filled with water, furnished interesting and important results. In developing detonation in a perfectly closed and sufficiently strong vessel, which is completely filled with water (in addition to the small detonative charge), the resistance offered by the water, *at the instant* of detonation, may be regarded as

similar to that which would be presented by a perfectly solid mass. Similarly if, instead of water only, the strong vessel be completely filled with a mixture of a solid substance (*e. g.* a fine powder or a fibre reduced to a fine state of division) and water, such a mixture should also, at the instant of detonation, behave as a very compact solid with reference to the resistance it opposes, at the instant of explosion, to the detonating charge which it encloses. If this be so, then a mixture of even the most finely divided gun-cotton fibre with water, if enclosed in a strong receptacle, such as a shell, should be in a condition readily susceptible of detonation, because, at the instant of explosion of the initiative charge, the particles of gun-cotton offer great resistance to mechanical motion. The correctness of this conclusion has been fully established by experiments, which have demonstrated that, while it is indispensable to employ gun-cotton in a very compact or highly compressed form in order to ensure its detonation under all other conditions, it may, if enclosed in strong vessels, such as shells, be employed with equal efficiency in a finely divided state, provided the spaces between the particles be *completely* occupied by water, the small detonating charge being immersed in the aqueous mixture. The following experimental illustrations of this will serve to compare with the shell-experiments given in Table III.

Spherical cast-iron shells $5\frac{1}{2}$ inches (·138 m.) in diameter, and weighing about 16 lb., were filled with mixtures of finely divided gun-cotton, or pulp, and water. In some instances the pulp (containing no more water than remained in it after thorough drainage in a centrifugal machine) was firmly rammed into the shell, a central cavity being formed in the mass to receive the "primer" of dry gun-cotton; water was then poured in, so as to fill the shell completely, and ample time was allowed for it to soak thoroughly into the rammed pulp and expel the air retained by the latter. The "primer," which consisted of a cylinder of air-dry gun-cotton weighing 1 ounce (31·2 grms.), fitted on to the ordinary "detonator," and coated with waterproofing material, was then inserted into the cavity provided for it (displacing part of the water contained in the latter). It was attached to the screw-plug, as in the experiments given in Table III., by which the shell was closed as perfectly as possible when the loading was completed. In other experiments the pulp was mixed with water to the consistency of thick paste, and poured into the shell in this state; in others it was introduced in a still more dilute condition, the fitting of the shell being completed, in all cases, as above described. The shells thus loaded were broken up by the explosion of the detonator into a very large number of fragments, of which from 350 to 400 were recoverable, a considerable proportion of the shell being, however, almost pulverized and buried in the oak casing of the chamber. Many of the larger fragments were also driven into the hard wood with great violence, being very difficult to extract; and the effects (both as regards disintegration and violent dispersion) furnished by from 2 to 4 ounces of gun-cotton used in this way were far greater than those produced with the full charge (about 1 lb.) of gunpowder.

These experiments conclusively demonstrated that gun-cotton in a fine state of divi-

sion may be exploded by detonation, with certainty, when mixed with water in different degrees of dilution, provided such mixtures be enclosed in a receptacle of sufficient strength. The detonation of 2 ounces of compressed gun-cotton, immersed in such mixtures, was without effect upon them when they were only partially confined; but 1 ounce applied in the same way developed detonation when the mixtures were completely confined in the cast-iron shells. This result is not attained by the employment of only 0·5 ounce of compressed gun-cotton as the detonator; but more sensitive explosive agents than gun-cotton are susceptible of detonation, under the same conditions, by much smaller initiative charges. Thus 2 ounces of mercuric fulminate, placed in a 16 Pr. cylindro-conoidal shell, which was then filled up with water, was detonated by means of about 15 grains (1 grm.) of the fulminate confined in a sheet-tin case. Although the loose fulminate was in a small heap at the bottom of the shell, being separated from the lower extremity of the detonating tube (attached to the screw-plug of the shell) by a water-space of 0·75 inch (0·018 m.), the shell was very uniformly broken up; 170 fragments were collected in the chamber, and 1 lb. 11 oz. (842·4 grms.) were not recovered, being dispersed in minute fragments.

The susceptibility of gun-cotton (and of other explosive bodies) to detonation in shells, when employed in admixture with water, as described, appears to have afforded the means of overcoming the hitherto insurmountable difficulty of employing violent but comparatively sensitive explosive agents in shells, with safety to the gun from which these are fired. Various plans have been devised, but hitherto without success, for preventing charges of gun-cotton, in shells, from being exploded through the agency of the concussion to which the latter are subjected when fired from a gun, and the attempted application of nitroglycerine-preparations in shells has failed for the same reason. But mixtures of gun-cotton with water may be fired from guns in shells with absolute freedom from liability to premature explosion; and it is now no longer doubtful that the desideratum of a simple and safe system of applying the violently destructive effects of gun-cotton in shells will be attained as soon as a safe adaptation of the mercuric detonating arrangement to the fuse of the shell has been perfected.

VII.—ON THE VELOCITY OF DETONATION, OR THE RATE AT WHICH DETONATION IS TRANSMITTED.

The very satisfactory results obtained by the Government Committee on explosive substances in employing the chronoscope, devised by Captain A. Noble, F.R.S., for determining the time occupied by a projectile in traversing different intervals in the bore of a gun, led me to avail myself of this instrument for estimating the velocity with which detonation is propagated or transmitted under various conditions. The construction and mode of using this chronoscope have been described in the preliminary Report of the Committee on Explosions issued by the War Office in February 1870. A few words may serve to give a sufficient explanation of the instrument for present purposes. A series of thin metal disks (36 inches in circumference), the edges of which

are covered with white paper coated with lamp-black, are fixed upon one common shaft, which is driven at high speed by means of a falling weight, continually wound up, and a series of very accurately constructed multiplying-wheels. The speed usually attained by these disks (the precise rate of which is ascertained by means of a stop-clock) is about 1000 inches per second linear velocity at their circumference; so that 1 inch of the latter, travelling at that rate, represents the one-thousandth part of a second; and as, in reading off the records obtained, 1 inch is divided into a thousand parts by the vernier used, a linear representation is thus obtained of intervals of time as minute as the one-millionth part of a second. The uniformity of rotation of the disks during the duration of an experiment is ascertained by a series of observations of the speed, by means of the stop-clock. Each revolving disk is brought into connexion with one of the secondary wires of an induction-coil, and the other wire is attached to an insulated discharger, fixed opposite the edge of its corresponding disk, and adjusted so that its point is just clear of the latter. If, therefore, the primary circuit of any one of the induction-coils is interrupted (*e. g.* by suddenly severing the conducting-wire) the induced current must leap across, at that instant, from the discharger to the circumference of the disk, and will produce a small but distinct white spot on the blackened paper. As all the disks revolve with the same velocity, it is obvious that if the primary circuits of their respective coils be simultaneously interrupted, the spots produced by the discharge of the secondary currents on all the disks will be in a straight line, but that, if they be successively interrupted, the spots on the several disks will form a curve, varying in character with the intervals of time which have elapsed between the successive development of the respective secondary currents. In using this chronoscope to measure the rate of progression of a projectile in a gun's bore, the shot is made to sever the primary wires of the coils which it passes in succession, by means of cutters which slightly project in the bore, and which it therefore brings to bear upon the wires as it passes over them. In applying the instrument to the measurement of the speed of detonation in open air, or of the rate of its transmission through tubes, the only difference in the *modus operandi* consisted in the manner of effecting the rupture of the primary wires.

The disks of gun-cotton used in the experiments for determining the rate at which detonation is propagated in open air were ranged in a row or train upon the ground (or, rather, upon the level surface presented by a support of very thin narrow deal board); they consisted generally of disks 3 inches in diameter, varying somewhat in thickness in different experiments; they were either ranged in a continuous row, each disk touching the one on either side, or with equal spaces intervening between them. At the commencement of the row, or train, the wire forming the primary circuit of the coil connected with the first chronoscope-disk was tightly stretched across the part where detonation was to be first established. To facilitate the sudden severance of the circuit by the explosion, this part was composed of a fine copper wire, insulated by means of a silk covering, which was held in close contact with the gun-cotton surface, across which it was stretched by being fastened round the insulated heads of two hook-

staples driven into the ground on either side of the piece of gun-cotton. The circuit-wire of the coil belonging to the last (eighth) chronoscope-disk was similarly fixed across the further extremity of the train, and the other wires were stretched across different parts at intervals of 1, 2, 4, and 6 feet in different experiments.

In determining the velocity of transmission of detonation through tubes, wrought-iron gas-pipe of 1·25 inch (·031 m.) diameter was used; these pipes had very small perforations bored into them at the desired intervals, so that the fine insulated wires could be passed transversely through the centre of the tube at those points. The wires were rigidly stretched by being wound round staples on each side of the tube, and the small disks of gun-cotton to which, and by which, detonation was to be transmitted were inserted into the tubes so as to be in close contact with the wires*.

a. *Detonation of dry gun-cotton, arranged in continuous rows.*

The first three results obtained with continuous rows or trains of gun-cotton disks laid on their flat sides, exhibited considerable want of uniformity, *i. e.* the rate at which velocity was transmitted from one point to another (through distances of 1–4 feet, ·304 to 1·216 m.) appeared to vary considerably in different parts of one and the same train, as well as in different experiments; but this was evidently due to the employment of too stout a wire as the means of severing the circuit by the explosion. A much finer wire was therefore substituted for it; but even with the employment of the finest procurable insulated wire, stretched as tightly as possible across the gun-cotton disks, it was obviously impossible to avoid very slight variations in the degree of rapidity with which the wires were broken by the detonation. Making allowance for this source of error, it will be evident from the following example that, in a continuous row of 170 disks (3 inches, =·075 m., in diameter and about 0·9 inch, =·0225 m., thick, and of the average weight of 2·6 ounces, 81·1 grms.), the detonation, measured at intervals of 6 feet in a length of 42 feet (=12·76 m.), was transmitted with uniform velocity; the measurement at the far extremity of the train was practically identical with the rate at which detonation was transmitted through the first 6 feet.

				Rate of progression of the detonation per second.
Between	0′	and	6′	=17466 feet (5309·664 m.).
,,	6′	,,	12′	=16815 ,, (5111·761 ,,).
,,	12′	,,	18′	=17972 ,, (5463·488 ,,).
,,	18′	,,	24′	=16252 ,, (4940·608 ,,).
,,	24′	,,	30′	=17511 ,, (5323·344 ,,).
,,	30′	,,	36′	=16099 ,, (4829·70 ,,).
,,	36′	,,	42′	=17738 ,, (5321·45 ,,).

Mean=17122 feet=5136·60 m.

* In carrying on these experiments I have received valuable assistance, at different times, from Captain Singer, R.N., Major Maitland, R.A., and Captains W. H. Noble and Jones, R.A.

In an earlier experiment with the same description of disks, the mean velocity with which detonation was transmitted along a train 30 feet (9·12 m.) in length, the rate of travel being measured at intervals of four feet, was 16,871 feet (=5061·3 m.) per second; the rate of travel in the first four feet was 18,527 (=5558·1 m.), and in the last four feet 18,442 feet (5532·6 m.) per second. A third similar experiment gave a mean velocity of 18,234 feet (5470·2 m.) per second.

In these experiments the separate 3-inch disks were just touching each other at two points of their circumference. The average weight of gun-cotton disks used was 2·6 ounces, =10·4 oz. per foot (324·48 grms. per ·304 m.) of the train. In another experiment a continuous train of *uniform* dimensions throughout was constructed of solid cylinders 1·25 inch (·031 m.) in diameter and 1·5 inch (·038 m.) long, which were laid on their sides, with the ends of each one in close contact with one end of another cylinder, so that the entire train was in the form of a continuous cylinder weighing 8 ounces per foot (249·6 grms. per ·307 m.). The mean rate of transmission of detonation in this experiment was 18,868 feet (5660·4 m.); the rate of travel in the first four feet was 18,180 (5454 m.), and in last four feet 18,950 feet (5685 m.) per second.

In order to ascertain whether the rate of transmission of detonation would be affected by a very considerable reduction in the amount of compressed gun-cotton included in a given length of the train, cylinders of 0·9 inch (·019 m.) diameter were arranged in a continuous row, as in the preceding experiment; their weight corresponded to 3 ounces (93 grms.) per foot (0·304 m.) of the train, or less than one third that of the disks used in the first experiments (and it corresponded to the weight of nitroglycerine used in an experiment to be presently described). In one experiment with these small disks the mean velocity of detonation was =18,546 feet (5638 m.) per second; in another it corresponded to 20,000 feet (6080 m.) per second. It will be seen that in these, and also in the preceding experiments with larger cylinders placed end to end, the velocity of transmission was higher than when the large disks were employed which rested on their broad surfaces, and only presented to each other comparatively small points of contact at their circumference. It was to be expected that the rapidity of detonation would be promoted by increasing the contact surfaces of the individual masses, and thus rendering the train as nearly as possible continuous. The individual records of velocities obtained at the different parts of the train (24 feet, =7·296 m., long) composed of the very small disks of gun-cotton presented greater variations than in the other experiments with larger masses; this was no doubt caused by a greater variation in the degree of suddenness with which the wires were fractured, by the comparatively less violent explosions.

In the second experiment with the very small disks, about one half of the train was constructed of cylinders having a central perforation of 0·25 inch (0·006 m.) diameter, those which composed the first half being perfectly solid. There were indications of a somewhat more rapid transmission of the detonation along the perforated part of the train.

The foregoing experiments demonstrate that the rate at which detonation is transmitted

from mass to mass of dry compressed gun-cotton (the individual masses being in actual contact with each other) is between seventeen and twenty thousand feet (5168 m. and 6080 m.) per second, and that the rapidity of transmission is affected by the compactness or rigidity of the masses of gun-cotton, but not importantly by a difference in the form and arrangement of the individual masses of gun-cotton, nor by very considerable variations in their weight.

b. *Detonation of dry gun-cotton, with spaces intervening between the masses.*

A few experiments were made for the purpose of ascertaining to what extent the rate of transmission of detonation was affected by leaving small spaces between masses of compressed gun-cotton of a particular size, the spaces being insufficient to arrest detonation. Disks 3 inches in diameter, and ranging in weight between 2·3 and 2·6 ounces (71·76 to 81·1 grms.), were placed in a line with intervening spaces of 0·5 inch (·013 m.) between each disk. The train was 28 feet (8·51 m.) long, but the detonation stopped short at 22 feet, there being at this point a somewhat light and comparatively spongy disk. The mean velocity of the detonation was =16,935 feet (5080·5 m.) per second, the rate of transmission being 15,606 feet (4681·8 m.) per second in the first four feet, and 16,573 (4971·9 m.) in the last four feet of the train. A second experiment with the same description of disks, selected so as to present no great difference in weight and compactness, the disks being placed, as before, 0·5 inch (·013 m.) apart, in a row 28 feet (8·51 m.) long, gave results closely agreeing with the above, the entire train being, however, detonated in this instance. The mean velocity of the detonation was =16,776 (5032·8 m.), the rate of transmission being 15,676 feet (4702·8 m.) per second in the first four feet and 16,218 (4865·4 m.) in the last four. The observations recorded of the progress of detonation along the last 16 feet of this train afforded an excellent illustration of the uniformity of the rate of transmission, even if the disks are spaced, and of the accuracy attainable by the method of observation employed.

	Rate of transmission of detonation per second.
Between 12′ and 16′	=16,815 feet (5044·5 m.).
„ 16′ „ 20′	=16,692 „ (5007·6 „).
„ 20′ „ 24′	=16,634 „ (4990·2 „).
„ 24′ „ 28′	=16,218 „ (4865·4 „).

In a third experiment, with a row of disks of the same dimensions as the foregoing, but the weight of which ranged between 3·75 (117 grms.) and 4 ounces (124·8 grms.), they were separated from each other by spaces of 0·75 inch (·0189 m.). Detonation was transmitted more rapidly than by the lighter disks (2·7 ounces=84·24 grms.), which were separated by spaces of 0·5 inch (·013 m.); the rate of travel in the last four feet happened to be identical with that in the first four, but the records obtained in the intermediate distances exhibited more considerable fluctuations than usual, as though

the uniformity of the rate of transmission were affected by the increased spaces which were allowed to intervene between the masses of gun-cotton. The fluctuations may, however, have been due in part to *slight* differences in the spaces, combined with small variations in the weight of the disks. The mean velocity of the detonation was =17,478 feet (5243·4 m.) per second. It would appear from these results with spaced disks that the separation of masses of compressed gun-cotton from each other by spaces insufficient to arrest detonation may retard the rate at which this is transmitted from mass to mass, the extent of such retardation being of course determined by the relation between the size of the individual masses and the extent of spaces intervening between them. The separation of masses of about 2·5 ounces (78 grms.) from each other by spaces of 0·5 inch (·013 m.) had a decided and uniform retarding effect upon the rate of travel of the detonation; but with masses at least a third larger, the further increase, by one third, of the space intervening between them did not maintain a similar uniform retardation in the rate of travel, although there were indications of such retardation at some points along the train or row of spaced disks.

c. *Detonation of moist and wet gun-cotton.*

Gun-cotton disks containing about 15 per cent. of water, ranged in a continuous row (the *dry* disks weighing the same as those used in previous experiments), were found to transmit detonation with the same velocity as the dry material; there appeared, indeed, some indication that the rate of travel was a little higher, the results obtained being *all* equal to the *highest* furnished by dry gun-cotton of the same density. Thus in one experiment the rate of travel in the first four feet was 18,416 (5524·8 m.), and in the last four 18,340 feet (5502 m.) per second, the mean velocity being =18,375 (5512·5 m.) per second. In the last 12 feet of the train the following rates were recorded:—

	Rate of travel of detonation per second.
Between 16′ and 20′	=18,880 feet (5664 m.).
„ 20′ „ 24′	=18,416 „ (5524·8 „).
„ 24′ „ 28′	=18,040 „ (5502 „).

In another experiment the mean velocity was =18,581 feet (5574·30 m.) per second, and in a third 18,433 feet (5529·9 m.) per second.

With gun-cotton disks which were *saturated* with water (containing about 30 per cent.), but in other respects similar to those used in the former experiments, the rate of transmission of detonation was *decidedly higher* than with the dry disks. In one experiment, with a train 30 feet long, the mean velocity of transmission was =19,213 feet (5763·9 m.) per second; and the records from which the mean result was derived included one *

* An interesting example was obtained, in the course of these experiments, of the manner in which the behaviour of gun-cotton, when exposed to the action of a detonation, as also the character of its detonation, is liable to be modified by a variation in the proportion of water with which it is impregnated. A number of disks ranging in weight from 2·5 to 2·7 ounces (78·0 to 84·24 grms.) when air-dry contained some of them

which had obviously been affected by some slight retardation in the breaking of the wire. The following were the recorded rates of travel :—

				Rate of transmission of detonation per second.
Between	0′	and	4′	=19,751 feet (5925·3 m.).
„	4′	„	8′	=19,751 „ (5925·3 „).
„	8′	„	12′	=19,006 „ (5701·8 „).
„	12′	„	16′	=20,281 „ (6084·3 „).
„	16′	„	20′	=18,928 „ (5678·4 „).
„	20′	„	24′	=17,608 „ (5282·4 „).
„	24′	„	28′	=19,169 „ (5750·7 „).

In another experiment the mean velocity was 19,664 feet (5899·20 m.) per second. The following records were obtained at 6-feet intervals, with a train of the wet disks 36 feet long :—

				Rate of travel of the detonation per second.
Between	0′	and	6′	=22,574 feet (6772·7 m.).
„	6′	„	12′	=18,404 „ (5521·2 „).
„	12′	„	18′	=19,916 „ (5974·8 „).
„	18′	„	24′	=20,036 „ (6010·8 „).
„	24′	„	30′	=19,516 „ (5854·8 „).
„	30′	„	36′	=19,240 „ (5772·0 „).
		Mean velocity		=19,948 „ (5984·4 „).

about 15, others 30 per cent. of water; they were arranged in a continuous train, of which the first 25 feet consisted entirely of the least wet disks, the remainder being composed of those saturated with water. When detonation was established (by means of an 8-ounce disk of dry gun-cotton), it was transmitted from disk to disk, in the first 25 feet, at a rate of 18,000 feet per second, but was arrested by the first disk containing the higher proportion of water, which, besides the two next following, was shattered and dispersed by the explosion. It has been shown that the detonation of gun-cotton containing 30 per cent. of water could not be accomplished by the employment of less than about 4 ounces of gun-cotton as the initiative detonator; in the above experiment, therefore, in which the train consisted of disks weighing only about 2·75 ounces (85·8 grms.), the detonation being transmitted by each disk to the one next in succession, it was consistent with former experiment that the last of those disks in the train which contained 15 per cent. of water should fail to transmit detonation to the first of the disks containing 30 per cent., although they were in contact. But in other experiments with trains which consisted *entirely* of disks containing 30 per cent. of water, and weighing only between 2·5 (78·0 grms.) and 2·7 ounces (84·24 grms.), detonation, established by means of an 8-ounce dry disk, was transmitted, without fail, throughout the longest trains experimented with (45 feet), provided the disks were in contact. Detonation was therefore transmitted to gun-cotton saturated with water by disks which contained *the same proportion of water*, but which were far too small to have produced this result had they been dry, or had they contained only 15 per cent. of water. These apparently anomalous results appear to indicate that the *quality* of detonation developed by gun-cotton is modified by the proportion of water which the latter contains.

These results demonstrate conclusively that when compressed gun-cotton is saturated with water, so that the air in the mass is replaced entirely by the comparatively incompressible liquid, detonation is transmitted at a decidedly more rapid rate than with equally compact dry gun-cotton.

d. *Detonation of nitrated gun-cotton.*

The considerable amount of a nitrate required to completely utilize the oxidizable constituents of gun-cotton should naturally tend to reduce the rapidity of explosion of that substance, when employed in the form of a mixture with any proportion of saltpetre approaching the theoretical requirement. Results obtained by the detonation of "nitrated" gun-cotton, in practical experiments, afforded decided evidence of less rapid action, which was confirmed by determinations of the rate at which detonation is transmitted along trains of the nitrated material. The disks employed were 3 inches (0·078 m.) in diameter, containing about two thirds the theoretical requirement of saltpetre (namely, 38 per cent.), and weighed about 4 ounces each. The mean velocity of detonation was =15,981 feet (4794·38 m.) per second, the rate of transmission generally ranging between 15,500 (4650 m.) and 16,000 feet (4800 m.) per second; it was therefore about 2000 feet per second below that of ordinary air-dry compressed gun-cotton.

e. *Detonation of dynamite.*

The material used in these experiments was that known as NOBEL'S dynamite (No. 1), and consisted of an intimate mixture of about seventy-three parts of nitroglycerine with twenty-seven parts of Kieselguhr*, made up into cylindrical cartridges 0·5 inch (0·013 m.) in diameter, and 8 inches (0·078 m.) long, by being firmly pressed into wrappers of stout parchment-paper. Thus prepared it resembles stiff clay, not sufficiently wet to be very plastic. The paper envelopes were removed from the cartridges for the purpose of these experiments, and the cylindrical masses (the average weight of which was 2·3 ounces =71·76 grms.) were placed end to end and pressed together; in this way perfectly continuous trains of dynamite, 30 feet and 42 feet (9·12 and 12·76 m.) long, were prepared; detonation was established by means of the ordinary "detonator" used with gun-cotton, which was inserted into a small cartridge of dynamite or a disk of gun-cotton, and placed upon one extremity of the train. The rate at which detonation was propagated was recorded at intervals of 4 feet and 6 feet (1·21 and 1·82 m.) along the trains; the mean velocities observed were 19,536 feet (5938·24 m.) and 21,592 feet (6563·96 m.) per second. As with gun-cotton, the rate of progression of the detonation was as high at the end of the longest train as at the commencement, and at one part of each of the trains it attained a velocity of 24,000 feet (7296 m.). These numbers show that the velocity with which detonation is transmitted by the plastic nitroglycerine mixture is decidedly higher than with dry compressed gun-cotton.

* The dynamite was obtained from the factory of the British Dynamite Company, near Glasgow, which has recently been established to meet the demands for NOBEL'S nitroglycerine preparations in the country.

Careful comparative submarine experiments, in which the explosive force was measured by the effects produced on crusher-gauges, have indicated that this dynamite (*i. e.* nitroglycerine diluted with one third its weight of inert material) is about equal, and sometimes a little inferior, in power to gun-cotton; but in open-air experiments (*e. g.* explosion of freely exposed charges against walls or timber) the dynamite has been observed to be sharper and somewhat more local in its action. These observations harmonize with the results of the velocity-determinations with the freely exposed trains.

Dynamite was, however, found to behave very differently from compressed gun-cotton when spaces were allowed to intervene between the masses composing a charge which is detonated in open air. It will be remembered that with pieces of gun-cotton 3 inches (0·076 m.) in diameter, weighing about 2·5 ounces (78 grms.) each, which were placed in a row with intervals of 0·5 inch (·013 m.), detonation was transmitted with a velocity ranging between 16,000 and 17,000 feet (4864 and 5168 m.) per second, and therefore not greatly inferior to the mean rate at which detonation was transmitted by similar disks arranged in continuous rows. But when charges of some of the *dynamite*, which had transmitted detonation at a rate of from 19,000 to 21,00 feet (5776 to 6384 m.), the cartridges forming a continuous row, were separated from each other by spaces of 0·5 inch (·013 m.), which were left at the same intervals as in the trains of gun-cotton (the weight of the individual masses of both substances being nearly alike), the mean rate of progression of the detonation was only 6239 feet (1896·65 m.) per second, or less than one third of the *lowest* velocity observed in the experiments with *continuous* trains of dynamite. The trustworthiness of this result was conclusively demonstrated by the uniformity of the observed velocities at different parts of a long train of the spaced dynamite cartridges, as is shown by the following numbers:—

				Rate of progression of the detonation per second.
Between	0′	and	4′ = 6591 feet	(2003·66 m.).
„	4′	„	8′ = 6133 „	(1864·43 „).
„	8′	„	12′ = 6159 „	(1872·23 „).
„	12′	„	16′ = 6394 „	(1943·77 „).
„	16′	„	20′ = 6552 „	(1991·80 „).
„	20′	„	24′ = 5789 „	(1759·85 „).
„	24′	„	28′ = 6059 „	(1841·93 „).

f. *Detonation of nitroglycerine.*

The quantity of nitroglycerine at my disposal for these experiments was limited to a few pounds; it was therefore necessary to devise arrangements for the attainment of trustworthy results with comparatively small quantities of material. V-shaped troughs, 14 feet long and about 2 inches deep in the centre, were constructed of thin sheet-metal, and the insulated wires at intervals of 2 feet were passed through, and cemented into,

small perforations in the sides of the trough, sufficiently near to the bottom to ensure their being covered by the nitroglycerine. In the first experiment the trough was filled to about one third its depth with the liquid, and the initiative detonation was developed at one extremity by means of a dynamite cartridge (and detonator) partly immersed in the nitroglycerine. Although pains were taken to fix the trough uniformly level upon the ground, there was a slight unavoidable elevation at one part—about 9 feet distant from the point of first detonation; the layer of nitroglycerine was at this point about half the depth of the remainder, and this sufficed to arrest detonation, which had been transmitted up to that part of the train at a rate of from 5200 to 6000 feet per second, the mean velocity being 5573 feet (1794 m.) per second. The layer or train of nitroglycerine used weighed 20 ounces (624 grms.), being at the rate of 1·4 ounce (43·68 grms.) per foot of the layer or train. In the next experiment a layer of fully double the depth was used (weighing about 3 ounces =93·6 grms.) per foot; the detonation proceeded along the entire length of the trough with undiminished velocity, which was, however, not higher than when the smaller quantity of nitroglycerine was employed, the mean rate of its progression being 5305 feet (1612 m.) per second, and the maximum rate, at any part, 5994 feet (1822 m.). The quantity of nitroglycerine employed, in a given length of the train, in this last experiment corresponded to that used in the two gun-cotton experiments, in which the smallest cylinders (=3 ounces per foot) were used, arranged in a continuous row, the rate of travel of the detonation ranging between 18,500 and 20,600 feet (5624 and 6262 m.) per second.

The very low results furnished by nitroglycerine, when detonated in open air, are obviously due to the physical peculiarity (*i. e.* to the liquid nature) of this explosive agent, the transmission of detonation being greatly retarded by the tendency of the liquid particles to escape from the blow of the detonation. In the first experiment it was demonstrated that when the mass of liquid subjected to the detonation was sufficiently reduced in quantity, it was actually dispersed, or made to yield so completely to the blow, that detonation was arrested while being transmitted at a rate of 5500 feet (1672 m.) per second. The very high velocity with which detonation may be transmitted by nitroglycerine, when that substance is in a condition or position enabling it to resist mechanical dispersion, was demonstrated by the results obtained with dynamite employed in the form of compressed cartridges. On the other hand, the remarkable manner in which the velocity of detonation of the latter was reduced by introducing such spaces between the cartridges as had no retarding effect upon the detonation of corresponding masses of the rigid gun-cotton, demonstrated that the somewhat plastic nature of the explosive material, and its consequent tendency to yield to force, affected the transmission of detonation very decisively when the conditions were not such as to ensure its transmission continuously from particle to particle.

These results confirm in an interesting manner the conclusions arrived at from ex-

periments in various other directions, regarding the influence exerted by the physical character or mechanical condition of an explosive body upon its susceptibility to detonation.

The comparatively limited quantities of nitroglycerine at my command have compelled me to suspend for a time further experiments with that substance; I hope, however, at some future time to have the opportunity of determining the rate at which detonation is transmitted by *confined* nitroglycerine, and of comparing it with gun-cotton under the same conditions, as well as with other explosive agents.

g. *Transmission of detonation by tubes.*

Several experiments were made for the purpose of comparing the rate of transmission of detonation by gun-cotton disks arranged as described in the foregoing, and by widely separated masses, through the agency of tubes. The mode of operating was as follows:—Iron gas-pipes, clean inside, and either 1·25 or 1·5 inch (1·031 or ·037 m.) diameter, had pairs of small holes drilled into them at intervals of 2 feet (·608 m.) and 3 feet 3 inches (1 metre) from each other, sufficiently large just to admit of the insertion of the fine insulated wire, and so placed that the latter, when thrust through both, was stretched across the interior of the tubes at its centre and at right angles to its length, being tightly secured to pegs fixed in the ground on either side of the tube. Disks of suitable dimensions to fit the tube were inserted, so that two, weighing together 1·5 ounce (46·8 grms.), were placed against each wire. In the wider (1·5 inch, ·037 m.) tubes the charges and wires were 2 feet apart, and in the 1·25 inch (·031 m.) tube they were 3 feet 3 inches (1 m.) from each other. The initiative detonation was produced at one end of the tubes by means of 2 ounces of gun-cotton. The rate at which it was transmitted from the starting-point (or initiative explosion) to the first charge in the tube was somewhat variable, ranging between 10,000 (3000 m.) and 13,000 (3900 m.) feet per second; the subsequent transmission along the tubes, from charge to charge, proceeded at a tolerably uniform, but considerably reduced rate, the average being 6000 feet (1800 m.) per second. The following is an example of the observations recorded, with employment of a tube 1·5 inch (·037 m.) diameter, the charges being separated by intervals of 2 feet:—

	Rate of transmission in feet per second.
From the initiative detonation to the charge 2 feet distant	9922 (2976·6 m.).
From the charge at 2 feet to that at 4 feet	=6693 (2607·9 „).
„ „ 4 „ „ 6 „	=5320 (1596·0 „).
„ „ 6 „ „ 8 „	=6957 (2087·1 „).
„ „ 8 „ „ 10 „	=6854 (2056·2 „).
„ „ 10 „ „ 12 „	=5648 (1694·4 „).
„ „ 12 „ „ 14 „	=5246 (1573·8 „).

There would appear to be some indication of a falling off in the rapidity of transmission in this experiment towards the end of the tube, but this was not borne out by other results with tubes of the same length.

In one experiment, with a tube of 1·5 inch (·037 m.) diameter, charges of only 1 ounce of gun-cotton were placed in the tube at intervals of 2 feet. The results obtained in the first 6 feet of the tube corresponded with those furnished by tubes of the same dimensions, when charges of 1·5 ounce (46·8 grms.) were employed; but the fourth and succeeding charges, though they exploded, did not detonate, no destructive effect being produced upon the tube; the wires were, however, severed by the explosions, and the records obtained indicated that the rate at which the *explosion* was transmitted from charge to charge, through the last three intervals of 2 feet each, was only between 1500 and 1800 feet (450 m. and 540 m.) per second.

It appears from the experiments with tubes that, when the relations between the amount of explosive agent, the diameter of the tube, and the space or length of tube intervening between the charges are such as to ensure the transmission of *detonation*, the rate of its transmission is about one third of that at which it travels, in open air, along a *continuous* mass, or row of masses, of the same material in the same condition.

VIII.—ON CIRCUMSTANCES WHICH INFLUENCE THE BEHAVIOUR OF EXPLOSIVE AGENTS WHEN EXPOSED TO HIGH TEMPERATURES.

In my former memoir on explosive agents I discussed* incidentally the influence of an accumulation of heat in a mass of gun-cotton in promoting violent explosion or detonation, if through any cause the materials become ignited, an influence which equally affects other explosive agents. This point has acquired much additional importance since the publication of that Paper, in consequence of its probable bearing upon the violent explosions which occurred at the Gun-cotton Works at Stowmarket in August 1871, and of the possibility thereby rendered manifest that violent explosions of gun-cotton, or other substances of analogous properties, may occur under circumstances which until lately were not considered to involve hazard.

The fact that the previous application of heat to an explosive agent increases the violence with which this will explode when flame, or a sufficiently powerful source of heat, is applied, admits of ready demonstration by simple experiments. Thus, if a small quantity of gun-cotton, wool, or thread be inserted and lightly pressed down into a test-tube, and ignited by application of a hot wire to the gun-cotton, or of a sufficient source of heat to the exterior of the tube, it will simply flash into flame rapidly, with but little indication of explosive violence; but if it be previously exposed to a heat of 90° to 100° C., until it has attained that temperature throughout, its ignition, while at that temperature, will be attended by decided evidence of greater explosive energy.

* Phil. Trans. 1869, vol. clix. pp. 495–6.

A small piece of compressed gun-cotton, similarly inserted in a tube, and ignited, while at the ordinary temperature, by platinum wire which was heated by an electric current, burned slowly, almost with the appearance of smouldering; but when another fragment was ignited in the same way, after having been raised for some time to 100° C., and while still heated to that temperature, it exploded with considerable violence and shattered the tube. The cause of this great difference in behaviour is evidently due in part to the circumstance that comparatively little heat had to be expended at the instant of ignition in raising the entire mass to the exploding point, and in part to the state of chemical tension, or predisposition to chemical change, into which the particles of the gun-cotton had passed by their continued submission to heat. Similar effects have been readily obtained by exposing gunpowder and other explosive agents to sufficient heat previous to their ignition.

In the case of a substance like gunpowder or mercuric fulminate, the mechanical condition (granulated or crystallized form) of which favours the rapid penetration of heat or flame throughout a mass, an explosion, of violence proportionate to the quantity which is ignited, must inevitably result from the inflammation of any portion. The circumstance that masses of compressed gun-cotton simply burn rapidly from the exterior to the centre, if ignited, imparts to this material at first sight an appearance of comparative safety (or of non-liability to violent explosiveness under ordinary conditions of exposure to flame or to an igniting temperature), which experience has shown to be in great measure delusive, inasmuch as it is true only so far as regards comparatively small quantities (a few hundred pounds) of the material, and is even then, to a considerable extent, dependent upon the circumstances under which the material is exposed to heat. This has been conclusively demonstrated by some experiments upon a considerable scale which have been carried out by the Government Committees on Explosive Agents and on Gun-cotton, of which the following is a brief account.

Single boxes of stout wood (0·75 inch in thickness) strongly made, filled with disks of compressed gun-cotton (28 lb. in each) and firmly closed with screws, have been surrounded by inflammable material, the burning of which exposed the case and its contents to considerable heat, eventually igniting it, and causing the box to burn fiercely. In every instance the gun-cotton in the box became inflamed after a few minutes and burned fiercely, the entire quantity being consumed in two or three seconds; but no explosion was brought about in any instance, and the box in which the gun-cotton was confined was only partially forced open by the pressure developed from within. Eight boxes of the same kind, each containing 28 lb. of gun-cotton, were afterwards placed in the centre of a pile of similar boxes, filled with earth to the same weight, and the contents of the innermost box in the heap were ignited by an electric fuse. They burned fiercely, and the flame penetrated to the gun-cotton in one other box; but in neither instance was an explosion produced, and the pile of boxes was scarcely disturbed; the remaining six which contained gun-cotton were charred but not otherwise injured. Another more severe experiment, in which one of the inner boxes

composing the heap was coated with tar and covered with wood shavings, these being afterwards set fire to, furnished a similar result; no explosion followed the eventual ignition of the gun-cotton contained in this and another box. These results were considered at the time to afford satisfactory proof that if even considerable quantities of gun-cotton, in the form of compactly compressed, homogeneous masses, were ignited when confined in strong wooden boxes, no explosion would occur, the gun-cotton merely burning rapidly; and that if such packages of compressed gun-cotton were exposed to the effects of fire from without, some portion must be inflamed by access of fire, and the combustion of the mass thus brought about before it can be raised to the temperature of explosion. Subsequent experiments, however, clearly demonstrated that the results above described must be considered *in relation to the quantity* of gun-cotton by which they are furnished, as well as to the degree of its confinement when subjected to the action of heat or fire. In the experiments just described, the largest quantity of gun-cotton employed was 224 lb. (contained in eight pakages); but some very different results were obtained in somewhat similar experiments conducted in each instance with three times that quantity (viz. 672 lb.) of gun-cotton. That amount of material, packed in 24 boxes of the kind used in the preceding trials, was, in the first instance, placed upon two tables in a light wooden shed; a heap of shavings and wood chips was then kindled immediately beneath the two piles of boxes, of which two were left partly open. The gun-cotton inflamed after the lapse of eight minutes, and continued to burn with increasing violence for about six seconds, when a powerful explosion occurred, the shed being blown to pieces and a deep crater formed in the ground. It was estimated, from comparative experiments with packages of gun-cotton purposely detonated, that the explosion occurred when only a small proportion had burned. In a second similar experiment the nature of the building only was varied, the 672 lb. of gun-cotton being placed (in boxes upon tables, and surrounded by inflammable material) in a strongly built brick magazine. In this instance also a violent explosion occurred after the gun-cotton had been burning, with increasing fierceness, for nine seconds. A third periment, conducted with an equal quantity of gun-cotton, was made for the purpose of ascertaining whether the result would be influenced by confining it less strongly. The 672 lb. were therefore packed in twenty-four deal boxes of lighter construction than those before used ($\frac{3}{8}$ inch instead of $\frac{1}{4}$ inch deal, and more lightly fastened together), and these were placed in light wooden sheds, the other details of the experiment being as before. The gun-cotton became inflamed about 36 minutes after the fire had been kindled in the hut, and burned fiercely for about 15 seconds; having subsided for a short time, there was a second burst of flame, and the gun-cotton was entirely burnt about 48 minutes after the first ignition, there being no explosion in this instance. A repetition of the experiment furnished the same negative result.

An experiment, similar to the first of the foregoing, was made some time afterwards with NOBEL'S dynamite (the mixture of nitroglycerine and Kieselguhr). 672 lb. of this material (containing about 500 lb. of nitroglycerine), enclosed in twelve stout wooden

packages, were placed upon tables in a light wooden building (8·5 feet square by 6·5 feet high), and a heap of inflammable material was placed between the tables. It was somewhat difficult to determine the precise time after the fire was first kindled in the building when the dynamite commenced burning, as the change in the appearance of the flame was less characteristic or sudden than with gun-cotton as observed from a distance. The rapidly increasing fierceness of the flame, however, about five minutes after the fire was kindled, showed that the nitroglycerine was burning; and after the lapse of ten minutes a violent explosion occurred, fragments of the wooden structure were thrown to great distances, and a large crater was formed in the ground.

These experiments, conducted with 672 lb. of gun-cotton and with 500 lb. of nitroglycerine in the form of dynamite, placed in confined spaces, demonstrated that if some portion of the explosive substance is ignited, the remainder of the material being pretty strongly confined, the very rapid increase in the intensity of the heat developed may soon combine to raise some portion of the *still confined* explosive to the inflaming point, and that then, the mass being already in a heated condition, the inflammation may proceed with such rapidity as to develop the pressure essential for establishing explosion while the substance is confined, which explosion would be instantaneously transmitted to the contents of the surrounding packages. It was also satisfactorily demonstrated that with equal quantities of explosive material confined in packages, a difference in the *strength* of confinement of the substance is productive of an important difference when the material is exposed to fire. The weaker the packages the more readily they are opened up by pressure from within; hence, when some portion of the contents of a box of light structure becomes raised to the inflaming-point, the pressure developed by the ignition is not sustained to a sufficient extent or for a sufficient time to bring about explosion. It need, however, be scarcely pointed out that the safeguard against explosion thus presented, in the case of compressed gun-cotton or of a nitroglycerine preparation, by the employment of packages of light structure, must be limited by the *quantity* of explosive material exposed to fire. Thus it is very probable that, although no explosion resulted from the exposure to fire and ultimate burning of 6 cwt. of gun-cotton contained in packages of light structure, while explosion was produced in experimenting with a similar quantity in strong packages, the difference in the strength of the boxes might not have ensured safety against explosion with a much larger quantity. Indeed it may be presumed that, if the quantity be sufficient, no further confinement than that afforded by the outer portions of a large pile of disks of compressed gun-cotton, or cartridges of dynamite, would be required, after the burning had proceeded for a sufficient time, for determining the explosion of some portion of the interior of the mass, in consequence of the resistance opposed to the escape of gas from the confined or inner portions to which the fire had reached.

The results furnished by the experiments just described were similar in their character, and the conditions which determined them, to those noticed on the occasion of the accident at the Stowmarket Gun-cotton Works in August 1871. The ignition of some

portion of the gun-cotton contained in one of three closely adjacent wooden store-sheds or magazines, containing together about 13 tons of the material, was unquestionably due primarily to the spontaneous decomposition of some very impure material, the existence of which in one of those stores was clearly demonstrated in the subsequent inquiry. This, and the gun-cotton stored with it, was packed in strong boxes of the same kind as those used in the above experiments. Several unusually hot days had preceded that of the explosion, and the contents of the boxes, confined in these wooden store-sheds, which they filled almost completely, must have become very warm throughout. A large volume of flame, apparently enveloping these sheds, was described by eye-witnesses at a considerable distance as having been observed for a short but very distinct interval previous to the explosion; and there appears no doubt that the ignition of some portion of gun-cotton must have been immediately followed by the inflammation of the entire contents of one box, the flame rapidly penetrating to other boxes, and in a very brief space of time determining the explosion of some portion of the confined gun-cotton in the manner already described. The explosion not only of the entire contents of the one shed but also of the two in close proximity was the inevitable result, the small brick partitions which separated the sheds from each other affording no impediment to the almost instantaneous transmission of the explosion. Among the buildings which were ignited by the flame, or burning *débris* from this principal explosion, there were two small store-houses or packing-sheds, containing gun-cotton packed in a number of boxes of very light construction as compared with the packing-cases already alluded to: these sheds and their contents were entirely consumed without any explosion. But there was a third of the same kind which contained some of the strong boxes filled with gun-cotton, and this, after having been some considerable time in flames, exploded with great violence.

In my Memoir "On the Stability of Gun-cotton"* I demonstrated by numerous experiments and by the results of observations upon a large scale, and extending over some considerable time, that water acts as a most perfect preservative of gun-cotton even under most severe conditions of exposure to heat, and that the material may be preserved for indefinite periods either in a moist state or completely immersed in water, without the slightest change, either chemical or physical, even if the damp or wet gun-cotton be continually exposed to daylight. The correctness of those statements has been fully confirmed by the further and extensive experience of the last five years. Not only have the samples which formed the subject of my former experiments been preserved and found unchanged upon recent examination, but several hundred pounds have been preserved in the damp state for the past seven years, and many thousand pounds in the form of compressed disks have been stored wet for more than two years. Not the slightest symptom of change has been observed in any single instance; and as the substance may thus be preserved in a perfectly uninflammable condition, this

* Phil. Trans. 1867, vol. clvii. p. 233.

method of storing gun-cotton in large quantities has been adopted by the Government, as tending to simplify very greatly the precautions needed in preserving large supplies of the material.

Some experiments have been made with the object of determining as accurately as possible the minimum percentage of water which would deprive compressed gun-cotton of its power of inflaming when brought into contact with a red-hot body. Some disks which contained about 15 per cent. of water were exposed to air in a room where they parted very slowly with the water by natural evaporation. Their weight was determined at the commencement of the experiment, and their power of inflaming was tested by pressing a red-hot iron rod in contact with one of their surfaces for a second or two, their weight being noted just before each trial. As a small proportion of the gun-cotton and water was vaporized each time the hot iron was applied, the weight of the disk was redetermined after each experiment. A period was at length reached at which the surface of the gun-cotton was inflamed, though not instantaneously, the flame spreading slowly from the point of contact with the source of heat, and being readily extinguished by a puff of breath. The proportion of water remaining in the disk when this point had been reached was 9·3 per cent. A disk which, though it smouldered slightly on the surface where the heated iron was applied, did not inflame, was found at that period to contain 10·7 per cent of water.

When removed from the mould of the hydraulic press in which the finely divided gun-cotton or pulp has been converted into homogeneous and very compact masses (of about the density of water) by the application of a pressure of about six tons upon the square inch, the material contains about 15 per cent. of water. In this condition it may be thrown on to a fire or held in a flame without exhibiting any tendency to burn; the material may be perforated by means of a red-hot iron, or with a drilling-tool, and the hard masses may with perfect safety be cut into slices by means of circular saws revolving with great rapidity. If placed upon a fire and allowed to remain there, a feeble and transparent flame passes over the surface from time to time as the exterior becomes sufficiently dry to inflame, and in this way a piece of compressed gun-cotton may be allowed to burn away very gradually indeed. In the same way a hank of gun-cotton yarn or a handful of the pulped material, which, after removal from the centrifugal wringing-machine used in the purifying operations, retains about 20 per cent. of water, burns away very slowly if thrown upon a fire and allowed to remain there. Boxes in which the damp material was packed have been exposed singly to fierce fires until the box itself has been burnt through, but the gun-cotton has only been charred or has burned slowly wherever, and as soon as, it has become sufficiently dry upon the surface to be inflammable. When conducting the large-scale experiments on the burning of dry gun-cotton, it was considered desirable to institute a similar experiment with the moist material for the purpose of ascertaining whether its long-continued exposure to fire when closely packed might result in the development of conditions favourable to the explosion of some portion. 6 cwt. (672 lb.) of compressed

gun-cotton containing about 20 per cent. of water was packed in the strong wooden boxes already described, which were placed upon tables in a light wooden shed. They were then surrounded by wooden shavings and chips, which were fired with the aid of some coal-tar naphtha. Eighteen minutes after the fire had been kindled, the shed itself was in flames, and not long afterwards the boxes could be seen burning upon the ground, the supports having been destroyed by the fire. The heap continued to burn for about three quarters of an hour, by which time the whole of the gun-cotton and woodwork had disappeared. At no stage during the experiments was any flame visible which could be positively identified as that of burning gun-cotton.

The apparent immunity of compressed gun-cotton, if sufficiently moist, from any tendency to explode when submitted in considerable quantities to the prolonged effects of a high temperature, needed confirmation by experiments upon a still larger scale and of a more severe nature, before it could be considered satisfactorily demonstrated that the conditions essential for developing the detonation of moist gun-cotton, namely the sufficient desiccation and subsequent detonation of some portion of the highly heated substance, might not possibly be brought about during the prolonged exposure of a great quantity to powerful heat. Two large experiments have therefore recently been instituted by the Government Committee on Gun-cotton, of which the following is a brief account:—Two small arched buildings, or magazines, were very strongly constructed of concrete and brickwork, the walls being 2 feet thick and the arched roofs 9 inches thick. Each magazine was provided with a wooden door, and with a staging 1 foot 8 inches high, which consisted of four brickwork pillars with pieces of railway-bar joining the two on each side together and supporting broad cross bars of wrought iron. In one magazine a tank of the dimensions now used for storing wet gun-cotton in, and constructed of stout pine coated internally with a pitch-composition, was placed upon the staging, and 4480 disks (or 2240 lb.) of gun-cotton were packed into it, the lid of the tank being then securely screwed down. A similar quantity of gun-cotton, packed in eighty stout and tightly closed boxes, each holding 28 lb., was placed in the other building, the boxes being piled on the staging as close together as possible. The gun-cotton, in both experiments, contained about 30 parts of water to 100 of the dry material. A large quantity of wood shavings and other inflammable materials was placed under the staging and round the gun-cotton packages in each magazine and set fire to, the doors being left ajar. The entire contents of both buildings had burned away, without any explosion, in rather less than two hours. The heat to which the gun-cotton, or portions of it, had been exposed was very great, as was demonstrated by the condition of the buildings, and the distortion of some of the wrought-iron bars, when the conflagration had subsided. The brilliant yellow flame characteristic of the burning of a mass of gun-cotton was not observed at any period throughout the experiment; but about one hour after the fires were kindled, considerable volumes of a pale yellow lambent flame, occasionally exhibiting a greenish tinge, issued from the doors. In the case of the magazine which contained the gun-cotton in separate boxes, there

were frequent increased outbursts of the flame, apparently caused by the successive fall of boxes from the staging into the fire beneath. When a tuft of gun-cotton wool is placed in a capacious vessel (*e. g.* a large glass beaker) and ignited, the flash of bright yellow flame first observed is followed by a pale yellow lambent flame, of the character above described, which lasts for a distinct interval, and is due to the burning of the inflammable gaseous products as air enters the vessel, the slight greenish tinge being caused by the presence of small quantities of nitrogen-oxides. The flame observed to issue from the magazines was quite similar to this, and was evidently produced by the burning of the gases gradually developed from the wet gun-cotton.

These last experiments demonstrate conclusively that even very severe exposure to heat of large packages or heaps of distinct masses of compressed gun-cotton, saturated, or nearly so, with water, is not attended by risk of explosion. Just as in smaller experiments, when the proportion of water has been expelled from the surface of the heated gun-cotton to the extent to reduce it to about 10 per cent., those portions of the mass burn quietly with a weak flame accompanied by the development of inflammable gas. No accumulation of dry or even of *nearly* dry gun-cotton can consequently take place, and no portion of the gun-cotton can be raised to the exploding temperature.

As compressed gun-cotton may, by employment of simple appliances, be made to exert its full explosive force when thoroughly saturated with water, and as it may therefore be applied in that condition to many of its more important uses, the non-liability of wet gun-cotton to explosion by simple exposure to heat, at any rate in such large quantities as have been experimented with, is a matter of considerable practical importance. The material may be easily preserved in store in an uninflammable condition, and employed at once in that state almost as readily, and with quite as much effect, as if it were dry; its storage in the wet state appears, moreover, to be an absolute safeguard against change, even when doubts exist as to the substance having been thoroughly purified. Therefore, although many years must still elapse before general confidence in the stability of *dry* gun-cotton is well established, very simple means now exist for dealing with this material as extensively as with gunpowder, and with unquestionably greater safety.

XI. *A Memoir on the Transformation of Elliptic Functions.*

By Professor CAYLEY, *F.R.S.*

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THE theory of Transformation in Elliptic Functions was established by JACOBI in the 'Fundamenta Nova' (1829); and he has there developed, transcendentally, with an approach to completeness, the general case, n an odd number, but algebraically only the cases $n=3$ and $n=5$; viz. in the general case the formulæ are expressed in terms of the elliptic functions of the nth part of the complete integrals, but in the cases $n=3$ and $n=5$ they are expressed rationally in terms of u and v (the fourth roots of the original and the transformed moduli respectively), these quantities being connected by an equation of the order 4 or 6, the modular equation. The extension of this algebraical theory to any value whatever of n is a problem of great interest and difficulty: such theory should admit of being treated in a purely algebraical manner; but the difficulties are so great that it was found necessary to discuss it by means of the formulæ of the transcendental theory, in particular by means of the expressions involving JACOBI's q $\left(\text{the exponential of } -\frac{\pi K'}{K}\right)$, or say by means of the q-transcendents. Several important contributions to the theory have since been made:—SOHNKE, "Equationes Modulares pro transformatione functionum Ellipticarum," CRELLE, t. xvi. (1836), pp. 97–130 (where the modular equations are found for the cases $n=3, 5, 7, 11, 13, 17, \&\ 19$); JOUBERT, "Sur divers équations analogues aux équations modulaires dans la théorie des fonctions elliptiques," Comptes Rendus, t. xlvii. (1858), pp. 337–345 (relating among other things to the multiplier equation for the determination of JACOBI's M); and KÖNIGSBERGER, "Algebraische Untersuchungen aus der Theorie der elliptischen Functionen," CRELLE, t. lxxii. (1870), pp. 176–275; together with other papers by JOUBERT and by HERMITE in later volumes of the 'Comptes Rendus,' which need not be more particularly referred to. In the present Memoir I carry on the theory, algebraically, as far as I am able; and I have, it appears to me, put the purely algebraical question in a clearer light than has hitherto been done; but I still find it necessary to resort to the transcendental theory. I remark that the case $n=7$ (next succeeding those of the 'Fundamenta Nova'), on account of the peculiarly simple form of the modular equation $(1-u^4)(1-v^4)=(1-uv)^4$, presents but little difficulty; and I give the complete formulæ for this case, obtaining them as well algebraically as transcendentally; I also to a considerable extent discuss algebraically the case of the next succeeding prime value $n=11$. For the sake of completeness I reproduce SOHNKE's modular equations, exhibiting them

for greater clearness in a square form, and adding to them those for the non-prime cases $n=9$ and $n=15$; also a valuable table given by him for the powers of $f(q)$; and I give other tabular results which are of assistance in the theory.

The General Problem.—Article Nos. 1 to 6.

1. Taking n a given odd number, I write

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left(\frac{P-Qx}{P+Qx}\right)^2,$$

where P, Q are rational and integral functions of x^2, $P\pm Qx$ being each of them of the order $\frac{1}{2}(n-1)$, or, what is the same thing, $(1\pm x)(P\pm Qx)^2$ being each of them of the order n; that is,

	$n=4p+1$,	$n=4p+3$,
Order of P in x^2 is	p ,	p,
„ Q „	$p-1$,	p;

whence in the first case No. of coefficients of P and Q is $(p+1)+p$, $=\frac{1}{2}(n+1)$, and in the second case No. is $(p+1)+(p+1)$, $=\frac{1}{2}(n+1)$, as before. Taking

$$P=\alpha+\gamma x^2+\varepsilon x^4+\ldots,$$
$$Q=\beta+\delta x^2+\zeta x^4+\ldots,$$

the formula is

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left(\frac{\alpha-\beta x+\gamma x^2-\ldots}{\alpha+\beta x+\gamma x^2+\ldots}\right)^2,$$

the number of coefficients being as just explained. Starting herefrom I reproduce in a somewhat altered form the investigation in the 'Fundamenta Nova,' as follows.

2. If the coefficients are such that the equation remains true when we therein change simultaneously x into $\frac{1}{kx}$ and y into $\frac{1}{\lambda y}$, then the variables x, y will satisfy the differential equation

$$\frac{M\,dy}{\sqrt{1-y^2.1-\lambda^2y^2}}=\frac{dx}{\sqrt{1-x^2.1-k^2x^2}}$$

(M a constant, the value of which, as will appear, is given by $\frac{1}{M}=1+\frac{2\beta}{\alpha}$); and the problem of transformation is thus to find the coefficients so that the equation may remain true on the above simultaneous change of the values of x, y.

In fact, observing that the original equation and therefore the new equation are each satisfied on changing therein simultaneously x, y into $-x, -y$, it follows that the equation may be written in the four forms

$$1-y=(1-x)\,A^2(\div),\quad 1+y=(1+x)\,B^2(\div),$$
$$1-\lambda y=(1-kx)C^2(\div),\quad 1+\lambda y=(1+kx)C^2(\div),$$

the common denominator being, say E, where A, B, C, D, E are all of them rational and integral functions of x; and this being so, the differential equation will be satisfied.

3. To develop the condition, observe that the assumed equation gives

$$y=\frac{x(P^2+2PQ+Q^2x^2)}{P^2+2PQx^2+Q^2x^4}, \quad =\frac{x\mathfrak{A}}{\mathfrak{B}} \text{ suppose,}$$

where $\mathfrak{A}$, $\mathfrak{B}$ are functions each of them of the degree $\frac{1}{2}(n-1)$ in x^2. (Hence, if with JACOBI $\frac{1}{M}$ denotes the value $(y\div x)_{x=0}$, we have $\frac{1}{M}=\left(1+\frac{2Q}{P}\right)_{x=0}, =1+\frac{2\beta}{\alpha}$, as mentioned.)

Suppose in general that U being any integral function $(1, x^2)^p$, we have

$$U^*=(k^2x^2)^p\left(1, \frac{1}{k^2x^2}\right)^p;$$

viz. let U^* be what U becomes when x is changed into $\frac{1}{kx}$ and the whole multiplied by $(k^2x^2)^p$.

Let y^* be the value of y obtained by writing $\frac{1}{kx}$ for x; then, observing that in the expression for y the degree of the numerator exceeds by unity that of the denominator, we have

$$y^*=\frac{1}{kx}\frac{\mathfrak{A}^*}{\mathfrak{B}^*},$$

whence

$$yy^*=\frac{1}{k}\frac{\mathfrak{A}\mathfrak{A}^*}{\mathfrak{B}\mathfrak{B}^*};$$

and the functions $\mathfrak{A}$, $\mathfrak{B}$ may be such that this shall be a constant value, $=\frac{1}{\lambda}$; viz. this will be the case if

$$\frac{\lambda}{k}=\frac{\mathfrak{B}\mathfrak{B}^*}{\mathfrak{A}\mathfrak{A}^*},$$

which being so, the required condition is satisfied.

4. I shall ultimately, instead of k, λ introduce JACOBI's u, v $(u=\sqrt[4]{k}, v=\sqrt[4]{\lambda})$; but it is for the present convenient to retain k, and instead of λ to introduce the quantity Ω connected with it by the equation $\lambda=k\Omega^2$; or say the value of Ω is $=v^2\div u^2$. The modular equation in its standard form is an equation between u, v, which, as will appear, gives rise to an equation of the same order between u^2, v^2; and writing herein $v^2=\Omega u^2$, the resulting equation contains only integer powers of u^4, that is of k, and we have an Ωk-form of the modular equation, or say an Ωk-modular equation, of the same order in Ω as the standard form is in v; these Ωk-forms for $n=3, 5, 7, 11$ will be given presently.

5. Suppose then, Ω being a constant, that we have identically

$$\mathfrak{A}=\frac{1}{\Omega k^{\frac{1}{2}(n-1)}}\mathfrak{B}^*;$$

this implies

$$\mathfrak{B}=\frac{\Omega}{k^{\frac{1}{2}(n-1)}}\mathfrak{A}^*.$$

(In fact if

$$\mathfrak{A} = a + cx^3 \ldots + qx^{n-3} + sx^{n-1},$$

$$\mathfrak{B} = b + dx^3 \ldots + rx^{n-3} + tx^{n-1},$$

then

$$\mathfrak{A}^* = s + qk^2x^3 \ldots + ck^{n-3}x^{n-3} + ak^{n-1}x^{n-1},$$

$$\mathfrak{B}^* = t + rk^2x^3 \ldots + dk^{n-3}x^{n-3} + bk^{n-1}x^{n-1},$$

and the assumed equation gives

$$a = \frac{1}{\Omega k^{(n-1)}} t, \quad c = \frac{k^2}{\Omega k^{(n-1)}} r, \ldots q = \frac{k^{n-3}}{\Omega k^{(n-1)}} d, \quad s = \frac{k^{n-1}}{\Omega k^{(n-1)}} b;$$

that is,

$$b = \frac{\Omega}{k^{(n-1)}} s, \quad d = \frac{\Omega k^2}{k^{(n-1)}} q, \ldots \quad r = \frac{\Omega k^{n-3}}{k^{(n-1)}} c, \quad t = \frac{\Omega k^{n-2}}{k^{(n-1)}} a,$$

and therefore $\mathfrak{B} = \frac{\Omega}{k^{(n-1)}} \mathfrak{A}^*$.)

From these equations $\frac{\mathfrak{B}\mathfrak{B}^*}{\mathfrak{A}\mathfrak{A}^*} = \Omega^2$, that is $= \frac{\lambda}{k}$, as it should be; so that Ω signifying as above, the required condition will be satisfied if only $\mathfrak{A} = \frac{1}{\Omega k^{(n-1)}} \mathfrak{B}^*$; or substituting for $\mathfrak{A}$, $\mathfrak{B}$ their values, if only

$$(P^2 + 2PQx^3 + Q^2x^6)^* = \Omega k^{(n-1)}(P^2 + 2PQ + Q^2x^6),$$

where each side is a function of x^3 of the order $\frac{1}{3}(n-1)$, or the number of terms is $\frac{1}{3}(n+1)$, the several coefficients being obviously homogeneous quadric functions of the $\frac{1}{3}(n+1)$ coefficients of P, Q. We have thus $\frac{1}{3}(n+1)$ equations, each of the form $U = \Omega V$, where U, V are given quadric functions of the coefficients of P, Q, say of the $\frac{1}{3}(n+1)$ coefficients $\alpha, \beta, \gamma, \delta$, &c., and where Ω is indeterminate.

6. We may from the $\frac{1}{3}(n+1)$ equations eliminate the $\frac{1}{3}(n-1)$ ratios $\alpha : \beta : \gamma \ldots$, thus obtaining an equation in Ω (involving of course the parameter k) which is the Ωk-modular equation above referred to; and then Ω denoting any root of this equation, the $\frac{1}{3}(n+1)$ equations give a single value for the set of ratios $\alpha : \beta : \gamma : \delta \ldots$, so that the ratio of the functions P, Q is determined, and consequently the value of y as given by the equation

$$\frac{1-y}{1+y} = \frac{(1-x)(P-Qx)^2}{(1+x)(P+Qx)^2}, \text{ or } y = \frac{x(P^2 + 2PQ + Q^2x^6)}{P^2 + 2PQx^3 + Q^2x^6}.$$

The entire problem thus depends on the solution of the system of $\frac{1}{3}(n+1)$ equations,

$$(P^2 + 2PQx^3 + Q^2x^6)^* = \Omega k^{(n-1)}(P^2 + 2PQ + Q^2x^6).$$

The Ωk-Modular Equations, $n = 3, 5, 7, 11$.—Article No. 7.

7. For convenience of reference, and to fix the ideas, I give these results, calculated, as above explained, from the standard or uv-forms.

	k^3	k	1	
Ω^4		$+1$		$=0$
Ω^3	-4			
Ω^2		$+6$		
Ω			-4	
Ω^0		$+1$		
	-4	$+8$	-4	

$n=3$

$\Omega=1$, we have $-4(k-1)^3=0$

	k^4	k^3	k^2	k	1	
Ω^5			$+1$			$=0$
Ω^4	-16		$+10$			
Ω^3			$+15$			
Ω^2			-20			
Ω			$+15$			
Ω^0			$+10$		-16	
			$+1$			
	-16		$+32$		-16	

$n=5$

$\Omega=1$, we have $-16(k^2-1)^5=0$

	k^6	k^5	k^4	k^3	k^2	k	1	
Ω^6				$+1$				$=0$
Ω^5	-64		$+56$					
Ω^4		-112		$+140$				
Ω^3			-112		$+56$			
Ω^2				$+70$				
Ω			$+56$		-112			
Ω^0				$+140$		-112		
					$+56$		-64	
				$+1$				
	-64	-112	0	$+352$	0	-112	-64	

$n=7$

$\Omega=1$, we have

$-16(k-1)^7(4k^2+3k+1)(k^2+3k+4)=0$

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$n=11$.

	k^{10}	k^9	k^8	k^7	k^6	k^5	k^4	k^3	k^2	k	1	
Ω^{12}						+ 1						+ 1
Ω^{11}	−1024		+ 1408		− 396							− 12
Ω^{10}		−5632		+ 4400		+ 1298						+ 66
Ω^{9}			−16192		+16368		− 396					−220
Ω^{8}				−18656		+19151						+495
Ω^{7}			−16016		− 1144		+16368					−792
Ω^{6}				+ 4400		− 7876		+ 4400				+924
Ω^{5}					+16368		− 1144		−16016			−792
Ω^{4}						+19151		−18656				+495
Ω^{3}					− 396		+16386		−16192			−220
Ω^{2}						+ 1298		+ 4400		−5632		+ 66
Ω^{1}							− 396		+ 1408		−1024	− 12
Ω^{0}						+ 1						+ 1
			−32208	−18656	− 1936	− 7876	− 1936	−18656	−32208			
			+ 1408	+ 8800	+32736	+40900	+32736	+ 8800	+ 1408			
	−1024	−5632	−30800	− 9856	+30800	+33024	+30800	− 9856	−30800	−5632	−1024	±94624

Equation-systems for the cases $n=3, 5, 7, 9, 11$.—Article Nos. 8 to 10.

8. $n=3$, cubic transformation. $k=u^4$, $\Omega=\frac{v^2}{u^3}$ (here and in the other cases).

$P=\alpha$, $Q=\beta$. The condition here is

$$k^2x^3\alpha^3+(2\alpha\beta+\beta^3)=\Omega k\{(\alpha^3+2\alpha\beta)+\beta^3x^3\},$$

and the system of equations thus is

$$k\alpha^3=\Omega\beta^3,$$

$$2\alpha\beta+\beta^3=\Omega k(\alpha^3+2\alpha\beta),$$

and similarly in the other cases; for these it will be enough to write down the equation-systems.

$n=5$, quintic transformation.

$P=\alpha+\gamma x^2$, $Q=\beta$.

$$k^2\alpha^3=\Omega\gamma^3,$$

$$2\alpha\gamma+2\alpha\beta+\beta^3=\Omega(2\alpha\gamma+2\beta\gamma+\beta^3),$$

$$\gamma^3+2\beta\gamma=\Omega k^2(\alpha^3+2\alpha\beta).$$

$n=7$, septic transformation.

$P=\alpha+\gamma x^3$, $Q=\beta+\delta x^3$.

$$k^3\alpha^3=\Omega\delta^3,$$
$$k(2\alpha\gamma+2\alpha\beta+\beta^3)=\Omega(\gamma^3+2\gamma\delta+2\beta\delta),$$
$$\gamma^3+2\beta\gamma+2\alpha\delta+2\beta\delta=\Omega k(2\alpha\gamma+2\beta\gamma+2\alpha\delta+\beta^3),$$
$$\delta^3+2\gamma\delta=\Omega k^3(\alpha^3+2\alpha\beta).$$

$n=9$, enneadic transformation.

$P=\alpha+\gamma x^3+\varepsilon x^4$, $Q=\beta+\delta x^3$.

$$k^4\alpha^3=\Omega\varepsilon^3,$$
$$k^3(2\alpha\gamma+2\alpha\beta+\beta^3)=\Omega(2\gamma\varepsilon+2\varepsilon\delta+\delta^3),$$
$$2\alpha\varepsilon+\gamma^3+2\alpha\delta+2\gamma\beta+2\beta\delta=\Omega(2\alpha\varepsilon+\gamma^3+2\gamma\delta+2\varepsilon\beta+2\beta\delta),$$
$$2\gamma\varepsilon+2\gamma\delta+2\varepsilon\beta+\delta^3=\Omega k^3(2\alpha\gamma+2\alpha\delta+2\gamma\beta+\beta^3),$$
$$\varepsilon^3+2\delta\varepsilon=\Omega k^4(\alpha^3+2\alpha\beta).$$

$n=11$, endecadic transformation.

$P=\alpha+\gamma x^3+\varepsilon x^4$, $Q=\beta+\delta x^3+\zeta x^4$.

$$k^3\alpha^3 \quad =\Omega\zeta^3,$$
$$k^3(2\alpha\gamma+2\alpha\beta+\beta^3)=\Omega(\varepsilon^3+2\varepsilon\zeta+2\delta\zeta),$$
$$k(2\alpha\varepsilon+\gamma^3+2\alpha\delta+2\gamma\beta+2\beta\delta)=\Omega(2\gamma\varepsilon+2\gamma\zeta+2\varepsilon\delta+2\beta\delta+\delta^3),$$
$$2\gamma\varepsilon+2\alpha\zeta+2\gamma\delta+2\varepsilon\beta+2\beta\zeta+\delta^3=\Omega k(2\alpha\varepsilon+\gamma^3+2\alpha\zeta+2\gamma\delta+2\varepsilon\beta+2\beta\delta),$$
$$\varepsilon^3+2\gamma\zeta+2\varepsilon\delta+2\delta\zeta=\Omega k^3(2\alpha\gamma+2\alpha\delta+2\gamma\beta+\beta^3),$$
$$2\varepsilon\zeta+\zeta^3=\Omega k^3(\alpha^3+2\alpha\beta),$$

and so on.

9. It will be noticed that if the coefficients of $P+Qx$ taken in order are

$$\alpha, \beta \ldots . \varepsilon, \sigma,$$

then in every case the first and last equations are

$$k^{\frac{1}{2}(n-1)}\alpha^3=\Omega\sigma^3,$$
$$2\varepsilon\sigma+\sigma^3=\Omega k^{\frac{1}{2}(n-1)}(\alpha^3+2\alpha\beta).$$

Putting in the first of these $k=u^4$, $\Omega=\frac{v^3}{u^6}$, the equation becomes

$$u^{2n}\alpha^3=v^3\sigma^3,$$

where each side is a perfect square; and in extracting the square root we may without loss of generality take the roots positive, and write $u^n\alpha=v\sigma$.

This speciality, although it renders it proper to employ ultimately u, v in place of k, Ω, produces really no depression of order (viz. the Ωk-form of the modular equation is found to be of the same order in Ω that the standard or uv-form is in v), and is in another point of view a disadvantage, as destroying the uniformity of the several equations: in the discussion of order I consequently retain Ω, k. Ultimately these are to

be replaced by u, v; the change in the equation-systems is so easily made that it is not necessary here to write them down in the new form in u, v.

10. The case $\alpha=0$ has to be considered in the discussion of order, but we have thus only solutions which are to be rejected; in the proper solutions α is not $=0$, and it may therefore for convenience be taken to be $=1$. We have then $\sigma=u^n+v$. The last equation becomes therefore

$$\frac{u^n}{v}\left(2\xi+\frac{u^n}{v}\right)=\frac{v^2}{u^2}u^{2n-2}(1+2\beta);$$

or recollecting that β is connected with the multiplier M by the relation $\frac{1}{M}=1+2\beta$, that is,

$$2\beta=\frac{1}{M}-1,$$

and substituting for $1+2\beta$ its value, the equation becomes

$$2\xi=v^3u^{n-4}\left(\frac{1}{M}-\frac{u^4}{v^4}\right);$$

that is, the first and last coefficients are $1, \frac{u^n}{v}$, and the second and penultimate coefficients are each expressed in terms of v, M. The cases $n=3$, $n=5$ are so far peculiar, that the only coefficients are α, β, or α, β, γ; in the next case $n=7$, the only coefficients are α, β, γ, δ, and we have in this case all the coefficients expressed as above.

The Ωk-form—Order of the Systems.—Article Nos. 11 to 22.

11. In the general case, n an odd number, we have Ω and $\frac{1}{2}(n+1)$ coefficients connected by a system of $\frac{1}{2}(n+1)$ equations of the form

$$\Omega=\frac{U}{U'}=\frac{V}{V'}=\ldots,$$

where U, V, .. U', V', ... are given quadric functions of the coefficients. Omitting the $(\Omega=)$, there remains a system of $\frac{1}{2}(n-1)$ equations of the form $\frac{U}{U'}=\frac{V}{V'}=$ .. or say

$$\begin{vmatrix} U, & V, & W, .. \\ U', & V', & W', .. \end{vmatrix}=0,$$

which determine the ratios $\alpha:\beta:\gamma\ldots$ of the coefficients; and to each set of ratios there corresponds a single value of Ω. The order of the system, or number of sets of ratios, is $=\frac{1}{2}(n+1).2^{\frac{1}{2}(n-1)}$, $=(n+1).2^{\frac{1}{2}(n-3)}$; and this is consequently the number of values of Ω, or order of the equation for the determination of Ω; viz. but for reduction the order in Ω of the Ωk-modular equation would be $=(n+1).2^{\frac{1}{2}(n-3)}$. In the case $n=3$, this is $=4$, which is right, but for any larger value of n the order is far too high; in fact, assuming (as the case is) that the order is equal to the order in v of the uv-form, the order should for a prime value of n be $=n+1$, and for a composite value not containing any square factor be $=$ the sum of the divisors of n. I do not attempt a general investigation, but confine myself to showing in what manner the reductions arise.

12. I will first consider the cubic transformation; here, writing for convenience $\frac{\alpha}{\beta}=\theta$, the equations give

$$\frac{k\theta^3}{2\theta+1}=\frac{1}{(k\theta^3+2\theta)^2}, \text{ that is, } k^3\theta^3(\theta+2)-(2\theta+1)=0,$$

and

$$k\theta^3=\Omega;$$

the equation in θ gives $(k^3\theta^4-1)^3-4\theta^3(k^3\theta^3-1)^3=0$, and we have thence

$$k(\Omega^3-1)^3-4\Omega(k\Omega-1)^3=0,$$

that is

$$k\Omega^4-4k^2\Omega^3+6k\Omega^2-4\Omega+k=0,$$

the modular equation; and then $k^3\theta^4-1+2\theta(k^3\theta^3-1)=0$, that is, $\Omega^3-1+2\theta(k\Omega-1)=0$, or $2\theta=-\frac{\Omega^3-1}{k\Omega-1}$, which is $=\frac{2\alpha}{\beta}$, say we have $\alpha=\Omega^3-1$, $\beta=2(1-k\Omega)$; consequently

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left\{\frac{\Omega^3-1+2(k\Omega-1)x}{\Omega^3-1-2(k\Omega-1)x}\right\}^3,$$

$$\lambda=\Omega^3k, \text{ and } \frac{1}{M}=\frac{\alpha^3+2\alpha\beta}{\alpha^3}=\frac{\theta+2}{\theta}=\frac{\Omega^3-4k\Omega+3}{\Omega^3-1},$$

which completes the theory.

13. Reproducing for this case the general theory, it appears *à priori* that Ω is determined by a quartic equation; in fact from the original equations eliminating Ω, we have an equation

$$\begin{vmatrix} U, & V \\ U', & V' \end{vmatrix}=0,$$

where U, U', V, V' are quartic functions of α, β; that is the ratio $\alpha:\beta$ has four values, and to each of these there corresponds a single value of Ω; viz. Ω is determined by a quartic equation.

14. Considering next the case $n=5$, the quintic transformation; the elimination of Ω gives the equations

$$\frac{U}{U'}=\frac{V}{V'}=\frac{W}{W'},$$

where U, U', &c. are all quadric functions of α, β, γ. We have thence $4\cdot4-2\cdot2$, $=12$ sets of values of $\alpha:\beta:\gamma$; viz. considering α, β, γ as coordinates *in plano*, the curves $UV'-U'V=0$, $UW'-U'W=0$ are quartic curves intersecting in 16 points; but among these are included the four points $U=0$, $U'=0$ (in fact the point $\alpha=0$, $\gamma=0$ four times), which are not points of the curve $VW'-V'W=0$; there remain therefore $16-4$, $=12$ intersections (agreeing with the general value $(n+1).2^{\frac{1}{2}(n-2)}$). Hence Ω is in the first instance determined by an equation of the order 12; but the proper order being $=6$, there must be a factor of the order 6 to be rejected. To explain this and determine the factor, observe that the equations in question are

$$k^3\alpha^3(2\alpha\gamma+2\beta\gamma+\beta^2)-\gamma^3(2\alpha\gamma+2\alpha\beta+\beta^2)=0,$$
$$k^4\alpha^3(\alpha+2\beta)\qquad -\gamma^3(\gamma+2\beta)\qquad =0;$$

the first of these has at the point $\alpha=0$, $\gamma=0$ a double point, the second a triple point; or there are at the point in question 6 intersections; but 4 of these are the points which give the foregoing reduction $16-4=12$; we have thus the point $\alpha=0$, $\gamma=0$, counting twice among the twelve points. Writing in the two equations $\beta=0$, the equations become $k^3\alpha^3\gamma-\alpha\gamma^3=0$, $k^4\alpha^4-\gamma^4=0$, viz. these will be satisfied if $k^3\alpha^3-\gamma^3=0$, that is, the curves pass through each of the two points ($\beta=0$, $\gamma=\pm k\alpha$), and these values satisfy (as in fact they should) the third equation,

$$k^3(2\alpha\gamma+2\alpha\beta+\beta^2)\alpha(\alpha+2\beta)-\gamma(\gamma+2\beta)(2\alpha\gamma+2\beta+\beta^2)=0;$$

it is moreover easily shown that the three curves have at each of the points in question a common tangent; viz. taking A, B, C as current coordinates, the tangent at the point (α, β, γ) of the second curve has for its equation

$$A(2\alpha^3+3\alpha^2\beta)k^4+B(k^4\alpha^3-\gamma^3)-C(2\gamma^3+3\gamma^2\beta)=0;$$

and for $\beta=0$, $\gamma=\pm k\alpha$ this becomes $2kA+B(k\mp1)\mp2C=0$, viz. this is the line from the point ($\beta=0$, $\gamma=\pm k\alpha$) to the point $(1, -2, 1)$. And similarly for the other two curves we find the same equation for the tangent.

Hence among the 12 points are included the point ($\gamma=0$, $\alpha=0$) twice, and the points ($\beta=0$, $\gamma=\pm k\alpha$) each twice: we have thus a reduction $=6$.

15. Writing in the equations $\gamma=0$, $\alpha=0$, the first and third are satisfied identically, and the second becomes $\beta^3=\Omega\beta^3$, that is the equations give $\Omega=1$; writing $\beta=0$, they become

$$k^3\alpha^3=\Omega\gamma^3, \quad \alpha\gamma=\Omega\alpha\gamma, \quad \gamma^3=\Omega k^3\alpha^3,$$

viz. putting herein $\gamma^3=k^3\alpha^3$, the equations again give $\Omega=1$; hence the factor of the order 6 is $(\Omega-1)^6$, and the equation of the twelfth order for the determination of Ω is

$$(\Omega-1)^6\{(\Omega, 1)^6\}=0,$$

where $(\Omega, 1)^6=0$ is the Ωk-modular equation above written down.

16. Reverting to the equation

$$\frac{1-y}{1+y}=\frac{(1-x)(P-Qx)^2}{(1+x)(P+Qx)^2},$$

it is to be observed that for $\alpha=0$, $\gamma=0$, that is $P=0$, this becomes simply $y=x$, which is the transformation of the order 1; the corresponding value of the modulus λ is $\lambda=k$, and the equation $\lambda=\Omega^2 k$ then gives $\Omega^2=1$, which is replaced by $\Omega-1=0$.

If in the same equation we write $\beta=0$, that is $Q=0$, then (without any use of the equation $\gamma^3=k^3\alpha^3$) we have $y=x$, the transformation of the order 1; but although this is so, the fundamental equation

$$(P^2+2PQx^2+Q^2x^4)^2=\Omega k^3(P^2+2PQ+Q^2x^2),$$

which, putting therein $Q=0$, becomes $(P^2)^2=\Omega k^3P^2$; that is $(k^3x^3\alpha+\gamma)^3=\Omega k^3(\alpha+\gamma x^3)^3$ is not satisfied by the single relation $\Omega-1=0$, but necessitates the further relation $\gamma^3=k^3\alpha^3$.

The thing to be observed is that the extraneous factor $(\Omega-1)^6$, equated to zero, gives for Ω the value $\Omega=1$ corresponding to the transformation $y=x$ of the order 1.

17. Considering next $n=7$, the septic transformation; we have here between $\alpha, \beta, \gamma, \delta$ a fourfold relation of the form

$$\left|\begin{array}{cccc} U, & V, & W, & Z \\ U', & V', & W', & Z' \end{array}\right|=0,$$

where, as before, U, U', &c. are quadric functions, and the number of solutions is here 8.2^3, $=32$; to each of these corresponds a single value of Ω, or Ω is in the first instance determined by an equation of the order 32. But the order of the modular equation is $=8$; or representing this by $\{(\Omega, 1)^8\}=0$, the equation must be

$$(\Omega, 1)^{24}\{(\Omega, 1)^8\}=0,$$

viz. there must be a special factor of the order 24.

18. One way of satisfying the equations is to write therein $\alpha=0$, $\delta=0$; the equations thus become

$$k\beta^3=\Omega\gamma^3,$$
$$\gamma^3+2\beta\gamma=\Omega k(2\beta\gamma+\beta^3);$$

or putting $\beta, \gamma=\alpha', \beta'$,

$$k\alpha'^3=\Omega\beta'^3,$$
$$\beta'^3+2\alpha'\beta'=\Omega k(2\alpha'\beta'+\alpha'^3),$$

which (with α', β' instead of α, β) are the very equations which belong to the cubic transformation; hence a factor is $\{(\Omega, 1)^4\}$.

Observe that for the values in question $\alpha=0$, $\delta=0$, $P=\beta' x^3$, $Q=\alpha'$,

$$(P\pm Qx)^3=x^3(\alpha'\pm\beta' x)^3, \quad =x^3(P'\pm Q'x)^3, \text{ if } P'=\alpha', \ Q'=\beta',$$

and therefore

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left(\frac{P'-Q'x}{P'+Q'x}\right)^3,$$

which is the formula for a cubic transformation.

19. The equations may also be satisfied by writing therein $\gamma=k\alpha$, $\delta=k\beta$; in fact substituting these values, they become

$$k^3\alpha^3=\Omega k^3\beta^3,$$
$$2k^3\alpha^3+k(2\alpha\beta+\beta^3)=\Omega k^3(\alpha^3+2\alpha\beta)+2\Omega k\beta^3,$$
$$k^3\alpha^3+2k(\beta^3+2\alpha\beta)=2\Omega k^3(\alpha^3+2\alpha\beta)+\Omega k\beta^3,$$
$$k^3(\beta^3+2\alpha\beta)=\Omega k^3(\alpha^3+2\alpha\beta);$$

the first and last of these are

$$k\alpha^3=\Omega\beta^3,$$
$$\beta^3+2\alpha\beta=\Omega k(\alpha^3+2\alpha\beta),$$

which being satisfied the second and third equations are satisfied identically; and these are the formulæ for a cubic transformation; that is, we again have the factor $\{(\Omega, 1)^4\}$.

Observe that for the values in question $\gamma=k\alpha$, $\delta=k\beta$, we have $P=\alpha(1+kx^3)$, $Q=\beta(1+kx^3)$; so that, writing $P'=\alpha$, $Q'=\beta$, we have for y the value

$$\frac{1-y}{1+y}=\frac{(1-x)(P'-Q'x)^2}{(1+x)(P'+Q'x)^2},$$

which is the formula for a cubic transformation.

20. It is important to notice that we cannot by writing $\alpha=0$ or $\delta=0$ reduce the transformation to a quintic one; in fact the equation $k^2\alpha^2=\Omega\delta^2$ shows that if either of these equations is satisfied the other is also satisfied; and we have then the foregoing case $\alpha=0$, $\delta=0$, giving not a quintic but a cubic transformation.

And for the same reason we cannot by writing $\alpha=0$, $\beta=0$, $\gamma=0$ or $\beta=0$, $\gamma=0$, $\delta=0$ reduce the transformation to the order 1. There is thus no factor $\Omega-1$.

21. As regards the non-existence of the factor $\Omega-1$, I further verify this by writing in the equations $\Omega=1$; they thus become

$$k^2\alpha^2=\delta^2,$$

$$k(2\alpha\gamma+2\alpha\beta+\beta^2)=\gamma^2+2\gamma\delta+2\beta\delta,$$

$$\gamma^2+2\beta\gamma+2\alpha\delta+2\beta\delta=k(2\alpha\gamma+2\beta\gamma+2\alpha\delta+\beta^2),$$

$$\delta^2+2\gamma\delta=k^2(\alpha^2+2\alpha\beta),$$

which it is to be shown cannot be satisfied in general, but only for certain values of k.

Reducing the last equation, this is $\gamma\delta=k^2\alpha\beta$, which, combined with the first, gives $\alpha\gamma=\beta\delta$; and if for convenience we assume $\alpha=1$, and write also $\theta=\pm\sqrt{k}$ (that is $k=\theta^2$), then the values of α, β, γ, δ are $\alpha=1$, $\beta=\gamma\theta^{-2}$, $\gamma=\gamma$, $\delta=\theta^2$; which values, substituted in the second and third equations, give two equations in γ, θ; and from these, eliminating γ, we obtain an equation for the determination of θ, that is of k. In fact the second equation gives

$$\theta^2(2\gamma+2\gamma\theta^{-2}+\gamma^2\theta^{-6})=\gamma^2+2\gamma\theta^2+2\gamma;$$

or, dividing by γ and reducing,

$$\gamma(1-\theta^4)=2\theta^2(\theta^2-1)(\theta^2-\theta+1), \text{ that is}$$

$$\gamma(1+\theta^2)=-2\theta^2(\theta^2-\theta+1),$$

or, as this may also be written,

$$(\gamma+\theta^2)(1+\theta^2)=-\theta^2(\theta-1)^2,$$

that is

$$(\gamma+\theta^2)=\frac{-\theta^2(\theta-1)^2}{\theta^2+1}.$$

Moreover the third equation gives

$$\gamma^2+2\gamma^2\theta^{-2}+2\theta^2+2\gamma=\theta^2(2\gamma+2\gamma^2\theta^{-2}+2\theta^2+\gamma^2\theta^{-6}),$$

that is,

$$\gamma^2(\theta^4-2\theta^2+2\theta-1)-2(\gamma+\theta^2)\theta^4(\theta^2-1)=0;$$

or dividing by θ^2-1, it is

$$\gamma^2(\theta-1)^2=2\theta^4(\gamma+\theta^2);$$

whence also

$$\gamma^2=\frac{-2\theta}{\theta^2+1}.$$

Also

$$4\theta^2(\theta^2-\theta+1)^3=\gamma^2(\theta^3+1)^3,$$

wherefore

$$2(\theta^2-\theta+1)^3=-\theta(\theta^3+1) \text{ or } 2(\theta^2-\theta+1)^3+\theta(\theta^3+1)=0,$$

or

$$\theta(\theta^3+1)^3+2(\theta^2-\theta+1)^3=0,$$

that is

$$2\theta^4-3\theta^3+6\theta^2-3\theta+2=0,$$

or finally

$$(2\theta^2-\theta+1)(\theta^2-\theta+2)=0.$$

We have thus $(2\theta^2+1)^2=\theta^2$, that is $4\theta^4+3\theta^2+1=0$ or $4k^2+3k+1=0$, or else $(\theta^2+2)^2=\theta^2$, that is $\theta^4+3\theta^2+4=0$ or $k^2+3k+4=0$; viz. the equation in k is

$$(4k^2+3k+1)(k^2+3k+4)=0,$$

these being in fact the values of k given by the modular equation on putting therein $\Omega=1$.

The equation of the order 32 thus contains the factor $\{(\Omega, 1)^4\}$ at least twice, and it does not contain either the factor $\Omega-1$, or the factor $\{(\Omega, 1)^6\}$ belonging to the quintic transformation; it may be conjectured that the factor $\{(\Omega, 1)^4\}$ presents itself six times, and that the form is

$$\{(\Omega, 1)^4\}^6(\Omega, 1)^8=0;$$

but I am not able to verify this, and I do not pursue the discussion further.

22. The foregoing considerations show the grounds of the difficulty of the purely algebraical solution of the problem; the required results, for instance the modular equation, are obtained not in the simple form, but accompanied with special factors of high order. The transcendental theory affords the means of obtaining the results in the proper form without special factors; and I proceed to develop the theory, reproducing the known results as to the modular and multiplier equations, and extending it to the determination of the transformation-coefficients $\alpha, \beta \ldots$

The Modular Equation.—Article Nos. 23 to 28.

23. Writing, as usual, $q=e^{-\frac{\pi K'}{K}}$, we have u, a given function of q, viz.

$$\begin{aligned} u&=\sqrt{2}q^{\frac{1}{4}}\frac{1+q^2.1+q^4.1+q^6..}{1+q.1+q^3.1+q^5..}\\ &=\sqrt{2}q^{\frac{1}{4}}(1-q+2q^2-3q^3+4q^4-6q^5+9q^6-12q^7+\ldots)\\ &=\sqrt{2}q^{\frac{1}{4}}f(q) \text{ suppose;} \end{aligned}$$

and this being so, the several values of v and of the other quantities in question are all given in terms of q.

The case chiefly considered is that of n an odd prime; and unless the contrary is stated it is assumed that this is so. We have then $n+1$ transformations corresponding to the same number $n+1$ of values of v; these may be distinguished as $v_0, v_1, v_2, \ldots v_n$;

viz. writing α to denote an imaginary n-th root of unity, we have

$$v_0 = (-)^{\frac{n^2-1}{8}}\sqrt{2}q^{\frac{n}{8}}f(q^n),\quad v_1 = \sqrt{2}(\alpha q^{\frac{1}{8}})^{\frac{1}{8}}f(\alpha q^{\frac{1}{8}}),\quad v_2 = \sqrt{2}(\alpha^2 q^{\frac{1}{8}})^{\frac{1}{8}}f(\alpha^2 q^{\frac{1}{8}})\ \&c.,$$

$$v_n = \quad \sqrt{2}q^{\frac{1}{8n}}f(q^{\frac{1}{n}}).$$

(Observe $(-)^{\frac{n^2-1}{8}} = +$ for $n = 8p \pm 1$, $-$ for $n = 8p \pm 3$.)

The occurrence of the fractional exponent $\frac{1}{8}$ is, as will appear, a circumstance of great importance; and it will be convenient to introduce the term "octicity," viz. an expression of the form $q^{\frac{f}{8}}F(q)$ ($f = 0$, or a positive integer not exceeding 7, $F(q)$ a rational function of q) may be said to be of the octicity f.

24. The modular equation is of course

$$(v - v_0)(v - v_1)\ldots(v - v_n) = 0;$$

say this is

$$v^{n+1} - Av^n + Bv^{n-1} - \ldots = 0,$$

so that $A = \Sigma v_0$, $B = \Sigma v_0 v_1$, &c. In the development of these expressions, the terms having a fractional exponent denominator n would disappear of themselves, as involving symmetrically the several n-th roots of unity, and each coefficient would be of the form $q^{\frac{f}{8}}F(q)$, F a rational and integral function of q. It is moreover easy to see that, for the several coefficients A, B, C, g will denote the positive residue (mod. 8) of n, $2n$, $3n$, ... respectively.

Hence assuming, as the fact is, that these coefficients are severally rational and integral functions of q, it follows that the form is

$$au^g + bu^{g+8} + cu^{g+16} + \ldots,$$

g having the foregoing values for the several coefficients respectively. And it being known that the modular equation is as regards u of the order $= n+1$, there is a known limit to the number of terms in the several coefficients respectively. We have thus for each coefficient an identity of the form

$$A = au^g + bu^{g+8} + \ldots,$$

where A and u being each of them given in terms of q, the values of the numerical coefficients a, b, .. can be determined; and we thus arrive at the modular equation.

25. It is in effect in this manner that the modular equations are calculated in SOHNKE's Memoir. Various relations of symmetry in regard to (u, v) and other known properties of the modular equation are made use of in order to reduce the number of the unknown coefficients to a minimum; and (what in practice is obviously an important simplification) instead of the coefficients Sv_0, Sv_0v_1, &c., it is the sums of powers Sv_0, Sv_0^2 &c. which are compared with their expressions in terms of u, in order to the determination of the unknown numerical coefficients a, b.. The process is a laborious one (although less so than perhaps might beforehand have been imagined), involving very high numbers; it requires the development up to high powers of q, of the high powers of the before mentioned function $f(q)$; and SOHNKE gives a valuable Table, which I reproduce here; adding to it the three columns which relate to ϕq.

ind. of q.	Φq	$\Phi^2 q$	$\Phi^3 q$	fq	f^2q	f^3q	f^4q	f^5q	f^6q	f^7q	f^8q	f^9q	$f^{10}q$	$f^{11}q$
	=	=	=	=	=	=	=	=	=	=	=	=	=	=
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	+2	+ 4	− 4	− 2	− 3	− 4	− 5	+ 6	− 7	− 8	− 9	− 10	+ 11	11
2	0	+ 4	+ 12	+ 5	+ 9	+ 14	+ 20	− 27	+ 35	+ 44	+ 54	+ 65	− 77	77
3	0	0	− 32	− 10	− 22	− 40	− 65	+ 98	− 140	− 192	− 255	− 330	+ 418	418
4	+2	+ 4	+ 76	+ 18	+ 48	+ 101	+ 185	− 309	+ 483	+ 718	+ 1026	+ 1420	− 1914	1914
5	0	+ 8	− 168	− 32	− 99	− 236	− 481	+ 882	− 1498	− 2400	− 3672	− 5412	+ 7733	7733
6	0	0	+ 352	+ 55	+ 194	+ 518	+ 1165	− 2330	+ 4277	+ 7352	+ 11997	+ 18765	− 28336	28336
7	0	0	− 704	+ 90	+ 363	+ 1080	+ 2665	− 5784	+ 11425	+ 20992	+ 36414	+ 60270	− 95931	95931
8	0	+ 4		+ 144	+ 637	+ 2162	+ 5830	− 13644	+ 28889	+ 55549	+ 103977	+ 181645	− 304068	304068
9	+2	+ 4		+ 226	+ 1155	+ 4180	+ 12320	− 30826	+ 69734	+ 145008	+ 281911	+ 518660	− 911240	911240
10	0	+ 8		+ 346	+ 1977	+ 7840	+ 24898	− 67107	+ 161735	+ 356368	+ 730953	+ 1413465	− 2501786	2501786
11	0	0		+ 522	+ 3312	+ 14328	+ 48860	− 141444	+ 362271	+ 844032	+ 1823059	+ 3597960	− 7120136	7120136
12	0	0		+ 777	+ 5443	+ 25591	+ 93665	− 289746	+ 786877	+ 1934534	+ 4390824	+ 9331565	− 18766739	18766739
13	0	+ 8		+ 1138	+ 8787	+ 44776	+ 176125	− 578646	+ 1662927	+ 4306368	+ 10256508	+ 22800030	− 47830486	47830486
14	0	0		+ 1648	+ 13968	+ 76918	+ 323685	− 1139527	+ 3428770	+ 9337704	+ 23303025	+ 54112825	− 118270746	118270746
15	0	0		− 2362	− 21894	− 129952	− 583798	+ 2159774	− 6913760	− 19771392	− 51631227	− 125990220	+ 284527793	284527793
16	+2	+ 4		+ 3348	+ 33873	+ 216240	+ 1035060	− 4052721	+ 13660346	+ 40965352	+ 111804966	+ 282998020	− 667553898	667553898
17	0	+ 8		− 4704	− 51795	− 354864	− 1806600	+ 7474806	− 26492361	− 83207976	− 237074748	− 623185010	+ 1530587236	1530587236
18	0	+ 4		+ 6554	+ 78345	+ 574958	+ 3108085	− 13108085	+ 50504755	+ 165944732	+ 493063403	+ 1348033540	− 3435796536	3435796536
19	0	0		− 9056	− 117418									
20	0	+ 8		+ 12425	+ 174033									
21	0	0		− 16932	− 239945									
22	0	0		+ 484										
23	0	0		− 592										
24	0	0		+ 728										
25	+2	+ 12		− 876										
26	0	+ 8		+ 1060										

ind. of q.	$f^{12}q$ =	$f^{13}q$ =	$f^{14}q$ =	$f^{15}q$ =	$f^{16}q$ =	$f^{17}q$ =	$f^{18}q$ =	$f^{19}q$ =	$f^{20}q$ =
0	1	1	1	1	1	1	1	1	1
1	− 12	− 13	− 14	− 15	− 16	− 17	− 18	− 19	− 20
2	+ 90	+ 104	+ 119	+ 135	+ 152	+ 170	+ 189	+ 209	+ 230
3	− 520	− 637	− 770	− 920	− 1088	− 1275	− 1482	− 1710	− 1960
4	+ 2523	+ 3263	+ 4151	+ 5205	+ 6444	+ 7888	+ 9558	+ 11476	+ 13665
5	− 10764	− 14651	− 19558	− 25668	− 33184	− 42330	− 53352	− 66519	− 82124
6	+ 41534	+ 59345	+ 82936	+ 113675	+ 153152	+ 203201	+ 265923	+ 343710	+ 439270
7	− 147720	− 221091	− 322828	− 461265	− 646528	− 890800	− 1208610	− 1617147	− 2136600
8	+ 490869	+ 768131	+ 1169847	+ 1739710	+ 2533070	+ 3619334	+ 5084478	+ 7034047	+ 9596460
9	− 1539472	− 2514551	− 3988292	− 6164345	− 9311664	− 13780540	− 20021534	− 28607673	− 40260300
10	+ 4592430	+ 7818200	+ 12896562	+ 20690964	+ 32387616	+ 49590581	+ 74438388	+ 109745767	+ 159174524
11	− 13111632	− 23233535	− 39809574	− 66222405	− 107299904	− 169812320	− 263104686		
12	+ 36006362	+ 66328964	+ 117921321	+ 203173760	+ 340436664	+ 556366922	+ 889020813		
13	− 95497116	+ 182681916	− 336630840	− 600165795	− 1039026144	− 1752038020	− 2884990266		
14	+ 245457000	− 487098378	+ 929461993	+ 1713196575	+ 3061896704	+ 5323089708	+ 9026077050		
15	− 613183064	− 1261118313	− 2489690882	− 4740491107	− 8739810688	− 15653783345	− 27314626158		
16	+1492474572	+ 3178449222	+ 6486711301	+12748926285	+ 24229115109	+ 44679433473	+ 80177033788*		
17	−3546915228	− 7815313766	−16475721276	−33400680615	− 65390485328	−124069449335	−228831885054		
18	+8245677110	+18783535199	+40874694490	+85415669230	+172155210320	+335888162944	+636376573943		

* So in SOHNKE, but a figure must have dropped out.

26. I give from SOHNKE the series of modular equations, adding those for the composite cases $n=9$ and $n=15$, as to which see the remarks which follow the Table.

$n=3$.

	v^4	v^3	v^2	v	1
u^4					-1
u^3		$+2$			
u^2					
u				-2	
1	$+1$				
	1	$+2$	0	-2	-1 $=(v+1)^2(v-1)$.

$n=5$.

	v^6	v^5	v^4	v^3	v^2	v	1
u^6							-1
u^5		$+4$					
u^4					-5		
u^3							
u^2			$+5$				
u						-4	
1	$+1$						
	1	$+4$	$+5$	0	-5	-4	-1 $=(v+1)^2(v-1)$.

$n=7$.

	v^8	v^7	v^6	v^5	v^4	v^3	v^2	v	1
u^8	0								$+1$
u^7		-8							
u^6			$+28$						
u^5				-56					
u^4					$+70$				
u^3						-56			
u^2							$+28$		
u								-8	
1	$+1$								0
	1	-8	$+28$	-56	$+70$	-56	$+28$	-8	$+1$ $=(v-1)^8$.

	v^{12}	v^{11}	v^{10}	v^9	v^8	v^7	v^6	v^5	v^4	v^3	v^2	v	1
w^{12}					0								+1
w^{11}				−16								+8	
w^{10}			+16								+10		
w^9		−16								−24			
w^8	0								+15				
w^7								+48					
w^6							−84						
w^5						+48							
w^4					+15								0
w^3				−24								−16	
w^2			+10								+16		
w		+8								−16			
1	+1								0				
	1	−8	+26	−40	+15	+48	−84	+48	+15	−40	+26	−8	+1

$n=9.$

$=(v-1)^{10}(v+1)^2$

	v^{12}	v^{11}	v^{10}	v^9	v^8	v^7	v^6	v^5	v^4	v^3	v^2	v	1
w^{12}					0								−1
w^{11}		+32								− 22			
w^{10}							−44						
w^9				+ 88								+22	
w^8	0								−165				
w^7						+132							
w^6			+44								−44		
w^5								−132					
w^4					+165								0
w^3		−22								− 88			
w^2							+44						
w				+ 22								−32	
1	+1								0				
	1	+10	+44	+110	+165	+132	0	−132	−165	−110	−44	−10	−1

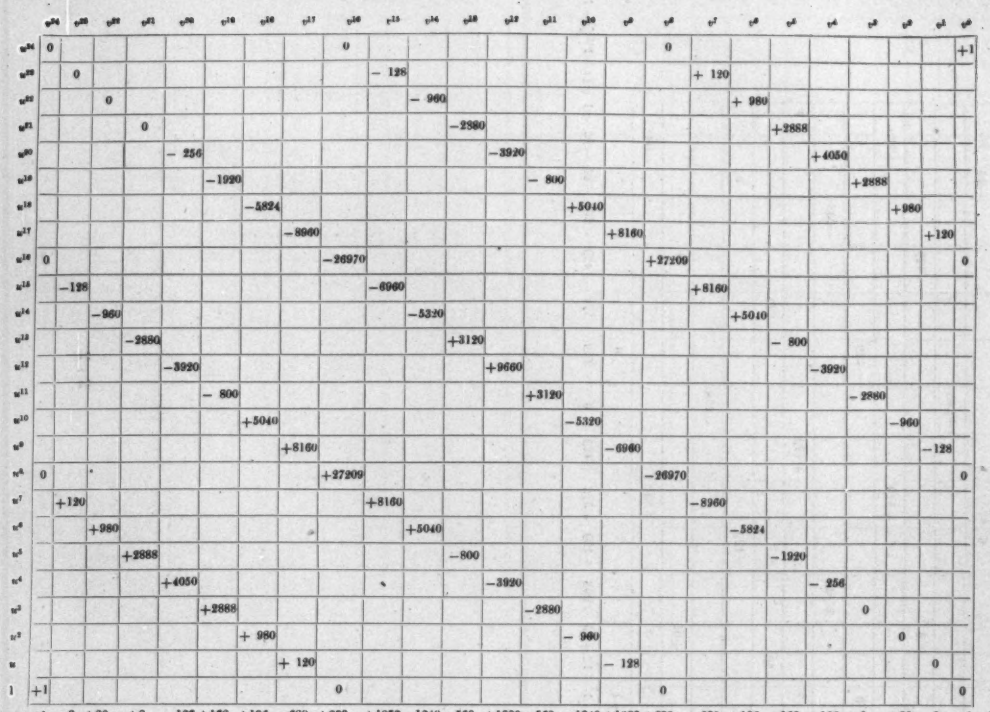
$n=11.$

$=(v+1)^{11}(v-1)$

	v^{14}	v^{13}	v^{12}	v^{11}	v^{10}	v^{9}	v^{8}	v^{7}	v^{6}	v^{5}	v^{4}	v^{3}	v^{2}	v	1
u^{14}							0								−1
u^{13}		+64								−52					
u^{12}					0								−65		
u^{11}								+208							
u^{10}			0								−429				
u^{9}						+520								+52	
u^{8}	0								−429						
u^{7}				+208								−208			
u^{6}							+429								0
u^{5}		−52								−520					
u^{4}					+429								0		
u^{3}								−208							
u^{2}			+65								0				
u						+ 52								−64	
1	+1								0						
	1	+12	+65	+208	+429	+572	+429	0	−429	−572	−429	−208	−65	−12	−1

$n=13.$

$=(v+1)^{1}(v-1).$


n=15.
+1 −8 +20 +8 −126 +168 +196 −680 +239 +1072 −1240 −560 +1820 −560 −1240 +1072 +239 −680 +196 +168 −126 +8 +20 −8 +1 = (v−1)^8 (v^8−1)^8.

$n = 17.$

	v^{18}	v^{17}	v^{16}	v^{15}	v^{14}	v^{13}	v^{12}	v^{11}	v^{10}	v^{9}	v^{8}	v^{7}	v^{6}	v^{5}	v^{4}	v^{3}	v^{2}	v	1
u^{16}			0								0								+1
u^{17}		−256								+ 272								− 34	
u^{16}	0								− 272								+425		
u^{15}								+ 1632								−2448			
u^{14}							− 4080								+7140				
u^{13}						+ 4896								−13464					
u^{12}					− 4080								+22446						
u^{11}				+1632								−33456							
u^{10}			−272								+44030								0
u^{9}		+272								−49164								+272	
u^{8}	0								+44030								−272		
u^{7}								−33456								+1632			
u^{6}							+22644								−4080				
u^{5}						−13464								+ 4896					
u^{4}					+7140								− 4080						
u^{3}				−2448								+ 1632							
u^{2}			+425								− 272								0
u		− 34								+ 272								−256	
1	+1								0								0		

$1 \quad -18 \quad +153 \quad -816 \quad +3060 \quad -8568 \quad +18564 \quad -31824 \quad +43758 \quad -48520 \quad +43758 \quad -31824 \quad +18564 \quad -8568 \quad +3060 \quad -816 \quad +153 \quad -18 \quad +1 = (v-1)^{18}.$

	v^{20}	v^{19}	v^{18}	v^{17}	v^{16}	v^{15}	v^{14}	v^{13}	v^{12}	v^{11}	v^{10}	v^{9}	v^{8}	v^{7}	v^{6}	v^{5}	v^{4}	v^{3}	v^{2}	v	1
u^{20}					0								0								−1
u^{19}		+512								− 608								+ 114			
u^{18}							+ 2432								− 2548						
u^{17}				−2432			*					+ 3344								−114	
u^{16}	0								+ 3952								−6859				
u^{15}						+5472								+2280							
u^{14}			−2432								−10488								−2584		
u^{13}								+25536								−2280					
u^{12}					−3952								−21242								0
u^{11}		−608								+20748								−3344			
u^{10}							+10488								−10488						
u^{9}				+3344								−20748								+608	
u^{8}	0								+21242								+3952				
u^{7}						+2280								−25536							
u^{6}			+2584								+10488								+2432		
u^{5}								− 2280								−5472					
u^{4}					+6859								− 3952								0
u^{3}		+114								− 3344								+2432			
u^{2}							+ 2584								− 2432						
u				− 114								+ 608								−512	
1	1								0								0				

$u=-19.$

$1 + 18 + 152 + 798 + 2907 + 7752 + 15504 + 23256 + 25194 + 16796 + 0 - 16796 - 25194 - 23256 - 15504 - 7752 - 2907 - 798 - 152 - 18 - 1 = (v+1)^{19}(v-1).$

Various remarks arise on the Tables. Attending first to the cases n a prime number; the only terms of the order $n+1$ in v or u are $v^{n+1} \pm u^{n+1}$, viz. $n \equiv 3$ or 5 (mod. 8) the sign is $-$, but $n \equiv 1$ or 7 (mod. 8) the sign is $+$. And there is in every case a pair of terms $v^n u^n$ and vu, having coefficients equal in absolute magnitude, but of opposite signs, or of the same sign, in the two cases respectively.

Each Table is symmetrical in regard to its two diagonals respectively, so that every non-diagonal coefficient occurs (with or without reversal of sign) 4 times; viz. in the case $n \equiv 1$ or 7 (mod. 8) this is a perfect symmetry, without reversal of sign; but in the case $n \equiv 3$ or 5 (mod. 8) it is, as regards the lines parallel to either diagonal, and in regard to the other diagonal, alternately a perfect symmetry without reversal of sign and a skew symmetry with reversal. Thus in the case $n=19$, the lines parallel to the dexter diagonal are -1 (symmetrical), $+114$, -114 (skew), 0, -2584, -6859, -2584, 0 (symmetrical), and so on. The same relation of symmetry is seen in the composite cases $n=9$ and $n=15$, both belonging to $n \equiv 1$ or 7, mod. 8.

If, as before, n is prime, then putting in the modular equation $u=1$, the equation in the case $n \equiv 1$ or 7 (mod. 8) becomes $(v-1)^{n+1} = 0$, but in the case $n \equiv 3$ or 5 (mod. 8) it becomes $(v+1)^n (v-1) = 0$.

27. In the case n a composite number not containing any square factor, then dividing n in every possible way into two factors $n=ab$ (including the divisions $n \,.\, 1$ and $1 \,.\, n$), and denoting by β an imaginary b-th root of unity, a value of v is

$$\pm \sqrt{2} (\beta q^{\frac{a}{b}})^{\frac{1}{8}} f(\beta q^{\frac{a}{b}});$$

so that the whole number of roots (or order of the modular equation) is $= \nu$, if ν be the sum of the divisors of n. Thus $n=15$, the values are

$$\sqrt{2} q^{\frac{1}{8}} f(q^{15}),\ -\sqrt{2} q^{\frac{1}{24}\cdot\frac{3}{5}} f(q^{\frac{3}{5}}),\ -\sqrt{2} q^{\frac{1}{24}\cdot\frac{5}{3}} f(q^{\frac{5}{3}}),\ \sqrt{2} q^{\frac{1}{24}\cdot\frac{15}{1}} f(q^{\frac{15}{1}})$$
$$1 \quad , \quad 3 \quad , \quad 5 \quad , \quad 15 \text{ roots};$$

and the order of the modular equation is $=24$. The modular equation might thus be obtained as for a prime number; but it is easier to decompose n into its prime factors, and consider the transformation as compounded of transformations of these prime orders. Thus $n=15$, the transformation is compounded of a cubic and a quintic one. If the v of the cubic transformation be denoted by θ, then we have

$$\theta^4 + 2\theta^3 u^3 - 2\theta u - u^4 = 0;$$

and to each of the four values of θ corresponds the six values of v belonging to the quintic transformation given by

$$v^6 + 4v^5\theta^1 + 5v^4\theta^2 - 5v^2\theta^4 - 4v\theta - \theta^5 = 0.$$

The equation in v is thus

$$(v^6 + 4v^5\theta_1 \,..\, - \theta_1^5)(v^6 \,..\, - \theta_2^5)(v^6 \,..\, - \theta_3^5)(v^6 \,..\, - \theta_4^5) = 0,$$

where θ_1, θ_2, θ_3, θ_4 are the roots of the equation in θ, viz. we have

$$\theta^4 + 2\theta^3 u^3 - 2\theta u - u^4 = (\theta - \theta_1)(\theta - \theta_2)(\theta - \theta_3)(\theta - \theta_4);$$

and it was in this way that the equation for the case $n=15$ was calculated. Observe that writing $u=1$, we have $(\theta+1)^3(\theta-1)=0$, or say $\theta_1=\theta_2=\theta_3=-1$, $\theta_4=+1$. The equation in v thus becomes $\{(v-1)^4(v+1)\}^3(v+1)^4(v-1)=0$, that is $(v-1)^{16}(v+1)^8=0$.

28. The case where n has a square factor is a little different; thus $n=9$, the values are

$$\begin{matrix}\sqrt{2}q^{\frac{9}{8}}f(q^9), & -\sqrt{2}q^{\frac{1}{8}\cdot\frac{3}{8}}f(q^3), & \sqrt{2}q^{\frac{1}{8}\cdot\frac{1}{8}}f(q^9),\\ 1\ , & 3\ , & 9\ , & \text{roots};\end{matrix}$$

but here ω being an imaginary cube root of unity, the second term denotes the three values,

$$\sqrt{2}q^{\frac{1}{8}}f(q),\ \sqrt{2}(q\omega)^{\frac{1}{8}}f(\omega q),\ \sqrt{2}(\omega^2 q)^{\frac{1}{8}}f(\omega^2 q),$$

the first of which is $=u$, and is to be rejected; there remain $1+2+9$, $=12$ roots, or the equation is of the order 12.

Considering the equation as compounded of two cubic transformations, if the value of v for the first of these be θ, then we have

$$\theta^4+2\theta^3u^3-2\theta u-u^4=0;$$

and to each of the four values of θ correspond the four values of v given by the equation

$$v^4+2v^3\theta^3-2v\theta-\theta^4=0.$$

One of these values is however $v=-u$, since the $v\theta$-equation is satisfied on writing therein $v=-u$; hence, writing

$$\theta^4+2\theta^3u^3-2\theta u-u^4=(\theta-\theta_1)(\theta-\theta_2)(\theta-\theta_3)(\theta-\theta_4),$$

we have an equation

$$(v^4+2v^3\theta_1^3-2v\theta_1-\theta_1^4)(v^4\ldots-\theta_2^4)(v^4\ldots-\theta_3^4)(v^4\ldots-\theta_4^4)=0,$$

containing the factor $(v+u)^4$, and which, divested hereof, gives the required modular equation of the order 12, which was in fact obtained in this manner.

Observe that writing $u=1$ we have $(\theta+1)^3(\theta-1)=0$, or say $\theta_1=\theta_2=\theta_3=-1$, $\theta_4=1$; the modular equation then becomes

$$\{(v-1)^3(v+1)\}^3(v+1)^4(v-1)\div(v+1)^4=0,$$

that is

$$(v-1)^{12}(v+1)^3=0.$$

The Multiplier Equation.—Article No. 29.

29. The theory is in many respects analogous to that of the modular equation. To each value of v there corresponds a single value of M; hence M, or what is the same thing $\frac{1}{M}$, is determined by an equation of the same order as v, viz. n being prime, the order is $=n+1$. The last term of the equation is constant, and the other coefficients are rational and integral functions of u^8, of a degree not exceeding $\frac{1}{2}(n-1)$; and not only so, but they are, $n\equiv 1 \pmod 4$, rational and integral functions of $u^8(1-u^8)$, and $n\equiv 3 \pmod 4$, alternately of this form, and of the same form multiplied by the factor $(1-2u^8)$.

The values are in fact given as transcendental functions of q; viz. denoting by M_0, M_1, $M_2 \ldots$, M_n the values corresponding to v_0, v_1, v_2, $\ldots$ v_n respectively, and writing

$$\varphi(q)=\frac{(1+q)(1+q^3)(1+q^5)\ldots(1-q^2)(1-q^4)(1-q^6)\ldots}{(1-q)(1-q^3)(1-q^5)\ldots(1+q^2)(1+q^4)(1+q^6)\ldots}$$

$$=1+2q+2q^4+2q^9+2q^{16}+\ldots,$$

then we have

$$M_0=\frac{(-)^{\frac{n-1}{2}}}{n}\frac{\varphi^3(q)}{\varphi^3(q^n)},$$

$$M_1=\frac{\varphi^3(q)}{\varphi^3(\alpha q^{\frac{1}{n}})},\ \ldots\ (\alpha \text{ an imaginary } n\text{th root of unity})$$

$$\vdots$$

$$M_n=\frac{\varphi^3 q}{\varphi^3(q^{\frac{1}{n}})}.$$

Hence, the form of the equation being known, the values of the numerical coefficients may be calculated; and it was in this way that JOUBERT obtained the following results. I have in some cases changed the sign of JOUBERT's multiplier, so that in every case the value corresponding to $u=0$ shall be $M=1$.

The equations are:—

$n=3$,

$$\frac{1}{M^4}$$
$$+\frac{1}{M^3}\cdot 0$$
$$+\frac{1}{M^2}\cdot -6$$
$$+\frac{1}{M}\cdot 8(1-2u^3)$$
$$-3=0.$$

$u=0$, this is

$$\left(\frac{1}{M}-1\right)^2\left(\frac{1}{M}+3\right)=0.$$

$u=1$, it is

$$\left(\frac{1}{M}+1\right)^2\left(\frac{1}{M}-3\right)=0.$$

$n=5$,

$$\frac{1}{M^6}$$
$$+\frac{1}{M^5}\cdot -10$$
$$+\frac{1}{M^4}\cdot +35$$
$$+\frac{1}{M^3}\cdot -60$$
$$+\frac{1}{M^2}\cdot +55$$
$$\frac{1}{M}\cdot -26+256u^6(1-u^3)$$
$$+5=0.$$

$u=0$ or 1, this is

$$\left(\frac{1}{M}-1\right)^2\left(\frac{1}{M}-5\right)=0.$$

$n=7$, $\frac{1}{M^8}$

$+\frac{1}{M^7}.0$

$+\frac{1}{M^6}.-28$

$+\frac{1}{M^5}.+112(1-2u^8)$

$+\frac{1}{M^4}.-210$

$+\frac{1}{M^3}.+224(1-2u^8)$

$+\frac{1}{M^2}.-140-21.256u^8(1-u^8)$

$+\frac{1}{M}.\{48+2048u^8(1-u^8)\}(1-2u^8)$

$+7=0.$

$u=0$, this is

$\left(\frac{1}{M}-1\right)^7\left(\frac{1}{M}-7\right)=0.$

$u=1$, it is

$\left(\frac{1}{M}+1\right)^7\left(\frac{1}{M}+7\right)=0.$

$n=11$, $\frac{1}{M^{12}}$

$+\frac{1}{M^{11}}.0$

$+\frac{1}{M^{10}}.-66$

$+\frac{1}{M^9}.+440(1-2u^8)$

$+\frac{1}{M^8}.-1485$

$+\frac{1}{M^7}.+3168(1-2u^8)$

$+\frac{1}{M^6}.-4620-3.11^2.256u^8(1-u^8)$

$+\frac{1}{M^5}.\{+4752+11.4096u^8(1-u^8)\}(1-2u^8)$

$+\frac{1}{M^4}.-3465-3.7.11.512u^8(1-u^8)$

$+\frac{1}{M^3}.\{+1760+11.83.2048u^8(1-u^8)\}(1-2u^8)$

$+\frac{1}{M^2}.-594-9.11.37.256u^8(1-u^8)-3.11.131072\{u^8(1-u^8)\}^2$

$+\frac{1}{M}\{120+15.4096u^8(1-u^8)-524288\{u^8(1-u^8)\}^2\}(1-2u^8)$

$-11=0.$

$u=0$, this is

$\left(\frac{1}{M}-1\right)^{11}\left(\frac{1}{M}+11\right)=0.$

$u=1$, it is

$\left(\frac{1}{M}+1\right)^{11}\left(\frac{1}{M}-11\right)=0.$

The Multiplier as a rational function of u, v.—Article Nos. 30 to 36.

30. The multiplier M, as having a single value corresponding to each value of v, is necessarily a rational function of u, v; and such an expression of M can, as remarked by KÖNIGSBERGER, be deduced from the multiplier equation by means of JACOBI'S theorem,

$$M^2 = \frac{1}{n}\frac{\lambda(1-\lambda^2)}{k(1-k^2)}\frac{dk}{d\lambda};$$

viz. substituting for k, λ their values u^2, v^2, and observing that if the modular equation be $F(u, v)=0$, then the value of $\frac{du}{dv}$ is $=-F'(v) \div F'(u)$, this is

$$M^2 = -\frac{1}{n}\frac{(1-v^2)vF'v}{(1-u^2)uF'u};$$

and then in the multiplier equation separating the terms which contain the odd and even powers, and writing it in the form $\Phi(M^2)+M\Psi(M^2)=0$, this equation, substituting therein for M^2 its value, gives the value of M rationally.

The rational expression of M in terms of u, v is of course indeterminate, since its form may be modified in any manner by means of the equation $F(u, v)=0$; and in the expression obtained as above, the orders of the numerator and denominator are far too high. A different form may be obtained as follows: for greater convenience I seek for the value not of M but of $\frac{1}{M}$.

31. Denoting, as above, by $M_0, M_1, \ldots M_n$ the values which correspond to $v_0, v_1, \ldots v_n$ respectively, and writing $S\frac{1}{M}=\frac{1}{M_0}+\frac{1}{M_1}\ldots+\frac{1}{M_n}$ &c., we have $S\frac{1}{M}, S\frac{v}{M}$, &c., all of them expressible as determinate functions of u; and we have moreover the theorem that each of these is a rational and integral function of u: we have thus the series of equations

$$S\frac{1}{M}=A,\ S\frac{v}{M}=B, \ldots, S\frac{v^n}{M}=H,$$

where A, B, ... H are rational and integral functions of u. These give linearly the different values of $\frac{1}{M}$; we have in fact

$$(v_0-v_1)\ldots(v_0-v_n)\frac{1}{M_0}=H-GSv_1+FSv_1v_2\ldots\pm Av_1v_2\ldots v_n,$$

where Sv_1, Sv_1v_2, &c. denote the combinations formed with the roots $v_1, v_2, \ldots v_n$ (these can be expressed in terms of the single root v_0); and we have also $(v_0-v_1)\ldots(v_0-v_n)=F'(v_0)$: the resulting equation is consequently $F'v_0\,\frac{1}{M_0}=R(u, v_0)$, R a determinate rational and integral function of (u, v_0); but as the same formula exists for each root of the modular equation, we may herein write M, v in place of M_0, v_0; and the formula thus is

$$F'v\,.\,\frac{1}{M}=R(u, v),$$

viz. we thus obtain the required value of $\frac{1}{M}$ as a rational fraction, the denominator being the determinate function $F'v$, and the numerator being, as is easy to see, a determinate function of the order n as regards v.

32. The method is applicable when M is only known by its expression in terms of q; but if we know for M an expression in terms of v, u, then the method transforms this into a standard form as above; and by way of illustration I will consider the case $n=3$, where the modular equation is

$$v^4+2v^3u^3-2vu-u^4=0,$$

and where a known expression of M is $\frac{1}{M}=1+\frac{2u^3}{v}$. Here writing S_{-1}, $S_0(=4)$, S, &c. to denote the sum of the powers -1, 0, 1, &c. of the roots of the equation, we have

$$S\frac{1}{M}=S_0+2u^3S_{-1}, \ =0 \ , \text{ as appears from the values presently given,}$$

$$S\frac{v}{M}=S_1+2u^3S_0 \ , \ =6u^3,$$

$$S\frac{v^2}{M}=S_2+2u^3S_1 \ , \ =0 \ ,$$

$$S\frac{v^3}{M}=S_3+2u^3S_2 \ , \ =6u;$$

and observing that v_0 being ultimately replaced by v, we have

$$Sv_1=Sv_0-v, \quad Sv_0v_1=Sv_0v_1-vSv_0+v^3, \ v_1v_2v_3=Sv_0v_1v_2-vSv_0v_1+v_2Sv_0-v^3,$$

that is

$$Sv_1=-2u^3-v, \ Sv_1v_2=2u^3v+v^3, \ v_1v_2v_3=2u-2u^3v^3-v^3,$$

we have

$$\begin{aligned} F'v\cdot\frac{1}{M}= \ & (S_3+2u^3S_3) \\ & +(2u^3+v)(S_2+2u^3S_1) \\ & +(2u^3v+v^3)(S_1+2u^3S_0) \\ & +(-2u+2u^3v^3+v^3)(S_0+2u^3S_{-1}), \end{aligned}$$

viz. this is

$$\begin{aligned} 2(2v^3+3v^3u^3-u)\frac{1}{M}= \ & v^3(S_0+2u^3S_{-1}) \\ & +v^2(S_1+4u^3S_0+4u^6S_{-1}) \\ & +v\,(S_2+4u^3S_1+4u^6S_0) \\ & +\ (S_3+4u^3S_2+4u^6S_1-2uS_0-4u^4S_{-1}). \end{aligned}$$

But we have

$$S_{-1}=-\frac{2}{u^3}, \ S_0=4, \ S_1=-2u^3, \ S_2=4u^6, \ S_3=6u-8u^9;$$

and the equation thus is

$$(2v^3+3v^3u^3-u)\frac{1}{M}=3(v^3u^3+2u^3v+1)u;$$

to verify which observe that, substituting herein for $\frac{1}{M}$ its value $1+\frac{2u^3}{v}$, the equation becomes

$$(2v^3+3v^3u^3-u)(v+2u^3)-3vu(v^3u^3+2u^3v+1)=0;$$

that is,

$$2v^4+4v^3u^3-4vu-2u^3=0, \text{ as it should do.}$$

33. Any expression whatever of M in terms of u, v is in fact one of a system of four expressions; viz. we may simultaneously change

					that is, signs are			
	u	v		$\frac{1}{M}$	$n\equiv 1$	$n\equiv 3$	$n\equiv 5$	$n\equiv 7 \pmod 8$
into	v,	$(-)^{\frac{v^2-1}{8}}u$,	$(-)^{\frac{n-1}{2}}$	nM	+ + +	+ − −	+ − −	+ + −
or	$\frac{1}{u}$,	$\frac{1}{v}$,		$\frac{v^4}{u^4M}$	+ + +	+ + +	+ + +	+ + +
or	$\frac{1}{v}$,	$(-)^{\frac{v^2-1}{8}}\frac{1}{u}$,	$(-)^{\frac{n-1}{2}}\frac{u}{v^4}$	nM;	+ + +	+ − −	+ − +	+ + −

Thus $n=3$, starting from $\frac{1}{M}=1+\frac{2u^3}{v}$, we have

$$\frac{1}{M}=1+\frac{2u^3}{v},\ -3M=1-\frac{2v^3}{u},\ \frac{v^4}{u^4M}=1+\frac{2v}{u^3},\ -\frac{u^4}{v^4}3M=1-\frac{2u}{v^3};$$

and of course if from any two of these we eliminate M, we have either an identity or the modular equation; thus we have the modular equation under the six different forms:

$$\begin{aligned}
&(1,2)\quad (v+2u^3)(u-2v^3)+3uv=0,\\
&(1,3)\quad v^3(v+2u^3)-u(u^3+2v)=0,\\
&(1,4)\quad (v+2u^3)(v^3-2u)+3u^4=0,\\
&(2,3)\quad (u-2v^3)(u^3+2v)+3v^4=0,\\
&(2,4)\quad v(v^3-2u)-u^3(u-2v^3)=0,\\
&(3,4)\quad (u^3+2v)(v^3-2u)+3u^3v^3=0.
\end{aligned}$$

34. Next $n=5$. Here, starting from $\frac{1}{M}=\frac{v-u^5}{v(1-uv^5)}$, the changes give

$$\frac{1}{M}=\frac{v-u^5}{v(1-uv^5)},\ 5M=\frac{u+v^5}{u(1+u^5v)},\ \frac{v^4}{u^4M}=\frac{v^3(v-u^5)}{u^4(1-uv^5)},\ \frac{u^4}{v^4}5M=\frac{u^3(u+v^5)}{v^4(1+u^5v)},$$

viz. the third and fourth forms agree with the first and second forms respectively; that is, there are only two independent forms, and the elimination of M from these gives

$$5uv(1-uv^3)(1+u^3v)-(v-u^3)(u+v^3)=0,$$

which is a form of the modular equation.

35. In the case $n=7$, starting from $\frac{1}{M}=\frac{-7u(1-uv)(1-uv+u^2v^2)}{u-v^7}$ (as to this see *post*, No. 43), the forms are

$$\frac{1}{M}=\frac{-7u(1-uv)(1-uv+u^2v^2)}{u-v^7}, \quad \ldots\ldots\ldots \quad (1)$$

$$-7M=\frac{-7v(1-uv)(1-uv+u^2v^2)}{v-u^7}, \quad \ldots\ldots\ldots \quad (2)$$

$$\frac{v^4}{v^4M}=\frac{-7v^4(1-uv)(1-uv+u^2v^2)}{v^3(u-v^7)}, \quad \ldots\ldots\ldots \quad (3)$$

$$-\frac{u^4}{v^4}7M=\frac{-7u^4(1-uv)(1-uv+u^2v^2)}{v^3(v-u^7)}; \quad \ldots\ldots\ldots \quad (4)$$

so that here again the third and fourth forms are identical with the second and third forms respectively; there are thus only two forms, and the elimination of M gives

$$(u-v^7)(v-u^7)+7uv(1-uv)^3(1-uv+u^2v^2)^3=0,$$

which is a form of the modular equation.

36. If in the foregoing equation,

$$Fv\,.\,\frac{1}{M}=R(u, v),$$

we make the change $u, v, \frac{1}{M}$ into $v, \pm u, \pm nM$, it becomes

$$\pm Fu\,.\,nM=R(v, \pm u);$$

and combining these equations, we have

$$\pm nM^2\,.\,\frac{Fu}{Fv}=\frac{R(v, \pm u)}{R(u, v)};$$

or substituting herein the foregoing value

$$M^2=-\frac{1}{n}\frac{(1-v^8)vFv}{(1-u^8)uFu},$$

this becomes

$$\mp\frac{v(1-v^8)}{u(1-u^8)}=\frac{R(v, \pm u)}{R(u, v)} \qquad \begin{matrix} +\text{for } n\equiv 3 \text{ or } 5 \pmod 8, \\ -\text{for } n\equiv 1 \text{ or } 7 \pmod 8, \end{matrix}$$

which must agree with the modular equation: thus in the last-mentioned case $n=3$, where we have

$$\tfrac{1}{3}Fv\,.\,\frac{1}{M}=3(v^3u^3+2u^4v+1)u,$$

or say

$$R(u, v)=(v^3u^3+2u^4v+1)u,$$

and therefore

$$R(v, -u)=(v^3u^3-2uv^4+1)v;$$

the equation is

$$+\frac{v(1-v^8)}{u(1-u^8)}=\frac{(v^3u^3-2uv^4+1)v}{(v^3u^3+2u^4v+1)u},$$

which is right; for JACOBI, p. 82, the modular equation, gives

$$1-u^8=(1-u^4v^4)(v^3u^3+2u^4v+1),\ 1-v^8=(1-u^4v^4)(v^3u^3-2uv^4+1).$$

Observe that the general equation

$$\mp\frac{v(1-v^2)}{u(1-u^2)}=\frac{R(v,\pm u)}{R(u,v)}$$

no longer contains the functions $F'v$, $F'u$, which enter into JACOBI'S expression of M^2.

Theorem in connexion with the multiplication of Elliptic Functions.
Article Nos. 37 to 40.

37. The theory of multiplication gives an important theorem in regard to transformation. Starting with the $n$thic transformation,

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left(\frac{\alpha-\beta x+\gamma x^2-\ldots}{\alpha+\beta x+\gamma x^2+\ldots}\right)^2,=\frac{1-x}{1+x}\left(\frac{P-Qx}{P+Qx}\right)^2,$$

we may form a like transformation,

$$\frac{1-z}{1+z}=\frac{1-y}{1+y}\left(\frac{\alpha'-\beta' y+\gamma' y^2-\ldots}{\alpha'+\beta' x+\gamma' y^2+\ldots}\right)^2,=\frac{1-y}{1+y}\left(\frac{P'-Q'y}{P'+Q'y}\right)^2,$$

such that the combination of the two gives a multiplication, viz. for the relation between y, z, deriving w from v as v from u, we have $w=u$; and instead of M we have M', $=\pm\frac{1}{nM}$; that is, we have

$$\frac{dx}{\sqrt{1-x^2}.\sqrt{1-u^2x^2}}=\frac{M\,dy}{\sqrt{1-y^2}.\sqrt{1-v^2y^2}}$$

$$\frac{dy}{\sqrt{1-y^2}.\sqrt{1-v^2y^2}}=\frac{M'dx}{\sqrt{1-x^2}.\sqrt{1-u^2x^2}}$$

and thence

$$\frac{dx}{\sqrt{1-x^2}.\sqrt{1-u^2x^2}}=\frac{\pm\frac{1}{n}\,dx}{\sqrt{1-x^2}.\sqrt{1-u^2x^2}};$$

or writing $x=\operatorname{sn}\theta$, we have $z=\pm\operatorname{sn}n\theta$; $\pm$ is here $(-)^{\frac{n-1}{2}}$, viz. it is $=-$ for $n=3$ or 7 (mod. 8), and $=+$ for $n=1$ or 5 (mod. 8).

Now in part effecting the substitution, we have

$$\frac{1-z}{1+z}=\frac{1-x}{1+x}\left(\frac{P-Qx}{P+Qx}\right)^2\cdot\left(\frac{P'-Q'y}{P'+Q'y}\right)^2,$$

where y denotes its value in terms of x.

And from the theory of elliptic functions, replacing $\operatorname{sn}n\theta$, $\operatorname{sn}\theta$ by their values $\pm z$, x, we have an equation

$$\frac{1-z}{1+z}=\frac{1-x}{1+x}\left(\frac{A-Bx+Cx^2-\ldots}{A+Bx+Cx^2+\ldots}\right)^2,$$

where $A-Bx+Cx^2-\ldots$, $A+Bx+Cx^2+\ldots$ are given functions each of the order $\frac{1}{2}(n^2-1)$; viz. the coefficients are given functions of k, or, what is the same thing, of u^4. Comparing the two results, we see that in the $n$thic transformation the sought-for function, $\alpha+\beta x+\gamma x^2+\ldots$ of the order $\frac{1}{2}(n-1)$, is a factor of a given function $A+Bx+Cx^2+\ldots$ of the order $\frac{1}{2}(n^2-1)$.

38. Considering the modular equation as known, then by what precedes we have

$$\alpha+\beta x+\gamma x^2+\ldots=\alpha\left\{1+\frac{\beta}{\alpha}x\ldots+\frac{u^n}{v}x^{\frac{1}{2}(n-1)}\right\};$$

that is, the given function $A+Bx+Cx^2+\ldots$ has a factor $1+\frac{\beta}{\alpha}x+\ldots+\frac{u^n}{v}x^{\frac{1}{2}(n-1)}$, of which one (the last) coefficient $\frac{u^n}{v}$ is known, and we are hence able theoretically to determine all the other coefficients rationally in terms of u, v; that is, the modular equation being known, we can theoretically complete the solution of the transformation problem. I do not, however, see the way to obtaining a convenient solution in this manner.

39. The formula in question for $n=3$ is

$$\frac{1+\operatorname{sn} 3\theta}{1-\operatorname{sn} 3\theta}=\frac{1-\operatorname{sn}\theta}{1+\operatorname{sn}\theta}\left(\frac{1+2\operatorname{sn}\theta-2k^2\operatorname{sn}^3\theta-k^2\operatorname{sn}^4\theta}{1-2\operatorname{sn}\theta+2k^2\operatorname{sn}^3\theta-k^2\operatorname{sn}^4\theta}\right)^2,$$

which, putting therein $x=\operatorname{sn}\theta$, $z=-\operatorname{sn} 3\theta$, and replacing k by u^4, may be written

$$1+z(\div)=(1+x)(1-2x+2u^2x^3-u^8x^4)^2(\div),$$

where the signs $(\div)$ indicate denominators which are obtained from the numerators by changing the signs of z, x respectively.

The theorem in regard to $n=3$, thus is, $1+\frac{u^3}{v}x$ is a factor of $1-2x+2u^8x^3-u^8x^4$; viz. writing in the last-mentioned function $x=-\frac{v}{u^3}$, we ought to have

$$0=1+2\frac{v}{u^3}-2\frac{v^3}{u}-\frac{v^4}{u^4},$$

that is

$$u^4+2uv-2u^3v^3-v^4=0,$$

which is in fact the modular equation.

40. And so for $n=5$ if $x=\operatorname{sn}\theta$, $z=\operatorname{sn} 5\theta$, and for $n=7$ if $x=\operatorname{sn}\theta$, $z=-\operatorname{sn} 7\theta$, the formulæ are:—

$n=5$,

$$1+z=(1+x)\{(\div)\}$$

1	
$+2$	x
-4	x^2
$-10u^2$	x^3
$+5u^2$	x^4
$+4u^2(3+2u^2)$	x^5
$+4u^2(1-u^2)$	x^6
$-4u^2(2+3u^2)$	x^7
$-5u^{16}$	x^8
$+10u^{16}$	x^9

$n=7$,

$$1+z=(1+x)\{(\div)\}$$

1	
-4	x
-4	x^2
$+4(2+7u^2)$	x^3
$-14u^2$	x^4
$-28u^2(3+2u^2)$	x^5
$+28u^2(4+u^2)$	x^6
$+4u^2(16+51u^2+8u^{16})$	x^7
$-u^2(144+305u^2+16u^{16})$	x^8
$-8u^2(4+25u^2+16u^{16})$	x^9

$$+\ 4u^{24}\ \ x^{10}$$
$$-\ 2u^{24}\ \ x^{11}$$
$$-\ \ u^{24}\ \ x^{12}\}^3 \div$$

Term in { } has factor

$$1+\frac{\beta}{\alpha}x+\frac{u^5}{v}x^2;$$

$u=1$, term in { } is

$$=(1+x)^7(1-x)^6.$$

$$+\ 8u^9(\ \ 8+\ 57u^8+\ 46u^{16})x^{10}$$
$$+56u^{16}(2+u^8)\qquad x^{11}$$
$$-\ 4u^{16}(\ 56+161u^8+\ 56u^{16})x^{12}$$
$$+56u^{24}(1+2u^8\qquad\ )x^{13}$$
$$+\ 8u^{24}(\ 46+\ 57u^8+\ \ 8u^{16})x^{14}$$
$$-\ 8u^{24}(\ 16+\ 25u^8+\ \ 4u^{16})x^{15}$$
$$-\ \ u^{24}(\ 16+305u^8+144u^{16})x^{16}$$
$$+\ 4u^{32}(\ \ 8+\ 51u^8+\ 16u^{16})x^{17}$$
$$+28u^{32}(\ \ 1+\ \ 4u^8)\qquad x^{18}$$
$$-28u^{32}(\ \ 2+\ \ 3u^8)\qquad x^{19}$$
$$-14u^{40}\qquad x^{20}$$
$$+\ 4u^{40}(7+2u^8)\qquad x^{21}$$
$$-\ 4u^{48}\qquad x^{22}$$
$$-\ 4u^{48}\qquad x^{23}$$
$$+\ \ u^{48}\qquad x^{24}\}^3$$
$$(\div)$$

Term in { } has factor

$$1+\frac{\beta}{\alpha}x+\frac{\gamma}{\alpha}x^2+\frac{u^7}{v}x^3;$$

$u=1$, term in { } is $(1+x)^{18}(1-x)^{14}$.

The transformations $n=3, 5, 7, 11$.—Article Nos. 41 to 51.

41. The cubic transformation, $n=3$.

I reproduce the results already obtained; since there are only two coefficients α, β, these are also the last but one and last coefficient ϱ, σ. Hence, from the values of α, β, ϱ, σ, we have

$$\alpha=1, \qquad 2\alpha=\frac{v^2}{u}\left(\frac{1}{M}-\frac{u^4}{v^4}\right),$$

$$2\beta=\frac{1}{M}-1, \qquad \beta=\frac{u^3}{v};$$

the two values of $\frac{1}{M}$ are thus $\frac{1}{M}=2\frac{u}{v^3}+\frac{u^4}{v^4}$, $=1+\frac{2u^3}{v}$, giving the modular equation

$$v^4+2v^3u^3-2vu-u^4=0;$$

and we then have

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left(\frac{v-u^3x}{v+u^3x}\right)^2.$$

42. The quintic transformation, $n=5$.

Here there are the three coefficients α, β, γ, or β, γ are the last but one and last coefficients ε, σ; we have

$$\alpha=1, \qquad 2\beta=v^3u\left(\frac{1}{M}-\frac{u^4}{v^4}\right),$$

$$2\beta=\frac{1}{M}-1, \qquad \gamma=\frac{u^5}{v}.$$

Comparing the two values of β, we have $\frac{1}{M}=\frac{v-u^5}{v(1-v^3)u}$, and then

$$\alpha=1,\ 2\beta=\frac{u(v^4-u^4)}{b(1-v^3u)},\ \gamma=\frac{u^5}{v},$$

so that only the modular equation remains to be determined.

The unused equation is

$$2\alpha\gamma+2\alpha\beta+\beta^2=\frac{v^2}{u^2}(2\alpha\gamma+2\beta\gamma+\beta^2),$$

which, putting therein $\alpha=1$, may be written

$$(2\gamma+\beta^2)(u^3-v^3)=2\beta(\gamma v^3-u^3);$$

attending to the value of β, this divides by u^3-v^3; in fact the equation may be written

$$2\gamma+\beta^2=-\frac{u(v^3+u^3)}{v(1-v^3u)}(\gamma v^3-u^3);$$

and then completing the substitution, and integralizing, this becomes

$$\{8vu^3(1-v^3u)^3+(v^4-u^4)^2\}=4uv(u^3+v^3)(1-u^3v)(1-uv^3),$$

viz. this is

$$4(1-v^3u)uv\{2u^3(1-v^3u)-(u^3+v^3)(1-vu^3)\}+(v^4-u^4)^2=0;$$

and the term in $\{\ \}$ being $=-(v^3-u^3)(1+vu^3)$, the whole again divides by v^3-u^3, and the equation thus becomes

$$(v^3+u^3)(v^4-u^4)-4uv(1-v^3u)(1+vu^3)=0,$$

which is the modular equation.

43. The septic transformation, $n=7$.

I do not propose to complete the solution directly from the fundamental equations for α, β, γ, δ, but resort to the known modular equation, and to an expression of M which I obtain by means thereof.

The modular equation is

$$(1-u^3)(1-v^3)-(1-uv)^3=0,$$

which may also be written

$$(v-u^7)(u-v^7)+7uv(1-uv)^3(1-uv+u^2v^2)^3,$$

as can be at once verified; but it also follows from CAUCHY's identity,

$$(x+y)^7-x^7-y^7=7xy(x+y)(x^2+xy+y^2)^3.$$

We then have

$$M^3=-\frac{1}{n}\frac{(1-v^8)vF'v}{(1-u^8)uF'u}.$$

Moreover

$$uF'u=-u^8(1-v^8)+uv(1-uv)^7$$
$$=\frac{-u^8}{1-u^8}(1-uv)^8+uv(1-uv)^7$$
$$=\frac{(1-uv)^7}{1-u^8}u(v-u^7);$$

and similarly

$$F'v=\frac{(1-uv)^7}{1-v^8}v(u-v^7),$$

whence

$$\frac{1}{M^3}=\frac{-7u}{v}\frac{(v-u^7)}{u-v^7}.$$

Writing this under the form

$$\frac{1}{M^3}=\frac{-7uv}{v^2}\frac{(v-u^7)(u-v^7)}{(u-v^7)^2},\quad =\frac{49u^3(1-uv)^6(1-uv+u^2v^2)^2}{(u-v^7)^2},$$

I find, as will appear, that the root must be taken with the sign $-$, and that we thus have $\frac{1}{M}=-\frac{7u(1-uv)(1-uv+u^2v^2)}{u-v^7}$, whence also $M=\frac{v(1-uv)(1-uv+u^2v^2)}{v-u^7}$.

44. Recurring now to the fundamental equations for the septic transformation, the coefficients are α, β, γ, δ, and we have

$$\alpha=1\quad,\quad 2\gamma=u^3v^3\left(\frac{1}{M}-\frac{u^4}{v^4}\right),$$
$$2\beta=\frac{1}{M}-1\,,\quad \delta=\frac{u^7}{v},$$

so that the coefficients are all given in terms of v, M. The unused equations are

$$u^8(2\alpha\gamma+2\alpha\beta+\beta^2)\qquad =v^8(\gamma^2+2\gamma\delta+2\beta\delta),$$
$$u^{-2}(\gamma^2+2\beta\gamma+2\alpha\delta+2\beta\delta)=v^2(2\alpha\gamma+2\beta\gamma+2\alpha\delta+\beta^2),$$

which, substituting therein for α, β, γ, δ the foregoing values, give two equations; from these, eliminating M, we should obtain the modular equation, and then M in terms of u, v.

Substituting in the first instance for α, δ their values, the equations are

$$u^8(2\beta+2\gamma+\beta^2)=v^8\left\{\gamma^2+2\frac{u^7}{v}(\beta+\gamma)\right\}$$

$$\gamma^2+2\beta\gamma+(2+2\beta)\frac{u^7}{v}=u^8v^8\left\{2\gamma+2\beta\gamma+2\frac{u^7}{v}+\beta^2\right\}.$$

The first of these is

$$4(1-uv)(2\beta+2\gamma)+4\beta^2-4\frac{v^8}{u^8}\gamma^2=0,$$

viz. this is

$$4(1-uv)\left(\frac{1}{M}-1+\frac{u^3v^3}{M}-\frac{u^7}{v}\right)+\left(\frac{1}{M}-1\right)^2-v^8\left(\frac{1}{M}-\frac{u^4}{v^4}\right)^2=0;$$

or observing that in this equation the coefficient of $\frac{2}{M}$ is

$$(1-u^3v^3)\{2-2uv+2u^3v^3-1-u^3v^3\},$$
$$=(1-u^3v^3)(1-uv)^3, \ =(1-uv)^3(1+uv),$$

the equation becomes

$$(1-v^3)\frac{1}{M^3}+\frac{2}{M}(1-uv)^4(1+uv)+1-u^3-4(1-uv)\left(1+\frac{u^7}{v}\right)=0.$$

45. This should be satisfied identically by the foregoing value of $\frac{1}{M}$; viz. it should be satisfied on writing therein

$$\frac{1}{M^3}=-\frac{7u}{v}\frac{v-u^7}{u-v^7},$$

$$\frac{1}{M}=-\frac{7u(1-uv)(1-uv+u^2v^2)}{u-v^7};$$

that is, we should have

$$-7\frac{u}{v}(v-u^7)(1-v^3)-14u(1-uv)^4(1+u^3v^3)$$
$$+(u-v^7)\left\{1-u^3-4(1-uv)\left(1+\frac{u^7}{v}\right)\right\}=0,$$

where observe that the $-$ sign of the second term is the sign of the foregoing value of $\frac{1}{M}$; so that the identity being verified, it follows that the correct sign has been attributed to the value of $\frac{1}{M}$.

46. Multiplying by v, the equation is

$$-7(1-u^3-\overline{1-uv})(1-v^3)-14uv(1-uv)^4(1+u^3v^3)$$
$$+\{1-v^3-\overline{1-uv}\}\{-8(1-uv)+1-u^3\}+4(1-uv)(v-u^7)(u-v^7)=0,$$

viz. this is

$$-7(1-u^3)(1-v^3)+7(1-uv)(1-v^3)-14uv(1-uv)^4(1+u^3v^3)$$
$$+\ (1-u^3)(1-v^3)-8(1-uv)(1-v^3)+\ 8(1-uv)^3$$
$$-1(1-uv)(1-u^3)+\ 4(1-uv)(v-u^7)(u-v^7)=0.$$

In the second column the coefficient of $1-uv$ is $2-u^3-v^3$, viz. this is

$$=(1-u^3)(1-v^3)+1-(uv)^3, \text{ or it is } =(1-uv)^3+1-(uv)^3.$$

Reducing also the other two columns by means of the modular equation, the equation thus becomes

$$-6(1-uv)^3-(1-uv)\{(1-uv)^3+1-(uv)^3\}-14uv(1-uv)^4(1+u^3v^3)$$
$$+\ 8\ (1-uv)^3$$
$$-28uv(1-uv)^3(1-uv+u^3v^3)^3=0.$$

This is in fact an identity; to show it, writing for convenience θ in place of uv, and observing that the terms

$$\begin{aligned} &-(1-\theta)(1-\theta^6)+8(1-\theta)^3, \\ =\ &(1-\theta)^3\{8-(1+\theta+\theta^2+\theta^3+\theta^4+\theta^5+\theta^6+\theta^7)\} \text{ are} \\ =\ &(1-\theta)^3(7+6\theta+5\theta^2+4\theta^3+3\theta^4+2\theta^5+\theta^6), \end{aligned}$$

the whole equation divides by $(1-\theta)^3$; or throwing out this factor, it is

$$\begin{aligned} &-6(1-\theta)^3-(1-\theta)^6+7+6\theta+5\theta^2+4\theta^3+3\theta^4+2\theta^5+\theta^6 \\ &\qquad -14\theta(1-\theta)(1+\theta^3)-28\theta(1-\theta+\theta^2)^2=0. \end{aligned}$$

The first line is $=14\theta(3-5\theta+6\theta^2-3\theta^3+\theta^4)$; whence, throwing out the factor 14θ, the equation is

$$3-5\theta+6\theta^2-3\theta^3+\theta^4-(1-\theta)(1+\theta^3)-2(1-\theta+\theta^2)^2,$$

that is

$$(1-\theta+\theta^2)(3-2\theta+\theta^2)-(1-\theta^2)(1-\theta+\theta^2)-2(1-\theta+\theta^2)^2=0;$$

or throwing out the factor $1-\theta+\theta^2$ the equation is

$$(3-2\theta+\theta^2)-(1-\theta^2)-2(1-\theta+\theta^2)=0,$$

which is an identity.

The other equation is

$$\gamma^2+2\beta\gamma+(2+2\beta)\frac{u^7}{v}=u^2v^2\left(2\gamma+2\beta\gamma+2\frac{u^7}{v}+\beta^2\right);$$

that is

$$\gamma^2+2\beta\gamma-u^2v^2\beta^2+2(1+\beta)\left(\frac{u^7}{v}-\gamma u^2v^2\right)-2u^2v=0,$$

which might also be verified, but I have not done this.

47. The conclusion is

$$\alpha=1,\ \beta=\tfrac{1}{2}\left(\frac{1}{\mathrm{M}}-1\right),\ \gamma=\tfrac{1}{2}u^2v^2\left(\frac{1}{\mathrm{M}}-\frac{u^4}{v^4}\right),\ \delta=\frac{u^7}{v},$$

where

$$\frac{1}{\mathrm{M}}=\frac{-7u(1-uv)(1-uv+u^2v^2)}{u-v^7},$$

and of course

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left(\frac{1-\beta x+\gamma x^2-\delta x^3}{1+\beta x+\gamma x^2+\delta x^3}\right)^3;$$

but the resulting form may admit of simplification.

48. The endecadic transformation, $n=11$.

I have not completed the solution, but the results, so far as I have obtained them, are interesting. The coefficients are $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta$; and we have, as in general,

$$\alpha=1 \quad , \quad 2\varepsilon=u^7v^3\left(\frac{1}{\mathrm{M}}-\frac{u^4}{v^4}\right),$$

$$2\beta=\frac{1}{\mathrm{M}}-1, \quad \zeta=\frac{u^{11}}{v}.$$

The unused equations then are

$$u^{14}(2\alpha\gamma+2\alpha\beta+\beta^2)=v^2(\varepsilon^2+2\varepsilon\zeta+2\delta\zeta),$$

$$u^6(\gamma^2+2\alpha\varepsilon+2\alpha\delta+2\beta\gamma+2\beta\delta)=v^2(2\gamma\varepsilon+2\gamma\zeta+2\delta\varepsilon+2\beta\zeta+\delta^2),$$

$$u^{-2}(2\gamma\varepsilon+2\alpha\zeta+2\gamma\delta+2\beta\varepsilon+2\beta\zeta+\delta^2)=v^2(\gamma^2+2\alpha\varepsilon+2\alpha\zeta+2\gamma\delta+2\beta\varepsilon+2\beta\delta),$$

$$u^{-10}(\varepsilon^2+2\gamma\zeta+2\delta\varepsilon+2\delta\zeta)=v^2(2\alpha\gamma+2\alpha\delta+2\beta\gamma+\beta^2);$$

but I attend only to the first and last, which, it will be observed, contain γ, δ linearly. If in the first instance we substitute only for α, ζ their values, the equations become

$$u^{12}\beta(2+\beta)-\frac{v^2}{u^2}\varepsilon\left(\varepsilon+2\frac{u^{11}}{v}\right) \qquad +u^{12}.2\gamma \qquad -vu^9.2\delta=0,$$

$$u^{-12}\varepsilon^2 \qquad -\frac{v^2}{u^2}\beta^2 \qquad +\left\{\frac{1}{uv}-\frac{v^2}{u^2}(1+\beta)\right\}.2\gamma+\left\{\frac{1}{uv}-\frac{v^2}{u^2}+u^{-12}\varepsilon\right\}.2\delta=0;$$

say, for a moment, these are

$$A+P.2\gamma+Q.2\delta=0,$$

$$B+R.2\gamma+S.2\delta=0,$$

giving

$$1:2\gamma:2\delta=PS-QR:QB-SA:RA-PB.$$

Here

$$PS-QR=\frac{u^{11}}{v}+\varepsilon-u^{12}v^2+u^2-u^7v^2(1+\beta)$$

$$=\tfrac{1}{2}\left\{\frac{2u^{11}}{v}+\left(u^7v^2\frac{1}{M}-\frac{u^{11}}{v}\right)-2u^{12}v^2+2u^2-2u^7v^2-\left(u^7v^2\frac{1}{M}-u^7v^2\right)\right\},$$

where the terms containing $\frac{1}{M}$ disappear of themselves, viz. this is

$$= \tfrac{1}{2}\left(\frac{u^{11}}{v}-2u^{12}v^2+2u^2-u^7v^2\right)$$

$$=-\tfrac{1}{2}\frac{u^7}{v}(v^4+2v^2u^2-2vu-u^4)$$

(observe that the term in ( ), equated to zero, gives the modular equation for the case $n=3$). It thus appears that γ and δ are given as fractions, having in their denominator this function $u^4+2uv-2u^2v^2-v^4$.

49. To complete the calculation, we have

$$QB-AS=-vu^9\left(\frac{\varepsilon^2}{u^{12}}-\frac{v^2}{u^2}\beta^2\right)$$

$$-\left\{u^{12}\beta(2+\beta)-\frac{v^2}{u^2}\varepsilon\left(\varepsilon+2\frac{u^{11}}{v}\right)\right\}\left\{\frac{1}{uv}-\frac{v^2}{u^2}+\frac{\varepsilon}{u^{12}}\right\};$$

viz. multiplying by 8, and substituting for 2β, 2ε their values, this is

$$8(QB-AS)=-2vu^9\left\{u^2v^2\left(\frac{1}{M}-\frac{u^4}{v^4}\right)^2-\frac{v^2}{u^2}\left(\frac{1}{M}-1\right)^2\right\}$$

$$-\left\{u^{12}\left(\frac{1}{M}-1\right)\left(\frac{1}{M}+3\right)-u^{12}v^2\left(\frac{1}{M}-\frac{u^4}{v^4}\right)\left(\frac{1}{M}+\frac{3u^4}{v^4}\right)\right\}\left(\frac{1}{uv}-\frac{2v^2}{u^2}+\frac{v^3}{u^5}\frac{1}{M}\right),$$

or, what is the same thing,

$$-\frac{8v}{u^7}(\mathrm{QB}-\mathrm{AS})=2\left\{u^4v^3\left(\frac{1}{\mathrm{M}}-\frac{u^4}{v^4}\right)^3-v^4\left(\frac{1}{\mathrm{M}}-1\right)^3\right\}$$

$$+\left\{\left(\frac{1}{\mathrm{M}}-1\right)\left(\frac{1}{\mathrm{M}}+3\right)-v^3\left(\frac{1}{\mathrm{M}}-\frac{u^4}{v^4}\right)\left(\frac{1}{\mathrm{M}}+3\frac{u^4}{v^4}\right)\right\}\left(u^4-2v^3u^3+\frac{v^4}{\mathrm{M}}\right),$$

viz. the left-hand side is

$$=2\left\{-v^4(1-u^4v^4)\frac{1}{\mathrm{M}^3}+2v^4(1-u^8)\frac{1}{\mathrm{M}}+u^{12}-v^4\right\}$$

$$+\left\{\frac{1}{\mathrm{M}^2}(1-v^8)+\frac{2}{\mathrm{M}}(1-u^4v^4)-3(1-u^8)\right\}\left(\frac{1}{\mathrm{M}}+u^3(u-2v^3)\right);$$

or say we have $-\frac{8v}{u^7}(\mathrm{QB}-\mathrm{SA})=\Pi$, where

$$\Pi=\ \frac{1}{\mathrm{M}^3}.v^4(1-v^8)$$

$$+\frac{1}{\mathrm{M}^2}.u^3(u-2v^3)(1-v^8)$$

$$+\frac{1}{\mathrm{M}}.4u^7v^7+v^4(1-3u^8)-4u^3v^3+2u^4$$

$$+\ .-2v^4+6v^3u^3(1-u^8)+u^4(-3+5u^8);$$

wherefore the value of 2γ is $=\frac{1}{4}\Pi\div(v^4+2v^3u^3-2vu-u^4)$. Similarly, writing

$$\Pi'=\ \frac{1}{\mathrm{M}^3}.v^4(1-v^8)$$

$$+\frac{1}{\mathrm{M}^2}.v(v^3-2u)(1-v^8)$$

$$+\frac{1}{\mathrm{M}}.4u^5v^5+v^4(3-u^8)+4uv-2u^4v^8$$

$$+\ \ v^4(-5+3u^8)+6vu(1-u^8)+2u^{12},$$

we find

$$2\delta=\frac{1}{4}\frac{u^3}{v}\Pi'\div(v^4+2v^3u^3-2vu-u^4);$$

in verification whereof observe that this being so, the first equation gives the identity

$$\left\{\left(\frac{1}{\mathrm{M}}-1\right)\left(\frac{1}{\mathrm{M}}+3\right)-v^3\left(\frac{1}{\mathrm{M}}-\frac{u^4}{v^4}\right)\left(\frac{1}{\mathrm{M}}+\frac{3u^4}{v^4}\right)\right\}(v^4+2v^3u^3-2vu-u^4)+\Pi-\Pi'=0.$$

50. The result is that, writing for the moment $v^4+2v^3u^3-2vu-u^4=\Delta$, the values of the coefficients are

$$\begin{array}{cccccc}\alpha, & \beta, & \gamma, & \delta, & \epsilon, & \zeta\\ =1, & \frac{1}{2}\left(\frac{1}{\mathrm{M}}-1\right), & \frac{1}{8}\frac{\Pi}{\Delta}, & \frac{1}{8}\frac{u^3}{v}\frac{\Pi'}{\Delta}, & \frac{1}{2}u^7v^3\left(\frac{1}{\mathrm{M}}-\frac{u^4}{v^4}\right), & \frac{u^{11}}{v}\end{array}$$

and

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left(\frac{1-\beta x+\gamma x^2-\delta x^3+\epsilon x^4-\zeta x^5}{1+\beta x+\gamma x^2+\delta x^3+\epsilon x^4+\zeta x^5}\right)^2;$$

the modular equation is known, and to complete the solution we require only an expression for M in terms of u, v.

51. We may herein illustrate the following theorem, viz. we may simultaneously change $u, v, \frac{1}{M}$, $\alpha:\beta:\gamma:\delta:\varepsilon:\zeta$ into $\frac{1}{u}, \frac{1}{v}, \frac{v^4}{u^4}\frac{1}{M}$, $\zeta:\varepsilon:\delta:\gamma:\beta:\alpha$.

Thus in the equation

$$\frac{\beta}{\alpha}=\tfrac{1}{2}\left(\frac{1}{M}-1\right),$$ making the change, we have

$$\frac{\varepsilon}{\zeta}=\tfrac{1}{2}\left(\frac{v^4}{u^4}\frac{1}{M}-1\right);$$ that is, $\tfrac{1}{2}\frac{v^4}{u^4}\left(\frac{1}{M}-\frac{u^4}{v^4}\right)=\tfrac{1}{2}\left(\frac{v^4}{u^4}\frac{1}{M}-1\right)$,

which is right.

So in the equation $\frac{\gamma}{\alpha}=\frac{1}{8}\frac{\Pi}{\Delta}$, if for a moment $(\Pi), (\Delta)$ are what Π, Δ become, the equation is $\frac{\delta}{\zeta}=\frac{1}{8}\frac{(\Pi)}{(\Delta)}$, that is, $\frac{1}{u^8}\frac{\Pi'}{\Delta}=\frac{(\Pi)}{(\Delta)}$, or $(\Pi)=\frac{1}{u^8}\frac{(\Delta)}{\Delta}\Pi'$; but obviously $\frac{(\Delta)}{\Delta}=-\frac{1}{u^4v^4}$; and the equation thus is $(\Pi)=-\frac{1}{u^{12}v^4}\Pi'$, or say $u^{12}v^4(\Pi)=-\Pi'$; that is

$$\begin{aligned}-\Pi'=u^{12}v^4\Big\{ &\frac{v^{12}}{u^{12}}\cdot\frac{1}{M^3}\cdot\frac{1}{v^4}\left(1-\frac{1}{v^3}\right)\\ &+\frac{v^8}{u^8}\cdot\frac{1}{M^2}\cdot\frac{1}{u^3}\left(\frac{1}{u}-\frac{2}{v^3}\right)\left(1-\frac{1}{v^3}\right)\\ &+\frac{v^4}{u^4}\cdot\frac{1}{M}\cdot\frac{4}{u^7v^7}+\frac{1}{v^4}\left(1-\frac{3}{u^3}\right)-\frac{4}{u^3v^3}+\frac{2}{u^4}\\ &-\frac{2}{v^4}+\frac{6}{v^3u^3}\left(1-\frac{1}{u^3}\right)+\frac{1}{u^4}\left(-3+\frac{5}{u^3}\right)\Big\},\end{aligned}$$

which is right.

The general theory by q-transcendents.—Articles Nos. 52 to 71.

52. I recur to the formula

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left(\frac{\alpha-\beta x+\gamma x^2\ldots+\sigma x^{\frac{1}{2}n(n-1)}}{\alpha+\beta x+\gamma x^2\ldots+\sigma x^{\frac{1}{2}n(n-1)}}\right)^2,$$

and seek to express the ratios $\alpha:\beta\ldots:\sigma$ in terms of q. Writing with JACOBI $\omega=\frac{m\mathrm{K}+m'i\mathrm{K}'}{n}$, we have in general

$$\alpha+\beta x+\gamma x^2\ldots+\sigma x^{\frac{1}{2}(n-1)}=\alpha\left(1+\frac{x}{\mathrm{snc}\,2\omega}\right)\left(1+\frac{x}{\mathrm{snc}\,4\omega}\right)\ldots\left(1+\frac{x}{\mathrm{snc}\,(n-1)\omega}\right)$$

$(\mathrm{snc}=\sin\mathrm{co\,am}$; viz. $\mathrm{snc}\,2\omega=\mathrm{sn}(\mathrm{K}-2\omega)$, &c.),

and the values of $\alpha, \beta, \ldots\theta$ which correspond to the moduli $v_0, v_1, \ldots v_n$, or say the values $(\alpha_0, \beta_0, \ldots\theta_0), (\alpha_1, \beta_1, \ldots\theta_1), \ldots(\alpha_n, \beta_n, \ldots\theta_n)$, are obtained by giving to ω the values

$$\begin{matrix}\omega_0, & \omega_1, & \omega_2\ldots & \omega_n\\ =\frac{2\mathrm{K}}{n}, & \frac{2\mathrm{K}+i\mathrm{K}'}{n}, & \frac{4\mathrm{K}+i\mathrm{K}'}{n}, & \ldots\frac{i\mathrm{K}'}{n},\end{matrix}$$

viz. the cases ω_0, ω_n correspond to JACOBI's first and second real transformations, and the others to the imaginary transformations.

I remark that $\omega=\omega_0$ gives for snc $2g\omega$ an expression which is rational as regards q, but $\omega=\omega_n$ an expression involving $q^{\frac{1}{n}}$, the real nth root of q; the other values, $\omega_1, \omega_2, \ldots$, give the like expressions, involving $\alpha q^{\frac{1}{n}}$, $\alpha^2 q^{\frac{1}{n}}$, ... (α an imaginary nth root of unity), the imaginary nth roots of q.

53. I consider first the expression

$$\frac{1}{\text{snc}\, 2g\omega_0}, \quad =\frac{1}{\text{sn}(K-2g\omega_0)}, \quad =\frac{\text{dn}\, 2g\omega_0}{\text{cn}\, 2g\omega_0}.$$

Here, writing $2g\omega_0=\frac{2K\xi}{\pi}$ (ξ for JACOBI's x, as x is being used in a different sense), that is

$$\xi=\frac{\pi}{2K}\cdot 2g\cdot\frac{2K}{n}, \quad =\frac{2g\pi}{n}$$

(and thence $e^{i\xi}=e^{g\frac{2\pi i}{n}}=\alpha^g$, $e^{2i\xi}=\alpha^{2g}$, if $\alpha=e^{\frac{2\pi i}{n}}$, an imaginary nth root of unity), we have (JACOBI, p. 86)

$$\frac{1}{\text{snc}\, 2g\omega_0}=\text{dn}\frac{2K\xi}{\pi}\div\text{cn}\frac{2K\xi}{\pi}$$

$$=\frac{C}{B}\cdot\frac{2e^{i\xi}}{1+e^{2i\xi}}\cdot\frac{(1+qe^{2i\xi})..(1+qe^{-2i\xi})..}{(1+q^2e^{2i\xi})..(1+q^2e^{-2i\xi})..}$$

$$\left(\frac{C}{B}=\left(\frac{(1+q^2)..}{(1+q)..}\right)^2=f^2(q)\right);$$

that is

$$\frac{1}{\text{snc}\, 2g\omega_0}=\frac{2\alpha^g}{1+\alpha^{2g}}\cdot f^2(q)\cdot\frac{(1+\alpha^{2g}q)..(1+\alpha^{n-2g}q)..}{(1+\alpha^{2g}q^2)..(1+\alpha^{n-2g}q^2)..},$$

where, for shortness, I write $(1+qe^{2i\xi})\ldots$ to denote the infinite product

$$(1+qe^{2i\xi})(1+q^2e^{2i\xi})(1+q^3e^{2i\xi})\ldots,$$

and similarly $(1+q^3e^{2i\xi})\ldots$ to denote the infinite product $(1+q^3e^{2i\xi})(1+q^4e^{2i\xi})(1+q^5e^{2i\xi})\ldots$, and the like for the terms in $e^{-2i\xi}$: the notation, accompanied by its explanation, is quite intelligible, and it would be difficult to make one which would be at the same time complete and not cumbrous; and then attributing to g the values $1, 2\ldots\frac{1}{2}(n-1)$, and forming the symmetric functions of these expressions, we have the values of $\frac{\beta}{\alpha}$, $\frac{\gamma}{\alpha}$ &c., or α being put $=1$, say the values of $\beta, \gamma, \ldots \sigma$.

54. I stop to notice a verification afforded by the value of β_0. Putting $u=0$, that is $q=0$, we have

$$\frac{1}{\text{snc}\, 2g\omega_0}=\frac{2\alpha^g}{1+\alpha^{2g}},$$

and thence

$$\beta_0=2\left\{\frac{\alpha}{1+\alpha^2}+\frac{\alpha^2}{1+\alpha^4}+\frac{\alpha^3}{1+\alpha^6}..+\frac{\alpha^{\frac{1}{2}(n-1)}}{1+\alpha^{n-1}}\right\},$$

we have $2\mathfrak{B}_0=\frac{1}{M_0}-1$; and putting as above $u=0$, the value of $\frac{1}{M_0}$ is $=(-)^{\frac{n-1}{2}}n$; whence

$$(-)^{\frac{1}{2}(n-1)}n-1=4\left\{\frac{\alpha}{1+\alpha^2}+\frac{\alpha^2}{1+\alpha^4}+\frac{\alpha^3}{1+\alpha^6}\cdots+\frac{\alpha^{\frac{1}{2}(n-1)}}{1+\alpha^{n-1}}\right\},$$

a theorem relating to the imaginary nth roots of unity, n an odd prime. In particular

$$n=3,\quad -4=4\left\{\frac{\alpha}{1+\alpha^2}\right\},\text{ at once verified by }\alpha^2+\alpha+1=0,$$

$$n=5,\quad 4=4\left\{\frac{\alpha}{1+\alpha^2}+\frac{\alpha^2}{1+\alpha^4}\right\},\text{ verified by }\alpha^4-1=0$$

(viz. the theorem is also true for the real root $\alpha=1$); in fact the term in $\{\ \}$ is

$$\left(\alpha(1+\alpha^4)+\alpha^2(1+\alpha^2)\right)\div(1+\alpha^2)(1+\alpha^4),\text{ that is }(\alpha+1+\alpha^3+\alpha^4)\div(1+\alpha^2+\alpha^4+\alpha),\ =1;$$

$$n=7,\quad -8=4\left\{\frac{\alpha}{1+\alpha^2}+\frac{\alpha^2}{1+\alpha^4}+\frac{\alpha^3}{1+\alpha^6}\right\},$$

which may be verified by means of $\alpha^6+\alpha^5+\alpha^4+\alpha^3+\alpha^2+\alpha+1=0$; and so on.

55. I further remark that we have

$$\frac{1}{M_0}=(-)^{\frac{1}{2}(n-1)}\left\{\frac{\operatorname{sn}2\omega_0\,.\,\operatorname{sn}4\omega_0\,..\,\operatorname{sn}(n-1)\omega_0}{\operatorname{snc}2\omega_0\,.\,\operatorname{snc}4\omega_0\,..\,\operatorname{snc}(n-1)\omega_0}\right\}^2.$$

But JACOBI (p. 86),

$$\operatorname{sn}2g\omega,\ =\operatorname{sn}\frac{2K\xi}{\pi},$$

$$=\frac{AK}{\pi i}\frac{e^{2\xi}-1}{e^{\xi}}\frac{(1-q^2e^{2\xi})\,..\,(1-q^2e^{-2\xi})\,..}{(1-qe^{2\xi})\,..\,(1-qe^{-2\xi})\,..},$$

where (p. 89)

$$\frac{AK}{\pi}=\frac{\sqrt[4]{q}}{\sqrt{k}}=\frac{1}{2}\left\{\frac{(1+q)(1+q^3)\,...}{(1+q^2)(1+q^4)...}\right\}^2=\frac{1}{2}f^{-2}(q),$$

that is,

$$\operatorname{sn}2g\omega=f^{-2}q\,.\,\frac{\alpha^{2g}-1}{2i\alpha^g}\frac{(1-\alpha^{2g}q^g)\,..\,(1-\alpha^{n-2g}q^g)\,..}{(1-\alpha^{2g}q)\,..\,(1-\alpha^{n-2g}q)\,..}.$$

Hence

$$\frac{\operatorname{sn}2g\omega_0}{\operatorname{snc}2g\omega_0}=\frac{\alpha^{2g}-1}{i(\alpha^{2g}+1)}\frac{1-\alpha^{2g}q^g\,..}{1+\alpha^{2g}q^g\,..}\frac{1+\alpha^{2g}q\,..}{1-\alpha^{2g}q\,..}\frac{1-\alpha^{n-2g}q^g\,..}{1+\alpha^{n-2g}q^g\,..}\frac{1+\alpha^{n-2g}q\,..}{1-\alpha^{n-2g}q\,..};$$

and giving to g the values $1, 2, \ldots \frac{1}{2}(n-1)$, and multiplying the several expressions, we have the value of $\frac{1}{M_0}$, viz. this is

$$\frac{1}{M_0}=(-)^{\frac{1}{2}(n-1)}\Pi\left\{\frac{(\alpha^{2g}+1)^2}{i^2(\alpha^{2g}+1)^2}\right\}R(q),$$

where $R(q)$ denotes the product of the several factors which contain q.

56. The (i^2) of the denominator gives a factor $i^{2\cdot\frac{n-1}{2}}=(-)^{\frac{n-1}{2}}$, which destroys the factor $(-)^{\frac{n-1}{2}}$. We have then a factor

$$\Pi\left(\frac{\alpha^{2g}-1}{\alpha^{2g}+1}\right)^2,\text{ which is }=(-)^{\frac{1}{2}(n-1)}n.$$

In fact, $n=3$, this is

$$\left(\frac{\alpha^2-1}{\alpha^2+1}\right)^3=-3,$$

viz. the numerator is $\alpha-2\alpha^2+1$, $=-3\alpha^2$, and the denominator is $(-\alpha)^3$, $=\alpha^3$.

So $n=5$, the formula is

$$\left(\frac{\alpha^2-1}{\alpha^2+1}\cdot\frac{\alpha^4-1}{\alpha^4+1}\right)^3=5, \text{ that is } \frac{(\alpha^2-1)^4}{(\alpha^4+1)^2}=5;$$

or

$$\frac{\alpha^8-4\alpha^6+6\alpha^4-4\alpha+1}{\alpha^8+2\alpha^4+1}=5,$$

viz. this is $5(1+\alpha^8+2\alpha^4)-(1-4\alpha-4\alpha^3+\alpha^8+6\alpha^4)=0$, which is right; and so in other cases.

We thus have

$$\frac{1}{M_0}=(-)^{\frac{1}{2}(n-1)}n\,.\,R(q),$$

which, on putting therein $u=0$, that is $q=0$, gives, as it should do, $\frac{1}{M_0}=(-)^{\frac{1}{2}(n-1)}n$.

57. As regards the expression of $R(q)$, observe that giving to g its different values, the factors $1-\alpha^{2g}q^g$ and $1-\alpha^{n-2g}q^g$ are all the factors other than $1-q^g$ of $1-q^n$, and so as to the other pairs of factors; viz. we have

$$R(q)=\left(\frac{1-q^n\,..\,1+q^n\,..\,1+q^2\,..\,1-q\,..}{1-q^2\,..\,1+q\,..\,1+q^n\,..\,1-q^n\,..}\right)^3,$$

viz. this is

$$=\left(\frac{1-q^n\,..\,1+q^n\,..}{1+q^n\,..\,1-q^n\,..}\right)^3\div\left(\frac{1-q^2\,..\,1+q\,..}{1+q^2\,..\,1-q\,..}\right)^3,$$

that is

$$\frac{1}{M_0}=(-)^{\frac{1}{2}(n-1)}n\,\frac{\varphi^3(q^n)}{\varphi^3(q)},$$

agreeing with a former result.

58. We have of course the identity $2\beta_0=\frac{1}{M_0}-1$; that is,

$$4S\frac{\alpha^g}{1+\alpha^{2g}}f^g(q)\cdot\frac{(1+\alpha^{2g}q\,)\,..\,(1+\alpha^{n-2g}q\,)\,..}{(1+\alpha^{2g}q^g)\,..\,(1+\alpha^{n-2g}q^g)\,..}=(-)^{\frac{1}{2}(n-1)}n\,\frac{\varphi^3(q^n)}{\varphi^3(q)}-1$$

$\left(g=1, 2, .. \tfrac{1}{2}(n-1)\right)$, which, putting therein $q=0$, is an identity before referred to; a form perhaps more convenient is obtained by dividing each side by $f^g(q)$.

59. I notice further that we have

$$v_0=u^n\{\operatorname{snc} 2\omega_0\,.\,\operatorname{snc} 4\omega_0\,.\,.\,.\,\operatorname{snc}(n-1)\omega_0\};$$

the term in $\{\ \}$ is

$$\Pi\frac{1+\alpha^{2g}}{2\alpha^g}f^{-g}(q)\,\frac{(1+\alpha^{2g}q^g)\,..\,(1+\alpha^{n-2g}q)^2\,..}{(1+\alpha^g\,q\,)\,..\,(1+\alpha^{n-2g}q\,)\,..},$$

where we have $\Pi\frac{1+\alpha^{2g}}{\alpha}=(-)^{\frac{1}{2}(n^2-1)}$. For example, $n=3$, the term is $\frac{1+\alpha^2}{\alpha}=-1$;

$n=5$, it is

$$\frac{(1+\alpha^2)(1+\alpha^4)}{\alpha.\alpha^2}, =\frac{1+\alpha^2+\alpha^4+\alpha}{\alpha^3}, =-1;$$

$n=7$, it is

$$\frac{(1+\alpha^2)(1+\alpha^4)(1+\alpha^6)}{\alpha.\alpha^3.\alpha^4}, =\frac{1+\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5+2\alpha^6}{\alpha^6}=1,$$

and so on. The term in question thus is

$$(-)^{\frac{1}{2}(n^2-1)}.\frac{1}{(\sqrt{2})^{n-1}}f^{-n+1}(q)\frac{1+q^n..1+q..}{1+q^2..1+q^n..},$$

that is

$$(-)^{\frac{1}{2}(n^2-1)}\frac{1}{(\sqrt{2})^{n-1}}f^{-n}(q)f(q^n).$$

This has to be multiplied by u^n, $=(\sqrt{2})^n q^{\frac{n}{2}}f^n(q)$, and we thus obtain

$$v_0=(-)^{\frac{1}{2}(n^2-1)}\sqrt{2}q^{\frac{n}{2}}f(q^n),$$

agreeing with a former result.

We have in what precedes a complete q-transcendental solution for the *transformatio prima*; viz. the original modulus $k^2(=u^2)$ being given as a function of q, then, as well the new modulus $\lambda_0^2(=v_0^2)$ and the multiplier M_0, as also the several functions which enter into the expression

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left\{\frac{\left(1-\frac{x}{\operatorname{snc}2\omega_0}\right)\cdots\left(1-\frac{x}{\operatorname{snc}(n-1)\omega_0}\right)}{\left(1+\frac{x}{\operatorname{snc}2\omega_0}\right)\cdots\left(1+\frac{x}{\operatorname{snc}(n-1)\omega_0}\right)}\right\}^2,$$

are all of them expressed as functions of q.

60. I consider in like manner the expression

$$\frac{1}{\operatorname{snc}2g\omega_n}\frac{1}{\operatorname{sn}(K-2g\omega_n)}, =\frac{\operatorname{dn}2g\omega_n}{\operatorname{cn}2g\omega_n}.$$

Here, writing $2g\omega_n=\frac{2K\xi}{\pi}$ (ξ instead of JACOBI's x as before), that is

$$\xi=\frac{\pi}{2K}.2g.\frac{iK'}{n}=\frac{g\pi iK'}{nK},$$

and thence

$$e^{i\theta}=e^{-\frac{g}{n}\frac{\pi K'}{K}}, =q^{\frac{g}{n}},$$

we have

$$\frac{1}{\operatorname{snc}2g\omega_n}=\operatorname{dn}\frac{2K\xi}{\pi}\div\operatorname{cn}\frac{2K\xi}{\pi}$$

$$=f^2(q).\frac{2q^{\frac{g}{n}}}{1+q^{\frac{2g}{n}}}.\frac{(1+q^{1+\frac{2g}{n}})..(1+q^{1-\frac{2g}{n}})..}{(1+q^{2+\frac{2g}{n}})..(1+q^{2-\frac{2g}{n}})..},$$

where the notations are as follows:

$(1+q^{1+\frac{2g}{n}})\ldots$ is the infinite product $(1+q^{1+\frac{2g}{n}})(1+q^{2+\frac{2g}{n}})(1+q^{3+\frac{2g}{n}})\ldots$,

and

$(1+q^{2+\frac{2g}{n}})\ldots$ the infinite product $(1+q^{2+\frac{2g}{n}})(1+q^{4+\frac{2g}{n}})(1+q^{6+\frac{2g}{n}})\ldots$;

and the like as to the expressions with exponents containing $-\frac{2g}{n}$.

And then attributing to g the values $1, 2, \ldots \frac{1}{2}(n-1)$, and forming the symmetric functions of these expressions, we have the values of $\frac{\beta}{\alpha}, \frac{\gamma}{\alpha}, \ldots \frac{\sigma}{\alpha}$; or α being put $=1$, say the values of $\beta, \gamma, \ldots \sigma$.

It is easy to see, and I do not stop to prove, that if instead of $\omega=\omega_n$ we have $\omega=\omega_1, \omega_2 \ldots$ or ω_{n-1}, we simply multiply $q^{\frac{1}{n}}$ by an imaginary nth root of unity; that is, replace the real nth root $q^{\frac{1}{n}}$ by an imaginary nth root of q.

In the case $u=0$, that is $q=0$, we have $\frac{1}{\operatorname{snc} 2g\omega_n}=0$, and thence $\beta=0$; and the like for the values $\omega_1, \omega_2, \ldots \omega_{n-1}$: the equation $2\beta=\frac{1}{M}-1$ gives consequently for $\frac{1}{M}$, n values each $=1$, agreeing with the multiplier equation.

61. We have for M_n the formula

$$\frac{1}{M_n}=(-)^{\frac{1}{2}(n-1)}\left\{\frac{\operatorname{sn}2\omega_n \operatorname{sn}4\omega_n \ldots \operatorname{sn}(n-1)\omega_n}{\operatorname{snc}2\omega_n \operatorname{snc}4\omega_n \ldots \operatorname{snc}(n-1)\omega_n}\right\}^2,$$

and, as before,

$$\operatorname{sn} 2g\omega_n=f'^{-2}(q).\frac{q^{\frac{2g}{n}}-1}{i}\frac{(1-q^{2+\frac{2g}{n}})\ldots}{2iq^{\frac{g}{n}}}\frac{(1-q^{2-\frac{2g}{n}})\ldots}{(1-q^{1+\frac{2g}{n}})\ldots}\frac{}{(1-q^{1-\frac{2g}{n}})\ldots};$$

hence

$$\frac{\operatorname{sn}2g\omega_n}{\operatorname{snc}2g\omega_n}=\frac{q^{\frac{2g}{n}}-1}{i(q^{\frac{2g}{n}}+1)}.\frac{(1-q^{2+\frac{2g}{n}})\ldots}{(1+q^{2+\frac{2g}{n}})\ldots}\frac{(1+q^{1+\frac{2g}{n}})\ldots}{(1-q^{1+\frac{2g}{n}})\ldots}\frac{(1-q^{2-\frac{2g}{n}})\ldots}{(1+q^{2-\frac{2g}{n}})\ldots}\frac{(1+q^{1-\frac{2g}{n}})\ldots}{(1-q^{1-\frac{2g}{n}})\ldots},$$

and we thence derive the value of $\frac{1}{M_n}$; viz. observing that we have in the denominator $(i^2)^{\frac{1}{2}(n-1)}$, $=(-)^{\frac{1}{2}(n-1)}$ which destroys this factor in the expression of $\frac{1}{M_n}$, this is

$$\frac{1}{M_n}=\Pi\left\{\frac{1-q^{\frac{2g}{n}}}{1+q^{\frac{2g}{n}}}\frac{(1-q^{2+\frac{2g}{n}})\ldots(1+q^{1+\frac{2g}{n}})\ldots(1-q^{2-\frac{2g}{n}})\ldots(1+q^{1-\frac{2g}{n}})\ldots}{(1+q^{2+\frac{2g}{n}})\ldots(1-q^{1+\frac{2g}{n}})\ldots(1+q^{2-\frac{2g}{n}})\ldots(1-q^{1-\frac{2g}{n}})\ldots}\right\}^2.$$

Now, giving to g its values, it is easy to see that we have

$$\Pi(1-q^{\frac{2g}{n}})(1-q^{2+\frac{2g}{n}})\ldots(1-q^{2-\frac{2g}{n}})\ldots=\frac{(1-q^{\frac{2}{n}})\ldots}{(1-q^{\frac{2}{n}})\ldots},$$

where $(1-q^{\frac{2}{n}})\ldots$ denotes $(1-q^{\frac{2}{n}})(1-q^{\frac{4}{n}})(1-q^{\frac{6}{n}})\ldots$, viz. it is the same function of $q^{\frac{1}{n}}$ that

$(1-q^s)\ldots$ is of q; also

$$\Pi(1+q^{1+\frac{2f}{n}})\ldots(1+q^{1-\frac{2f}{n}})\ldots=\frac{(1+q^{\frac{1}{n}})\ldots}{(1+q)\ldots},$$

where $(1+q^{\frac{1}{n}})\ldots$ denotes $(1+q^{\frac{1}{n}})(1+q^{\frac{3}{n}})(1+q^{\frac{5}{n}})\ldots$, viz. it is the same function of $q^{\frac{1}{n}}$ that $(1+q)\ldots$ is of q; and the like as to the denominator factors: we thus have

$$\frac{1}{M_n}=\left\{\frac{(1-q^{\frac{2}{n}})\ldots}{(1-q^2)\ldots}\,\frac{(1+q^{\frac{1}{n}})\ldots}{(1+q)\ldots}\,\frac{(1+q^{\frac{3}{n}})\ldots}{(1+q^{\frac{2}{n}})\ldots}\,\frac{(1-q)\ldots}{(1-q^{\frac{1}{n}})\ldots}\right\}^2,$$

viz. this is

$$=\left\{\frac{(1-q^{\frac{2}{n}})\ldots}{(1+q^{\frac{2}{n}})\ldots}\,\frac{(1+q^{\frac{1}{n}})\ldots}{(1-q^{\frac{1}{n}})\ldots}\right\}^2\div\left\{\frac{(1-q^2)\ldots(1+q)\ldots}{(1+q^2)\ldots(1-q)\ldots}\right\}^2;$$

or we have

$$\frac{1}{M_n}=\varphi^2(q^{\frac{1}{n}})\div\varphi^2(q),$$

agreeing with a former result.

We have

$$2\beta_n=\frac{1}{M_n}-1, \text{ that is}$$

$$2\left\{\frac{1}{\operatorname{snc}2\omega_n}+\frac{1}{\operatorname{snc}4\omega_n}\ldots+\frac{1}{\operatorname{snc}(n-1)\omega_n}\right\}=\frac{\varphi^2(q^{\frac{1}{n}})}{\varphi^2(q)}-1,$$

a result which, substituting on the left-hand side the foregoing values of the several functions, must be identically true.

62. We have also

$$v_n=u^n\{\operatorname{snc}2\omega_n\operatorname{snc}4\omega_n\ldots\operatorname{snc}(n-1)\omega_n\},$$

where the term in { } is

$$=\Pi f^{-3}(q)\frac{(1+q^{\frac{2f}{n}})}{2q^{\frac{f}{n}}}\,\frac{(1+q^{2+\frac{2f}{n}})\ldots(1+q^{2-\frac{2f}{n}})\ldots}{(1+q^{1+\frac{2f}{n}})\ldots(1+q^{1-\frac{2f}{n}})\ldots};$$

or observing that the sum of the exponents $\frac{f}{n}$ is $\frac{1}{n}\{1+2\ldots+\frac{1}{2}(n-1)\}=\frac{n^2-1}{8n}$, this is

$$=f^{-n+1}(q)\cdot\frac{1}{(\sqrt{2})^{n-1}q^{\frac{n^2-1}{8n}}}\,\frac{(1+q^{\frac{2}{n}})\ldots(1+q)\ldots}{(1+q^2)\ldots(1+q^{\frac{1}{n}})\ldots};$$

or, the last factor being $f(q^{\frac{1}{n}})\div f(q)$, the expression is

$$f^{-n}(q)\frac{1}{(\sqrt{2})^{n-1}}q^{-\frac{n}{8}+\frac{1}{8n}}f(q^{\frac{1}{n}});$$

or, multiplying by u^n, $=(\sqrt{2})^n q^{\frac{n}{8}}f^n(q)$, we have

$$v_n=\sqrt{2}q^{\frac{1}{8n}}f(q^{\frac{1}{n}}),$$

agreeing with a former result.

We have in what precedes the complete q-transcendental solution for the *transformatio secunda*; viz. the original modulus $k(=u^4)$ being given as a function of q, then, as well the new modulus $\lambda_n(=v_n^4)$ and the multiplier M_n, as also the several functions which enter into the formula

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left\{\frac{\left(1-\frac{x}{\operatorname{snc} 2\omega_n}\right)\cdots\left(1-\frac{x}{\operatorname{snc}(n-1)\omega_n}\right)}{\left(1+\frac{x}{\operatorname{snc} 2\omega_n}\right)\cdots\left(1+\frac{x}{\operatorname{snc}(n-1)\omega_n}\right)}\right\}^2,$$

are all expressed in terms of q. The expressions all contain $q^{\frac{1}{n}}$, and by substituting for this an imaginary nth root of q, we have the formulæ belonging to the several $(n-1)$ imaginary transformations.

63. As an illustration of the formulæ for the *transformatio secunda* I write $n=7$; and putting for greater convenience $q=r^7$, that is $r=q^{\frac{1}{7}}$, then we have

$$v_7=\sqrt{2}r^{\frac{1}{4}}f(r), \quad \frac{1}{M_7}=\frac{\varphi^8(r)}{\varphi^8(r^7)},$$

$$\frac{1}{\operatorname{snc} 2\omega_7}=2f^2(r^7)\mathrm{A}, \quad \frac{1}{\operatorname{snc} 4\omega_7}=2f^2(r^7)\mathrm{B}, \quad \frac{1}{\operatorname{snc} 6\omega_7}=2f^2(r^7)\mathrm{C},$$

where

$$\mathrm{A}=r\cdot\frac{5\,.\,19\ldots\ 9\,.\,23\ldots}{2\,.\,16\ldots 12\,.\,26\ldots},$$

$$\mathrm{B}=r^3\cdot\frac{3\,.\,17\ldots\ 11\,.\,25\ldots}{4\,.\,18\ldots\ 10\,.\,24\ldots},$$

$$\mathrm{C}=r^5\cdot\frac{1\,.\,15\ldots\ 13\,.\,27\ldots}{6\,.\,20\ldots\ 8\,.\,22\ldots},$$

where the numerator of A denotes $(1+r^5)(1+r^{19})\ldots(1+r^9)(1+r^{23})\ldots$, and so in other cases, the difference of the exponents being always $=14$. And we have, as mentioned, the identical equation

$$f^2(r^7)(\mathrm{A}+\mathrm{B}+\mathrm{C})=\tfrac{1}{4}\left\{\frac{\varphi^8 r}{\varphi^8 r^7}-1\right\}.$$

The values of the several expressions up to r^{56} are as follows. Mr. J. W. L. Glaisher kindly performed for me the greater part of the calculation.

Ind. of r.	A	B	C	Sum.	Multiplied by $f^3(r^2)$.	$\varphi^3(r) \div \varphi^3(r^2)$.
0				0	0	+ 1
1	+ 1			+ 1	+ 1	+ 4
2		+ 1		+ 1	+ 1	+ 4
3	− 1		+ 1	0	0	0
4			+ 1	+ 1	+ 1	+ 4
5	+ 1	+ 1		+ 2	+ 2	+ 8
6	+ 1	− 1		0	0	0
7	− 1			− 1	− 1	− 4
8	− 1			− 1	− 3	− 12
9	+ 1	− 1	− 1	− 1	− 3	− 12
10	+ 2	+ 1	− 1	+ 2	+ 2	+ 8
11	− 1		− 1	− 2	− 4	− 16
12	− 2	− 1	− 1	− 4	− 8	− 32
13		+ 2		+ 2	+ 2	+ 8
14	+ 2	− 1		+ 1	+ 3	+ 12
15	+ 1	− 1	+ 1	+ 1	+ 8	+ 32
16	− 2	+ 2	+ 2	+ 2	+ 9	+ 36
17	− 2	− 2	+ 2	− 2	− 6	− 24
18	+ 1	+ 1	+ 2	+ 4	+ 13	+ 52
19	+ 2	+ 2	+ 2	+ 6	+ 24	+ 96
20		− 3	+ 1	− 2	− 6	− 24
21	− 2	+ 2	− 1	− 1	− 8	− 32
22	− 2	+ 1	− 2	− 3	− 20	− 80
23	+ 2	− 4	− 3	− 5	− 24	− 96
24	+ 3	+ 3	− 4	+ 2	+ 16	+ 64
25		− 1	− 4	− 5	− 33	− 132
26	− 4	− 3	− 3	−10	− 62	− 248
27	− 2	+ 5	− 1	+ 2	+ 16	+ 64
28	+ 4	− 3	+ 1	+ 2	+ 19	+ 76
29	+ 5	− 1	+ 3	+ 7	+ 46	+ 184
30	− 3	+ 6	+ 5	+ 8	+ 56	+ 224
31	− 7	− 6	+ 7	− 6	− 40	− 160
32	+ 1	+ 1	+ 7	+ 9	+ 77	+ 308
33	+ 9	+ 5	+ 4	+18	+144	+ 576
34	+ 3	− 8	+ 1	− 4	− 38	− 152
35	− 9	+ 5	− 1	− 5	− 42	− 168
36	− 7	+ 2	− 5	−10	− 99	− 396
37	+ 7	− 9	− 9	−11	−122	− 488
38	+11	+10	−11	+10	+ 88	+ 352
39	− 4	− 3	−10	−17	−168	− 672
40	−13	− 8	− 7	−28	−310	−1240
41	− 2	+13	− 3	+ 8	+ 82	+ 328
42	+13	− 8	+ 3	+ 8	+ 88	+ 352
43	+ 8	− 3	+ 9	+14	+204	+ 816
44	−11	+14	+14	+17	+252	+1008
45	−14	−14	+16	−12	−182	− 728
46	+ 5	+ 4	+15	+24	+344	+1376
47	+17	+11	+12	+40	+632	+2528
48	+ 3	−20	+ 5	−12	−168	− 672
49	−17	+13	− 5	− 9	−175	− 700
50	−13	+ 5	−14	−22	−401	−1604

64. As already mentioned, the foregoing expressions of the coefficients in terms of q may be applied to the determination of the coefficients as rational functions of u, v.

Representing by θ any one of the coefficients $\alpha, \beta, \gamma \ldots \sigma$, consider the sum

$$S\frac{v^{r}\theta}{a},$$

f a positive integer, and the summation extending as before to the $n+1$ values of v, and corresponding values of $\frac{\theta}{\alpha}$. This is a rational function of u, and it is also integral. As to this observe that the function, if not integral, must become infinite either for $u=0$ (this would mean that the expression contained a term or terms $Au^{-\kappa}$) or for some finite value of u. But the function can only become infinite by reason of some term or terms of $Sv^f\frac{\theta}{\alpha}$ becoming infinite; viz. some term $\frac{1}{\operatorname{snc} 2f\omega}$ must become infinite; or attending to the equation

$$v=u^n\{\operatorname{snc} 2\omega \operatorname{snc} 4\omega \ldots \operatorname{snc}(n-1)\omega\},$$

it can only happen if $u=0$, or if $v=\infty$; and from the modular equation it appears that if $v=\infty$, then also $u=\infty$: the expression in question can therefore only become infinite if $u=0$, or if $u=\infty$. Now $u=0$ gives the ratios $\frac{\beta}{\alpha}, \frac{\gamma}{\alpha}, \ldots$, each of them a determinate function of n, that is finite; and gives also $v=0$, so that the expression does not become infinite for $u=0$; hence it does not become infinite either for $u=0$ or for any finite value of u; wherefore it is integral. The like reasoning applies to the sum $Sv^{-f}\frac{\theta}{\alpha}$; viz. this is a rational function of u; and it is quasi-integral, viz. there are no terms having a denominator other than a power of u, the highest denominator being u^{nf}; viz. the expression contains negative and positive integer powers of u, the lowest power (highest negative power) being $\frac{1}{u^{nf}}$.

65. It is to be observed, further, that writing the expression in the form

$$v_0^f\frac{\theta_0}{\alpha_0}+S'v^f\frac{\theta}{\alpha}$$

(where S' refers to the values $v_1, v_2, \ldots v_n$ of the modulus), and considering the several quantities as expressed in terms of q, then in the sum S' every term involving a fractional power $q^{\frac{h}{n^2}}$ acquires by the summation the coefficient $(1+\alpha+\alpha^2\ldots+\alpha^{n-1})$, and therefore disappears; there remains only the radicality q^1 occurring in the expressions of the v's; and if $nf\equiv\mu \pmod 8$, $\mu=0$, or a positive integer less than 8, then the form of the expression is $q^{\frac{\mu}{n}}$ into a rational function of q. Hence this, being a rational and integral function of u, must be of the form

$$Au^\mu+Bu^{n+\mu}+Cu^{n+16}+\&c.$$

66. We have thus in general

$$Sv^f\frac{\theta}{\alpha}=Au^\mu+Bu^{n+\mu}+\&c.;$$

and in like manner

$$Sv^{-f}\frac{\theta}{\alpha}=A'u^{-nf}+B'u^{-nf+\mu}+\&c.$$

We may in these expressions find a limit to the number of terms, by means of the before mentioned theorem that we may simultaneously interchange u, v; $\alpha, \beta, \ldots \varrho, \sigma$ into $\frac{1}{u}, \frac{1}{v}$; $\sigma, \varrho, \ldots \beta, \alpha$. Starting from the expression of $Sv^f \frac{\theta}{\alpha}$, let φ be the corresponding coefficient to θ; viz. in the series $\alpha, \beta \ldots \theta \ldots \varphi \ldots \varrho, \sigma$, let φ be as removed from σ as θ is from α; then the equation becomes

$$Sv^{-f} \frac{\phi}{\sigma} = Au^{-\mu} + Bu^{-\mu-\nu} + \&c.,$$

where $\frac{\varphi}{\sigma} = \frac{\varphi}{\alpha} \frac{\alpha}{\sigma} = \frac{v}{u^n} \frac{\varphi}{\alpha}$; the equation thus is

$$Sv^{1-f} \frac{\varphi}{\alpha} = Au^{n-\mu} + Bu^{n-\mu-\nu} + \&c.;$$

and by what precedes the series on the right-hand side can contain no negative power higher than $\frac{1}{u^{n(f-1)}}$; that is, the series of coefficients A, B, C... goes on to a certain point only, the subsequent coefficients all of them vanishing.

In like manner from the equation for $Sv^{-f} \frac{\theta}{\alpha}$ we have

$$Sv^{f+1} \frac{\varphi}{\alpha} = A'u^{(n+1)\nu} + B'u^{(n+1)\nu-\nu} + \&c.,$$

where the indices must be positive; viz. the series of coefficients A', B', .. goes on to a certain point only, the subsequent coefficients all of them vanishing.

67. The like theory applies to the expression $\frac{1}{M}$. We have, putting as before $nf = \mu \pmod 8$,

$$S\, v^f \frac{1}{M} = Au^{\mu} + Bu^{\mu+\nu} + \ldots,$$

$$Sv^{-f} \frac{1}{M} = A'u^{-\mu'} + B'u^{-\mu'+\nu} + \ldots$$

and we find a limit to the number of terms by the consideration that we may simultaneously change $u, v, \frac{1}{M}$ into $\frac{1}{u}, \frac{1}{v}, \frac{v^n}{u^n M}$; the equations thus become

$$Sv^{n-f} \frac{1}{M} = Au^{n-\mu} + Bu^{n-\mu-\nu} + \ldots$$

(where, if $f =$ or < 4, there must be on the right-hand side no negative power of u; but if $f > 4$, then the highest negative power must be $\frac{1}{u^{(f-4)n}}$), and

$$Sv^{4+f} \frac{1}{M} = A'u^{\mu'+4} + B'u^{\mu'-4} + \ldots,$$

where on the right-hand side there must be no negative power of u.

68. It is to be remarked that β, ϱ being always given linearly in terms of $\frac{1}{M}$, it is the same thing whether we seek in this manner for the values of β, ϱ or for that of $\frac{1}{M}$; but the latter course is practically more convenient. Thus in the cases $n=5$, $n=7$ we require only the value of $\frac{1}{M}$.

In the case $n=11$, where the coefficients are $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta$, it has been seen that γ, δ are given as cubic functions of $\frac{1}{M}$: seeking for them directly their values would (if the process be practicable) be obtained in a better form, viz. instead of the denominator $(\mathrm{F}v)^3$ there would be only the denominator $\mathrm{F}(v)$.

69. I consider for $\frac{1}{M}$ the cases $n=3$ and 5:

$$n=3, f=0, 1, 2, 3, \text{ then } \mu=0, 3, 6, 1;$$

and we write down the equations

$$\mathrm{S}\frac{1}{M}=\mathrm{A} \quad \text{giving} \quad \mathrm{S}\frac{v^4}{M}=\mathrm{A}u^4,$$

$$\mathrm{S}\frac{v}{M}=\mathrm{A}'u^3 \quad \text{,,} \quad \mathrm{S}\frac{v^3}{M}=\mathrm{A}'u,$$

$$\mathrm{S}\frac{v^2}{M}=0 \quad \text{,,} \quad \mathrm{S}\frac{v^3}{M}=0;$$

viz. if we had in the first instance assumed $\mathrm{S}\frac{1}{M}=\mathrm{A}+\mathrm{B}u^3+\ldots$, this would have given $\mathrm{S}\frac{v^4}{M}=\mathrm{A}u^4+\mathrm{B}u^{-4}+\ldots$, whence B and the succeeding coefficients all vanish; and so in other cases. We have here only the coefficients A, A'; and these can be obtained without the aid of the q-formulæ by the consideration that for $u=1$ the corresponding values of $v, \frac{1}{M}$ are

$$v=1, -1, -1, -1,$$

$$\frac{1}{M}=3, -1, -1, -1,$$

whence $\mathrm{A}=0$, $\mathrm{A}'=6$; or we have the equations

$$\mathrm{S}\frac{1}{M}=0, \quad \mathrm{S}\frac{v}{M}=6u^3, \quad \mathrm{S}\frac{v^2}{M}=0, \quad \mathrm{S}\frac{v^3}{M}=6u,$$

giving as before

$$(2v^3+3v^2u-u)\frac{1}{M}=3(v^4u^3+2u^3v+1)u,$$

reducible by means of the modular equation to $\frac{1}{M}=1+\frac{2u^3}{v}$.

70. $n=5$. Corresponding to $f=0, 1, 2, 3, 4, 5$, we have $\mu=0, 5, 2, 7, 4, 1$,

and we find

$$S\frac{1}{M}=A \quad \text{giving} \quad S\frac{v^4}{M}=Au^4,$$

$$S\frac{v}{M}=0 \quad \text{,,} \quad S\frac{v^5}{M}=0,$$

$$S\frac{v^2}{M}=A'u^2 \quad \text{,,} \quad S\frac{v^3}{M}=A'u^3,$$

$$S\frac{v^5}{M}=A''u+B''u^9 \quad \text{,,} \quad Sv^{-1}\frac{1}{M}=A''u^3+B''u^{-3}.$$

But for $u=1$ the corresponding values of $v, \frac{1}{M}$ are

$$v=1,\ -1,\ -1,\ -1,\ -1,\ -1,$$

$$\frac{1}{M}=5,\quad 1,\quad 1,\quad 1,\quad 1,\quad 1;$$

whence $A=A'=10$, $A''+B''=0$, or say the value of $S\frac{v^5}{M}$ is $=A''u(1-u^8)$.

The value of A'' is found very easily by the q-formulæ, viz. neglecting higher powers of q, we have

$u=q^{\frac{1}{2}}\sqrt{2}$, $v_0=q^{\frac{1}{2}}\sqrt{2}$, $\frac{1}{M_0}=5$; $v_1=q^{\frac{3}{2}}\sqrt{2}$, $\frac{1}{M_1}=1$; hence

$S\frac{v^5}{M}=\frac{v_0^5}{M}+S'\frac{v^5}{M}$, $=5q^{\frac{1}{2}}(\sqrt{2})^5=A''q^{\frac{1}{2}}\sqrt{2}$; that is $A''=20$, and the equations are

$$S\frac{1}{M}=10,\ S\frac{v}{M}=0,\ S\frac{v^2}{M}=10u^2,\ S\frac{v^3}{M}=0,\ S\frac{v^4}{M}=10u^4,\ S\frac{v^5}{M}=20u(1-u^8);$$

whence

$$\begin{aligned}F'v.\frac{1}{M}=&20u(1-u^8)\\&-10u^4(Sv_0-v)\\&-10u^2(Sv_0v_1v_2-vSv_0v_1+v^2Sv_0-v^3)\\&-10\ \ (Sv_0v_1v_2v_3v_4-vSv_0v_1v_2v_3+v^2Sv_0v_1v_2-v^3Sv_0v_1+v^4Sv_0-v^5),\end{aligned}$$

where Sv_0, &c. are the coefficients of the equation

$$v^5+4v^4u^4+5v^3u^3-5v^2u^4-4vu-u^5=0,$$

viz. Sv_0, v_0v_1, $v_0v_1v_2$, $v_0v_1v_2v_3$, $v_0v_1v_2v_3v_4$

are $-4u^4$, $+5u^3$, 0, $-5u^4$, $4u$;

or the equation is

$$\begin{aligned}F'v.\frac{1}{M}=&\ 20u(1-u^8)\\&-10u^4(-4u^4-\ \ v)\\&-10u^2(\ \ \ -5u^3v-4u^5v^3-\ v^2)\\&-10\ (4u\ \ +5u^4v\ \ \ \ -5v^3u^3-4v^4u^5-v^5),\end{aligned}$$

or say

$$\tfrac{1}{2}\mathrm{F}'v\,\frac{1}{\mathrm{M}}=5\{v^8+\ 4v^6u^2+\ 6v^4u^4+4v^2u^6+\ vu^8-2u(1-u^8)\},$$

where

$$\tfrac{1}{2}\mathrm{F}'v\ =\ 3v^8+10v^6u^2+10v^4u^4\qquad -5vu^8-2u.$$

Hence also, reducing by the modular equation,

$$\tfrac{1}{2}v\mathrm{F}'v\,\frac{1}{\mathrm{M}}=5u\{v^4u+4v^3u^4+6v^2u^3+2v(1+u^8)+u^3\},$$

the one of which forms is as convenient as the other.

71. Making the change $u,\ v,\ \frac{1}{\mathrm{M}}$ into $v,\ -u,\ -5\mathrm{M}$, we have

$$-\tfrac{1}{2}\mathrm{F}'u\,.\,5\mathrm{M}=5\{-u^8+4v^2u^4-6v^4u^2+4v^6u^2-v^8u-2v(1-v^8)\};$$

and comparing with the equation

$$5\mathrm{M}^3=-\frac{(1-v^8)v\mathrm{F}'v}{(1-u^8)u\mathrm{F}'u},$$

we obtain

$$\frac{v(1-v^8)}{u(1-u^8)}=\frac{-2v(1-v^8)-v^4u+4v^7u^2-6v^2u^3+4v^5u^6-u^9}{-2u(1-u^8)+u^4v+4u^7v^3+6u^5v^5+4u^3v^7+v^9}.$$

Writing for a moment $\mathrm{M}=u^4+6u^2v^2+v^4$, $\mathrm{N}=u^2+v^2$, this is

$$-\frac{v(1-v^8)}{u(1-u^8)}=\frac{-2v(1-v^8)-u\mathrm{M}+4v^2u^2\mathrm{N}}{-2u(1-u^8)+v\mathrm{M}+4v^3u^3\mathrm{N}},$$

that is

$$-4uv(1-u^8)(1-v^8)-\{u^3(1-u^8)-v^3(1-v^8)\}\mathrm{M}+4v^2u^3\{u^3(1-v^8)+v^3(1-u^8)\}\mathrm{N}=0.$$

But we have

$$u^3(1-u^8)-v^3(1-v^8)=(u^3-v^3)\{1-u^3-u^6v^3-u^4v^4-u^3v^6-v^3\},$$
$$u^3(1-v^8)+v^3(1-u^8)=(u^3+v^3)\{1-u^3v^3(u^4-u^3v^3+v^4)\}.$$

Hence, replacing M, N by their values, this is

$$-4uv(1-u^8)(1-v^8)$$
$$-(u^3-v^3)(1-u^3-u^6v^3-u^4v^4-u^3v^6-v^3)(u^4+6u^2v^2+v^4)$$
$$+4u^3v^3(u^3+v^3)^2\{1-u^3v^3(u^4-u^3v^3+v^4)\}=0;$$

viz. writing $u^3-v^3=\mathrm{A},\ uv=\mathrm{B}$, this is

$$-4\mathrm{B}\{1-\mathrm{A}^4-4\mathrm{A}^2\mathrm{B}^2-2\mathrm{B}^4+\mathrm{B}^8\}$$
$$-\ \mathrm{A}\{1-\mathrm{A}^4-5\mathrm{A}^2\mathrm{B}^2-3\mathrm{B}^4\}(\mathrm{A}^2+8\mathrm{B}^2)$$
$$+4\mathrm{B}^3(\mathrm{A}^2+4\mathrm{B}^2)\{1-\mathrm{A}^2\mathrm{B}^2-\mathrm{B}^4\}=0,$$

that is

$$-4\mathrm{B}\{(1-\mathrm{A}^4-4\mathrm{A}^2\mathrm{B}^2-2\mathrm{B}^4+\mathrm{B}^8)-\mathrm{B}^2(\mathrm{A}^2+4\mathrm{B}^2)(1-\mathrm{A}^2\mathrm{B}^2-\mathrm{B}^4)\}$$
$$-\ \mathrm{A}(1-\mathrm{A}^4-5\mathrm{A}^2\mathrm{B}^2-3\mathrm{B}^4)(\mathrm{A}^2+8\mathrm{B}^2)=0;$$

viz.

$$-4\mathrm{B}(1-\mathrm{A}^4-5\mathrm{A}^2\mathrm{B}^2-3\mathrm{B}^4)(1-\mathrm{B}^4)$$
$$-\ \mathrm{A}(1-\mathrm{A}^4-5\mathrm{A}^2\mathrm{B}^2-3\mathrm{B}^4)(\mathrm{A}^2+8\mathrm{B}^2)=0;$$

or throwing out the factor $-(1A^4-5A^2B^2-3B^4)$, this is

$$A(A^2+8B^2)+4B(1-B^2)=0,$$

the modular equation, which is right.

The four forms of the modular equation, and the curves represented thereby.
Article Nos. 72 to 79.

72. The modular equation for any value of n has the property that it may be represented as an equation of the same order ($=n+1$, when n is prime) between u, v or between u^2, v^2, or between u^4, v^4, or between u^8, v^8. As to this, remark that in general an equation $(u, v, 1)^m=0$ of the order m gives rise to an equation $(u^2, v^2, 1)^{2m}=0$ of the order $2m$ between u^2, v^2; viz. the required equation is

$$(u, v, 1)^m(u, -v, 1)^m(-u, v, 1)^m(-u, -v, 1)^m=0,$$

where the left-hand side is a rational function of u^2, v^2 of the form $(u^2, v^2, 1)^{2m}$; or again starting from a given equation $(u, v, w)^m=0$, and transforming by the equations $x:y:z=u^2:v^2:w^2$, the curve in (x, y, z) is of the order $2m$; in fact the intersections of the curve by the arbitrary line $ax+by+cz=0$ are given by the equations $(u, v, w)^m=0$, $au^2+bv^2+cw^2=0$, and the number of them is thus $=2m$. Moreover, by the general theory of rational transformation, the new curve of the order $2m$ has the same deficiency as the original curve of the order m. The transformed curve in $x, y, z, =u^2, v^2, w^2$ may in particular cases reduce itself to a curve of the order m twice repeated; but it is important to observe that here, taking the single curve of the order m as the transformed curve, this has no longer the same deficiency as the original curve; and in particular the curves represented by the modular equation in its four several forms, writing therein successively u, v; u^2, v^2; u^4, v^4; u^8, v^8, $=x, y$, are not curves of the same deficiency.

73. The question may be looked at as follows: the quantities which enter rationally into the elliptic-function formulæ are k^2, $\lambda^2=u^8$, v^8; if a modular equation $(u, v)^\nu=0$ led to the transformed equation $(u^8, v^8)^{8\nu}=0$, then to a given value of u^8 would correspond 8 values of u, therefore 8ν values of v, giving the same number, 8ν, values of v^8; that is, the values of v^8 corresponding to a given value of u^8 would group themselves in eights corresponding to the 8 values of u. There is in fact no such grouping; the equations are $(u, v)^\nu=0$, $(u^8, v^8)^\nu=0$; to a given value of u^8 correspond 8 values of u, and therefore 8ν values of v, but these give in eights the same value of v^8, so that the number of values of v^8 is $=\nu$.

74. I consider the case $n=3$: here, writing x, y for u, v, we have here the sextic curve

I. $y^4-x^4+2xy(x^2y^2-1)=0$;

and it is easy to see that the remaining forms wherein x, y denote u^2, v^2; u^4, v^4; and u^8, v^8 respectively, are derived herefrom as follows; viz.

II. $(y^2-x^2)^2-4xy(xy-1)^2=0$, that is

$$y^4+6x^2y^2+x^4-4xy(x^2y^2+1)=0;$$

III. $(y^3+6xy+x^2)^3-16xy(xy+1)^3=0$, that is

$$y^4+6x^2y^3+x^4-4xy(4x^3y^3-3x^3-3y^3+4)=0;$$

IV. $(y^3+6xy+x^2)^3-16xy(4xy-3x-3y+4)^3=0$, that is

$$y^4-762x^3y^3+x^4-4xy\{64x^3y^3-96x^3y-96xy^3+33x^3+33y^3-96x-96y+64\}=0,$$

where it may be noticed that the process is not again repeatable so as to obtain a sextic equation between x, y standing for u^{16}, v^{16} respectively.

The curve I. has a dp (fleflecnode) at the origin, viz. the branches are given by $y^3-2x=0$, $-x^3-2y=0$; and it has 2 cusps at infinity, on the axes $x=0$, $y=0$ respectively; viz. the infinite branches are given by $y+2x^3=0$, $-x+2y^3=0$ respectively. These same singularities present themselves in the other curves.

The curve II. has the four dps $(x^3-y^3=0,\ xy-1=0)$, that is

$$(x=y=1),\ (x=y=-1),\ (x=i,\ y=-i),\ (x=-i,\ y=i).$$

Corresponding hereto we have in the curve III. the 2 dps $x=y=1$, $x=y=-1$, and in the curve IV. the dp $x=y=1$.

The curve III. has besides the 4 dps $y^3+6xy+x^3=0$, $xy+1=0$, that is

$$(1+\sqrt{2}, 1-\sqrt{2}),\ (1-\sqrt{2}, 1+\sqrt{2}),\ (-1-\sqrt{2}, -1+\sqrt{2}),\ (-1+\sqrt{2}, -1-\sqrt{2});$$

and corresponding hereto in the curve IV. we have the 2 dps

$$(3+2\sqrt{2}, 3-2\sqrt{2}),\ (3-2\sqrt{2}, 3+2\sqrt{2}).$$

The curve IV. has besides the 4 dps $y^3+6xy+x^3=0$, $4xy-3x-3y+4=0$, or say $(2x-\frac{3}{2})(2y-\frac{3}{2})+\frac{7}{4}=0$, $2(x+\frac{3}{4})^3+2(y+\frac{3}{4})^3-\frac{117}{16}=0$. Hence the 4 curves have respectively the dps and deficiency following:—

	dps.		dps.	Def.
2, 1	=		3	7
2, 1, 4	=		7	3
2, 1, 2, 4	=		9	1
2, 1, 1, 2, 4	=		10	0;

viz. the curve IV. representing the equation between u^8 and v^8 is a unicursal sextic.

It may be noticed that except the fleflecnode at the origin, and the cusps at infinity, the dps in question are all acnodes (conjugate points).

75. The foregoing equations may be exhibited in the square diagrams:—

I.

	y^4	y^3	y^2	y	1
x^4					−1
x^3		+2			
x^2					
x				−2	
1	+1				

1 2 0 −2 −1
$=(y+1)^2(y-1)$

II.

	y^4	y^3	y^2	y	1
					+1
		−4			
			+6		
				−4	
	+1				

1 −4 +6 −4 +1
$=(y-1)^4$

III.

	y^4	y^3	y^2	y	1
					+1
		−16		+12	
			+6		
		+12		−16	
	+1				

1 −4 +6 −4 +1
$=(y-1)^4$

IV.

	y^4	y^3	y^2	y	1
					+1
		−256	+384	−132	
		+384	−762	+384	
		−132	+384	−256	
	+1				

1 −4 +6 −4 +1
$=(y-1)^4$

where the subscript line, showing in each case what the equation becomes on writing therein $x=1$, serves as a verification of the numerical values.

The curve IV. being unicursal, the coordinates may be expressed rationally in terms of a parameter; and we in fact have

$$x=\frac{\alpha^3(2+\alpha)}{1+2\alpha},\quad y=\frac{\alpha(2+\alpha)^3}{(1+2\alpha)^3}.$$

These values give

$$\begin{aligned}
16xy &= 16\alpha^4(2+\alpha)^4 && \div(1+2\alpha)^4,\\
4+4xy-3x-3y &= (4,\ 8,\ 12,\ 32,\ 50,\ 32,\ 12,\ 8,\ 4)(1,\ \alpha)^8 && \div(1+2\alpha)^4,\\
x^3+6xy+y^3 &= 4\alpha^3(2+\alpha)^3(4,\ 8,\ 12,\ 32,\ 50,\ 32,\ 12,\ 8,\ 4)(1,\ \alpha)^8 && \div(1+2\alpha)^6,
\end{aligned}$$

and the equation of the curve is thus verified.

76. Considering in like manner the modular equation for the quintic transformation, we derive the four forms as follows:—

I. $x^6y^6+5x^3y^3(x^3-y^3)+4xy(1-x^4y^4)=0.$

II. $\{x^3-y^3+5xy(x-y)\}^3-16xy(1-x^3y^3)^3=0$, that is
$x^9+15x^4y^3+15x^3y^4+y^9-2xy(8-5x^4+10x^3y^3-5y^4+8x^4y^4)=0.$

III. $(x^3+15x^2y+15xy^2+y^3)^3-4xy(8-5x^3+10xy-5y^3+8x^3y^3)^3=0$, that is
$x^9+655x^4y^3+655x^3y^4+y^9-640x^3y^3-640x^4y^4$
$+xy(-256+320x^3+320y^3-70x^4-660x^3y^3-70y^4+320x^4y^3+320x^3y^4-256x^4y^4)=0.$

IV. $(x^3+655x^3y+655xy^3+y^3-640xy-640x^3y^3)^3$
$-xy(-256+320x+320y-70x^3-660xy-70y^3+320x^3y+320xy^3-256x^3y^3)^3=0$:
or expanding the two terms separately, this is

x^5y		− 65536	=0
x^4y		+163840	
x^3y		+163840	
x^2y		−138240	
x^2y^2	+ 409600	−542720	
xy^2		−138240	
x^2y	− 1280	+ 44800	
x^2y^2	− 838400	+631040	
x^2y^3	− 838400	+631040	
xy^4	− 1280	+ 44800	
x^2	+ 1		
x^2y	+ 1310	− 4900	
x^2y^2	+ 430335	−297200	
x^2y^3	+1677252	−986072	
x^2y^4	+ 430335	−297200	
xy^5	+ 1310	− 4900	
y^6	1		
x^2y^2	− 1280	+ 44800	
x^2y^3	− 838400	+631040	
x^2y^4	− 838400	+631040	
x^2y^5	− 1280	+ 44800	
x^2y^3		−138240	
x^2y^4	+ 409600	−542720	
x^2y^5		−138240	
x^2y^4		+163840	
x^2y^5		+163840	
x^2y^5		− 65536	

77. The square diagrams are:—

I.

	y^6	y^5	y^4	y^3	y^2	y	1
x^6							−1
x^5		+4					
x^4					−5		
x^3							
x^2			+5				
x^1						−4	
1	+1						

1 +4 +5 0 −5 −4 −1

$=(y+1)^5(y-1)$.

II.

y^6	y^5	y^4	y^3	y^2	y	1
						+1
	−16				+10	
				+15		
			−20			
		+15				
	+10				−16	
+1						

1 − 6 +15 −20 +15 − 6 +1

$=(y-1)^6$.

III.

	y^6	y^5	y^4	y^3	y^2	y	1
x^6							+1
x^5		−256		+320		− 70	
x^4			−640		+655		
x^3		+320		−660		+320	
x^2			+655		−640		
x		− 70		+320		−256	
1	+1						
	+1	− 6	+ 15	− 20	+ 15	− 6	+1

$=(y-1)^6$.

IV.

	y^6	y^5	y^4	y^3	y^2	y	1
x^6							+1
x^5		− 65536	+163840	−138240	+ 43520	− 3590	
x^4		+163840	−133120	−207360	+133135	+ 43520	
x^3		−138240	−207360	+691180	−207360	−138240	
x^2		+ 43520	+133135	−207360	−133120	+163840	
x		− 3590	+ 43520	−138240	+163840	− 65536	
1	+1						
	+1	− 6	+ 15	− 20	+ 15	− 6	+1

$=(y-1)^6$.

where the subscript line, showing in each case what the equation becomes on writing therein $x=1$, serves as a verification of the numerical values.

78. The curve I. has at the origin a dp in the nature of a flecnode, viz. the two branches are given by $x^3+4y=0$, $-y^3+4x=0$ respectively; and there are two singular points at infinity on the two axes respectively, viz. the infinite branches are given by $-y-4x^5=0$, $x-4y^5=0$ respectively. Writing the first of these in the form $-yz^4-4x^5=0$, we see that the point at infinity on the axis $x=0$ (*i. e.* the point $z=0$, $x=0$) is $=6$ dps; and similarly writing for the other branch $xz^4-4y^5=0$, the point at infinity on the axis $y=0$ (*i. e.* the point $z=0$, $y=0$) is $=6$ dps*.

Moreover, as remarked to me by Professor H. J. S. Smith, the curve has 8 other dps;

* These results follow from the general formulæ in the paper "On the Higher Singularities of Plane Curves," C. & D. M. J. t. vii. (1865) pp. 212–222; but they are at once seen to be true from the consideration that the curve $yz^4-x^5=0$, which has only the singularity in question, is unicursal; the singularity is thus $=6$ dps.

viz. writing ω to denote an eighth root of -1, $(\omega^8+1=0)$, then a dp is $x=\omega$, $y=\omega^5$. To verify this observe that these values give

$6x^5$	$=\overbrace{+6}^{\omega^5}$	$-6y^5$	$=\overbrace{+6}^{\omega}$
$+20x^3y^3$	-20	$+10x^4y$	-10
$-10xy^4$	-10	$-20x^3y^3$	-20
$+4y$	$+4$	$+4x$	$+4$
$-20x^4y^5$	$+20$	$-20x^3y^4$	$+20$

or the derived functions each vanish. Thus I. has in all $1+12+8$, $=21$ dps.

In II. we have in like manner $1+12+4$, $=17$ dps; viz. instead of the 8 dps, we have the 4 dps $x=\omega^3$, $y=\omega^3$, $(\omega^8+1=0)$, or, what is the same thing, $x=\omega$, $y=-\omega$, where $\omega^4+1=0$. But we have besides the 12 dps given by

$$x^3-y^3+5xy(x-y)=0,\ 1-x^3y^3=0,$$

viz. we have in all $1+12+4+12$, $=29$ dps.

In III. we thence have $1+12+2+6$, $=21$ dps; and, besides, the 12 dps given by

$$x^3+15x^2y+15xy^2+y^3=0,\ 8-5x^3+10xy-5y^3+8x^3y^3=0,$$

in all $1+12+2+6+12$, $=33$ dps.

And in IV. we thence have $1+12+1+3+6$, $=23$ dps; and, besides, the 12 dps given by

$$x^3+655x^2y+655xy^2+y^3-640xy-640x^3y^3=0,$$

$$-256+320x+320y-70x^3-660xy-70y^3+320x^2y+320xy^2-256x^3y^3=0$$

(these curves intersect in 16 points, 4 of them at infinity, in pairs on the lines $x=0$, $y=0$ respectively; and the intersections at infinity being excluded, there remain $16-4$, $=12$ intersections); there are thus in all $1+12+1+3+6+12$, $=35$ dps.

Or arranging the results in a tabular form and adding the values of the deficiency, we have

	dps.	dps.	Def.
I.	$1+12+8$	$=21$,	$=15$,
II.	$1+12+4+12$	29,	7,
III.	$1+12+2+6+12$	33,	3,
IV.	$1+12+1+3+6+12$	35,	1,

so that the curve IV. is a curve of deficiency 1, or bicursal curve. It appears by JACOBI's investigation for the quintic transformation (Fund. Nov. pp. 26–28) that we can in fact express x, y, that is u^5, v^5, rationally in terms of the parameters α, β connected by the equation

$$\alpha^3=2\beta(1+\alpha+\beta),$$

which is that of a general cubic (deficiency $=1$); we in fact have

$$\frac{2-\alpha}{\alpha-2\beta}=\frac{v^4}{u^4},\ \beta=\frac{u^5}{v},$$

that is,

$$u^8(=x)=\beta^3\left(\frac{2-\alpha}{\alpha-2\beta}\right),\ v^8(=y)=\beta^3\left(\frac{2-\alpha}{\alpha-2\beta}\right)^8,$$

where α, β satisfy the relation just referred to. The actual verification of the equation IV. by means of these values would be a work of some labour.

79. In the general case p an odd prime, then in I. we have at the origin a dp (in the nature of a flecnode) and at infinity 2 singular points each $=\frac{(p-1)(p-2)}{2}$ dps. I infer, from a result obtained by Professor SMITH, that there are besides $(p-1)(p-3)$ dps; but I have not investigated the nature of these. And the Table of dps and deficiency then is

		dps.	Def.
I.	$1+(p-1)(p-2)+\ (p-1)(p-3)$	$2p^2-7p+6$,	$4p-5$
II.	$1+(p-1)(p-2)+\frac{1}{2}(p-1)(p-3)+\frac{1}{2}(p^2-1)$	$2p^2-5p+4$,	$2p-3$
III.	$1+(p-1)(p-2)+\frac{1}{3}(p-1)(p-3)+\frac{1}{3}(p^2-1)+\frac{1}{3}(p^2-1)$	$2p^2-4p+3$,	$p-2$
IV.	$1+(p-1)(p-2)+\frac{1}{4}(p-1)(p-3)+\frac{1}{4}(p^2-1)+\frac{1}{4}(p^2-1)+\frac{1}{4}(p^2-1)$	$2p^2-\frac{7}{4}p+\frac{3}{2}$,	$\frac{1}{4}p-\frac{3}{4}$;

viz. his values of the deficiencies being as in the last column, the total number of dps must be as in the last but one column.

XII. *Studies on Biogenesis.* By William Roberts, *M.D., Manchester.* *Communicated by* Henry E. Roscoe, *F.R.S.*

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Introduction.

The question of the origin of *Bacteria* and *Torulæ* lies so deeply at the root of some of the most important problems, not only of biology, but of pathology and practical therapeutics, that I make no apology for bringing forward the fruits of another investigation on the subject.

The main question in controversy is whether these organisms originate *de novo* in the media where they grow, or whether they spring, like higher beings, from germs or parents like themselves.

On the one hand it is contended that there exist in ordinary air and water (in addition to their proper elements) multitudes of germinal particles, and that the quasi-spontaneous production of *Bacteria* and *Torulæ* in organic media is in reality due to infection by these particles. On the other hand the existence of these supposed germs is doubted or denied; and it is affirmed that these organisms can and do arise where infection by preexisting germs is impossible.

It is important to bear in mind at the outset that these two theories (Panspermism and Abiogenesis) are not necessarily wholly destructive of each other. It is conceivable that, while the ordinary and common origin of *Bacteria* and *Torulæ* is by procession from preexisting germs, there may also be conditions in which they arise *de novo*. At any rate it appears very desirable to establish the fundamental propositions of the panspermic theory, as broadly expressed in the preceding paragraph, on independent grounds, and without prejudice to the question of abiogenesis.

The point of view here indicated is adhered to throughout this paper. It resembles the attitude assumed by pathologists in regard to contagious diseases. No pathologist doubts, for example, the contagiousness of small-pox, nor that the ordinary production and spread of the disease is due to infection. And this belief is not inconsistent with the notion, very commonly held, that in some previous age small-pox did arise *de novo*; nor would it now be shaken, nor the practical deductions therefrom set aside, if it were proved that under certain rare etiological combinations small-pox might still arise *de novo*.

The inquiry is divided into three sections. The first section is devoted to the examination of the conditions under which organic liquids and mixtures are rendered barren by heat. In the second section is investigated the question whether the normal

juices and tissues of animals and plants are capable of producing organisms without infection by extraneous germs. In the third section the facts adduced in the two previous sections are considered in their bearing on the origin of *Bacteria* and *Torulæ*, and some of the alleged cases of abiogenesis are tested experimentally.

The experiments were all contrived on a plan which favoured not only the birth but the continuous growth of any organisms which made their appearance. The materials were both maintained at a suitable temperature and furnished with a free supply of air, so that the changes initiated might have an opportunity of going on until their nature became undoubted.

In judging of the absence or presence of organisms, the microscope was, of course, the principal test. The magnifying-power generally employed was 500 diameters, controlled sometimes by a magnifying-power of 1200 diameters. In addition to this, however, note was always taken of the naked-eye appearances, of the presence or absence of turbidity, of a film on the surface, and of a deposit at the bottom, as well as of the reaction and odour. Signs of growth and multiplication were regarded as the only indefeasible evidence of living organisms.

The term *Bacteria* is used in the comprehensive sense adopted by Cohn to include the various organisms described as vibrios, micrococci, microzymes, and schistomycetes. The terms *Torulæ* and *fungoid vegetations* are used to designate organisms belonging to the type of the yeast-plant and the *Penicilium glaucum*. The word *germ* is used simply in the general sense of a particle endowed with the power of provoking germination in a suitable medium.

Section I.—On the Sterilization by Heat of Organic Liquids and Mixtures.

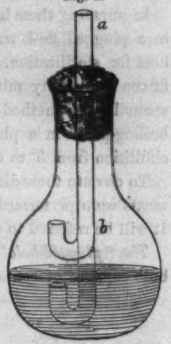
When beef-tea or a decoction of turnip is boiled for a few minutes in a flask, of which the neck is plugged with cotton-wool, the liquid passes into a state of permanent sterility. It can be kept for months and even years exposed to the most favourable conditions of warmth and light, with a constantly renewed supply of air; but so long as the cotton-wool plug remains undisturbed, neither *Bacteria* nor *Torulæ*, nor any other organisms, make their appearance in it. The liquid has not, however, lost its fitness to nourish and promote the growth of these organisms; for if the cotton-wool plug be withdrawn so as to admit unfiltered air into the flask, or if a drop of ordinary water be introduced, *Bacteria* or *Torulæ* speedily make their appearance, and grow and multiply with the utmost luxuriance.

This experiment, which will be referred to as the "plugged-flask" experiment, may be instructively varied in the following manner, which is a simplification of Pasteur's bent-tube experiment. A glass-tube (*a b*, fig. 1), 4 inches long, is bent at one end into a U-shape. The longer limb of the tube is then tightly wrapped round with cotton-wool so as to form a plug. This plug is inserted into the neck of a flask * half

* The flasks used in this and the "plugged-flask" experiments were the ordinary four-ounce flasks used by chemists.

filled with beef-tea or a decoction of turnip, as represented in fig. 1. A plug of cotton-wool is also inserted into the upper end of the glass tube at *a*. The flask is then boiled over the flame for five minutes. When the flask is quite cold, the plug at *a* is gently withdrawn. The liquid in a flask thus prepared remains permanently barren. In a week or two the condensed steam collected at the bend of the tube dries up, and the tube becomes an open channel into the flask. If the flask be examined at the end of two or three months, the contained liquid will be found still perfectly transparent. The experiment may then be carried a step further. If the little tube (*a b*) be pressed downward through the cotton-wool plug until the U-shaped portion is completely submerged, as represented by the dotted lines in the figure, the liquid in the flask is brought into contact with the particles of air-dust collected in the bend of the tube. The result of this contact is speedily seen; for in a few days patches of mould appear on the surface, or the liquid becomes turbid from *Bacteria* *.

Fig. 1.



Bent-tube experiment simplified.

These experiments admit of an easy explanation on the panspermic theory. The living germs and organisms contained within the flasks are killed by the heat during ebullition; and the fresh supplies of air which enter the flasks on cooling are deprived of their germs—in the first case by filtration through the cotton-wool plug; in the second case they are arrested in the bend of the tube *a b*, the shorter limb of which they are unable to ascend against the force of gravity; and thus the necrosis at first effected by the heat is succeeded by a state of permanent sterility through want of living germs to start the process of germination.

The condition of "permanent sterility" here described is essentially characterized by loss of the power of *originating* organisms with conservation of the power of *nourishing and promoting the growth* of organisms.

The degree of heat required to induce this state of permanent sterility varies greatly according to the nature of the materials operated upon.

Pasteur long ago pointed out that milk required more heat to sterilize it than sweetened yeast-water; more recently Dr. Bastian and others have shown that turnip-infusion with cheese and some other mixtures cannot be sterilized, as an ordinary decoction can, by boiling for five or ten minutes; and experiments to be presently

* The barrenness in these experiments is not so absolutely permanent as in the simple plugged-flask experiment, as may be seen from the following observation:—On June 19th, 1872, I put up a flask of beef-tea in the manner above described, but in the process of boiling some of the beef-tea frothed over into the bend of the tube. On March 27th, 1873, the liquid in the flask was still quite unaltered, but I could see minute specks of mould creeping up the short limb of the tube *a b*, and about to drop into the liquid below. A few days after specks of mould began to appear on the surface of the liquid in the flask; these speedily grew until they covered the entire surface.

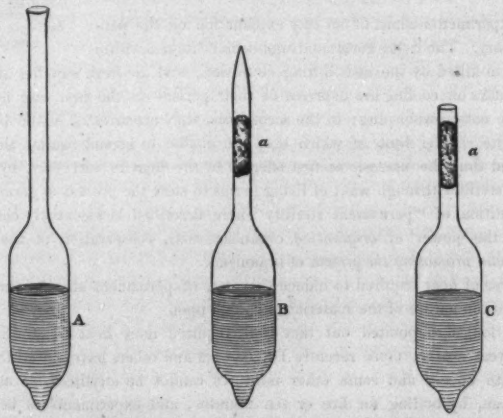
described prove that there are liquids which germinate after exposure to the heat of boiling water for even two and three hours, or after exposure to a heat of 9° Cent. above that of boiling water for more than half an hour.

In pursuing these inquiries it was found that the method of direct boiling over a flame in a plugged flask was unsuited to liquids which required a prolonged application of heat for sterilization. In the first place, the evaporation that ensued on boiling for fifteen or twenty minutes seriously altered the concentration of the materials; and, secondly, this method did not permit an accurate gradation of temperature; for the boiling-point in a plugged flask was observed to rise above the normal temperature of ebullition from 3° to 6° Cent. according to the tightness of the plugging.

To obviate these disadvantages another plan was adopted; and, as most of my experiments were performed by this method, it is necessary to describe it more particularly. It will be referred to as the "plugged-bulb" method.

The "plugged-bulb" Method.—An ordinary delivery-pipette, having on it an oblong bulb capable of containing 30 to 50 cub. centims., was sealed hermetically at one end

Fig. 2. The plugged-bulb experiment.



A. The bulb charged. B. The bulb charged, plugged, and sealed ready for heating. C. The bulb with its neck filed off, and set aside to see if it will germinate. The figures are drawn about half the actual size.

(see fig. 2, A). The materials of the experiment were then introduced into the bulb until it was two thirds full. The inside of the neck of the bulb was next wiped dry, and a plug of cotton-wool (*a*) was inserted about its middle. Lastly, the neck was drawn out above the plug and sealed in the flame, as represented in fig. 2, B.

When the bulb was thus charged and sealed, it was weighted with a leaden collar and submerged in the semi-upright position (so as to prevent the wetting of the cotton-wool plug) in a can full of water. The can was next placed over a source of heat and boiled for the required time. The bulb was then withdrawn; and, when quite cold, its neck was filed off above the cotton-wool plug as represented in fig. 2, C. Finally, it was set aside in the upright position, and maintained at a suitable temperature.

When it was desired to test the effects of a higher temperature than that of boiling water, the can was filled with brine or oil instead of water.

By this method the materials of the experiment could be exposed for any desired time to a desired heat without evaporation, and without the disturbing effects of ebullition; and by subsequently filing off the neck of the bulb above the cotton-wool plug, free access of filtered air was provided, so that the conditions most favourable to germination were maintained for an indefinite time*.

In the last four years I have performed several hundred experiments by the "plugged-flask" and the "plugged-bulb" methods, on a great variety of organic liquids and mixtures. The flasks and bulbs, after heating, were either placed on a marble slab covering a warm-water cistern, which stands in a corner of my room, or they were kept in a warm greenhouse. In the former case their temperature ranged from 15° to 27° Cent., and in the latter from 15° to 32° Cent.

The following summary exhibits the general results of the experiments. The values here given must not, however, be regarded as absolute, but rather as comparative and approximative values, which hold good only for the precise quantities, materials, and methods of experimenting adopted. The experiments are thrown into three groups according to the comparative facility of sterilization exhibited by the various liquids and mixtures employed.

Group I. *Substances sterilized by five or ten minutes' boiling in a plugged flask.*—The easiest substances to sterilize were infusions of animal or vegetable tissues raised to the boiling-point for a few seconds and then filtered. These were, strictly speaking, decoctions rather than infusions; they were sterilized, if quite fresh, by three or four minutes' boiling. In the same category stood solutions of organic salts—citrates, acetates, and tartrates, and healthy and diabetic urine.

Infusions made slowly at blood-heat, and not raised to the boiling-point, generally (but not always) required to be boiled for five or ten minutes. Many infusions so made let fall a sediment on boiling, and the non-removal of this by filtration appeared to increase their resistance to sterilization. It appeared also that the longer the infusions were kept before boiling, the greater, as a rule, proved their resistance to sterilization.

* My experience does not agree with that of Dr. Bastian, that liquids sealed in ebullition are more favourably circumstanced for germination than the same liquids under the ordinary pressure of the atmosphere. In repeating his experiments I found that the liquids which germinated after being sealed in ebullition, also germinated (and much more abundantly) when air was admitted to them through cotton-wool.

This was attributed to the gradual diminution of the acidity of the infusions when they were long kept, and to the commencing multiplication of organisms in them.

The following were the infusions actually experimented on and found conformable to the above rules:—infusions of beef, mutton, pork, codfish, mussel, carrot, turnip, hay, malt, pear, apple, cucumber, cabbage, lettuce, tomatoes, vegetable marrow, and parsnep.

It was found that pieces of carrot, turnip, apple, cucumber, &c. floating in water were nearly as easily sterilized as filtered infusions of these substances. But chopped *green* vegetables floating in water (such as kidney-beans, asparagus, cabbage and lettuce, and green peas) were very difficult to sterilize by boiling in a plugged flask. Such mixtures could be boiled for fifteen or twenty minutes or longer, and yet they almost invariably germinated a few days after. Chopped boiled white of egg floating in water also behaved in the same way. The singular resistance of green vegetables to sterilization appeared to be due to some peculiarity of surface, perhaps their smooth glistening epidermis, which prevented complete wetting of their surfaces; for I found that if they were previously thoroughly crushed in a mortar they were sterilized much more easily. The difficulty in the case of egg-albumen was presumably due to the alkaline reaction of that substance.

Group II. *Substances sterilized by exposure for not less than twenty to forty minutes to the heat of boiling water in a plugged bulb.*—To this group belonged:—mixtures of chopped green vegetables with water (with or without alkali), pieces of flesh-meat, or fish, or boiled egg with water, blood, dropsical fluids, milk, albuminous urine, and turnip-infusion with cheese. Experiments performed in this fashion often yielded beautiful preparations. Owing to the absence of the disintegrating turmoil of ebullition the pieces of vegetable or flesh retained their original appearance in the bulbs, and the supernatant water preserved its transparency. In the case of egg-albumen and dropsical fluids it was found better, instead of mixing the materials at first with water, to proceed as follows:—About two drachms of egg-albumen or dropsical fluid were conveyed to the bottom of the bulb by means of a long-necked funnel. The lower part of the bulb was then immersed in boiling water until the albumen coagulated into a solid cake, then water was introduced and the bulb plugged, sealed, and boiled in the usual way. When the experiment was thus performed, the supernatant water remained brilliantly limpid and colourless.

Group III. *Substances sterilized by exposure for not less than one or two hours to the heat of boiling water in a plugged bulb.*—Only one member of this group was encountered, namely, superneutralized hay-infusion. Of all the substances examined by me this proved to be the most difficult to sterilize. As a large number of experiments were made with hay-infusion, and as different specimens of hay differ a good deal from each other, a bundle of good meadow-hay was secured, and all the infusions were made with this hay, so as to insure as great a uniformity as possible in the materials of the experiment. The infusion was always made in the same way. The hay was soaked in a mini-

mum of water for four hours at blood-heat. The infusion was then filtered and diluted with ordinary water until it had a specific gravity of 1006. Prepared in this way, hay-infusion had the colour of sherry; it was slightly acid in reaction, and it boiled without forming any sediment. When neutralized with ammonia or liquor potassæ, it threw down a copious sediment which increased on boiling; this sediment subsided completely on standing, and left a perfectly transparent supernatant liquor. This perfect transparency permitted the detection of the slightest turbidity from the development of *Bacteria*.

Unneutralized hay-infusion, when quite fresh, was easily sterilized by five minutes' boiling in a plugged flask; but when it was slightly alkalized by ammonia or liquor potassæ, its resistance to sterilization by heat was increased to a most marvellous degree. It could no longer be sterilized even by fifteen or twenty minutes' boiling in a plugged flask, which almost reduced the infusion to dryness by evaporation. By means of the plugged-bulb method, however, alkalized hay-infusion could be sterilized without difficulty; but it required more than an hour's exposure to the heat of boiling water to effect this.

It was found that the maximum resistance to sterilization resided in infusions alkalized with about five drops of liquor potassæ per ounce (about one per cent.); two or three drops less than this, or four or five drops more than this, considerably diminished this resistance.

The following experiments will serve to illustrate the high resisting power of alkalized hay-infusion to sterilization by heat. In every case the necks of the bulbs were filed off above the plug after the experiment, and the bulbs were afterwards placed in a warm place.

1. *March* 27, 1873.—Seven plugged bulbs, charged with alkalized hay-infusion, were boiled in a can of water for fifty minutes. On March 31st, four days after, all were turbid and covered with a thick film of *Leptothrix*-filaments.

2. *March* 31, 1873.—Three plugged bulbs, charged with alkalized hay-infusion, were boiled in a can of water for two hours. On April 3rd all three were turbid, and covered with an abundant film.

3. *April* 4, 1873.—Five plugged bulbs (*a*, *b*, *c*, *d*, and *e*) were charged with hay-infusion alkalized with 0·7 per cent. of liquor potassæ. They were boiled in a can of water for various lengths of time: *a* was boiled for an hour and became turbid in four days; *b* was boiled for two hours, and *d* and *e* for seven hours; these three remained permanently barren. An accident happened to *c* just before it was put into the can; it was therefore re-charged with the same infusion, but *alkalized a little more strongly*: this one was boiled for three hours, nevertheless it became turbid and covered with a film in a few days. This was the most extreme example met with of resistance to sterilization with the heat of boiling water.

4. *July* 15, 1873.—Five plugged-bulbs, charged with hay-infusion alkalized with 1·4 per cent. of liquor potassæ, were boiled together in a can of water. One of the bulbs was taken out at the end of every hour. The first bulb, withdrawn after being

boiled an hour, became turbid from *Bacteria* in three days; the other four, boiled for periods varying from two to five hours, remained permanently sterile.

5. *July* 15, 1873.—Six plugged bulbs, charged with hay-infusion alkalized with 1·4 per cent. of liquor potassæ, were boiled together in a can of saturated brine for various periods from fifteen to ninety minutes. One of the bulbs was withdrawn from the boiling brine every quarter of an hour. The boiling-point of the brine was 109° Cent. The first bulb, withdrawn at the end of the first quarter of an hour, became fertile, all the rest remained permanently barren.

6. *July* 30, 1873.—Nine plugged bulbs, divided into sets of three each, were charged with hay-infusion alkalized to three different degrees. The first set was alkalized with one per cent., the second with two per cent., and the third with three per cent. of liquor potassæ. These were all boiled together in a can of saturated brine. One bulb of each set was withdrawn from the boiling brine at the end of fifteen minutes, one of each set at the end of twenty-five minutes, and the remainder at the end of thirty-five minutes. The bulbs of the first set (those which were alkalized with one per cent.) all proved fertile in three days; the other six remained permanently barren.

This last experiment showed that hay-infusion alkalized with one per cent. of liquor potassæ possessed an immensely greater power of resistance to sterilization by heat than infusions alkalized with two and three per cent.

7. *July* 14, 1873.—Six plugged bulbs, charged with hay-infusion alkalized with 1·4 per cent. of liquor potassæ, were slowly heated in a can of oil up to a temperature of 125° Cent., and maintained at that temperature for periods varying from five to thirty minutes. All of these remained permanently barren.

In all the above-described experiments of the three groups, the sterilized liquids and mixtures, whether they were sterilized by a few minutes' boiling in a plugged flask, or by one or more hours exposure in a plugged bulb to the heat of boiling water, or by exposure to the still higher temperature of boiling brine or heated oil, immediately, and without exception, recovered their fertility when exposed to the contamination of unfiltered air or of ordinary water. Their susceptibility in this respect did not differ whether the preparations had been kept for months or even years, or whether they had only been kept a few days.

The following were the general conclusions drawn from the experiments:—

1. Organic liquids and mixtures are capable of being permanently sterilized by the heat of boiling water. Some are sterilized by an exposure to this heat for a few minutes, others require an exposure of twenty to forty minutes, and others an exposure of one, two, or several hours. A slight difference in the aggregation of the materials or a slight difference in their reaction was sufficient to alter very greatly the amount of heat required for their sterilization.

2. There appeared to be two factors of equal importance in the process of sterilization, namely, the *degree* of heat and the *duration* of its application. These two factors were mutually compensatory, in such fashion that a longer exposure to a lower tempe-

rature was equivalent to a shorter exposure to a higher temperature. For example, speaking roughly, an exposure for an hour to a heat of 100° Cent. was equivalent to an exposure for fifteen minutes to a heat of 109° Cent.

3. Slightly alkaline liquids were always more difficult to sterilize than slightly acid liquids. This was probably due to the fact that albuminoid matter is more easily and more completely coagulated by a given heat in acid than in alkaline solutions.

4. No organic liquid or mixture, subjected for however short a time to the heat of boiling water, ever produced (provided there was no fresh infection) any fungoid or torulaceous organisms*. If germination took place, the organisms produced invariably belonged to the great group of *Bacteria*. This appears to indicate that *Torulæ* and their germs are more easily destroyed by heat than *Bacteria* and their germs. No development of *Bacteria* into fungoid vegetations was ever observed.

5. Sterilization generally proved to be permanent if germination did not occur within four days. Sometimes, however, germination took place on the sixth or eighth day, and very rarely even as late as the tenth and twelfth day. Differences in the degree of warmth at which the preparations were maintained (after the heating) accounted for most of these cases of retarded germination, but not for all. Some I explained in this way:—the flasks were often moved about and disturbed, and possibly their contents were thus brought into contact with unsterilized portions which had spurted about the sides of the flask during ebullition. Neither of these explanations, however, appeared applicable to some of the cases of retarded germination. There was certainly no constant relation between retarded germination and the amount of heat used in the experiment. As a rule, germination, if it took place at all, was as prompt in preparations which had been boiled for an hour as in those which had only been boiled for five minutes. Something further will be said of these cases of retarded germination at the end of the third section.

Section II.—ON THE CAPACITY OF THE JUICES AND TISSUES OF ANIMALS AND PLANTS TO GENERATE *BACTERIA* AND *TORULÆ* WITHOUT EXTRANEOUS INFECTION.

It cannot be doubted that if the juices and tissues of healthy plants and animals could be placed, without extraneous contamination, in circumstances favourable to germination, their behaviour under such circumstances would furnish important data in the controversy respecting the origin of *Bacteria* and *Torulæ*. If, for example, it was found that the blood, flesh, milk, and urine of an animal, the contents of an egg, and the juices and tissues of a plant remained permanently barren, although adequately supplied with moisture, air, warmth, and light—so long as extraneous infection was prevented—such a result would furnish the strongest argument hitherto adduced in favour of the view that the appearance of *Bacteria* and *Torulæ* in organic infusions was

* Mention will be made subsequently of exceptions to this rule, or rather apparent exceptions, for they evidently belong to a totally different category.

due to the agency of imported germs, and not to any act of so-called spontaneous generation.

The importance of this path of inquiry was distinctly indicated by PASTEUR; but he does not appear to have pursued it, except in regard to urine, which he succeeded in keeping unaltered without heating*.

More recently Dr. BURDON SANDERSON†, in his admirable studies on contagion, incidentally encountered this question, and arrived at some very remarkable results. He satisfied himself that blood, muscle, urine, saliva, milk, egg-albumen, healthy pus, and blister-serum had no power of breeding *Bacteria* except when infected with ordinary water. Mr. LISTER tested this question more directly, and succeeded in showing that blood, milk, and urine remained permanently barren when preserved from the contact of extraneous germs.

Dr. SANDERSON'S experiments were planned on the supposition entertained by him that *Bacteria*-germs, while abundant in water, were almost absent from the air. In repeating some of his experiments, I failed in obtaining similar results. The air of the rooms wherein I worked was evidently highly charged with *Bacteria*-germs. In pursuing the inquiry, therefore, I sought to avoid air-contamination as sedulously as water-contamination‡.

The experiments which follow were all carried out on a plan which was, in principle, the same in every case. The materials of the experiment were enclosed in sterilized glass bulbs or tubes, which were plugged at one end with cotton-wool and hermetically sealed at the other. They were then set aside in a warm place to see if they would germinate.

The bulbs and tubes were prepared in the following manner:—They were first drawn out at the lower ends into capillary points (fig. 3, *b*, *b*) and sealed in the flame; the upper ends were plugged at *a*, *a* with cotton-wool.

* Annales de Chimie et de Physique, 1862, p. 66.

† Thirteenth Report of the Medical Officer of the Privy Council, p. 65.

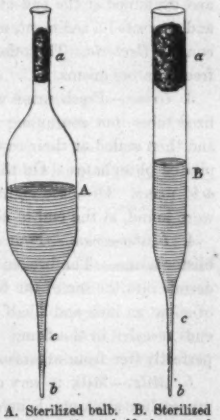
‡ The avoidance of air-contamination is important for another reason. The air is admitted, by most observers, to be highly charged with fungoid germs, and the growth of fungi has appeared to me to be antagonistic to that of *Bacteria*, and *vice versâ*. I have repeatedly observed that liquids in which the *Penicillium glaucum* was growing luxuriantly could with difficulty be artificially infected with *Bacteria*; it seemed, in fact, as if this fungus played the part of the plants in an aquarium, and held in check the growth of *Bacteria*, with their attendant putrefactive changes. On the other hand, the *Penicillium glaucum* seldom grows vigorously, if it grow at all, in liquids which are full of *Bacteria*.

It has further seemed to me that there was an antagonism between the growth of certain races of *Bacteria* and certain other races of *Bacteria*.

On the panspermic theory it may be assumed that what takes place when an organic liquid is exposed to the contamination of air or water is this:—A considerable variety of germinal particles are introduced into it, and it depends on a number of conditions (composition of the liquid, its reaction, precedence and abundance of the several germs) which of these shall grow and take a lead, and which shall partially or wholly lie dormant and unproductive. There is probably in such a case a struggle for existence and a survival of the fittest. And it would be hazardous to conclude because a particular organism was not found growing in a fertile infusion, that the germs of that organism were really absent from the contaminating media.

The next step was to destroy any germs adhering to the interior of the bulbs and tubes or floating in the air within them—in other words, to sterilize them. This was done in the case of the bulbs by introducing water into them before the plugging, and boiling over the flame for ten minutes. The tubes were sterilized by passing and repassing them through the flame of the spirit-lamp until they were quite hot, as shown by commencing charring of the cotton-wool plug. If the conditions of the experiment required that there should be some water in the tubes, their capillary ends were snipped off (after sterilization) and boiling water was sucked into them, and the ends again resealed. Bulbs and tubes thus prepared might be regarded as "sterilized chambers"—that is to say, chambers deprived of germinal particles by the destructive power of heat, and having free access of air, also deprived of germs by filtration through the cotton-wool plug. A number of such sterilized bulbs and tubes (some containing water and some empty) were prepared and laid aside for future use.

Fig. 3.



A. Sterilized bulb. B. Sterilized tube. About half the actual dimensions.

Experiments were made on the following substances:—

1. *Egg-albumen.*—Two sets of experiments were made with egg-albumen.

First set.—At the end of March, 1873, eight sterilized bulbs (fig. 3, A) containing water were charged with egg-albumen, in the following manner:—

A fresh egg was fixed in a convenient support, and a small piece of the shell was chipped off, care being taken to leave the subjacent membrane uninjured. Then a sterilized bulb was taken and the capillary portion (*b c*) immersed for a few seconds in boiling water in order to destroy any adhering germs. The sealed end was then rapidly snipped off, and the capillary portion plunged into the interior of the egg. About two cubic centimetres of the albumen were then sucked up by the mouth into the bulb. When this was accomplished, the bulb was quickly withdrawn, and its capillary end sealed in the flame.

At first the albumen formed a distinct layer below the water in the bulbs; but in a few days the soluble portions diffused into the water above, and formed a transparent colourless solution, while the scanty insoluble portions separated in a hazy cloud or flakes which occupied the lower strata of the fluid.

These eight bulbs were kept through the ensuing summer and autumn, and were finally examined on the 1st of October. Six of them were quite unaltered in appearance, and no trace of any organism could be detected under the microscope. The other two were evidently changed. They had become turbid a few days after being put up, and had become increasingly so for a few weeks, after which they underwent no further

change. Under the microscope spherical *Bacteria* were seen abundantly, and a few cells resembling *Torulæ*. There were, however, no staff-shaped *Bacteria* and none of the usual signs of putrefaction.

Second set.—On the 17th of December, 1873, seven sterilized tubes containing water (fig. 3, B) were charged with egg-albumen in the manner above described. Of these, two became turbid with *Bacteria*; the other five continued transparent, and when examined at the end of two months were found quite free from organisms.

2. *Blood.*—The end of the finger was thoroughly cleansed, and blood drawn therefrom by needle-punctures. The blood was sucked into sterilized tubes containing water, and then sealed in the flame with the precautions above described. About two drops of blood were conveyed into each tube. In these experiments there was obviously considerable risk both of air- and of water-contamination. Ten tubes were put up in this way, and examined at the end of four weeks. Four of them had assumed a bright red colour and deposited a sediment, and were more alkaline than the others. These were found to contain *Bacteria*. The other six had an amber tint and no sediment; these were quite free from orgarisms.

3. *Urine.*—Fresh urine was received into a large superheated test-tube. Six sterilized tubes, not containing any water, were charged with the urine in the usual way, and then sealed at their capillary ends. The urine was alkaline and turbid from precipitated phosphates. On the following day two more tubes were similarly charged with acid urine. Of these eight tubes one became turbid from *Bacteria*, the other seven were found, at the end of eight weeks, perfectly unaltered and free from organisms.

4. *Blister-serum.*—Four sterilized tubes, not containing any water, were charged with blister-serum. The broken capillary ends were thrust directly through the raised epidermis into the sac of the blister, and the serum was sucked into the tubes to the depth of about an inch and a half. The tubes were then quickly withdrawn, and their capillary ends resealed in the flame. At the end of ten weeks all four were found unaltered and perfectly free from organisms.

5. *Milk.*—Milk is very difficult to obtain free from extraneous contamination. In my earlier attempts I used superheated test-tubes plugged with cotton-wool. Test-tubes so prepared were taken into the cowhouse at milking-time, and charged with milk by momentary removal and reinsertion of the plugs. Milk so obtained, however, invariably curdled in a week and swarmed with *Bacteria*.

I afterwards adopted the following plan with somewhat better success:—A glass tube (*a b*, fig. 4) was drawn out at each end to a narrow orifice. The lower portion of this was tightly wrapped round with cotton-wool and inserted as a plug into a large test-tube (*c*) containing water to the depth of about an inch. A cap of cotton-wool was also tied over the narrow orifice at *a*. The water in the test-tube was then briskly boiled, and the boiling was continued almost to dryness. When the apparatus was cold I took it into the cowhouse and, seizing a teat, I pulled off quickly the cotton-wool cap

at *a*, and pushed the narrow point (*a*) into the duct of the teat. Holding it firmly in this position, I milked into the test-tube until sufficient milk had been obtained. I then drew the test-tube away from the little tube, pressing in the cotton-wool around it as I did so, until the latter was entirely withdrawn from the test-tube. Operating in this way I trusted to obtain milk, if not entirely free from extraneous germs, at least containing but few of them.

Fig. 4.

About half the actual size.

From a test-tube thus filled I charged ten empty sterilized tubes in the manner already described, and resealed their capillary orifices. Of these ten, three remained unchanged. When examined from three to six weeks afterwards the milk in them was perfectly sweet to the taste, its reaction was neutral or faintly acid, like that of fresh milk, there was no curdling, and no signs of organisms under the microscope. The other seven changed within ten or twelve days. Some of them curdled and others putrefied; all became highly acid, and, under the microscope, *Bacteria*, either staff-shaped or spherical, were found in them.

6. *Grape-juice.*—Eleven sterilized tubes, six empty and five containing water, were charged with grape-juice in the following manner:—A fresh grape was firmly seized with the fingers and thumb, and a spot on its surface was pressed for a few seconds against the flame of a spirit-lamp so as to destroy any adhering germs. The point of the sterilized tube, also heated for a moment in the flame, and quickly snipped off by an assistant, was then thrust into the grape at the heated spot. Compression was now made on the grape until a sufficient quantity of the turbid juice was forced into the tube. The tube was then withdrawn and its point sealed in the flame. The eleven tubes thus charged remained permanently unchanged. When examined at various periods, from five to eight weeks, they were all found free from organisms, and the taste and reaction of their contents were indistinguishable from those of the fresh grape-juice.

7. *Orange-juice.*—Eight sterilized tubes, four empty and four containing water, were charged with orange-juice. The oranges were fresh from the trees*. A portion of the rind was stripped off, and the capillary end of one of the tubes, momentarily heated in the flame, was snipped off and thrust into the orange. The orange was then compressed until a sufficiency of the juice was squeezed into the tube, which was then withdrawn and sealed in the flame. The tubes thus charged were examined at various intervals, from four to eight weeks; all were found perfectly free from organisms, and preserved unaltered the reaction and taste of fresh orange-juice.

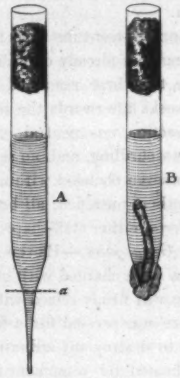
8. *Tomato-juice.*—Three sterilized tubes, containing water, were charged with the

* This and all the other experiments in this section, except the first set with egg-albumen, were performed at San Remo during November, December, and January, 1873-74.

juice of half-ripe tomatoes. The proceeding followed was the same as with oranges. At the end of two months the supernatant liquor in the tubes was perfectly transparent, and no appearance of organisms was found under the microscope.

9. *Turnip.*—I found it much less easy to deal with the solid tissue of the turnip and potato than with the liquid pulp of the grape and orange; but the results obtained, though less uniform, were not less conclusive. The following was the plan adopted:—A sterilized tube (fig. 5, A) containing water was nicked with a file near the base of the capillary part at *a*, where the tube had a diameter of about two millimetres. A fresh oblong turnip was then fractured across, and the tube, snapped off at the nicked point, was quickly thrust into the substance of the turnip. A narrow cylinder of turnip-tissue, about an inch long, was thus forced into the column of water in the tube. The tube was then detached, and its end sealed with melted sealing-wax (fig. 5, B).

Fig. 5.



Fourteen tubes were charged in this way, and examined from time to time in the course of the ensuing two months. Ten remained sterile and four became fertile. The water in the former continued perfectly limpid, and no organisms could be discovered in it when examined microscopically. The water in the latter became turbid, and was found to swarm with *Bacteria*.

10. *Potato.*—Seven sterilized tubes, containing water, were charged with potato-tissue in the way above described for turnip. Of these, four remained barren, and three became fertile with *Bacteria*.

Conclusions.—As already stated, the ideal conditions of the experiments could not in any case be carried out with absolute stringency. Some risk of extraneous infection was always encountered in conveying the materials of the experiments into the sterilized bulbs and tubes. The results obtained are therefore not altogether uniform; but as this was in accordance with the expectation of the experiments, it adds to, rather than detracts from, their validity. Where the conditions of the experiments could be carried out in almost ideal perfection, as with egg-albumen, urine, blister-serum, grape- and orange-juice, the results were nearly uniform; but when, on the contrary, the risk of extraneous infection was obviously considerable, as with blood, milk, turnip, and potato, the results were less uniform, though even in these cases, with the single exception of milk, the sterile tubes were in a majority.

The following Table exhibits a summary of the results:—

Liquid or tissue experimented on.	Number of experiments.	Results.	
		Remained sterile.	Became fertile.
Egg-albumen....	15	11	4
Blood.....	10	6	4
Urine.....	8	7	1
Blister-serum ..	4	4	0
Milk	10	3	7
Grape-juice	11	11	0
Orange-juice....	8	8	0
Tomato-juice....	3	3	0
Turnip-tissue....	14	10	4
Potato-tissue....	7	4	3
Totals.	90	67	23

Thus out of 90 experiments on the juices and tissues of plants and animals, exposed to conditions favourable to germination, absolute sterility was observed 67 times, and development of organisms only 23 times. It was scarcely possible to obtain a more clear demonstration of the general conclusion that the normal tissues and juices have no inherent power to originate organisms, and that when organisms appear therein their development is due to germs imported from without.

SECTION III.—ON THE BEARING OF THE FACTS ADDUCED IN THE PRECEDING SECTIONS ON THE ORIGIN OF *BACTERIA* AND *TORULÆ*, AND ON THE REAL EXPLANATION OF SOME OF THE ALLEGED CASES OF ABIOGENESIS.

We have seen that organic liquids and mixtures sterilized by heat, and the normal fluids and tissues of plants and animals, remain permanently barren, under the most favourable conditions of air, moisture, warmth and light, so long as they are protected from extraneous infection; but if unfiltered air or ordinary water be brought into contact with them, this barrenness is immediately and invariably succeeded by fertility.

Such a sequence of events can only be explained by the supposition that there exist in ordinary air and water, in addition to their proper elements, incredible multitudes of particles capable of provoking germination.

The exact nature of these germinal particles is, however, not a matter of actual knowledge. It is evident that they are organic, because of their easy destructibility by heat. It is further evident that the atmospheric germs are solid particles, for they can be mechanically filtered from the air, and they are incapable of ascending against gravity. It may be assumed that they consist partly of true spores and partly of the organisms themselves, floating amid the dust of the atmosphere or mingled with the molecular matter always present in ordinary water. But it cannot be said that they

have ever been actually seen and identified. The ingenious attempts of PASTEUR and others to demonstrate germs in the air are manifestly illusory. Like them I have repeatedly collected air-dust and found abundance of molecules, circles, spheres, and particles of various kinds under the microscope; but these could not be identified as true spores, nor distinguished from particles of inert dust. Indeed the objects sought after are so minute and so wanting in characteristic forms, that such a search, with our present instruments, appears well-nigh hopeless*.

But although an obscurity hangs over the precise nature of these particles, the reality of their existence is not doubtful; nor is it doubtful that the ordinary development of *Bacteria* and *Torulæ* is directly due to their agency. These fundamental propositions of the panspermic theory may be regarded as the expression of acquired facts in science.

I come now to deal with those cases which have hitherto proved a stumbling-block to the general acceptance of the panspermic theory—namely, the development of *Bacteria* (without the possibility of fresh infection) in liquids and mixtures which have been subjected to a boiling heat.

It is truly, as Dr. HUGHES BENNETT has remarked, a "violent assumption" that living particles can survive a boiling heat. Nevertheless it is an assumption that may be justified both on grounds already supplied in the preceding sections and by direct experiment. The *à priori* argument stands thus:—Take milk. It has been shown that pure milk, drawn without extraneous contamination from the teat, has no germinating power. The germination of ordinary milk is therefore due to imported germs; and if the same milk still germinate after boiling in closed vessels, this must be due to the survival of these germs. Or take the case of egg-albumen. Uncontaminated egg-albumen does not germinate when mixed with sterilized water. But if pieces of boiled white of egg be mixed with ordinary water, the mixture invariably germinates after boiling (in a plugged flask) for ten or fifteen minutes. This can only be explained by supposing that the germs contained in the ordinary water have survived the boiling.

But this point was tested by direct experiments with hay-infusion. Two series of experiments were made—A and B.

A. In the first section it has been shown that alkalized hay-infusion will germinate after exposure for more than an hour to the heat of boiling water, but if the heat be continued for two or more hours permanent sterilization is effected. In the course of my experiments a considerable number of bulbs containing alkalized hay-infusion, sterilized in this way, had accumulated on my hands. These bulbs were plugged with

* A striking illustration of the minuteness of the spores of certain organisms is given in a remarkable paper on the life-history of a cercomonad by DALLINGER and DRYSDALE in the Microscopical Journal for August 1873. The spores of this organism were so minute that they could not be individually recognized even with the $\frac{1}{25}$ objective. They could only be recognized with this high power as seen in enormous aggregation and motion in a mass. With a magnifying-power of 2500 diameters they were only just visible as the minutest dots.

cotton-wool, and had been kept in a warm place for several months. The liquids in them were perfectly transparent, and showed no signs of germination. The contents of these bulbs seemed especially favourable for testing the possibility of germs surviving a boiling heat. These bulbs, sixteen in number, were divided into three sets.

The first set (six bulbs) were treated thus:—The plugs were withdrawn and the bulbs were exposed in the vertical position for six hours to the contact of unfiltered air; they were then replugged and boiled briskly over the flame for five minutes. Of these, three germinated within four days, and three remained permanently barren.

The second set (six bulbs) were also unplugged, and a little ordinary water (from one to sixty drops) was introduced into the bulbs; they were then replugged and boiled for five minutes. Four of these germinated and two remained barren.

The third set, consisting of four bulbs, were unplugged, and a few drops of an alkalized hay-infusion turbid from *Bacteria* were added to each; these were then replugged and boiled for five minutes. All of these germinated.

Lastly, the five barren bulbs of the first and second sets, after remaining barren for a month, were unplugged again and infected with alkalized hay-infusion turbid with *Bacteria*. They were then replugged and boiled for five minutes. All germinated in four days.

These experiments prove directly and positively that there exist in ordinary air and water particles which, in certain liquids, are capable of preserving their germinal activity after exposure to a boiling heat for five minutes. They also prove that certain types of *Bacteria*, or their germs, are capable of surviving a similar heat.

If these experiments are regarded in conjunction with those published by Dr. Bastian* and F. Cohn†, it would seem that the vital resistance to heat of *Bacteria* and their germs varies greatly according to the nature of the liquid in which they subsist, and probably also according to the species or type of *Bacteria*. It does not seem unreasonable to suppose that different races of *Bacteria*, or different phases of their development, are capable of offering very different degrees of resistance to the destructive influences of heat‡.

B. We have seen that unneutralized hay-infusion is sterilized by exposure to a boiling heat for five minutes; but if the infusion be slightly alkalized by ammonia or potash, it germinates after exposure to the same heat for more than an hour. How does the

* Proc. Roy. Soc., March 20, 1873 (vol. xxi. p. 224).

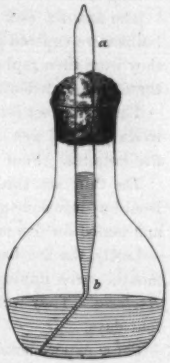
† Beiträge zur Biologie d. Pflanzen, 2^tes^ Heft, p. 217.

‡ I may refer again on this point to the paper of Dallinger and Drysdale already cited. These observers found that while adult cercomonads were killed at a temperature of 60° C. their spores survived a heat of 127° C.

T. Pace (Maandblad v. Natuur-Wettenschappen, May 1873) found that *Bacteria* in turnip-infusion with cheese retained their normal appearances and movements after being heated in closed tubes up to 160° C.; but as their subsequent power of growing and multiplying was not tested, these experiments must be regarded as inconclusive. The same objection lies against the experiments of the late Dr. Crace-Calvert (Proc. Roy. Soc. 1871, vol. xix. p. 468).

alkali produce this extraordinary effect? It must be in one of two ways: either it increases the vital resistance to heat of the germs contained in the infusion, or it increases the abiogenic aptitude of the infusion itself. Which of these explanations is the true one? The question was tested in the following manner:—

Fig. 6.



Ten flasks were charged with unneutralized hay-infusion. Five of these were simply plugged with cotton-wool, and boiled over the flame for five minutes. The other five were also plugged with cotton-wool; but through the centre of each plug there passed a hermetically sealed glass tube, of the shape represented in the annexed figure (fig. 6, *a b*), and containing the proper quantity of liquor potassæ to neutralize the infusion in the flask. These tubes had been previously (after being charged with liquor potassæ and sealed) heated in oil up to 121° Cent., in order to destroy any germs they might contain. The flasks thus prepared were then also boiled over the flame for five minutes.

At the end of a fortnight the contents of the ten flasks remained unchanged; they were evidently permanently sterilized. I next proceeded to alkalize the infusions in the flasks without introducing any fresh germs, that is, without removing the plugs.

The flasks of the first set were placed under a bell-jar in company with a beaker containing liquor ammoniæ. In the course of two hours the volatile alkali had diffused through the plugs and had neutralized the infusions, as was evidenced by the formation of abundant deposits in them. These flasks were then removed and set aside in a warm place to see if they would germinate.

The flasks of the second set were each treated as follows:—The little tube *a b* was pressed down against the bottom of the flask until it broke at the bend *b*, then the flame of the blowpipe was cautiously directed against the upper end, *a*. The expansion of the contained air thus induced was sufficient to expel the liquor potassæ into the liquid below; immediate turbidity ensued. The flasks were then placed beside their companions of the first set.

Not one of these ten flasks germinated; at the end of two months they were still barren. Now this result was strictly conformable to the view that the effect of the alkali was to increase the power of survivance of the germs, but it was wholly unconformable to the alternative view. According to the former view, the germs contained in the infusions were destroyed by the preliminary boiling, and no subsequent addition of alkali could, of course, restore their vitality. But on the opposing theory there was no reason why the alkali should not have been equally effective in promoting germination, whether added before or after the short preliminary boiling*.

* In fact this was so. It will be remembered that in preparing the hay-infusions they were first made very

By carrying the experiments a step further it was shown that, although the contents of the flasks had not acquired the power to germinate, they had acquired the property of enabling freshly introduced germs to survive a boiling heat; for when the flasks were unplugged and infected with ordinary air or water and then replugged and boiled five minutes, their contents in every instance germinated in a few days.

These experiments appear to warrant the following conclusions:—

1. That the germinal particles of air and water (or some of them) are capable of surviving the heat of boiling water in certain media.

2. That when we speak of different liquids and mixtures as possessing different degrees of resistance to sterilization by heat, it would be more exact to say that the germinal particles of air and water possess varying degrees of vital resistance to heat according to the nature of the media in which they subsist.

The issue of the foregoing inquiry has been to confirm in the fullest manner the main propositions of the panspermic theory, and to establish the conclusion that *Bacteria* and *Torulæ*, when they do not proceed from visible parents like themselves, originate from invisible germs floating in the surrounding aërial and aqueous media.

Nevertheless I cannot withstand the impression that this general and common mode of origin may, under rare conditions, be supplemented by another and an abiogenic mode of origin.

The facts on which this impression rests are only isolated and quasi-exceptional units among countervailing hundreds; but the record of this inquiry would be incomplete without an account of them.

The facts alluded to consist, first, of cases of retarded germination of *Bacteria*; and, secondly, of two cases in which fungoid vegetation, of a type new to me, appeared in plugged bulbs after being heated in boiling water.

In studying the sterilization of organic liquids by heat, I found, as a rule (as already stated), that if germination did not take place within four days it did not take place at all; that is, the media proved to be permanently sterilized. But occasional exceptions to this rule were encountered, in which the media remained sterile for six, eight, and, very rarely, for ten and twelve days, and then germinated. The most remarkable of these exceptions were, however, the two following:—

1. *November* 21, 1871.—Some dried marrow-fat peas were soaked in water for thirty hours, until they had fully recovered their original plumpness. About a dozen of these were enclosed, with water, in a plugged flask and boiled over the flame for ten minutes. A second flask was charged and boiled in the same way, except that the

strong, and afterwards diluted down with water to the standard density of 1006. But on several occasions I was obliged to boil the infusions before diluting and alkalizing, in order to keep them sweet until my engagements permitted me to go on with the further steps of the experiments. In these instances the resistance to sterilization was just as great as when there was no preliminary boiling.

peas were first finely crushed in a mortar. Both these flasks remained unchanged for a period of seven months; but in the beginning of June 1872 the contents of the first flask became turbid and covered with a film, and, when examined under the microscope, revealed abundance of short staff-shaped *Bacteria*. The reaction, originally neutral, was now faintly alkaline. The second flask remained apparently unchanged for sixteen months; and when the contents were examined at the end of that time no trace of organisms could be found under the microscope, and the reaction was neutral.

2. *May* 4, 1872.—Two plugged flasks, charged with very thin gruel, were gently boiled over the flame for fifteen minutes. These flasks remained apparently unaltered for six weeks, and then one of them began to exhibit rapid changes. Its contents became more diffluent and yellower and covered with a pellicle. On July the 13th this flask was opened for examination. Its contents were faintly acid (originally neutral), and had a smell of sour dough (all the starch had disappeared), and abundance of spherical *Bacteria* were seen with the microscope. The other flask, also opened for examination on the 13th of July, was quite unaltered. Its contents were neutral, odourless, reacted strongly with iodine, and contained no trace of living organisms.

Results of this kind, standing alone, would not have produced much impression on my mind, because, although exceptional in some respects, they were of the same order as those witnessed in defective sterilization, the organisms discovered being of the same kind as those ordinarily produced by extraneous infection. They might therefore have been regarded simply as faulty experiments*.

But the two following results very strongly arrested my attention. Not only did organisms appear under exceptional conditions, but they were of a species distinctly different from the organisms with which I was so familiar as the result of extraneous infection.

1. *July* 17, 1872.—A plugged bulb, charged with a highly albuminous urine, was boiled in a can of water for twenty minutes. When cold the neck of the bulb was filed off above the cotton-wool, and it was set aside in a warm place. An abundant precipitation of albumen had occurred, and the supernatant fluid was perfectly transparent.

This remained unaltered until the beginning of March 1873, a period of nearly eight months. At this time I noticed the appearance of numerous whity-brown specks, like small pins' heads, scattered on the sides of the glass at and near the surface of the fluid. To the naked eye they looked like specks of solidified fat, but with the aid of a lens they were seen to have a radiated structure. These specks steadily, though slowly, grew and multiplied, until at the end of six weeks some of them were as large as hempseeds. The supernatant fluid still continued perfectly transparent. On the 28th of April the bulb was opened for examination. The urine was, as at first, acid. Under the

* It seemed possible that in some of these cases minute insects might have crept into the flasks or bulbs through the cotton-wool plugs, and thus communicated infection to their contents.

microscope the little masses were found to consist of an interlacement of fibres, exactly resembling the mycelium of the *Penicilium glaucum* or of the sugar-fungus, except that the fibres had only about half the diameter of the former. In structure the fibres more resembled those of the sugar-fungus than those of the *Penicilium glaucum*, for they had globular enlargements on them here and there. Amid these fibres were innumerable very minute spores, much smaller than the *Torulæ* usually found in urine. I could not clearly make out any aërial fructification. No *Bacteria* were found.

Several other specimens of albuminous urine treated in the same way were found wholly barren at the end of eighteen months.

2. *February* 25, 1873.—A plugged bulb, containing pieces of Swedish turnip, floating in water alkalized with 0·2 per cent. of sod. bicarb., was boiled in a can of water for thirty minutes. When cold the neck was filed off above the cotton-wool, and the bulb was placed in a warm greenhouse. On the 7th of March the supernatant liquid had become turbid, but on the 21st of March it had again become perfectly transparent, and an immense woolly growth had rapidly sprung from the surface of the liquid and filled up the entire upper part of the bulb, even extending into its neck. Under the microscope this woolly mass was found to consist of an interlacement of fibres exactly resembling the mycelium of the *Penicilium glaucum*. Amid the fibres were numerous minute dots like very young *Torulæ*. There was no proper aërial fructification. No *Bacteria* were found.

In both these instances the organisms evidently belonged to the class of fungoid vegetations represented by the *Penicilium glaucum* and the sugar-fungus. But their mode of growth was so entirely different, that it was equally evident that they were not identical with either of those species, and, further, that they were totally different from each other. These were the only two instances, out of many hundred experiments, in which I witnessed the development of a fungoid vegetation in a liquid which had been exposed, for however short a time, to the heat of boiling water.

The facts just enumerated are far too few, and of too exceptional a character, to permit a deduction in favour of abiogenesis; but they certainly impose a reserve which is highly significant. If future investigations should establish the occurrence of abiogenesis, this would not overturn the panspermic theory, it would only limit its universality; and it may be predicted with some confidence that if abiogenesis exist the conditions of its occurrence can only be determined by an inquirer who is fully alive to the truth and penetrating consequences of the panspermic theory.

XIII. The Bakerian Lecture.—*Researches in Spectrum-Analysis in connexion with the Spectrum of the Sun.*—No. III. *By* J. Norman Lockyer, *F.R.S.*

Received November 20,—Read November 27, 1873.

Introduction.

The Researches in Spectrum-Analysis in which I have been engaged are opening out into so many lines of work that I think it desirable to communicate to the Royal Society the present state of the inquiry on its most general aspect, and also to enter somewhat into detail on some of the points to which my attention has specially been directed, the more so as the methods employed are such as can be, and I sincerely trust will be, taken up by other workers.

To commence, then, by a general statement, I may remark that I have in the first place endeavoured to determine whether the new method of spectroscopic observation, which I have before described to the Royal Society, is really as competent as it promised to be in the quantitative direction, what are the conditions essential to its successful employment, and how far it would take us.

As the time I could devote to this inquiry was limited, I confined myself to mixtures of two metals. The results of the experiments which I have to report in the present paper, and in a separate one which will be communicated by Mr. W. C. Roberts, Chemist of the Mint, and myself, are in more ways than one satisfactory, the history of science having shown that a great increase in theoretical work may be hoped for as soon as an instrument such as the spectroscope gets into practical use in the arts.

It may be said that while the qualitative spectrum-analysis depends upon the *positions* of the lines, the quantitative analysis depends not upon their position but upon their *length*, *brightness*, *thickness*, and *number*, as compared with the number visible in the spectrum of a pure vapour.

The character of a spectrum when observed by the new method is so entirely different from that which results from following the integrating method, that maps of the various spectra, in which not only the relative lengths of the various lines, but their individuality shall be faithfully recorded once for all, are an absolute necessity before much further progress can be made in the research; while, from what has been stated in my former paper, it is clear that for this purpose volatilization of the vapours of the metals in the electric arc is necessary, as by this means the densest vapour can readily be procured. I have therefore rendered the vapours I have investigated incandescent by a current from a battery of 30 pint Grove's cells.

The difficulties of eye-observations were found to be very great, not only in identifying

the lines, but in retaining their characteristics in the memory until a record of length, thickness, and brilliancy could be placed on paper. Further, I have determined that when it is a question of mapping the various characteristics of the lines which the new method reveals in the spectrum of the vapour of a metal in the electric arc, and not the position only, on an average ten lines an hour is the progress made. At this rate the work I have proposed to execute would be interminable. I have therefore attempted to bring in the aid of photography.

But there was another reason. I have long had cause to think that many, if not all, of the coincidences which ÅNGSTRÖM, THALÉN, and others have chronicled—coincidences which made the allocation of the FRAUNHOFER lines so difficult and, worse still, doubtful—had no real physical existence, but resulted from impurities; and it seemed to me that this question could be at once settled by confronting photographs of the spectra of those vapours in which the coincidences have been recorded; because if the coincidences were due to impurities, the lines would be longest and thickest in the spectrum to which they really belonged.

The portion of the spectrum on which the attempt has at present been made is that from H to F. I may add that I have grounds for supposing that the least refrangible part of the spectrum thus left unrecorded is, in the case of the metallic elements, the least important one to study, and that I am making arrangements for photographing the part of the spectrum more refrangible than H.

I have been fortunate enough not only to devise a very convenient method of thus confronting spectra, but of applying it under almost the best possible conditions, since, by the kindness of Dr. FRANKLAND, I have been permitted the use of a room in his laboratory, where I have been enabled to photograph the spectra of the vapours of the solar metals, confronting these spectra with the solar spectrum and the spectra of other allied metals on the same plate.

As this branch of the research has necessitated the use of a battery of 30 or 40 cells, it would have been impossible to carry it on in my dwelling-house, where all my previous work has been conducted.

The construction of a Table of all the named FRAUNHOFER lines, showing the length and thickness of the lines of the metallic vapours to the absorption of which they are probably due, has also been taken in hand. This Table enabled me, before the photographic work was commenced, to allocate upwards of 50 lines in the solar spectrum, which had, I presume, been overlooked by ÅNGSTRÖM and THALÉN.

It is, however, incomplete, inasmuch as I was not in a position to record the lengths of all the lines of iron and titanium with the induction-spark; and since the photographic work has been going on such a flood of light has been thrown on the true origin of so many of the unnamed FRAUNHOFER lines, that the Table in question may be said to be already useless for the part of the spectrum from H to F.

This Table was made as a preliminary to the construction, on a much larger scale than ÅNGSTRÖM'S, of a new photographic map of the part of the spectrum from H to F, in which

I proposed to myself to attempt to clear away all difficulties touching coincidences, and to give below it complete maps of all the solar elements with the long and short lines, showing at a glance, therefore, those which are and are not reversed at the present epoch. I am glad to say that the map is making good progress.

I have also commenced a preliminary search for other elements in the sun; and this has necessitated an inquiry into the absorption-spectra of the metalloids by means of photography, which is not yet complete.

Of the above-named branches of the research I propose on the present occasion to refer more particularly to the following:—

I. The experiments made on a possible quantitative spectrum-analysis.
II. The method of photographing spectra.
III. The coincidences of spectral lines.
IV. The preliminary inquiry into the existence of elements in the sun not previously traced.

I. THE EXPERIMENTS MADE ON A POSSIBLE QUANTITATIVE SPECTRUM-ANALYSIS.

Some time ago I gave an account of some experiments with some tin and magnesium amalgams, and I remarked* :—"It is possible to begin with an alloy which shall only give us the longest line or lines in the spectrum of the smallest constituent, and by increasing the quantity of this constituent the other lines can be introduced in the order of their length. This reaction is so delicate that I learnt from it a thing I had not before observed, that the least refrangible line of *b*, the triple line of magnesium, is really a little longer than its more refrangible companion; for the spectrum of magnesium was reduced to this one line in an alloy in which special precautions had been taken to introduce the minimum of magnesium.

"It follows from this statement that not only is the spectrum-analysis almost infinitely more delicate than it has hitherto been supposed to be in the case of the elements in which the difference between the longest and the shortest lines is least†, but that in time it may become quantitative." On a second occasion‡ I went further and laid before the Society for their inspection maps of the spectra of certain alloys, pointing out that the lines of any constituent of a mechanical mixture disappeared from the spectrum as its percentage was reduced.

After the second paper was sent in to the Royal Society I commenced a series of observations, the object of which was to study not only the disappearance of the lines, but other general changes which might supervene; and for this purpose I mounted a micrometer eyepiece on the observing-telescope of the spectroscope. This enabled me

* Philosophical Transactions, 1873, p. 261.

† "The great lengths of the lines of sodium, lithium, &c. at once account for the delicacy of their spectrum reactions."

‡ Philosophical Transactions, 1873, p. 655.

to notice the following additional phenomena, in which a change in the lines which remained was brought about by a change of composition.

I. The lines varied in their lengths as the percentage of the element to which they were due varied.

II. Some of the lines appreciably varied in their thickness and brightness, or both, in the same way.

III. In cases where the brightness of a line was estimated through a considerable range of percentage composition by comparison with an air-line, the latter was observed to grow faint and then to disappear when the brightness of the line compared with it increased.

IV. In cases where the brightness or thickness of the line of one element was estimated by comparison with the adjacent line of the other constituent of the alloy, the point of equal brightness was observed to ascend or descend (I used this method in order to avoid the uncertainty of micrometric measurements of the tips of the lines in consequence of their variation in length due to the unequal action of the spark).

V. In some cases where the percentage of a constituent was so small that none of its lines were visible, there yet seemed to be an effect produced upon the lines of the other constituent as compared with those of the spectrum of the same vapour of the opposite pole.

These conclusions were derived from observations of the alloys which I had made in the manner indicated in my second paper; and I saw that it would be important to observe series in which the change of percentage composition between the specimens was not so great, and of which actual assays had been made.

I therefore begged Mr. C. Freemantle, the Deputy Master of the Mint, to allow me the use of specimens of the gold-copper and silver-copper alloys prepared for the coinage, as in them I had exactly what the research required—namely, ranges with small variations and of undoubted accuracy. I have here to express my deep obligations to that gentleman, who at once, with the greatest promptitude and courtesy, acceded to a request made to him in the interests of science.

Before, however, I proceed to consider the cases of the Mint alloys, it will be well to briefly notice some experiments with alloys in which the variations in the proportions of constituents were greater.

As an instance of a moderately small difference, an attempt was made with a portion of a half-sovereign to which $\frac{1}{1000}$ of its weight of copper was added. This was compared with another portion of the same coin; but no difference was detected, possibly on account of a failure in making an alloy on so small a scale without some loss of the smaller and more oxidizable constituent.

An alloy of 5·5996 grms. lead and 0·6221 grm. silver was then made, the silver being dissolved in the lead, which was fused in a bent hard-glass tube in a current of hydrogen.

The percentage composition of this alloy was 89 per cent. lead and 11 per cent. silver. The silver lines at w. l. 5470, 5464, 5209 were recognized in its spectrum, and another line at 5401 was uncertain.

The opposite pole was composed of some supposed pure lead, and it was observed that its lines were longer than those of the lead in the alloy. A very faint line was observed in its spectrum opposite to the silver line 5290; but this was most probably the faint lead line 5206, as the longer silver lines did not exhibit themselves in its spectrum.

The opposite pole was now replaced by one of common sheet lead (it had at first been pure assayers' sheet), and the line in question was not stronger than in the pure lead; hence it was almost certainly the line 5206 of lead.

In order to see how much silver was required to render its spectrum visible, and as the common sheet lead was suspected to contain traces of silver, an alloy was made which contained 0·01 per cent. of silver. In this no silver lines were visible; and the same result appeared when ·05 per cent. silver was alloyed with the lead. The same was the case when 0·11 per cent. was added; and even the use of a large jar failed to cause the silver lines to appear.

Finally, an alloy containing 1·0 per cent. was made, and in this the silver lines appeared very distinctly, and three in number; 5464 was the longest, 5209 next, and 5470 the shortest.

Another alloy with 0·5 per cent. was now made, and in this the two lines of silver at w.l. 5464, 5209 were at last discovered, but they were very short and faint. Continued work showed that the line 5464 dies out between ·05 per cent. and 0·2 per cent. of silver (for it was at last discovered in the ·05 per cent. alloy), and also convinced me of the extreme irregularity of the various portions of the alloy, though they had been made with the greatest accuracy which the means at my command permitted.

Experiments were also made with alloys of tin and cadmium, the latter forming 0·154 per cent. of the alloy. The longest cadmium line at w.l. 5085 alone remained permanently visible, exceedingly faint, but unchanged in length; it had a short bright stump. When the line at w.l. 4799 was observed it appeared as a stump only, and neither 4677 nor any other cadmium lines could be found. Another alloy of tin with 1·0 per cent. cadmium showed 5085 distinct and bright, as was 4799; 4677 was likewise distinctly visible, and the tip of the least refrangible winged line 5377 commenced to appear. The other winged cadmium line, 5338, could not be identified, as it is nearly coincident with a tin line. After this, with the increased percentages of 5 per cent. and 10 per cent. of cadmium, the effect was mainly to lengthen and brighten the longest lines.

In order to determine whether these results would be affected by the presence of other metals in the alloy, one was made of 89·11 per cent. lead, 9·90 per cent. zinc, ·09 per cent. cadmium, and 0·85 per cent. tin; and the cadmium line showed no appreciable difference from its appearance in an alloy containing 0·1 per cent. cadmium with 99·9 per cent. of a single other metal.

In the Mint specimens with their small variations I found the same phenomena *en*

petit which I was already familiar with *en grand*, and the smaller variations in the phenomena enabled me to understand them better.

I found that in the gold-copper standards an increase of a 1000th part in the gold brought the lines down, while a similar increase in the copper carried them up, *i. e.* increased the height of the vapour from the pole.

I found, on the other hand, that in the silver-copper standards an increase of a 1000th part in the silver carried the lines up, while a similar increase in the copper brought them down in the field of view, *i. e.* reduced the height of the vapour from the pole.

After registering these facts, I saw at once that all the phenomena might be explained by assuming a change of volatility; by assuming, in fact, that alloys differing a 1000th part are different physical things, and that the spark acts upon the alloy as a whole as well as upon each vapour separately.

Thus in the cases referred to, in which copper is common to both, we find the melting-points to be as follows:—

Gold . . . 1200° (Pouillet).
Copper between 1200° and 1000°, precise point not determined.
Silver . . . 1000° (Pouillet).

And the intermediate position which copper occupies at once explains the different actions on its lines brought about by the addition of gold and of silver.

II. THE METHOD OF PHOTOGRAPHING SPECTRA.

The camera employed in the attempts already made is one for the use of which I am indebted to Lord Lindsay. It is furnished with a 3-inch lens by Dallmeyer, of about 23 inches focal length, and carries a plate 5×5 inches. The camera has replaced the observing-telescope of the spectroscope described in my former paper. Three prisms of 45° and one of 60° have been employed.

With these arrangements the whole spectrum from beyond H to the red falls upon a 5-inch plate. In order to show how abundantly qualified the lens used is for its work, I may state that the solar spectrum from w. l. 3900·0 to 4500·0 has been obtained at once in tolerably fair focus.

In order to obtain photographs of the solar spectrum of the very best kind, it is necessary to limit the beam passing through the prisms to very small dimensions—a method employed with such admirable results by Mr. Rutherfurd.

In the attempts to photograph the long and short lines of metallic spectra, it was found that this object could not be well obtained with the electric lamp in its usual position (with vertical poles), as the central column of dense vapour, as a rule, extended across the arc, *i. e.* from pole to pole, and gave all the short lines.

In order to obviate this I determined to make use of a horizontal arc. This was accomplished by placing the lamp on its side and firmly securing it in that position. The image of the horizontal arc was then thrown on the vertical slit in the usual

manner. This method was found to be perfectly successful. The central portion of the spectrum due to the dense core of the arc is found to consist of all the lines which this part of the arc alone can give, the longer lines given by the outer less dense strata extending beyond the spectrum of the core to the various distances from the centre to which the vapour capable of giving them extends. (See Plate XL.)

The lines thus photographed are pointed at either end, and disappear from the centre in the order of their length, so that a double or duplicate determination exquisitely symmetrical of their lengths is thus obtained.

Although the lengths, thicknesses, and intensities of the lines are thus readily recorded, we have so far no scale by which to fix their positions. In order to obviate this objection, I resolved to photograph the solar spectrum on each plate immediately above or below the metallic spectrum under examination. To accomplish this an extension of the ordinary method of working was introduced, depending upon the following considerations.

It is obvious that when we observe a spectrum its breadth will depend upon the length of the slit. If we could at the same time illuminate different portions of the slit with rays proceeding from different vapours, the spectra of the different light-sources could be seen at once. But we cannot do this. What we can do when we introduce photography is to illuminate *successively* different portions of the slit, the effect being that on different portions of the photographic plate the various spectra will successively record themselves.

Acting on this principle I first covered up the upper half of the slit, allowing the image of the horizontal arc to fall centrally on the slit, so that in this way I got impressed an image of half the thickness of the horizontal arc. After this was accomplished I covered up the half of the slit first used, opening that which was previously closed, and through this newly opened portion I admitted sunlight. But to do this effectively certain precautions were necessary. A description of the method used will show how these have been taken.

The laboratory in which the work has been carried out has two windows, one nearly due (magnetic) south, the other nearly (magnetic) west. Outside each window level slate slabs have been erected as supports for a heliostat. Either window can be used at pleasure. The spectroscope is supported on a platform on rollers, the height of the platform being such that the horizontal beam from the heliostat is coincident with the axis of the collimator. In addition to the lens placed between the lamp and the slit to throw an image of the arc on the latter, another lens is now introduced between the heliostat and the lamp—heliostat, lenses, lamp, and collimator being of course in the same straight line. The action of the newly interpolated lens is to throw an image of the sun between the poles of the lamp, so that when the spectrum of the arc is properly focused by the camera lens on to the photographic plate, the solar spectrum, when subsequently thrown in, is also in focus.

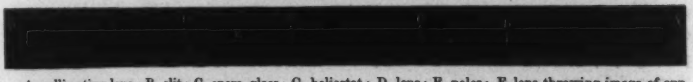
By these means we not only obtain a photographic record of the long and short lines with the individuality of each, but we get the solar spectrum as a scale.

The accompanying diagrams show the arrangements adopted in the cases mentioned.

Fig. 1.—Arrangement for obtaining solar spectrum alone.

Fig. 2.—Arrangement for obtaining long and short lines.

Fig. 3.—Arrangement for obtaining and comparing lines with solar spectrum.



A, collimating lens; B, slit; C, opera-glass; G, heliostat; D, lens; E, poles; F, lens throwing image of sun between poles.

In that branch of the research which deals with the causes of the coincidences of the lines in the various spectra, it is not absolutely essential that we should refer the lines to the solar spectrum each time; thus if we study the cases of aluminium and calcium impurities, it is better to photograph the suspected spectra side by side and confront them.

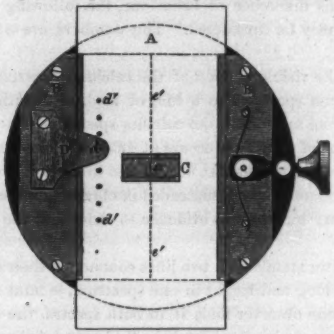
To do this all that is necessary is to extend the application of the principle already referred to. In fact the only practical limit to the number of spectra we can get on to one plate is the time the plate takes to dry, and instead of uncovering half of the slit at one time we may uncover any smaller portion. (See Plate XXXIX.)

This I have effected by means of a brass shutter with a square opening cut through it, which slides in grooves in front of and up and down the slit. On one side of the shutter holes are bored, the distance between each hole being the same as the height of the opening in the shutter. A short pin fixed to a spring falls into each hole in succession as the shutter is moved up or down, and so ensures that there shall be neither superposition of spectra nor gaps between them. The accompanying diagram (fig. 4) will make the arrangement quite clear.

I find that, by the use of the shutter, four or more spectra showing positions and thicknesses (but not lengths of the lines) can readily be obtained on one plate; and by

having the solar spectrum in the middle it serves as a scale (Plate XXXVIII.). Such spectra directly compared are shown in the accompanying Plates (XXXVIII.–XL.).

Fig. 4.—Slit plate with sliding shutter (actual size).



A, shutter; B B, grooves in which it slides; C, opening; D *d*, spring; *d'*, holes to regulate motion of shutter; *e*, slit seen through opening; *e'*, slit behind shutter.

III. ON THE COINCIDENCES OF SPECTRAL LINES.

In former references to this subject I have shown:—

1st. That the lines of metallic spectra do not all extend to the same distance from the pole which is emitting the light. In other words, some lines are short, others are long.

2nd. That in those cases where in the solar spectrum the whole number of lines given by a metal do not appear, those that do thus appear are the longest lines.

3rd. That it was possible to ascertain roughly by the number of lines left in a spectrum when a metal was alloyed, the percentage of it existing in the alloy.

4th. That greater tension of the incandescent vapour giving a spectrum would probably increase the number of lines in a spectrum.

The examination of the various spectra of metals and alloys undertaken in connexion with these researches afforded evidence not only of the great impurity of most of the metals used, but of the fact that many, if not all, of the coincidences observed by THALÉN and others might be explained in the light of former work.

An examination of ÅNGSTRÖM'S map of the solar spectrum shows many cases in which a line has been observed to be common to two or more spectra; and this is especially the case with the lines of iron, titanium, and calcium, nearly every other solar metallic spectrum exhibiting one or more cases of coincidence with the latter.

Now it is first to be remarked that these cases of coincidence can scarcely be accidental.

In those cases which have been examined, it has frequently been found that a line

coincident in different spectra is long and bright in only one of them, and that in others it is short, or faint, or both; or it may even, in certain specimens of the substances, be altogether absent from the spectrum.

As an instance of this difference of behaviour, the following cases in the spectra of calcium and strontium may be considered. The numbers are wave-lengths taken from THALÉN's list.

The longest line in the visible portion of the calcium spectrum, wave-length 4226·3, is found in the strontium spectrum as a line of medium length. 4607·5, one of the longest lines of strontium, appears in the calcium spectrum as a short line.

Another very long line of strontium occurs at 4215·3, in close proximity to the longest calcium line, and, according to THALÉN, occurs also in the spectrum of that metal.

I have, however, never, until lately, succeeded in obtaining any evidence of its presence in the calcium spectrum; but there is evidence to show that the metal I employed was very pure.

We have here, then, two metals with two lines common to their spectra; and it is found that the line which was long and bright in one spectrum, is faint in the other; and with regard to a third line, one observer finds it in both spectra, the other in one only, and after many attempts succeeds in observing it in the second, *but only in a specimen known to be contaminated with the first.*

The simplest explanation of the case, bearing in mind the facts already established in my first paper, is that the calcium used to produce the spectrum was contaminated to a certain extent with strontium, the strontium in turn containing calcium, a state of things which a moment's consideration will show to be not only possible but most probable, the close chemical relation of the two metals, and the extreme difficulty of making even an approximate separation when mixed, being well known.

Even if we knew nothing of the probability of mixture occurring in the cases of the two metals in question, the behaviour of the line at w. l. 4215·3 is sufficient to show what is the true cause of the coincidences.

The longest lines of calcium at wave-lengths 4226, 3968, and 3933 occur also in iron, cobalt, nickel, barium, &c., and assume very considerable proportions, equalling or surpassing in length many undoubted lines of those elements; on the other hand, the iron lines at wave-lengths 4071, 4063, and 4045 occur in calcium, strontium, and barium, and in other metals.

Again, the longest lines of aluminium at wave-lengths 3961 and 3943 occur usually in the spectrum of iron as longish lines, and are to be found in the spectra of cobalt, nickel, calcium, strontium, and barium, and in other metals, where they are even longer than some of the true lines of the metals in which they occur.

As a result of these considerations the following general statements may be hazarded, premising that it is possible that further inquiry may modify them.

1st. If the coincident lines of the metals are considered, those cases are rare in which the lines are of the first order of length in all the spectra to which they are common:

those cases are much more common in which they are long in one spectrum and shorter in the others.

2nd. As a rule, in the instances of those lines of iron, cobalt, nickel, chromium, and manganese which are coincident with lines of calcium, the calcium lines are long, while the lines as they appear in the spectra of the other metals are shorter than the longest lines of those metals. Hence we are justified in assuming that short lines of iron, cobalt, nickel, chromium, and manganese, coincident with long and strong lines of calcium, are really due to traces of the latter metal occurring in the former as an impurity.

3rd. In cases of coincidences of lines found between the lines of various spectra the line may be fairly assumed to belong to that one in which it is longest and brightest.

In order to show what a fair promise there is that all these questions will in time be set at rest by photography, and set at rest in the direction I have indicated, I beg permission to refer to one of the very earliest photographs taken by the method which has been described in a former part of this paper. It is a confronting of the spectra of the metals calcium and strontium.

A simple comparison of the two spectra shows that there are three strong and thick lines common to the two at w. l. 3968, 3943, the two H lines, and 4226·3, the large calcium line near G. The spectrum of strontium shows three lines which are very much thicker than the corresponding lines in the (calcium) spectrum; they are situated near wave-lengths 4029 and 4077 and at 4215·3. The latter line has been ascribed by THALÉN to calcium, and is coincident with a strong solar line. An inspection of the photograph, however, at once showed me that this line is really a strontium line, since it is thickest in the spectrum of that metal; so that this single plate was at once sufficient, in the light of these researches, to establish the presence of strontium in the reversing layer of the sun.

It is seen, then, that a comparison of a photograph of any spectrum with the photographs of the other spectra in which coincident lines occur, will be sufficient to show to which spectrum a disputed line belongs. It will be also noticed that the three calcium lines first mentioned are nearly as thick in the lower (strontium) spectrum as in that of calcium itself, while the difference between the thick lines of strontium and the corresponding lines also visible in the calcium spectrum is very great. All these facts are easily explained on the supposition that the calcium was very much less impregnated with strontium than the strontium with calcium. In fact I had such faith in the efficacy of the method, and in the opinion that coincidences are merely due to impurities, that I did not even consider it necessary to change the poles, but proceeded at once to place the strontium salt on that which had just before served for the ignition of the calcium. This at once accounts for the greater impurity of the former. In nearly pure strontium the same lines are seen, but they are then thinner and shorter.

I may remark that the lines at 4045, 4063, and 4071 are due to an iron impurity; these are the longest lines of iron in that portion of the spectrum photographed.

I may conclude this statement by remarking that the above facts have been given as

instances of the results to be obtained by the confronting of the spectrum of each solar metal with every other, a task which it will be necessary to accomplish before the Table and Maps referred to can be presented to the Royal Society.

IV. PRELIMINARY INQUIRY INTO THE EXISTENCE OF ELEMENTS IN THE SUN NOT PREVIOUSLY TRACED.

In a paper communicated to the Royal Society on December 12, 1872 (Phil. Trans. 1873, p. 253), I have shown that the test formerly relied on to decide the presence or absence of a metal in the sun (namely, the presence or absence of the brightest and strongest lines of the metal in question in the average solar spectrum) was not a final one, and that the true test was the presence or absence of the longest lines of the metal, this longest line being that which remains longest in the spectrum when the pressure of the vapour is reduced.

Of the test in question I have said, in the paper already mentioned, "It is one, doubtless, which will shortly enable us to determine the presence of new materials in the solar atmosphere, and it is seen at once that to the last published table of solar elements (that of THALÉN) must be added zinc, aluminium, and possibly strontium as a result of the new method."

In order to pursue the inquiry under the best conditions, complete maps of the long and short lines of all the elements are necessary. It was, however, not absolutely necessary for the purposes of a preliminary inquiry to wait for such a complete set of maps, for the lists of lines given by the various observers may be made to serve as a means of differentiating between the longest and shortest lines, because I have also shown that the lines given at a low temperature, by a feeble percentage composition, or by a chemical combination of the vapour to be observed are precisely those lines which appear longest when the complete spectrum of the pure dense vapour is studied.

Now with regard to the various lists and maps published by various observers, it is known (1) that very different temperatures were employed to produce the spectra, some investigators using the electric arc with great battery-power, others the induction-spark with and without the jar; (2) that some observers employed in certain cases the chlorides of the metals the spectra of which they were investigating, others used specimens of the metals themselves.

It is obvious, then, that these differences of method could not fail to produce differences of result; and accordingly, in referring to various maps and tables of spectra, we find that some include large numbers of lines omitted by others. A reference to these tables, in connexion with the methods employed, shows at once that the large lists are those of observers using great battery-power or metallic electrodes, the small ones those of observers using small battery-power or the chlorides. If the lists of the latter class of observers be taken, we shall have only the longest lines, while those omitted by them and given by the former class will be the shortest lines.

In cases, therefore, in which I had not mapped the spectrum by the new method of

observation referred to in my paper, I have taken the longest lines as thus approximately determined; for it seemed desirable, in view of the very large number of unnamed lines, to search at once for the longest elemental lines in the solar spectrum without waiting for a complete set of maps.

A preliminary search having been determined on, I endeavoured to get some guidance by seeing if there was any quality which differentiated the elements already traced in the sun from those not traced; and to this end I requested my assistant, Mr. R. J. FRISWELL, to prepare two lists showing broadly the chief chemical characteristics of the elements traced and not traced. This was done by taking a number of the best known compounds of each element (such, for instance, as those formed with oxygen, sulphur, chlorine, bromine, or hydrogen), stating after each whether the compounds in question were unstable or stable. Where any compound was known not to exist, that fact was indicated.

Two Tables were thus prepared, one containing the solar, the other the more important non-solar elements (according to our knowledge at the time).

These Tables gave me, as the differentiation sought, the fact that in the main the known solar elements formed stable oxygen-compounds.

I have said in the main, because the differentiation was not absolute; but it was sufficiently strong to make me commence operations by searching for the outstanding strong oxide-forming elements in the sun.

The result up to the present time has been that *strontium*, *cadmium*, *lead*, *copper*, *cerium*, and *uranium**, in addition to those elements in THALÉN'S last list, would seem with considerable probability to exist in the solar reversing layer. Should the presence of *cerium* and *uranium* be subsequently confirmed, most of the iron group of metals will thus have been found in the sun.

As another test, certain of those elements which form unstable compounds with oxygen were also sought for, gold, silver, mercury being examples. None of these were found.

The same result occurred when the lines due to the jar-spark taken in chlorine, bromine, iodine, and those of some of the other non-metals were sought, these being distinguishable as a group by formation of compounds with hydrogen.

Now other researches, not yet completely ready for publication, have led me to the following conclusions:—

I. The absorption of some elementary and compound gases is limited to the most refrangible part of the spectrum when the gases are rare, and creeps gradually into the visible violet part, and finally to the red end of the spectrum, as the pressure is increased.

II. Both the general and selective absorption of the photospheric light are greater (and therefore the temperature of the photosphere of the sun is higher) than has been supposed.

III. The lines of compounds of a metal and iodine, bromine, &c. are observed generally in the red end of the spectrum, and this holds good for absorption in the case of aqueous vapour.

* Potassium has since been added.

Such spectra, like those of the metalloids, are separated spectroscopically from those of the metallic elements by their columnar or banded structure.

IV. There are, in all probability, no compounds ordinarily present in the sun's reversing layer.

V. When a metallic compound vapour, such as is referred to in III., is dissociated by the spark, the band spectrum dies out, and the elemental lines come in, according to the degree of temperature employed.

Again, although our knowledge of the spectra of stars is lamentably incomplete, I gather the following facts from the work already accomplished with marvellous skill and industry by SECCHI of Rome.

VI. The sun, so far as the spectrum goes, may be regarded as a representative of class (β) intermediate between stars (α) with much simpler spectra of the same kind and stars (γ) with much more complex spectra of a different kind.

VII. Sirius, as a type of α, is (1) the brightest (and therefore hottest?) star in our northern sky; (2) the blue end of its spectrum is open; it is only certainly known to contain hydrogen, the other metallic lines being exceedingly thin, thus indicating a small proportion of metallic vapours; while (3) *the hydrogen lines in this star are enormously distended*, showing that the chromosphere is largely composed of that element.

There are other bright stars of this class.

VIII. As types of γ the red stars may be quoted, the spectra of which are composed of channelled spaces and bands, and in which naturally the blue end is closed. Hence the reversing layers of these stars probably contain metalloids, or compounds, or both, in great quantity; and in their spectra not only is hydrogen absent, but the metallic lines are reduced in thickness and intensity, which in the light of V., *antè*, may indicate that the metallic vapours are being *associated*. It is fair to assume that these stars are of a lower temperature than our sun.

I have asked myself whether all the above facts cannot be grouped together in a working hypothesis which assumes that in the reversing layers of the sun and stars various degrees of "celestial dissociation" are at work, which dissociation prevents the coming together of the atoms which, at the temperature of the earth and at all artificial temperatures yet attained here, compose the metals, the metalloids, and compounds.

On this working hypothesis, the so-called elements not present in the reversing layer of a star will be in course of formation in the coronal atmosphere and in course of destruction as their vapour-densities carry them down; and their absorption will not only be small in consequence of the reduced pressure of that region, but what absorption there is will probably be limited wholly or in great part to the invisible violet end of the spectrum in the case of such bodies as the pure gases and their combinations and chlorine (see I. *antè*).

The spectroscopic evidence as to what may be called the plasticity of the molecules of the metalloids, including of course oxygen and nitrogen, but excluding hydrogen, is so overwhelming, that even the absorption of iodine, although generally it is transparent

to violet light, may (as I have found in a repetition of Dr. ANDREWS's experiments on the dichroism of iodine, in which I observed the spectrum) in part be driven into the violet end of the spectrum, for iodine in a solution in water or alcohol at once gives up its ordinary absorption properties and stops violet light*.

A preliminary comparison of the ordinary absorption-spectrum of a stratum of 6 feet of chlorine renders it not improbable that chlorine at a low temperature is the cause of some of the FRAUNHOFER lines in the violet, although, as said before, I have not yet obtained certain evidence as to the reversal of the bright lines of chlorine seen in the jar-spark.

There is also an apparent coincidence between some of the faint FRAUNHOFER lines and some of the lines of the low temperature absorption-spectrum of iodine.

Should subsequent researches strengthen the probability of this working hypothesis, it seems possible that iron meteorites will be associated with the metallic stars and stony meteorites with metalloidal and compound stars. Of the iron group of metals in the sun, iron and nickel are those which exist in greatest quantity, as I have determined from the number of lines reversed. Other striking facts, such as the presence of hydrogen in meteorites, might also be referred to.

An interesting physical speculation connected with this working hypothesis is the effect on the period of duration of a star's heat which would be brought about by assuming that the original atoms of which a star is composed are possessed with the increased potential energy of combination which this hypothesis endows them with. From the earliest phase of a star's life the dissipation of energy would, as it were, bring into play a new supply of heat, and so prolong the star's light.

May it not also be that if chemists take up this question which has arisen from the spectroscopic evidence of what I have before termed the plasticity of the molecules of the metalloids taken as a whole, much of the power of variation which is at present accorded to metals may be traced home to the metalloids? I need only refer to the fact that, so far as I can learn, all so-called changes of atomicity take place when metalloids are involved, and not when metals alone are in question.

As instances of these, I may refer to the triatomic combinations formed with chlorine, oxygen, sulphur, &c. in the case of tetrad or hexad metals.

May we not from these ideas be justified in defining a metal, provisionally, as a substance, the absorption-spectrum of which is *generally* the same as the radiation-spectrum, while the metalloids are substances the absorption-spectrum of which, generally, is not the same? In other words, in passing from a cold to a comparatively hot state, the plasticity of these latter comes into play, and we get a new molecular arrangement. Hence are we not justified in asking whether the change from oxygen to ozone is but a type of what takes place in all metalloids?

My best thanks are due to Mr. R. J. FRISWELL for the valuable aid he has afforded me in these investigations.

* I have since obtained the same result by observing the absorption of I vapour in a white-hot tube.

Explanation of the Plates.

PLATE XXXVIII.

Illustrating use of slit plate.

Comparison of the spectra of Calcium and Barium with the solar spectrum.

Upper spectrum	. . . . .	Barium, H to beyond G.
Middle „	. . . . .	Solar „ „ „
Lower „	. . . . .	Calcium „ „ „

Note.—The Calcium contains Aluminium, Strontium, and Iron as impurities. The barium contains the same impurities in addition to calcium.

PLATE XXXIX.

Illustrating use of slit plate.

Confronting of spectra of Manganese, Nickel, Lenarto meteorite, and Iron containing calcium and other impurities.

Upper spectrum	. . . . .	Manganese . . H to beyond G.
		Nickel . . . „ „ „
		Lenarto meteorite „ „ „
Lower „	. . . . .	Impure iron . „ „ „

Note.—The Manganese spectrum includes the longest lines of Calcium, Aluminium, and Iron. Nickel shows the same impurities in addition to manganese, while the longest nickel lines are to be traced in the spectrum of the Lenarto meteorite.

PLATE XL.

Illustrating use of horizontal arc.

Long and short lines near F of impure iron. The lower part gives the spectrum of the core of the arc; the upper part the spectrum of the extreme limits of the vapour.

XIV. *On the Quantitative Analysis of certain Alloys by means of the Spectroscope.*

By J. Norman Lockyer, *F.R.S., and* W. Chandler Roberts, *Chemist of the Mint.*

Received November 20,—Read November 27, 1873.

In certain researches on Spectrum-Analysis, the results of which have already been communicated by one of us to the Royal Society, it has been shown that the spectroscope may be employed to distinguish minute differences in the composition of certain alloys. We have therefore instituted a series of experiments with the view to ascertain the degree of accuracy of which the method is capable, and now submit the results which we have already obtained.

In the researches above referred to, an electric-spark-current was used to volatilize the alloy under examination, and the image of the spark was thrown by means of a lens on the slit of a spectroscope; the phenomena observed indicated that a change in the percentage of any constituent of a metallic alloy ordinarily causes a change in the character of lines exhibited in the resulting spectrum, in length, brightness, or thickness; and the percentages of the constituents, it was stated, might be determined by comparison with the spectra of pieces of metal of known composition.

It may be remarked that changes are exhibited in the spectra in various ways, thus:—

1. Changes in the lengths of lines of one constituent.
2. Changes in the position of the point at which lines belonging to different metals in the alloy show equal brightness or thickness.
3. Changes in the relative lengths of a pair of lines belonging severally to the constituents of the alloy.

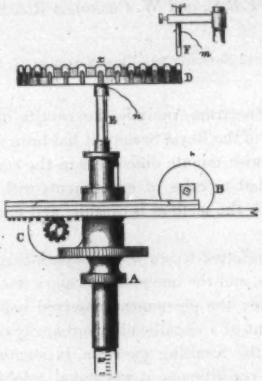
Any of the above changes may give the requisite information, and all are due to the same cause, which evidently is the difference of alloys as regards volatility. Thus in the case of gold and copper, the former being less volatile than the latter, there will be a minimum of metallic vapour with pure gold and a maximum with pure copper; and when alloys are employed, there will be a regular gradation between these two limits.

In the experiments about to be detailed, unknown alloys and check pieces of known composition having been arranged on a suitable stand (as shown in the diagram, p. 496), they were in turn brought immediately under the fixed electrode F (of aluminium or iridio-platinum).

We were soon convinced that it was necessary to regulate with extreme accuracy the length of the spark by which the incandescent vapour of the alloy was produced; and

the following experiments were made with a view to determine what details of manipulation would give the most satisfactory results.

In the first attempts a fragment of the alloy of a more or less irregular shape was simply held in a suitable clip, and a spark-current passed from the alloy to a similar



A. Milled head of elevating-screw.
B. C. Horizontal adjusting-screws.
D. Revolving table.
E. Insulating glass rod.
F. Fixed terminal.
x. Series of assay-pieces.
m, *n*. Wires from induction-coil.

fragment of aluminium or other metal. We then adopted a method orally suggested to one of us by Professor STOKES, which consists in the passage of the spark-current in a vertical line across the shortest distance between two cylinders arranged in a horizontal position, with their axes at right angles to each other.

The use of these cylinders was attended with many difficulties, and we substituted for them strips of metal cut in the form indicated in the figure (at *x*).

The distance between the poles was at first adjusted by a gauge, and subsequently by a cathetometer; but the most accurate results were obtained by placing the portion of alloy in the field of a microscope furnished with a 3- or 4-inch objective, a simple mechanical arrangement, shown above, bringing the surface from which the spark would pass to the point of intersection of two spider-lines in the eyepiece of the microscope.

We were careful to select alloys which were homogeneous in character; and our attention was first devoted to observations on the zinc-cadmium alloy, one of a series of

alloys termed by MATTHIESSEN "solidified solutions of one metal in another." We next operated upon the gold-copper alloy, as its molecular arrangement appeared to render it peculiarly suited for the purpose of the research, the use of this alloy being attended with the additional advantage that the ordinary method of assay has rendered it possible to determine its composition with accuracy to the $\frac{1}{10,000}$ part of the original weight of the assay-piece, a degree of precision which will appear remarkable to all who are familiar with the ordinary methods of quantitative analysis.

We give in the accompanying curves (Plate XLI.) the results of our experiments with alloys of known composition, in which experiments the precautions to which we have referred have been taken. The results are represented graphically, the coordinates being the composition of the alloys as determined by the ordinary method of assay, and by the varying points of equal brightness as measured on the micrometer-scale of the lines at the wave-lengths stated in the diagrams. The micrometer is fixed to the eyepiece end of the observing-telescope. It is an ordinary wire-micrometer with movable horizontal wires. The micrometer-readings are represented by spots placed opposite the compositions as determined in the ordinary manner, a mean curve being subsequently constructed.

We have found that there are several variations possible in the method of observation, and we are yet uncertain which of these variations will afford the most satisfactory results.

Our method of procedure was upon each occasion to prepare such a curve by means of accurately known standards; and having this curve, to determine, by means of the micrometer-readings, the positions which various specimens of unknown composition would occupy on it. By carrying the eye from the curve to the side on which, as in those communicated to the Society, the parting assay determinations were shown, we were enabled to compare the places assigned to the specimens, on the curve, by the spectroscope with those determined by the parting assay which we then learnt for the first time. We do not think it necessary to give these curves, but may say that, as the result of many series of experiments with alloys of unknown composition, the difference between the spectroscopic and parting assays was but small so long as the conditions under which we experimented continued to be uniform.

Descriptions of Curves.

Curve No. 1.

A number of alloys of zinc and cadmium were synthetically prepared, and from these a series of five, the percentages of cadmium in which increased by 1 per cent. from 50 per cent. to 54 per cent., were carefully selected for the experiments. It is interesting to note that both the metals employed in the alloy are very fusible (zinc melting at 433° C. and cadmium at 228° C.); and observations by means of the spectroscope at once enabled us to arrange these alloys in their correct order.

Curve 2.

This comprises a series of alloys ranging from 900·0 to 922·5 parts of gold in 1000 of the alloy. It was obtained by measurements made by placing the micrometer-wires in a vertical position and measuring the base of the lowest visible portions of the spectroscopic lines under examination, a method which appeared to be better suited to wide ranges than observations made in the upper portions of the spectral lines; these latter were, however, found more suited to the study of narrow ranges, such as that shown in Curve 3. We should remark that, at this early stage of our inquiry, our attention was mainly devoted to ascertaining whether the curves would be continuous, and to determining at what point a curve deduced from any given lines in the spectrum would cease to be sensitive.

Curve 3.

The Curve 3 was constructed by means of observations made upon several specimens of gold-copper alloy, the composition of which had been determined to the $\frac{1}{10.000}$ part in the ordinary manner. Micrometer-readings for these specimens were taken, which, together with their known approximate compositions, furnished the coordinates to the curve.

It will be seen at a glance that with regard to the lower part of the curve a change of composition of $\frac{1}{10.000}$ gives a much greater change in the micrometer-readings than it does in the upper part. It is also to be remarked that, as in this more sensitive portion a change of $\frac{1}{10.000}$ part is represented by a change of 200 in the micrometer-readings, a true mean curve can only be constructed when the actual composition of the alloy is stated with much greater accuracy than that afforded by the present method of assay.

We should observe that as in several cases duplicate and even triplicate readings were made with the same specimens, the number of spots placed opposite any figure in the vertical column does not necessarily indicate the number of pieces of that composition under examination. If we examine the Mint specimens, of which the composition is stated to be 916·4, it is seen that the maximum departure from the curve is less than $\frac{1}{10.000}$ part; and in this case it is impossible to say that any error has been made by the spectroscope, because, were the composition of all the alloys in question to be what the spectroscope states them to be, they would still be called 916·4 on the parting assay. Again, if we consider the specimens 916·6, it will be seen that while in those two with micrometer-readings 150 the departure is not more than that previously stated, $\frac{1}{10.000}$ part, on the other hand the composition of the two the micrometer-readings of which were 250 would carry them into a region of the curve which would spectroscopically indicate their composition to be less than 916·5. The departure in this case was so great that it was considered advisable to make a fresh determination of their composition by the ordinary method; and it was satisfactory to find that the parting assay determination, as revised, was almost identical with that given by the spectroscope.

Curve 4.

The range of alloys is the same as those given in the preceding curve; but the method used was that employed in obtaining Curve 2, and it is interesting as illustrating the change introduced into the form of the curve by varying the method by which the spectral lines were observed.

Practical Considerations.

It is impossible to foresee to what analytical operations the new method may be found to be applicable; but as the experiments hitherto have been mainly directed towards the development of a new method of assaying gold, it is advisable that attention should be limited to the relative advantages of the old and new methods.

We ascertained by careful experiment that the amount of alloy actually volatilized during an observation in no case exceeds 0·0001 grm.; and it is interesting to compare this with the amount of metal usually employed in assaying gold, which varies from 0·5 to 1 grm.

It may be objected that the amount of metal employed in the new method is very small; but granting the accuracy of the method and the homogeneity of the alloy, there is of course no reason why the composition of a gold ingot may not be ascertained by it as accurately as by the old method.

It is now well known, as previously mentioned, that the existing method is usually accurate to the $\frac{1}{10,000}$ part of the portion of metal operated upon, the process possessing in addition to its accuracy many incidental advantages, not the least of which is the possibility which it affords of detecting, at different stages of the work, the presence of metallic impurities, such as iridium.

On the other hand, it should be observed that the method now in use comprises six distinct operations, and two hours are occupied in ascertaining the final result of the assay; and as it is frequently important to determine the value of an ingot of gold with rapidity, it will be obvious that the new method possesses marked advantages in this respect, for by its aid a result may be obtained in a few minutes.

In conclusion, we would submit that experience alone can show whether this new process may be made as trustworthy as the existing method; and we would draw attention to the fact that hitherto there has not been absolute identity in the conditions under which the several experiments were made; for instance, the intensity of the current employed to volatilize the metal varied from time to time; and this, together with other defects, mainly arose from the fact that we have as yet considered the problem solely from its scientific aspect, and have not provided ourselves with instruments well adapted for satisfying all the conditions which may arise in practice.

Whether this new method be found preferable to the old one or not, the fact has been clearly established that it is possible to detect, by its means, differences in the composition of the gold-copper alloy as minute as the $\frac{1}{10,000}$ part of the whole mass.

XV. *On Attraction and Repulsion resulting from Radiation* *.

By WILLIAM CROOKES, *F.R.S. &c.*

Received August 12,—Read December 11, 1873.

1. IN a paper "On the Atomic Weight of Thallium," presented to the Royal Society June 18, 1872, after describing a balance with which I was enabled to perform weighings of apparatus &c. in a vacuum, I noted a peculiarity in relation to the effect of heat in diminishing the apparent weight of bodies. I said, "That a hot body should appear to be lighter than a cold one has been considered as arising from the film of air or aqueous vapour condensed upon or adhering to the surface of the colder body, or from the upward currents of air caused by the expansion of the atmosphere in the vicinity of the heated body. But neither hypothesis can be held when the variation of the force of gravitation occurs in a vacuum as perfect as the mercurial gauge will register, and under other conditions which I am now supplying, and which I purpose embodying in a paper to be submitted to the Royal Society during a subsequent session" †.

With the vacuum-balance mentioned above I carried out many experiments, but was unable to obtain results which were at all concordant; and it was soon found necessary to investigate the phenomena with smaller and less complicated apparatus.

2. Most chemical manuals warn beginners against the errors occasioned by weighing substances while hot; and, up to a moderately high degree of exhaustion, I was prepared to find a piece of glass apparatus, when hot, apparently lighter than the weights which should balance it were the whole system at the same temperature. But instead of the interfering causes diminishing as the rarefaction proceeded, they seemed rather to increase, or at all events to become irregular in their action, sometimes appearing to oppose, and at others to supplement the force of gravity. In such a vacuum as a good air-pump would produce, the actions of the ascending current of air and of the adhering film, it might be presumed, should cease to exert an influence; and I could think of no other disturbing cause except the lengthening of the beam, owing to the heat radiated from the apparatus below it. An increase in the length of the beam should make a mass suspended at its extremity appear heavier; but whilst I frequently noticed an action which might be due to this cause, I occasionally obtained results which were so anomalous as to convince me that some cause which I had not hitherto recognized was at work (49), and to lead me to hope that perhaps I might succeed in tracing a connexion between heat and the force of gravity.

* This paper was read before the Royal Society, and the abstract was published in the 'Proceedings' (vol. xxii. p. 37) under the title "On the Action of Heat on Gravitating Masses."

† Philosophical Transactions, 1873, vol. clxiii. p. 287.

3. Many physicists have worked on the subject of repulsion by heat. I give here a brief *résumé* of the state of knowledge on this subject up to the time of my commencing these experiments, premising, however, that much of this historical information was unknown to me until some of the experiments here recorded were finished and I commenced putting my notes together. The earliest mention I can find is by the Rev. A. BENNET, F.R.S., who in the year 1792 published a paper* on "A new suspension of the magnetic needle intended for the discovery of minute quantities of magnetic attraction: also an air-vane of great sensibility; with new experiments on the magnetism of iron filings and brass." Mr. BENNET used a spider's thread as a means of suspension. This he found by experiment to be absolutely free from torsion. I quote the following experiments from his paper:—

"Experiment IV. A bristle was suspended horizontally by a spider's thread, somewhat stronger than the last, and after turning the wheel till it produced 4800 revolutions, it shortened the thread from 3 inches to 1 inch; yet either end of the bristle would move towards any warm substance which was presented to it either with or against the direction of the twist†.

"Experiment V. Several other light substances were suspended by fine spiders' threads and placed in a cylindrical glass about 2 inches in diameter, as the thinnest part of the wing of a dragon-fly, thistledown, and the down of dandelion; of these the last appeared most sensible to the influence of heat; for when this down was fastened to one end of a fine gold wire, suspended horizontally on to one end of two bits of straw joined together in the form of the letter T inverted, it would turn towards any person who approached it at the distance of 3 feet, and would move so rapidly towards wires heated by my hand, as very much to resemble magnetic attraction.

"Experiment VI. A bottle filled with cold water was brought near the glass cylinder standing in a warm room, and soon after the down of dandelion appeared to be repelled by the bottle by turning away from it. The bottle was removed to the other side, and the dandelion again moved towards the opposite side.

"Experiment VII. A piece of paper was tied over the mouth of a glass jar, about 4 inches in diameter. Two holes were made in the paper opposite to each other, and near the edge of the glass. The jar was placed upon a table, and suffered to stand a considerable time to cool in a room without fire. I then sat near it on the side where one of the holes in the paper was in the nearer and the other in the farther end of the diameter. I next filled another glass with smoke, and placed it with its mouth over the two holes in the paper. The smoke was now seen to descend through the farthest hole, and mixing with the air in the lower jar, plainly showed that the air moved slowly towards the side of the glass warmed by the heat of my body.

"Experiment X. To the end of a fine gold wire 3 inches long, and suspended by a spider's thread in a cylindrical glass, was fastened a small circular bit of writing-paper;

* Philosophical Transactions, 1792, p. 81.

† For a rediscovery of this fact, seventy-nine years after, see par. 14.

light was admitted through a small hole, and also the focus of a large lens was thrown upon the paper, with the intention of observing whether it would be moved by the impulse of light; but though these experiments were often repeated, and once with the paper suspended in an exhausted receiver, yet I could not perceive any motion distinguishable from the effects of heat. Perhaps sensible heat and light may not be caused by the influx or rectilinear projection of fine particles, but by the vibrations made in the universally diffused *caloric* or matter of heat or fluid of light. I think modern discoveries, especially those of electricity, favour the latter hypothesis."

4. In his 'Elementary Treatise on Heat'* Professor Balfour Stewart, F.R.S., cites this experiment of Mr. Bennet's as one of the arguments against the emissive and in favour of the undulatory theory of light and heat. Bearing in mind the overwhelming proofs we now possess that the undulatory theory more nearly expresses the truth than does the emissive theory, it is not likely that the very different results I have succeeded in obtaining (56, 57, 58), by the employment of instruments of a delicacy unattainable eighty years ago, will have any weight in modifying the accepted theories of light and heat.

5. The next mention of the dynamic action of heat is by Laplace, who, in his 'Mécanique Céleste'†, speaks of the "repulsive force of heat" as subsisting among the particles of a fluid, but observes that experiment shows it has no other effect on capillary attraction than what results from its diminishing the density of the fluid.

6. In the year 1824 Libri‡ published some experiments on the movement of translation experienced by a drop of liquid suspended to a metallic wire, one of the ends of which is heated. This he inferred was due to repulsion produced by the heat between the wire and the particles of the liquid. The Rev. Baden Powell, F.R.S., says§ that trying to repeat Libri's experiment he has never been able to succeed, except in producing a slight apparent motion in the drop, which seems explicable from the mere effect of evaporation on the side next the heat.

7. In the 'Annales de Chimie et de Physique' for 1825‖ are two papers by Fresnel, in which he gives an account of an experiment on the repulsion exerted by heat. To the two extremities of a fine magnetic needle, suspended by a cocoon-fibre, he attached vertical disks of foil and mica, so as to test with the same apparatus an opaque and a transparent body. The fixed body which was to repel the torsion-balance was another disk of foil. The whole was covered with the receiver of an air-pump, and a vacuum, up to 1 or 2 millimetres, was obtained. The whole was then taken into sunshine, and turned so that the needle was kept slightly out of the magnetic meridian by pressure of the fixed disk against one of the movable disks. On concentrating the sun's rays on either of these disks, they instantly separated, sometimes to the extent of a millimetre. On withdrawing the lens, the torsion-balance only gradually returned to its

* Oxford, at the Clarendon Press, 1866, pp. 161, 352.
† Suppl. livr. x. p. 75, A.D. 1799–1805.
‡ Mem. Accad. Torino, xxviii.
§ Philosophical Transactions, 1834, p. 485.
‖ Vol. xxix. pp. 57, 107.

original position. To see if these phenomena were due to the residual gas, air was gradually let in; and on repeating the experiment when the density of the enclosed air was fifteen or twenty times greater than at first, it was found that the repulsion had not sensibly increased, as it should have done had it been due to currents of heated air. Under some conditions, indeed, the movement was not so great as in a vacuum. Sometimes FRESNEL observed an action of attraction, the disks adhering when heated, and separating when the lens was removed. With pieces of copper suspended to the magnetic needle the attraction was very apparent; when the movable and fixed disks were near together they approached on applying heat. Reasons are given why these effects cannot be due to electricity or magnetism, but the author does not seem to have tried any further experiments.

8. M. SAIGEY in 1827 * described an experiment with a needle of lead delicately suspended at different distances from a bar of copper. He found the number of oscillations in a given time decrease with the distance. From his experiments he arrived at the following results:—All bodies exert between themselves a feeble repulsive action under ordinary circumstances. A very marked attraction may be observed between a cold and a heated body, or between two bodies of different temperatures, whether screens be interposed or not. SAIGEY concludes that in many cases results obtained without the appreciable development of magnetism or electricity have been attributed to these forces.

9. In the year 1834 Professor J. D. FORBES † published an elaborate research on the vibrations which Mr. A. TREVELYAN had found to take place between metallic masses having different temperatures. The general conclusion at which he arrived is that there is a repulsive action exercised in the transmission of heat from one body into another which has a less power of conducting it. These repulsions only take place between bodies having a certain amount of conducting-power, below which some metals fall; it must be excitable in a most minute space of time, and is energetic in proportion to the difference of conducting-power of the substances and to their difference of temperature.

10. The Rev. BADEN POWELL, in the same year ‡, published a paper "On the Repulsive Power of Heat." He employed an arrangement somewhat similar to FRESNEL'S (7), the disks being two small plates of glass with truly plane surfaces. He found that if in the first instance they were pressed together so as to adhere, heat always overcame the attraction, and the movable disks sometimes receded to a sensible distance; but Professor POWELL says that this effect (and perhaps also that in FRESNEL'S experiment) appeared to him in a great measure due to another cause than repulsion, viz. the slight curvature which will be given to the plate of glass by the greater expansion of the more heated surface producing a convexity towards the heat.

11. By pressing the disks closely together, the coloured rings formed would give a

* Bulletin Mathématique, tom. ix. pp. 89, 167, 239, tom. xi. no. 167; Bull. Sci. Nat. viii. p. 287.

† Trans. Roy. Soc. Edinburgh, vol. xii. p. 429. ‡ Philosophical Transactions, 1834, p. 485.

test of the interval between the disks. Professor POWELL found that the tints invariably *descended* in the scale when heat was applied, showing that the interval between the disks increased, and proving the existence of a repulsive power exerted between heated surfaces at small though sensible distances—the *warping*, or change of figure, if any, in the glasses by heat being readily seen to be such as ought to cause the rings to *enlarge* at the first instant. From experiments made by the contact of a lens with different substances, Professor POWELL inferred that whatever tends to increase the rapidity of communication of heat, tends to increase the observed effect. The effect is increased when water, instead of air, is introduced between two lenses.

12. In 1838 Professor POWELL* gave some additional notes on the same subject, but no new form of experiment was tried.

13. Dr. JOULE, F.R.S.†, gave an account in 1863 of a new and extremely sensitive thermometer. It was based upon the disturbing effect of currents of air upon finely suspended magnetic needles. By diminishing the directive force of the needle, the instrument was made sufficiently delicate to move to the heat radiated from a small pan containing a pint of water heated to 30°, placed 3 yards off, and also to give evidence of the heat of the moon. I have little doubt that these movements were not so much due to currents of air as to the mechanical effects of radiation described in this paper.

14. It is right that I should mention here that in September 1871 I received a letter from Mr. J. REYNOLDS, mentioning that he had constructed a little instrument which would turn to the hand, to a fire, or to any source of heat. It consisted of a thin slip of deal suspended by a filament of spider's web, and enclosed in a thin glass flask. This little instrument was more sensitive than any I had then constructed, as the spider's web was much freer than cocoon-silk from torsion, and Mr. REYNOLDS kindly allowed me to experiment with it.

15. I cannot do better, in bringing this historical summary to a conclusion, than draw attention to a passage written in 1868 by Professor GUTHRIE, F.R.S.‡, in which he distinctly points out a probable relation between heat and gravity. He says:—"If the ætherial vibrations which are supposed to constitute radiant heat resemble the aerial vibrations which constitute radiant sound, the heat which all bodies possess, and which they are all supposed to radiate in exchange, will cause all bodies to be urged towards one another."

16. Were it such a relation between heat and gravity of which I had been getting glimpses, it was evident that a much more delicate apparatus would be necessary to render it distinct, and I accordingly commenced a series of experiments with the view of ascertaining what form of apparatus would be most sensitive to the action sought.

The first requisite was to get rid of the error arising from the expansion of the beam by heat; and since, in working with hot bodies, the metallic masses used as

* Phil. Mag. vol. xii. April 1838. † Chemical News, vol. vii. p. 150.

‡ Proceedings of the Royal Society, vol. xix. p. 35.

weights would themselves become warm, and since the action I sought to establish was only likely to be due to a *difference* in temperature between the hot body on one arm of the balance and the cold weights on the other arm, both being under the influence of the same force of gravity, I endeavoured to obtain the desired results by means of a spring-balance, one in which the variations of gravity should be measured, not against gravity itself, but against the tension of a spring.

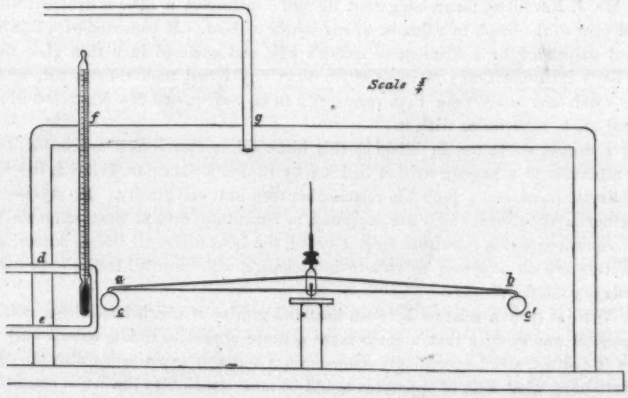
17. I tried many forms of spring-balance, and obtained with them results which, at the time, I thought sufficiently satisfactory. The sources of error were, however, so numerous, and the manipulation was so difficult, that I ultimately gave up that form of balance in favour of the one usually employed.

In order to obtain a very high degree of rarefaction, without the trouble and uncertainty attending the use of an ordinary air-pump, it was necessary to have the balance sufficiently small to enable it to be exhausted by the Sprengel pump.

Before proceeding to the forms of apparatus finally adopted and the experiments made therewith, it will, I think, be useful, for the sake of other experimentalists, if I briefly describe some of the arrangements successively tried and rejected, with the reasons for so doing.

18. A light beam (*a b*, fig. 1) was made of two pieces of fine flattened brass wire

Fig. 1.



120 millims. long, 5 millims. apart at the point of suspension, and joined together at each end. A pair of very fine needle-points, one on each side of the beam, represented knife-edges, and worked on glass plates cemented horizontally to upright pillars. The

centre of gravity could be altered by a screw, and a pointer projecting upwards enabled me to see a movement of the beam. A brass ball (c, c') 6 millims. diameter, was soldered to each end of the beam. The balance was adjusted so that it made one oscillation in about five seconds.

19. The case consisted of a rectangular box of brass, 10 mm. thick, the front of which was replaced by plate glass. At one end two holes were drilled (d, e) about 20 mm. apart, and a curved piece of glass tube was cemented in, so that both ends were outside. At the top a hole (f) was drilled to receive a thermometer, and another (g) to receive the tube attaching the instrument to the Sprengel pump. By sending a current of hot water through the bent glass tube, heat could be communicated to one of the brass balls, and the movement, if any, of the beam could be seen by a micrometer in front. Many experiments were tried with this apparatus, and the result appeared to be that warming the ball caused it to sink (49). This action might, however, be due to the expansion of the brass beam by heat; so to obviate this source of error, I sought for a material wherewith to make a beam which should be as little affected as possible in this manner.

20. A stick of fine-grained charcoal was worked up into the shape of a beam, and fitted at each end and in the centre with appropriate metallic collars for the needle-points and brass balls. To get rid of absorbed gases, which experience showed were evolved unequally in a vacuum, and thereby threw the beam out of adjustment, it was first heated strongly in an exhausted tube, and then soaked in an alcoholic solution of shellac. When quite dry the charcoal was heated till the shellac fused. After much difficulty the beam was adjusted, and being enclosed in the brass case described above, exhaustion was effected. With this charcoal beam I also found that heat generally caused the brass ball to sink (49).

21. These results were opposed to what I had before noticed, but many anomalies were observed (49). Thus the diminution of gravity did not appear to vary as the rarefaction increased; and the position of the hot body, in relation to the brass ball, seemed to have considerable influence on the direction and amount of movement. It was also difficult to make the brass case, with its numerous joints, sufficiently tight to hold a Sprengel vacuum*, even by painting it over with gold-size when partially exhausted, and I therefore decided to form the beam of some other material.

22. A mica beam was at first tried, but it was found to be liable to split across. Magnesium possesses the advantages of lightness and rigidity, but was inapplicable,

* As I shall have to speak of various kinds of vacua, it will be best to name them distinctively to avoid periphrasis. I shall call the best vacuum which my air-pump will give an *air-pump vacuum*—this is one or two millimetres below the barometer. The ordinary vacuum produced by the Sprengel pump I shall call a *Sprengel vacuum*—in this the gauge is appreciably level with the barometer. A so-called "perfect" vacuum, produced by potash and carbonic acid, as subsequently described, or by similar means, I shall call a *chemical vacuum*. I object to the term *perfect*, as applied to any vacuum at present known, as I believe that where force can travel we are not justified in assuming the absence of matter—imponderable it may be, and unaffected by ordinary forms of force—but none the less *matter*.

owing to its expanding by heat. A gridiron beam of zinc and iron or of zinc and glass, and a beam formed of two reversed thermometers, their bulbs forming the gravitating masses, were thought of, so as to have self-compensation for changes of temperature; but after a few experiments the weight of such beams was found to be an insuperable objection, even had the difficulties of adjustment been overcome. Altogether glass seemed to offer most advantages, as being sufficiently rigid, whilst its low conducting-power for heat rendered it little liable to introduce errors from expansion.

A straight glass beam was drawn from a piece of rectangular glass tube, and it was fitted with needle-point centres and brass balls at each end, similar to the plan adopted with the brass and charcoal beams. This was fitted in the small brass case, and many experiments were tried with it. The results were still anomalous, the apparent action of heat being sometimes in one direction and sometimes in another (49).

Greater delicacy was still required, and the brass balls at the end of the beam were accordingly replaced with magnesium balls; and instead of enclosing the balance in the brass case, I sealed it up, after exhaustion, in a glass tube. With this a large number of experiments were tried; but as the subsequent results, after I had discovered one of the laws governing the phenomena, were much more satisfactory, I forbear from occupying space in describing these preliminary experiments.

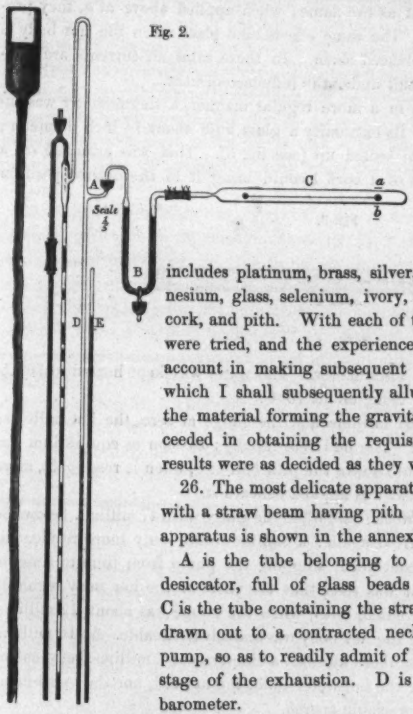
23. Afterwards I experimented successively with a glass beam having no special weight at the ends, and with the same beam with terminal knobs of glass fused on; this appeared to increase the delicacy somewhat, but the weight was still too great to allow of accurate results being obtained; and I finally adopted straw as the material for the beam, varying the gravitating masses at the end as experience dictated. Straw possesses many advantages: it is exceedingly light yet rigid; it dries easily, and evolves no vapour in a vacuum; moreover it is not likely to introduce errors by altering in shape under the influence of the moderate degrees of temperature to which it is subjected in these experiments.

24. The method of supporting the straw beam in the centre, so as to secure the maximum sensitiveness without the liability to get out of order, was difficult at first. After trying suspension by fibres of cocoon-silk from a glass frame, suspension on a fine glass axis resting on thin glass rods, and many other devices, I finally adopted the following mode of support.

The pointed half of a small sharp needle is broken off about half a millimetre shorter than the internal diameter of the glass tube; the blunt end is then ground very sharp on Arkansas stone. The straw, about 7 inches long, having its gravitating masses at the ends, is then balanced on a knife-edge so as to let it roll over to a stable position and to find its centre; and the needle is then run through it at right angles, at such a distance above the horizontal centre of the straw that the centre of gravity of the whole system is a little below the centre of suspension. The beam being slipped into the glass tube (sealed at one end), the needle is supported very delicately against the sides of the glass by its points, and with the least possible amount of friction. It is best now

to exhaust temporarily, heating the straw by passing a spirit-flame along the tube, so as to drive off moisture. If, as is almost certain to be the case, one end becomes heavier than the other, equilibrium can be restored, without much difficulty, by holding the spirit-flame for a few seconds under the heavier end, so as to slightly char the straw or other material. When in good adjustment and sufficiently sensitive the balance is ready for experiment.

Fig. 2.



25. The material with which I formed the masses at the ends includes platinum, brass, silver, lead, bismuth, aluminium, magnesium, glass, selenium, ivory, charcoal of different kinds, straw, cork, and pith. With each of these a large series of experiments were tried, and the experience gained with each was turned to account in making subsequent apparatus*. Certain differences, which I shall subsequently allude to, were noticed according to the material forming the gravitating mass; but as soon as I succeeded in obtaining the requisite degree of delicacy, the chief results were as decided as they were unexpected.

26. The most delicate apparatus for general experiment is made with a straw beam having pith masses at the end. The general apparatus is shown in the annexed figure (fig. 2):—

A is the tube belonging to the Sprengel pump†. B is the desiccator, full of glass beads moistened with sulphuric acid. C is the tube containing the straw balance with pith ends. It is drawn out to a contracted neck at the end connected with the pump, so as to readily admit of being sealed off if desired at any stage of the exhaustion. D is the pump-gauge, and E is the barometer.

27. The whole being fitted up as here shown, and the apparatus being full of air to begin with, I passed a spirit-flame across the lower part of the tube at *b*, observing the movement by a low-power micrometer; the pith ball (*a b*) descended slightly, and then immediately rose to considerably above its original position. It seemed as if the

* It is only fair to acknowledge here the assistance which I have received during the progress of these experiments from my young friend and pupil, Mr. Charles H. Gimingham. Without his skill with the blow-pipe and delicacy of manipulation with complicated apparatus, it would have been difficult for me to have carried out this investigation during the limited time I am able to devote to original research.

† For a full description of this pump, with diagrams, see Phil. Trans. 1873, vol. clxiii. p. 295.

true action of the heat was one of attraction, instantly overcome by ascending currents of air. A hot metal or glass rod and a tube of hot water applied beneath the pith ball at *b* produced the same effect as the flame; when applied above at *a*, they produced a slight rising of the ball. The same effects take place when the hot body is applied to the other end of the balanced beam. In these cases air-currents are sufficient to explain the rising of the ball under the influence of heat.

28. In order to apply the heat in a more regular manner, a thermometer was inserted in a glass tube, having at its extremity a glass bulb about 1½ inch diameter; it was filled with water and then sealed up (see fig. 3). This was arranged on a revolving stand, so that by means of a cord I could bring it to the desired position

Fig. 3.

without moving the eye from the micrometer. The water was kept heated to 70° C., the temperature of the laboratory being about 15° C.

29. The barometer being at 767 millims. and the gauge at zero, the hot bulb was placed beneath the pith ball at *b*. The ball rose rapidly; as soon as equilibrium was restored, I placed the hot-water bulb above the pith ball at *a*, when it rose again, more slowly, however, than when the heat was applied beneath it.

30. The pump was again set to work; and when the gauge was 147 millims. below the barometer, the experiment was tried again; a similar result, only more feeble, was obtained. The exhaustion was continued, stopping the pump from time to time to observe the effect of heat, when it was seen that the effect of the hot body regularly diminished as the rarefaction increased, until, when the gauge was about 12 millims. below the barometer, the action of the hot body was scarcely noticeable. At 10 millims. below it was still less; whilst when there was only a difference of 7 millims. between the barometer and the gauge, neither the hot-water bulb, the hot rod, nor the spirit-flame caused the ball to move in an appreciable degree.

The inference was almost irresistible that the rising of the pith was only due to currents of air, and that at this near approach to a vacuum the residual air was too highly rarefied to have power in its rising to overcome the inertia of the straw beam and the pith balls. A more delicate instrument would doubtless show traces of movement at a still nearer approach to a vacuum; but it seemed evident that when the last trace of air had been removed from the tube surrounding the balance—when the balance was suspended in empty space only—the pith ball would remain motionless, wherever the hot body were applied to it.

31. I continued exhausting. On next applying heat, the result showed that I was

far from having discovered the law governing these phenomena; the pith ball rose steadily, and without that hesitation which had been observed at lower rarefactions. With the gauge 3 millims. below the barometer, the ascension of the pith when a hot body was placed beneath it was equal to what it had been in air of ordinary density; whilst with the gauge and barometer level its upward movements were not only sharper than they had been in air, but they took place under the influence of far less heat—the finger, for example, instantly repelling the ball to its fullest extent.

To verify these unexpected results, air was gradually let into the apparatus, and observations were taken as the gauge sank. The same effects were produced in inverse order, the point of neutrality being when the gauge was about 7 millims. below a vacuum.

32. When the balance was in air of ordinary density, and the hot body was placed *above* the pith ball in the position *a* (see fig. 2), it will be remembered that the action was to cause the ball to rise; the rising was, however, less decided than when the heat was applied below (27, 29). On re-exhausting the balance-tube and taking a series of observations, placing the hot bulb above the pith ball in the position *a* (fig. 2) instead of below it, the ascending tendency of the pith got less and less. Several millimetres below the previously ascertained point of neutrality the hot bulb at *a* ceased to exert an action, and when the neutral point was exceeded by some millimetres I could still detect no movement. However, at 2 millims. below a vacuum, I detected a tendency of the pith to sink when the experiment was tried; and in a good Sprengel vacuum there was an unmistakable repulsion exerted between the two bodies, the pith sinking in obedience to the radiation from above almost as strongly as it rose when the heat was applied beneath.

33. I now wished to ascertain the effect of cold on the balanced pith balls, and for this purpose a lump of ice was employed.

The experiments were first tried with the balance-tube full of air, cold being applied either above or below the pith ball; a lump of ice generally produced an upward movement of the pith, but it was very faint, and sometimes the motion appeared to be in the opposite direction. It was evident that the true action of a cold body, whatever it might be, was here masked by currents of air; and I therefore exhausted the apparatus and tried the effect of ice at the previously ascertained neutral point, viz. at about 7 millims. below the vacuum. It was absolutely inert. I then carried the exhaustion to the fullest extent, testing the balance with ice during its progress. As the gauge approached nearer and nearer to the height of the barometer, the ice commenced to attract the pith; and at last, when the gauge and barometer were level, the attraction of the ice, whether applied above or below, was very marked, being exactly opposite but equal to the action of the bulb of hot water.

34. In trying some of these experiments in a Sprengel vacuum an action was noticed which led me to think that some of these movements might be due to electricity. When a hot glass rod is held motionless against the lower side of the exhausted tube, the repul-

sion of the pith ball takes place in a perfectly regular manner; but if the glass rod has been passed once or twice through the fingers, or if it is rubbed a few times sideways along the exhausted tube, the beam immediately moves about in a very irregular manner, sometimes being repelled from and at others attracted to the side of the tube, where it sticks until the electrical excitement subsides. When the finger is rubbed against the exhausted glass tube, the same electrical interference takes place; attractions and repulsions occur by fits and starts, the pith sticks to the tube, and does not regain its ordinary state for some hours. When a small spirit-flame is passed beneath the pith end of the balance in the vacuum, a similar but much fainter electrical effect is noticed. This, however, is not sufficient to interfere with the repulsion due to radiation unless the vacuum is very good and free from aqueous vapour.

35. The end of a glass beam in a Sprengel vacuum was found to be attracted by either pole of an induction-coil, when the other pole was not well insulated.

To ascertain whether electricity exerted any special action in the ordinary repulsions in vacuum, the following experiments were tried:—

36. A straw beam with pith extremities was enclosed in a tube (24) and exhausted to the full power of the Sprengel pump. After adjustment by heat, till it was in equilibrium and very delicate to slight radiation, it was re-exhausted and hermetically sealed up. The tube was then completely surrounded with wet blotting-paper, with the exception of a small aperture through which the movement of the beam could be observed; the blotting-paper was connected to earth by a wire soldered on to a gas-pipe.

On bringing a piece of warm copper beneath the pith ball, it rose as readily as if the outside of the tube had been dry and insulated. The finger moistened with warm water also repelled the pith; and when cooled with melting ice, and then applied dripping wet either above or below the pith ball, there was attraction. The same result, but more strongly marked, took place when a piece of ice was used instead of the cold finger.

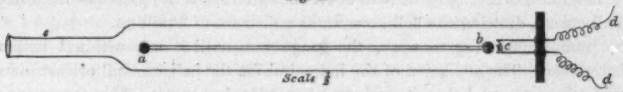
A straw beam furnished with brass balls at each end* was suspended in the usual manner on a double-pointed needle; and the brass balls and needle were placed in metallic connexion by means of a very fine platinum wire. The needle did not rest on the sides of the glass tube but in steel cups, to which was soldered a platinum wire passing through the glass tube and connected to earth. The tube was then exhausted, and the usual experiments tried with hot and cold bodies, both with and without a wet blotting-paper cover. In all cases the brass balls behaved normally, being repelled by heat and attracted by cold.

These experiments show that electricity is not a chief agent in these attractions and repulsions, however much it may sometimes interfere with and complicate the phenomena.

* Preliminary experiments showed me that the brass balls on the straw beam acted in every respect the same as the pith balls, with regard to hot and cold bodies externally applied, both at ordinary pressures and in a vacuum; but they moved in a more sluggish manner.

37. I now wished to see the effect of applying to the gravitating masses a hot body inside the exhausted balance-tube instead of outside, and accordingly constructed an apparatus shown in fig. 4.

Fig. 4.



a, b are two balls suspended on a straw beam; c is a platinum spiral fastened to two stout copper wires cemented into holes drilled through a plate of glass, which can be cemented to the end of the tube; d, d are the battery-wires; e is the extremity of the tube, drawn out for attachment to the Sprengel pump. A single Grove's cell served to heat the platinum spiral to redness. By careful management and turning the tube round the necessary degree, I could place the equipoised brass ball either over, under, or at the side of the source of heat. A contact-key enabled me to heat the spiral without removing the eye from the micrometer. With this apparatus I wished to learn more about the behaviour of the balance during the progress of the exhaustion, both below and above the point of no action, and also to ascertain the pressure corresponding with this critical point.

In air of ordinary density the action of the hot spiral was one of attraction both above and below. Not wishing, however, to complicate the action by air-currents, I exhausted with the Sprengel pump until the gauge stood about 40 millims. belo v the barometer. I then tried the following experiments:—

38. The brass ball was placed so that its position when in equilibrium was about 1 millim. above the spiral, and the latter was rendered incandescent. The ball was immediately drawn down to the spiral, sometimes touching and then rebounding with considerable force.

39. The brass ball was then arranged so that it was about 1 millim. below the spiral. On turning on the battery-current the ball rose to the hot platinum. This latter action might be due to air-currents; but it is difficult to imagine that air-currents could drag the ball down to the hot spiral when the latter was beneath it (38).

40. The ball was arranged so that the platinum spiral was opposite the end, but a little above, as shown in fig. 4 A. On igniting the spiral the movement was very slightly

Fig. 4 A.

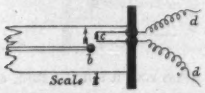
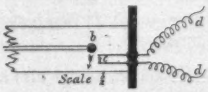


Fig. 4 B.



upwards. When the spiral was rather below the ball, as shown in fig. 4 B, the ball moved downwards when contact with the battery was made; the tendency in each of

these cases was to bring the centre of gravity of the brass ball as near as possible to the source of heat.

41. The pump was then worked until the gauge had risen to 5 millims. of the barometric height. On arranging the ball above the spiral and making contact, the attraction was still strong, drawing the ball downwards a distance of 2 millims.

The pump continuing to work, the gauge rose until it was within 1 millim. of the barometer. The attraction of the hot spiral for the ball was still evident, drawing it down when placed below it, and up when placed above it. The movement was, however, much less decided than before, and in spite of previous experience (30, 31) the inference was very strong that the attraction would gradually diminish until the vacuum was absolute, and that then, and not till then, the neutral point would be reached. Within 1 millim. of a Sprengel vacuum there appeared to be no room for a change of sign.

42. The gauge rose until there was only half a millimetre between it and the barometer. The metallic hammering heard when the rarefaction is close upon a vacuum commenced, and the falling mercury only occasionally took down a bubble of air. On turning on the battery-current, there was the faintest possible movement of the brass ball in the direction of attraction.

43. The working of the pump was continued. On next making contact with the battery, no movement of the ball could be detected. The red-hot spiral neither attracted nor repelled. I had arrived at the critical point. On looking at the gauge I saw it was level with the barometer.

44. The pump was now kept at full work for an hour. The gauge did not rise perceptibly; but the metallic hammering increased in sharpness, and I could see that a bubble or two of air had been carried down. On igniting the spiral, I found that the critical point had been passed. The sign had changed, and the action was faint but unmistakable *repulsion*. The pump was still kept going, and an observation was taken from time to time during several hours. The repulsion continued to increase. The tubes of the pump were now washed out with oil of vitriol*, and the working was continued for an hour.

45. The action of the incandescent spiral was now found to be energetically repellent, whether it was placed above or below the brass ball (figs. 4 c, 4 d). The finger

Fig. 4 c.

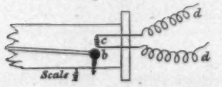
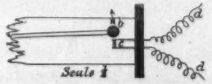


Fig. 4 d.



exerted a repellent action, as did also a warm glass rod, a spirit-flame, and a piece of hot copper.

* This can be effected without interfering with the exhaustion. (See Philosophical Transactions, 1873, vol. clxiii. p. 296.)

46. I now heated the sulphuric acid, in the U-tube desiccator attached to the pump, until it boiled. A little aqueous vapour was driven off; the sharpness of the hammering of the mercury changed to a duller sound; and on trying an experiment once more with the ignited spiral, the action was seen to have returned to one of attraction.

On leaving the pump to itself for an hour or two, the sulphuric acid, as it cooled, again took up the aqueous vapour; and as the absorption proceeded I could trace the action of the hot spiral from attraction, through the point of neutrality, up to decided repulsion.

47. The critical point, in the case of brass balls, is much higher than with pith balls. In the case of pith, applying heat outside, I obtained neutrality when the gauge was about 7 millims. below the barometer, and decided repulsion when there was still a difference of 2 millims. With the apparatus just described, however, using brass balls and an internal source of heat, the critical point is very near the Sprengel vacuum. By close observation I can just distinguish that the gauge is a fraction of a millimetre below the barometer when the hot spiral ceases to exert an attractive action on the brass ball; but my unassisted eye is not able to detect a difference on the gauge between a Sprengel vacuum in which the ignited spiral is neutral to, and one in which it strongly repels, the brass ball.

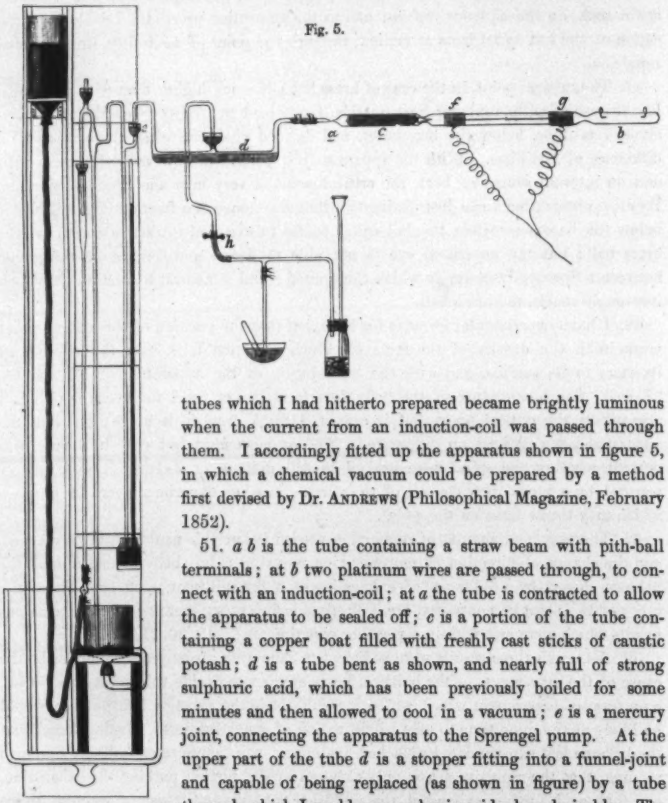
48. I have experimental grounds for believing that the position of the critical point varies with the density of the mass on which radiation falls, with the relation of its mass to its surface, and with the temperature of the apparatus (62, 63, 64, 65, 67, 75). The temperature of the body used to attract or repel the brass or pith ball also affects the critical point, but it cannot, I think, modify it much; for with the apparatus last described no difference in kind of movement, but only in degree, was noticed, whether the spiral were ignited to full redness, or whether it were merely warmed by a momentary contact of the wires. Further experiments are in progress which may throw light on this point.

49. The very high amount of rarefaction needed before this neutral point is reached, and the change of direction of movement on applying a hot body to one arm of the balance, caused by a difference of exhaustion of a few millimetres on one side or the other of the point of neutrality, are, I think, a sufficient explanation of the anomalous results which were met with in my earlier experiments (2, 19, 20, 21, 22).

50. Although these results were sufficient to show that air-currents could not be the cause of the movements of the balance, I was anxious to decide this point once for all, by a form of experiment which, whilst it would settle the question indisputably, would be likely at the same time to afford information of much interest. Having found that the balance last experimented with had its neutral point close to an ordinary Sprengel vacuum, that the repulsive action only came on by still further pushing the exhaustion, and that, as I further exhausted, the repulsion got stronger, it was of interest to see what would take place in a vacuum so nearly perfect that it would not carry a current from a RUHMKORFF'S coil.

In small tubes, and taking certain special precautions, I could prepare a vacuum with my Sprengel pump which would hardly allow an induction-current to traverse it, or would only show a faint, cloud-like discharge; but it was impossible to effect this in the large tubes required for these experiments, and, in fact, all the Sprengel vacuum balance-

Fig. 5.



tubes which I had hitherto prepared became brightly luminous when the current from an induction-coil was passed through them. I accordingly fitted up the apparatus shown in figure 5, in which a chemical vacuum could be prepared by a method first devised by Dr. Andrews (Philosophical Magazine, February 1852).

51. *a b* is the tube containing a straw beam with pith-ball terminals; at *b* two platinum wires are passed through, to connect with an induction-coil; at *a* the tube is contracted to allow the apparatus to be sealed off; *c* is a portion of the tube containing a copper boat filled with freshly cast sticks of caustic potash; *d* is a tube bent as shown, and nearly full of strong sulphuric acid, which has been previously boiled for some minutes and then allowed to cool in a vacuum; *e* is a mercury joint, connecting the apparatus to the Sprengel pump. At the upper part of the tube *d* is a stopper fitting into a funnel-joint and capable of being replaced (as shown in figure) by a tube through which I could pass carbonic acid when desirable. The carbonic acid was prepared by the action of hydrochloric acid on marble; when not being passed into the exhausted tube, the gas was kept bubbling through mercury,

where a tubeful could be collected from time to time (as shown in the figure) to test with potash. It was found necessary to keep the evolution going on all the time pretty briskly, to prevent air diffusing in. The joints were made of double caoutchouc tubing, the smaller one tightly wired on and coated with glycerine before the larger tube was slipped over it. The whole was then tightly bound with wire. To prevent air creeping down between the mercury and the glass, glycerine was poured over all the mercury joints, except the one at the top of the mercury fall-tube, which was kept for oil of vitriol, with which the pump was lubricated from time to time.

52. The apparatus being exhausted of air, the balance was adjusted by heating the ends so as to slightly char the one which happened to be the lower. f and g are two collars of silver foil encircling the tube where the heat is to be applied, and connected with earth by wire. At a very high rarefaction the flame of a spirit-lamp excites so much electrical disturbance in the balance, that its adjustment becomes well nigh impossible. This arrangement was adopted in the endeavour to carry off the electricity; it is, however, only partially successful, and the electrification of the balance at the highest rarefactions is still very troublesome.

The air having been removed from the apparatus as perfectly as the Sprengel pump would effect it, carbonic acid was let into the tube by cautiously opening the tap h. Exhaustion was again effected, and carbonic acid passed in a second time. This was then pumped out, and the apparatus was filled a third time. This alternate filling with carbonic acid and exhaustion was continued until the gas collected at the bottom of the mercury fall-tube of the pump was entirely absorbed by potash. When this was found to be the case, the exhaustion was allowed to proceed to the highest possible point *. The pump was then stopped; an induction-current now being passed between the wires at the end b showed the usual white light of a carbonic-acid vacuum (a trace of red shows atmospheric nitrogen).

The sticks of potash in the copper boat in c were then heated to incipient fusion, and the whole was allowed to cool for some hours. The tube was then sealed off by applying a spirit-flame to the contracted part a; the potash was then heated again, and the whole was set aside to give the potash time to absorb the residual carbonic acid.

53. By testing from time to time with an induction-coil, the progress of the absorption could be traced; and when the current ceased to pass through the tube, but preferred to strike across in air the full length of the spark, the vacuum was considered nearly perfect. Warming the potash with a spirit-flame, at any time, will cause it to give off sufficient aqueous vapour to allow the spark to pass as a cloud-like luminosity. This will be gradually absorbed until, in the course of from a few days to a few weeks, the vacuum will again cease to conduct.

54. It is very difficult to get the vacuum in large tubes so nearly absolute as not to

* When the pump is working in a very good vacuum, the friction of the falling mercury produces a very beautiful effect in the dark. Brilliant points of light flash about wherever the mercury-drops are splashed from side to side, and the pump is frequently illuminated with a phosphorescent glow filling all the tubes.

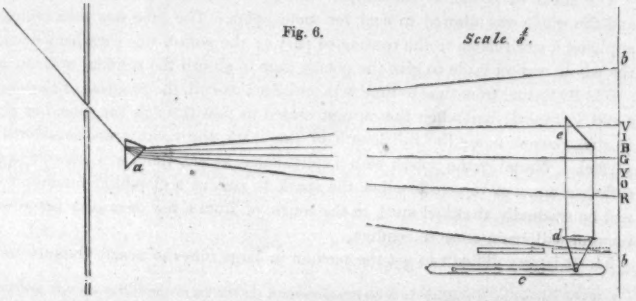
allow a white, cloud-like discharge to pass. I have tried experiments with balances sealed up in vacua of both kinds prepared in this manner, and I always find the repulsion on the approach of a hot body is very decided, although I do not think it is more energetic than with the same kind of balance enclosed in the best Sprengel vacuum I could prepare. These experiments have therefore set at rest the doubts which might have arisen had I only worked with the Sprengel vacua, that air-currents were the cause of the phenomena. They have decided the important point, that in a chemical vacuum which will not carry an induction-current the repulsion by radiation is decided and energetic.

55. Besides the straw and pith balance mentioned above (51, 52), I have prepared and sealed up in chemical vacua balances made of a glass beam and platinum terminals, and also of glass entirely, the ends being flat glass plates. They act in all respects as the straw and pith balance, only being somewhat less sensitive.

56. The delicacy of one of the straw and pith balances in a good vacuum is very great, as may be comprehended by the following experiment:—The balance-tube was supported on a stand, and immediately beneath one of the pith balls was placed the face of a bismuth-antimony thermopile. On connecting the terminal wires of the pile with a single cell-battery, so that the current flowed in one direction, heat was produced, and the pith ball was repelled. On reversing the current, cold was produced and the pith ball was attracted.

57. The rays of the sun allowed to shine on the terminal pith ball of one of these delicate balances *in vacuo* repel it strongly. If concentrated by a lens, the focal point beats the ball away, as if it were struck with a material agent. If the sun's light be filtered through coloured glasses before concentrating them on the pith ball, the action is slightly diminished. Passing the light through two very clear plates of alum with parallel sides, and having an aggregate thickness of 8 millims., had but little action.

Fig. 6.



58. A beam of sunlight was passed through a prism *a* (fig. 6), and the spectrum was projected horizontally on to a screen (*b*) 18 feet off. In front of the screen, a little

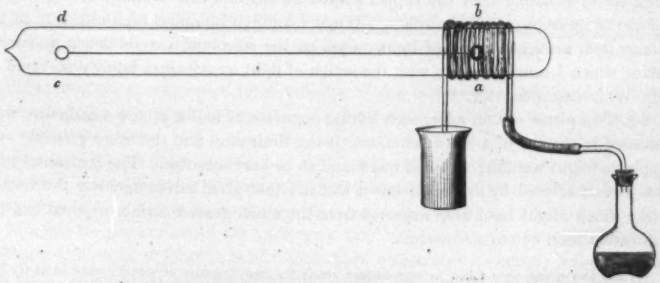
below it, was arranged the balance-tube *c*, having fixed above it a lens *d*, so that one of the pith balls was in the focus of the lens. A small right-angled prism (*e*) was then held in the different parts of the spectrum, so as to reflect any desired ray on to the lens and concentrate it on to the pith ball. By turning the right-angled prism sideways, the ray could be thrown off and on the ball, and the action readily noted. In the ultra-violet rays the repellent action was slight; the effect increased as I made the reflecting prism traverse the spectrum from the violet to the red. The maximum action was in the extreme visible red, and it gradually faded away as the invisible rays beyond the red were reflected down. An appreciable action of repulsion was, however, observed a full half spectrum in length below the least refrangible visible rays. The plates of alum, interposed in the path of the rays, cut off a small portion only of their action.

59. The following experiments were tried with a very sensitive pith-ball balance sealed up in a good Sprengel vacuum:—

The beam was turned over, so that the centre of gravity was a very little above the centre of suspension; the balance consequently set on either side. In this condition the ball touching the tube could be made to rise by placing the finger beneath it, the balance then oversetting on the other side. The higher ball could also be made to sink in the same manner by placing the finger on the top of the tube just over it.

60. A piece of glass tube, bent in the form of a spiral (fig. 7), was supported on a stand, so that it could be slipped over one end of the balance-tube, which it fitted

Fig. 7.



loosely. Steam was then passed through the glass coil, and a woollen cover was put on it, so that the end of the balance-tube could be kept at 100° C. When the temperature became uniform and steady the pith ball took up a position, as nearly as could be judged, in the axis of the spiral, and then remained stationary. On raising the hot coil, so as to bring the lower part close to the tube, the pith ball rose a little, and on lowering the spiral the pith ball sank.

On applying the warm finger to the cool extremity of the balance-tube, either above or below the pith ball, the latter was repelled by it; and by employing a lump of ice in the same way in each position the pith ball was attracted. The balance seemed to be as sensitive when one end was heated in this manner as it was when both ends were of the same temperature; but the rise and fall were not to so great an extent, owing to the controlling action of the hot spiral.

61. The following experiments were tried with a view of ascertaining the conditions of greatest sensitiveness. In all cases the balances were in a good Sprengel vacuum.

A ball of ivory was balanced against a ball of brass on a straw beam; the ivory was more sensitive to radiation than the brass.

62. A pith ball and one of platinum of the same weight were balanced on a straw beam. The pith was very sensitive, being readily repelled by the finger; but the platinum was sluggish, and required a spirit-flame to move it.

63. Two pith balls of the same weight, one gilt and the other plain, were fixed to the ends of a straw beam; they appeared equally sensitive to radiation.

Two rectangular blocks of silver and bismuth, each weighing $4\frac{1}{2}$ grains, were balanced against one another on a straw beam*; they were each repelled by a warm body applied above or below. The bismuth was a little more sensitive than the silver to the action of heat, but it exposed a little more surface for the rays to impinge upon.

64. A selenium ball was balanced against a copper ball: the selenium was more sensitive to radiation than the copper; but I do not think it was more so than would be due to its more extended surface. When I allowed luminous rays, either from the sun or from artificial sources of light, to fall on the selenium, I could detect no special action which I could correlate with the action of light on selenium lately discovered by Mr. Willoughby Smith†.

65. Two pieces of thin mica, each having a surface of half a square centimetre, were fastened to the end of a straw beam, one being horizontal and the other vertical. On applying slight warmth, each end was found to be very sensitive. The horizontal mica was easiest affected by the heat; but it had not that great advantage over the vertical piece which might have been expected from the much greater surface exposed had the movement been due to air-currents.

* These two metals were taken as representing nearly the two extremes of metallic conductors for heat and electricity, and also being the best (silver) and the worst (bismuth) in the list of "vibrators" given by Professor Forbes in his "Experimental Researches regarding certain vibrations which take place between metallic masses having different temperatures" (Transactions of the Royal Society of Edinburgh, 1834, vol. xii. pp. 429–461).

† Telegraphic Journal, vol. i. p. 78.

Received August 18,—Read December 11, 1873.

66. A piece of flat plate-glass, 15 millims. wide and 1·5 millim. thick, was heated in the middle before the blowpipe till quite soft, and then drawn out till a long ribbon of glass was produced, the width and thickness of which retained the proportions of the original piece. From the middle of this a portion 150 millims. long, 4 millims. wide at the ends, and 3 millims. wide in the centre, was cut off. A double-pointed needle (24) was then secured to the centre by binding with platinum wire and fusing the latter to the glass. This little balance was adjusted until it was very delicate, and was then enclosed in a tube containing potash at one end and platinum wires sealed in at the other end; it was then arranged, in connexion with the Sprengel pump and carbonic-acid apparatus, so as to produce a chemical vacuum (51, 52, 53).

When the flame of a spirit-lamp was passed under one end of this balance, the apparatus being full of air at the ordinary density, the result was decided attraction (27, 37, 38, 40, 41). On exhausting and testing from time to time in the same manner*, I found the attraction, or rather the sinking of the heated end towards the spirit-flame, to keep at about the same strength until the gauge had risen about 500 millims. After this there was a gradual decline in the downward movement of the glass end of the balance when heat was applied, until the gauge stood about 100 millims. below the barometer. On testing the movement at this pressure I at first thought it was very slight; but on keeping the flame of the lamp for about half a minute below the end of the balance the latter commenced to sink, and then the downward movement was almost as great as ever. With a difference of 95 millims. the phenomena were similar, the lamp, however, requiring to be kept under the balance end longer, and the ultimate movement not being so great. At 90 millims. the same thing occurred, the time of heating having to be still longer.

When the gauge was raised to 85 millims. of the barometric height, and the lamp was applied, the first movement was one of repulsion. The glass end rose instantly, but to a very slight extent. On keeping the lamp under for about a minute, the end of the balance slowly came down, until it had sunk a little below its original position.

At 80 millims. difference between gauge and barometer the effect was almost the same as at 85 millims. The preliminary rise was, if any thing, a little more marked. For fear of injuring the connexions of the apparatus, I did not like to apply the spirit-flame for a longer time than a minute, as the glass then commenced to soften; I therefore, at the higher exhaustions, paid most attention to the initial movement of the glass beam, merely keeping the lamp beneath long enough to see if the continued heating drew the beam down again.

At a difference of 45 millims. between the gauge and the barometer, the neutral point was about reached. The glass beam was not quite motionless on applying the spirit-

* It is perhaps scarcely necessary to say that the apparatus was allowed full time to acquire the ordinary temperature of the laboratory between each of these experiments.

lamp: it rose very slightly at first, and then descended to the same distance below the original level on continuing the heat.

At 35 millims. the initial rise was immediate and decided. Continued heat only lowered the end a trifle, but did not bring it down to any thing near its original position.

From this point the attraction towards the heat ceased to be perceptible. The upward movement increased in strength and amplitude until the Sprengel vacuum was reached *.

At 10 millims. below the barometer the glass beam was repelled by a glass rod or a lump of copper heated to 100° C.; at a difference of 5 millims. between the gauge and the barometer the finger applied below repelled the beam, and at 2 millims. difference the beam was repelled when the finger was applied above.

In the Sprengel vacuum the beam was very sensitive to the approach of a warm body, a touch of the finger sending it away to the fullest extent.

67. I consider that the difference observed between the behaviour of this glass beam, the straw beam with brass ends (37 to 40), and the straw beam with pith ends (30, 31, 32) is to be accounted for by the different materials forming the balances, the different way in which heat was applied in the three cases, and the extent of surface exposed in proportion to mass. There is, however, much to be found out in respect to the position of the critical point, and I am still at work on the subject.

Carbonic acid was then let into the apparatus, and it was re-exhausted. Not wishing to tax the potash too severely, I did not spend much time during this exhaustion; a few observations were, however, taken at different heights of the gauge, and the position of the neutral point was apparently a little raised. The general results were, however, the same as with air. In a carbonic-acid Sprengel vacuum the repulsion by heat was exactly the same as in an air Sprengel vacuum.

68. The remainder of the process for producing a chemical vacuum was then gone through; the tube was set aside till the potash had done its work, and experiments were then tried with it. To the heat of a spirit-lamp, a warm glass or metal rod, the fingers, or the image of a gas-flame concentrated by a lens, the balance was very sensitive, being instantly repelled to a distance varying with the intensity of the heat. Experiments were tried with different rays of the solar spectrum with a similar result to that described in par. 58. The vacuum was so nearly perfect that an induction-spark would not pass, but preferred to strike across its full distance in air.

69. If one of the most sensitive balances (straw and pith (24), or the glass one last described (66)) in a well-exhausted tube is carefully turned on its side, after a little

* In these and many of the other observations accurate measurements were taken of the extent of movement by means of a micrometer-eyepiece, and the time was also accurately noted by a seconds' watch. As, however, the equal and uniform delicacy of the instrument could not be depended upon, I think it best only to give the results in general terms rather than to mislead by an affectation of accuracy not justified by the instrumental means employed.

practice the beam can be balanced on one point of the suspending-needle, which will be nearly vertical. In this position the beam has a horizontal movement; and by carefully adjusting the level of the tube the delicacy of the beam can be made very much superior to what it would be suspended in the ordinary way. By bringing a warm body near one end of the balance, it is now driven away to the utmost extent, and a piece of ice attracts it with equally marked energy.

70. On trying this experiment in air of ordinary density, the approach of a hot body causes unmistakable attraction, and a cold body repulsion. In a vacuum this mode of arranging the apparatus did not at first appear to offer advantages over the plan already adopted; but as from the direction of movement it was not likely that air-currents could interfere, or at all events not to any great extent, I have arranged apparatus for obtaining the movements of repulsion and attraction in a horizontal instead of a vertical plane, so as to examine the action in air.

Instead of supporting the beams on needle-points, so that they could only move up and down, I suspend them by the centre to a long fibre of cocoon-silk in such a manner that the movements would be in a horizontal plane. With apparatus of this kind, using very varied materials for the index, enclosing them in tubes and bulbs of different sizes, and experimenting in air and gases of different densities up to Sprengel and chemical vacua, I have carried out a large series of experiments, and have obtained results which, whilst they entirely corroborate those already described, carry the investigation some steps further in other directions. I propose shortly to submit an account of this second series of researches to the Society.

71. I have more recently instituted experiments to ascertain how far the action of gravitation in CAVENDISH'S celebrated experiment is likely to be modified under the influence of heat. For many months I have been experimenting with apparatus devised for this purpose. The investigation is not sufficiently advanced to justify further details, but I may perhaps be permitted to give here an outline of one of the results.

72. I find that a heavy metallic mass, when brought near a delicately suspended light ball, attracts or repels it under the following circumstances:—

I. *When the ball is in air of ordinary density.*

a. If the mass is *colder* than the ball, it *repels* the ball.

b. If the mass is *hotter* than the ball, it *attracts* the ball.

II. *When the ball is in a vacuum.*

a. If the mass is *colder* than the ball, it *attracts* the ball.

b. If the mass is *hotter* than the ball, it *repels* the ball.

73. The density of the medium surrounding the ball, the material of which the ball is made, and a very slight difference between the temperatures of the mass and the ball exert so strong an influence over the attractive and repulsive force, and it has been so difficult for me to eliminate all interfering actions of temperature, electricity, &c., that I have not yet been able to get distinct evidence of an independent force (not being of the nature of heat or light) urging the ball and the mass together.

Experiment has, however, shown me that, whilst the action is in one direction in dense air, and in the opposite direction in a vacuum, there is (as I have already pointed out with the balances) an intermediate pressure at which differences of temperature appear to exert little or no interfering action. By experimenting at this critical pressure, and at the same time taking all the precautions which experience shows are necessary, it would seem that such an action as was obtained by CAVENDISH, REICH, and BAILY should be rendered evident.

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74. Throughout the course of these investigations I have endeavoured to keep in my mind the possible explanations which may be given of the actions observed, and I have tried, by selecting some circumstances and excluding others, to put each hypothesis to the test of experiment. The most obvious explanation is that the movements of the beam, or of the horizontal index (69, 70), are due to the currents formed in the residual gas, which, theoretically, must be present to some extent even in those vacua which are most nearly absolute.

In favour of this explanation it may be urged that a highly rarefied gas may be much more mobile than when it is denser, and therefore the more rapid impingement of its particles, when set in ascension by warmth, would increase their mechanical action. Increased momentum may counterbalance diminished number.

That the residual gas in an air-pump vacuum is capable of exerting considerable mechanical action, may be assumed by the phenomena attending the passage of meteorites through the upper regions of the atmosphere, their friction against the air at an average height of 65 miles above the earth's surface raising them to incandescence, although at that height the attenuation of the air probably surpasses that of most artificial vacua.

On the other hand, it is most difficult to believe that the residual air in a Sprengel vacuum, where the gauge and barometer are appreciably level, can exert, when gently warmed by the finger, an upward force capable of instantly overcoming the inertia of a mass of matter weighing several grains, and setting it in motion. It must be remembered that the upward current supposed to do this is simply due to the diminished weight of a portion of the gas, caused by its increase in volume by the heat applied.

75. Another argument in favour of the air-current explanation can be drawn from the fact that when a light beam, having equal weights of pith and platinum suspended at the ends, is sealed up in a Sprengel vacuum, the application of warmth below causes the pith to rise more readily than the platinum (62); the pith obviously offers a much more extended surface than the platinum does to the impact of air-particles.

This, moreover, is not an isolated instance. Throughout the whole of these experiments the law appears to be that the force exerted is in proportion to the extent of surface exposed (48, 62, 63, 64, 65, 67) rather than in proportion to the mass. Much

surface and extreme lightness are the requisites in selecting materials for the beam, index, or gravitating mass; and when the masses have the same specific gravity and extent of surface, their position in respect to the source of heat determines the extent of movement. Thus a cylinder of pith is more sensitive when arranged for the heat to act on its side than on its end; and the film of mica in experiment 65 was more affected when the heat struck its flat surface than its edge, although the difference was not so great as might have been expected had air-currents been the cause of motion.

76. But these facts can equally well be used on the opposite side; for assuming that the movement is due to a repulsive action of radiation, it is reasonable to suppose that extended surface, weights being equal, would have an advantage. The repulsion by radiation only acts on the surface of bodies, and does not seem to act on the molecules which constitute thickness. When radiant heat gets below the surface of a body, it spends itself in doing mechanical work of another kind, viz. dilatation or expansion.

77. However strong may be the reasons in favour of the air-current explanation, they are, I think, answered irrefragably by the phenomena themselves. An air-current produced by heat can cause the beam of a balance to rise, can drive a suspended index sideways, and, by a liberal assumption of eddies and reflections, can perhaps be imagined to cause these movements to take place sometimes in the opposite directions; but as rarefaction proceeds these actions will certainly get less, and they will cease to be appreciable some time before a vacuum is attained: a point of no action or neutrality will be reached. But this neutral point should certainly be nearer a vacuum when a light body exposing much surface, such as pith, is under experiment than when the mass acted on is heavy like brass; whereas in practice the contrary obtains. Pith and thin glass balances, which should be sensitive to highly attenuated air-currents, cease to respond to heat at a rarefaction of 7 millims. (30) and 45 millims. (66), whilst brass only ceases to be affected when the gauge and the barometer are appreciably level (43).

But even could the phenomena up to neutrality be explained by air-currents, these are manifestly powerless to act after this critical point is passed. If a current of air within 7 millims. of a vacuum cannot move a piece of pith, certainly the residual air in a Sprengel vacuum should not do so; and, *à fortiori*, the residual air in a chemical vacuum could not move a piece of platinum (55).

It is, however, abundantly demonstrated that in all cases, after this critical point is reached, the repulsion by radiation is most apparent, and it increases in energy as the vacuum approaches perfection.

78. Again, the movements not only reappear on passing a particular point of atmospheric density, but they take place in the opposite direction (31, 32, 33, 44, 46, 66). Thus in all cases, when the atmospheric density is between the neutral point and a vacuum, the action of a body hotter than the moving beam or index is to repel it, whilst the action of a colder body is to attract. Now it is very probable that were it not for the interference of air-currents the action in air of greater density would always be for the hot body to attract. This is actually the case in many experiments (27, 37,

38, 40, 41, 66, 70, 72); and observations more recently made, but only alluded to in par. 72, have proved that in air of ordinary density a cold body repels.

On the supposition that air-currents are the motive power, the effects noticed when the source of heat is internal to the tube, and applied above the moving beam (37, 39, 40, 41, 45), are inexplicable, whilst they are easily comprehended on the repulsion-by-radiation hypothesis.

If an additional argument is necessary to show that air-currents are not the cause of the repellent action of a hot body, I bring forward the fact that the movement attains its maximum when there is no air at all present (54, 55, 68).

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79. Effects probably due to this repulsive action of radiation are constantly met with. I will instance the following:—

Cohesion and adhesion are diminished by heat. This naturally follows if increased temperature augments the force of repulsion between the molecules.

The phenomenon of the spheroidal state is probably due in some measure to a repulsive force exerted between closely approximated bodies, one of which is at a very high temperature. This action is generally supposed to take place only when one of the bodies is volatile, and the rapidly formed skin of vapour is held to be a sufficient cause of non-contact. I venture to anticipate that a condition similar to the spheroidal state will be found to obtain between non-volatile bodies.

Many finely divided chemical precipitates, when incandescent in a platinum crucible, assume a remarkable mobility and flow about like water. Precipitated silica is an instance which will occur to chemists. A space can readily be distinguished between the powder and a hot capsule containing it. Electricity may, however, play some part in this action; for precipitates, when heated, sometimes become sufficiently electrical by stirring with a glass rod to fly out of the basin containing them; oxalate of lime possesses this property in a remarkable degree*.

80. It must, however, be remembered that my experiments show the action of hot bodies in air to be that of attraction, and that the repulsion by heat only becomes evident near upon a vacuum. It is seen, therefore, that radiant light or heat has an attractive or repulsive action, according to the medium in which it acts, corresponding results being furnished by cold. There appears to be an interfering action of air other than that of the currents caused in it by heat, which masks or overcomes the true action of heat; but in a vacuum this interfering cause is absent, and radiant heat is free to exert its full repellent action, whilst cold or negative heat acts in the opposite direction. Heat and cold, heat present and heat absent—molecular activity and molecular rest—are therefore antagonistic in their action on a body free to move in empty space. The molecules of matter whose mode of motion constitutes heat are drawn together and

* Faraday's 'Experimental Researches in Electricity,' vol. ii. p. 163.

condensed as these vibrations diminish in amplitude, whilst heat drives them apart, expanding a solid, changing a solid into a liquid and a liquid into a gas.

The masses used in my experiments are likewise repelled by heat and drawn together by cold. And it is with no weak force or feeble action that I have been dealing. It is so decided that in some of my balances the approach of a finger will completely overturn them, whilst the radiant warmth of the body affects them 6 feet off; and at higher temperatures and with larger masses the action must be still more energetic.

81. It is not unlikely that in the experiments here recorded may be found the key of some as yet unsolved problems in celestial mechanics. In the sun's radiation passing through the quasi vacuum of space we have the radial repulsive force, possessing successive propagation, required to account for the changes of form in the lighter matter of comets and nebulæ; and we may learn by that action, which is rapid and apparently fitful, to find the cause in those rapid bursts which take place in the central body of our system; but until we measure the force more exactly we shall be unable to say how much influence it may have in keeping the heavenly bodies at their respective distances.

So far as repulsion is concerned, we may argue from small things to great, from pieces of pith up to heavenly bodies; and we find that repulsion shown between a cold and warm body will equally prevail, when for melting ice is substituted the cold surface of our atmospheric sea in space, for a lump of pith a celestial sphere, and for an artificial vacuum a stellar void.

Attraction being developed by radiant heat under influences connected with air, it is not easy to conceive how it will be produced for cosmical purposes by heat; the upper surface of our atmosphere must present a very cold front, and from this we might argue repulsion by the sun, unless we fill space with a body acting like air, when we should have attraction. We might readily find conditions for both, but how to harmonize them is a difficulty.

Although the force of which I have spoken is clearly not gravity solely as we know it, it is attraction developed from chemical activity, and connecting that greatest and most mysterious of all natural forces, action at a distance, with the more intelligible acts of matter. In the radiant molecular energy of solar masses may at last be found that "agent acting constantly according to certain laws" which NEWTON held to be the cause of gravity.

XVI. *On Electrotorsion.* By GEORGE GORE, *F.R.S.*

Received November 26, 1873,—Read January 8, 1874*.

WIEDEMANN has experimentally examined† the influence of magnetism upon the mechanical torsion of iron wire, and has shown that an iron wire hung in the centre of a helix and twisted is more or less untwisted when a current traverses the helix. But as the torsion in his experiments was produced by the combined influence of a voltaic current and previous mechanical twist, and is quite a distinct phenomenon from that produced by the combined influence of electric currents only, which forms the subject of this communication, and as no one appears to have discovered the particular class of phenomena which are treated of in this investigation, I take an opportunity of making known my experiments and the new facts I have found.

[Since the publication of the abstract of this paper in the 'Proceedings of the Royal Society,' vol. xxii. p. 57, January 8, 1874, Professor WIEDEMANN (to whom I had sent a copy of that abstract) has kindly written to me as follows:—"You have found independently some results which I had already published in the year 1862 in POGGENDORFF'S 'Annalen,' vol. cxvii. p. 208. A short abstract of these experiments is also given in my 'Treatise on Galvanism, &c.' (1st edition, vol. ii. p. 445; 2nd edition, vol. ii. p. 565), where you will find my complete theory of the relations between magnetism and torsion."]

1. *Apparatus employed.*

I took a voltaic helix, 2·45 metres long and 1·5 centimetre internal diameter, of insulated copper wire 1·8 mm. thick, coiled as a double conductor and in two layers. To form a tube upon which to coil the wire, two half-round strips of wood, with a semicircular groove in each, were glued together, and the superfluous glue removed from the interior whilst still warm by means of a rope soaked in hot water. A brass tube may be used instead of the wooden one, as it does not prevent the torsions. The helix was fixed in an upright pillar of wood (Plate XLII. fig. 1), and a string $2\frac{1}{2}$ metres long, supporting a leaden bullet, was attached to a pin near the top of the coil to enable the apparatus to be placed vertical.

A straight rod of soft iron, 2·6 metres long and 11 mm. diameter, was entirely enclosed, except its ends, within the helix, and securely suspended by a brass clamp.

* The passages enclosed in square brackets were added in July 1874.

† WIEDEMANN'S 'Die Lehre von Galvanismus und Elektromagnetismus,' vol. ii. p. 559; Pogg. Ann. vol. ciii. p. 571, May 1856, and vol. cvi. p. 161, 1859; Annales de Chimie, vol. liii. 1858, p. 379, and vol. lvi. p. 373, 1859.

screw attached to its upper end and fixed to the top of the wooden support. To the lower and free end of the bar was fixed by a screw a movable and rigid horizontal pointer of brass, 47 centims. long, weighing about 300 grammes (=10½ ounces), and balanced by an adjustable counterpoise. In a few experiments, where rapid and continuous *vibration* was required, the free end of the pointer was tipped with a thick piece of platinum wire.

With this apparatus the torsional movements were sufficiently large not to require the employment of the method of detecting and measuring them by means of a mirror and telescope, or luminous image reflected to a distant graduated scale. The lower end of the iron rod had fixed to it a vertical piece of thick platinum wire about 3 centims. long, which dipped into mercury contained in a little platinum cup separately supported by a horizontal rod of copper to act as a conductor; or it had sometimes attached to it by means of a screw, B (fig. 2), a strong loop of copper, A, for the purpose of supporting a weight, and provided with a stout platinum wire (C) to dip into the mercury. Instead of the axial bar, a wire of iron, about 2 mm. thick, was generally employed, and the torsions obtained with it were much greater. The helix was excited by means of 6 or 12 GROVE'S elements, each of about half a litre (or one pint) capacity, and having platinum plates 10 centims. long and 8 centims. wide.

The current from 4 or 6 of those cells, in double series, being now passed in an uninterrupted state through the coil, and simultaneously with it, or during its passage, the one from the remaining 6 or 8 cells, in double row, being passed through the bar or wire, the index moved either to the right hand or the left, according to the directions of the currents.

By fixing a platinum-tipped horizontal brass screw upon a separate support, so that the screw just touched the side of the platinum end of the pointer, and completing the axial circuit through it, continued vibration of the pointer was obtained, provided the screw was placed on the proper side and the index was sufficiently rigid.

The more conspicuous effects may be successfully obtained with a much smaller apparatus.

In all cases the torsion was accompanied by elongation, and the resulting motion was in a diagonal direction, and the maximum of vibration was therefore obtained by placing the contact-screw so as to touch the index between its side and upper surface. By placing the screw in such a position that the vertical movement alone did not break the contact, rapid vibration, due to torsion only, could be maintained during any length of time; and by placing it in contact with the upper surface of the pointer and passing the current through the coil and index only, continued vibration, due entirely to elongation, could also be readily obtained.

2. *Does an axial electric current lengthen an iron bar?*

To ascertain this I passed the current through the bar only; but not even a tendency to vertical vibration could be perceived. I also tried a thinner rod, and a wire strained

by a weight, and applied the contact-screw to the *under* side of the index (as well as to the upper), in order to detect any shortening, but obtained no positive results; probably an extremely minute degree of shortening occurred, and was counterbalanced by expansion produced by heat of conduction-resistance.

3. *Methods and conditions of obtaining the torsions.*

The torsions may be produced:—(1st) by the passage of axial currents alternately in opposite directions, (2nd) by the alternate passage of coil-currents and axial ones, (3rd) by the simultaneous passage of both, and (4th) by the temporary passage of an axial current during the continuance of a coil one, or *vice versâ*. The 1st and 2nd methods yield only very small torsions, and the 3rd and 4th produce exceedingly large ones.

The best conditions are:—a slender wire of the softest iron, with but little weight attached to it, and free from mechanical twist, enclosed nearly its entire length in a powerful voltaic coil; a downward current in the iron wire simultaneous with a coil-current producing a north* pole below; and repeatedly applying the pair of currents synchronously with the movements of the index (the two currents may be made simultaneous in their action by causing them either to leave the two batteries or return to them by a single wire). Or better, a continuous current in the coil, producing a south pole below and a temporary one in the axial wire, and reversing the direction of the latter synchronously with each half-vibration of the pointer: a very convenient plan is to pass the entire current through the coil, and during its continuance divert, by means of a reverser, such portion of the current as will flow through the axial wire, and reverse the direction of that portion at the end of each half-vibration. A quantity of electricity passing through the axial wire of about four times that which circulates through the coil is a suitable proportion.

With a soft-iron wire 2·6 metres long and 1·75 mm. thick, four Grove's cells being arranged as two attached to the coil, and eight as four to the axial wire, a single contact produced a movement of 23 mm. of the end of the pointer, and by synchronous contacts a swing of 80 centims. (or 31 inches=102 degrees), or more than a quarter of a circle, was easily obtained. [With a shorter and thinner wire and suitable helix, and the current of a Noë's thermopile of 80 elements connected as 20×4, a swing of 36 inches was produced.] The torsional movements, when first observed, were so small as to require the aid of a magnifying-glass in order to detect them; they are all attended by audible sounds in the iron.

The currents employed in these experiments being large and of low tension, the following reverser offering but little resistance was employed. Upon a wooden base were fixed four binding-screws, A, B, C, and D (Plate XLII. fig. 3). To A and B were fixed two flat and very flexible brass springs (E and F), united by a cross piece of

* Throughout this paper I call that a north pole which points to the south, and that a south pole which seeks the north.

ivory, G. To the opposite end of the base-board were also fixed by screws (H and I) two similar springs (J and K) similarly united by the ivory, L. The screw A was connected with I by a stout copper wire beneath the board; and B was similarly connected with H. To the free end of each of the four springs was attached, by means of a small screw, a vertical thick wire of platinum which projected beneath the spring for the purpose of dipping into one of two small cavities (M and N) containing mercury. The screws C and D were also provided with similar wires for dipping into the mercury.

4. *Is a magnetic metal necessary?*

I tried wires of platinum, silver, copper, lead, tin, cadmium, zinc, magnesium, aluminium, brass, and German silver, also a zinc rod 2·75 m. long and 11 mm. thick, and applied the currents in various ways, but obtained no signs of torsion. A cord of gutta percha, strained by a weight, was also subjected to the influence of the coil-current, but without effect.

5. *General cause of the torsions.*

The torsions are not due to momentary induction-currents of electricity, because they continue as long, and in many cases only as long, as the currents, and because a soldered brass tube interposed between the whole length of the coil and the axial wire did not prevent or diminish them.

The most probable cause of the phenomena appears to be the combined influence of ordinary induced longitudinal magnetic polarity and of a direction of induced magnetism at right angles to that, producing molecular motion and change of position of the particles, the two directions of induced magnetism being caused by the coil and axial electric currents respectively. We know by the alterations which occur in the length and diameter of a soft iron rod, when it is subjected to the influence of a coil-current (discovered by Dr. Joule), and the sounds then produced in it, that a molecular movement and change of position of its particles then occur; and it appears equally certain, from the torsions and sounds produced, that whenever an electric current traverses the axis of a longitudinally magnetized piece of iron, it also causes a molecular movement and change of position of the particles.

[The phenomena may be explained upon the supposition that a coil-current lengthens an iron rod by causing the axes of its molecules to place themselves parallel to the axis of the rod; an axial one places them tangentially; and the two currents acting together place them in a direction intermediate between those two, *i. e.* in a spiral direction—the mass of the iron being in each case lengthened in the direction of the axes of the molecules, and shortened in a direction at right angles to that.]

6. *Law of direction of electrotorsion.*

It might be theoretically anticipated that the torsional movements occur in definite directions with relation to the electric currents. This has been found to be the case; and the law of the phenomenon in iron is as follows:—A. *With an axial current.* A

current flowing from a south to a north pole produces a left-handed torsion, and one from a north to a south pole produces a right-handed one*. B. *With a coil-current.* A coil-current with its south pole below, circulating round a vertical rod of iron through which an electric current proceeds upwards, produces left-handed torsion, and a reverse one produces right-handed torsion, in the sense already employed*.

Each of these laws, but the latter one the most frequently, is affected in a very limited proportion of cases by the order of succession in which the currents are applied.

7. *Will a coil-current alone produce torsion?*

I passed the current from 12 GROVE's cells through the coil surrounding the iron bar 11 mm. thick (see page 529), in each direction. In each case a very small movement of the pointer to the *left* hand (= right-handed torsion*) occurred on connecting the battery, and to the right on disconnecting it. By including the index (but not the bar) in the circuit, continued vibration could be obtained by placing the contact-screw against the right-hand side of the pointer in such a position that the vertical movement alone did not break the contact, but not by placing it at the left-hand side. Similar but smaller movements in the same direction were obtained with a thinner rod, by passing the current either way. With a thin iron wire, strained by a weight, no torsions could be obtained by such means. By employing a thin tube of soft iron 10 mm. outer diameter, having the same weight of 5½ kilogrammes attached, much larger torsions occurred; but each of the movements in this case was to the right on connecting the battery, and to the left on disconnecting, irrespective of the direction of the current. It is probable that the difference of direction of torsion in the bars and tube was due to difference of internal mechanical strain, because I have found that twisting a bar before passing the coil-current enables that bar to exhibit small torsions; WIEDEMANN also (see note, p. 529) had previously investigated that circumstance. Torsions produced by electric currents only are reversed by reversing either of the currents, but those produced by the combined influence of a coil-current and previous mechanical twist were not.

8. *Will a parallel current produce torsion?*

I suspended an iron wire 1·7 mm. diameter, provided with the pointer, in the axis of a brass tube 2·6 metres long, the tube being in one case 11·7 mm. diameter, and in another case only 7 mm. diameter. The current from the 12 GROVE's cells, arranged as 3, was in each case passed up the tube, also down it; but no perceptible torsion was manifested.

I substituted for the brass tube a thin one of soft iron 10 mm. external diameter, and for the iron wire a cotton-covered copper one 1·7 mm. thick, with a lead weight suspended from it to keep it straight, the pointer being attached to the tube and the

* By "right-handed" torsion, in the foregoing sentences, I mean twist in the direction of the thread of an ordinary screw; but a right-handed movement of the index (sometimes also called torsion) in all other parts of this paper means the reverse of this.

battery arranged as above. By passing the current down the wire a small movement of the index, equal to 1 mm., to the left hand occurred; and by passing it up an equal amount of movement to the right hand took place. I also repeated the experiment with a copper wire 2·75 mm. diameter; the movements produced by each first current agreed in direction with those stated above, but by each subsequent current in the same direction opposite torsions of small extent occurred. The transmission of these currents diminished the residuary magnetism of the tube if the lower end was a north pole. These results show that an axial electric current in a separate conductor inside the iron acts inductively, and produces a small amount of torsion.

9. *Production of torsion by axial currents.*

To determine if an axial current in the bar or wire itself would produce torsion, I excluded the helix from the circuit, and passed the current from 12 cells in one direction, first up the bar (of 11 mm. diameter), making several successive contacts, and then down it several times. In each case a decided torsion took place on the first contact, and the index returned a small portion of the distance on stopping the current; and feeble torsions occurred in the same direction by each after-contact, the pointer returning a similar small distance on disconnecting.

10. *Will a previous coil-current enable an axial one to produce torsion?*

The above-mentioned torsions produced by an axial current alone were, to a certain extent, dependent upon the iron being previously magnetic. To ascertain this I converted the lower end of the bar, first into a *south* pole by momentary application of the coil-current and then passed the axial current *down*wards several times, then converted it into a *north* pole and again passed the axial current *down*wards several times. I then applied the coil-current to form a *south* pole, and passed the axial current *up*wards repeatedly; and then to form a *north* pole, and again passed the axial current *up*wards. In each case the *first* application of the axial current produced a comparatively large torsion, the pointer returning a small distance on stopping the current, and each subsequent contact produced in the same direction the smaller movements already mentioned; the *large* deflections, therefore, could not be produced in the same direction without intervening reversal. The directions of movement were such that an axial current proceeding from a south to a north pole imparted a left-handed twist to the iron, and one in the opposite direction produced a right-handed torsion, as shown in figs. 1, 2, 3, 4, Class A, Plate XLII.

With the thin iron tube of 10 mm. diameter, using 12 GROVE's cells arranged as 6, I obtained much larger movements than with the bar. Both with the bar and tube the magnitudes of the torsions in opposite directions were generally alike.

It appears from these results that to produce electrotorsion freely requires the application of a coil-current and of an axial one in the iron itself, and that although the former alone only slightly twists a bar or tube of iron which has been previously sub-

jected to mechanical torsion, or does not twist it at all if it is free from such previous strain (see p. 533), it leaves a residuary condition in the iron which renders the bar capable of being afterwards freely twisted in opposite directions by opposite electric currents passed axially through it; and the opposite magnetic polarities conferred by opposite directions of the coil-current enable an electric current passed axially in one direction through the bar to produce opposite directions of torsion. It follows from this that the direction of longitudinal magnetic polarity can be ascertained by means of the direction of electrotorsion.

As in each of these experiments with an axial current in the iron itself the *first* movement of torsion in either direction was a *large* one, and the needle only slightly returned towards zero on the cessation of the current, the temporary action of an axial current succeeding a coil one leaves an iron bar in a twisted state, and the direction of the twist is opposite with opposite directions of the axial current. The results also show that when torsions are produced by this method, detorsion is prevented by some coercive or retaining influence within the bar itself, and thus potential mechanical power becomes stored up in the iron.

11. *Magnitudes of torsions produced by axial currents.*

The actual magnitudes of the torsions produced by this method were small, and varied with the length and thickness of the iron, its degree of magnetic polarity, and the strength of the battery. With iron wires 2·6 m. long, and from 1·75 to 2·17 mm. thick, possessing weak south poles at their lower ends, alternately reversed axial currents from 12 cells arranged as 3 produced 3 to 4 mm. of movement of the end of the pointer, the pointer being 47 centims. long from its point of suspension to its extremity. With a wire 3 mm. thick, and a weak south pole below, sustained by terrestrial magnetic influence only, alternately reversed axial currents produced from 1·5 to 3·5 mm. of movement; but when these currents succeeded coil ones, the movements varied from 2·25 to 5·5 mm.: and with one 3·77 mm. thick, supporting 5½ kilogrammes, and having a weak south pole below sustained by terrestrial influence only, alternately opposite axial currents from the aforesaid battery produced 1·75 to 1·83 mm. of movement; but when those currents succeeded coil ones, the movements varied from ·75 to 2·75 mm. The magnitudes of the torsions diminished with increase of thickness of the iron.

12. *Are the torsions related to electromagnetic sounds?*

Numerous axial currents in various series were transmitted through a wire 2·17 mm. diameter (up currents, down ones, and alternate up and down ones), and the sounds observed with the aid of a stethoscope, the circuit in each instance being completed by the mutual contact of wires, not by mercury, because of the interference of the snapping sound at the surface of that metal.

In every case a more or less distinct sound occurred at the commencement of each current, and little or none at its cessation. Those emitted by currents which succeeded

others in the *same* direction, and which produced the small elastic torsions only, were more metallic, and those which succeeded currents in an *opposite* direction, and produced the large inelastic torsions, were more dull. Those produced by repetition currents in the same direction did not appear to diminish in loudness in proportion to the diminution of magnitude of the torsions. Axial currents succeeding coil ones were also tried; each yielded a feeble sound at its commencement and none at its termination. In every instance in which torsion occurred there was sound emitted, and in every case where no sound was produced no torsion took place; probably therefore the two phenomena are mutually related, and the torsion is dependent upon the cause which produces the sound.

[We know that by the passage of coil-currents alone around an iron wire, or axial currents alone through it, sounds without torsion occur; and the experiments of De la Rive (Phil. Trans. 1847, p. 40; Phil. Mag. vol. xxxi. 1847, p. 328, and vol. xxxv. 1849, p. 428) have shown that such currents produce sounds in various non-magnetic substances, and those substances do not exhibit electrotorsion (see Section 4); and as the object of this research was to examine the torsion and not the sounds, I have not investigated the latter phenomena except in such cases as they appear to be connected with the former.]

It is manifest, from a consideration of the torsions and sounds, that an electric current passed through iron in a more or less magnetized state produces a molecular movement and a change of position of its particles, and that the new position continues as long as the current. Also that, on the cessation of the current, the particles make very little movement, and return only a small extent towards their original positions, because little or no sound occurs, and only a small amount of detorsion then takes place. Also that an electric current in the opposite direction produces a similar set of changes, except that the changes are in the reverse direction.

The small detorsions which occur on the cessation of every single axial current succeeding a coil one appear to be due to the reaction of the ordinary mechanical elasticity of the metal; but that is a point I have not examined.

13. *Distribution of coil-current influence in the helix.*

In order to ascertain whether the torsional influence of the helix was less at the middle of the coil than at its extremities, I tried its effect upon an iron wire 61 centims. (=24 inches) long and 1·37 mm. diameter, in the two positions, the remainder of the axial conductor being formed of thick copper wire upon which the two currents have little or no torsional effect. The current from 12 Grove's cells was applied in the most effective way, and in the same manner in each case. A series of four torsions was produced in each position, those produced by the wire at the middle of the coil being obtained after the others; the former averaged 6·44 mm. and the latter 6·11 mm. If, therefore, a small allowance be made for the gradual weakening of the current, the degrees of torsional influence at the middle part of the coil and at its ends are about

alike. The magnitudes of the torsions in each case appeared to be proportional to the length of iron within the helix.

14. *Interference of terrestrial magnetism.*

It must not be forgotten that in all these experiments terrestrial magnetism operates, and more or less influences the results; also that the iron is liable to have its residual magnetism weakened by the repeated passage of an axial current in either direction through it; and if the lower end of it is a *north* pole, the polarity is often reversed. A single contact of the battery is sufficient to remove the polarity, if the current is powerful and the iron wire a small one.

In consequence of not being able to maintain a bar or wire of iron perfectly free from longitudinal magnetism during the passage of an axial current through it whilst in a vertical position, and the difficulty of detecting minute torsions with the bar in a horizontal one, I was unable to ascertain if an axial current alone would produce torsion, by experimenting with a demagnetized iron bar at right angles to the magnetic meridian. But as the magnitudes of the torsional effects of an axial current increase with the strength of the longitudinal magnetic polarity of the iron, and opposite longitudinal magnetic polarities enable each axial current to produce opposite torsions, it is highly probable that if a vertical iron rod or wire could be maintained in as perfectly an unmagnetized state in a longitudinal direction as it can in a transverse one, an axial current alone, like a coil one alone (see p. 533), would produce little or no torsion.

15. *Will axial currents remove the residuary effect of coil ones?*

Some special experiments were made with the thin wrought-iron tube of 10 mm. diameter to determine the above question, or whether the previous application of a coil-current would enable any number of torsions in alternately opposite directions to be obtained by continually reversing the axial stream. The current from 12 cells arranged as 6 was employed, and no weight was attached to the tube.

1. *With a south pole below.*—Any number of torsions could be obtained by such means, and the longitudinal magnetic polarity of the iron was not entirely removed by many passages of the current, because terrestrial magnetism continually renewed it, and because the axial current was small in relation to the conducting-power of the tube, and therefore produced less disturbing effect upon the longitudinal magnetism.

2. *With a north pole below.*—The first two downward currents, and then two upward ones, produced movements in the normal direction, the last of these being only deficient in magnitude; but all the succeeding currents caused torsions agreeing in direction with the existence of a *south* pole below, and the longitudinally magnetic polarity, tested by means of a small compass-needle, was found reversed after several axial currents had passed.

16. *Axial currents a test of residuary longitudinal magnetism in iron and steel.*

As the electromagnetic torsions constituted a new means by which residuary longitudinal magnetism might be detected, I endeavoured to ascertain more exactly and completely the effect of repeated axial currents upon that condition. I suspended in the helix a soft-iron wire 2·6 metres long and 2·22 mm. diameter, and measured the various torsions produced by a current from 12 cells arranged as 3.

A. *After making a north pole below* by means of the battery.—The current was passed 14 times through the wire, alternately in opposite directions, commencing with an upward one. The following are the magnitudes of the several torsions, expressed in millimetres of movement of the end of the pointer (the vertical arrows indicate the directions of the axial currents, and the horizontal ones those of the movements):—1. ↓ ←4·0; 2. ↑ →2·0; 3. ↓ →1·0; 4. ↑ ←2·75; 5. ↓ →2·5; 6. ↑ ←3·75; 7. ↓ →3·75; 8. ↑ ←4·33; 9. ↓ →4·25; 10. ↑ ←4·75; 11. ↓ →4·25; 12. ↑ ←4·75; 13. ↓ →4·0; 14. ↑ ←4·9. On applying a compass-needle, the lower end of the wire was found magnetically neutral at the 3rd contact, and a feeble south-pole at the 10th. On examining the directions of the torsions, it will be observed that a change took place between the 2nd and 3rd currents, the directions of those previous to that change agreeing with the existence of a *north* pole below, and of those after it with a *south* one; and on comparing the magnitudes of the movements, we find that previous to that change of polarity they diminished, and after it those of the torsions produced by the *up*ward currents increased to the 10th contact only, whilst those of the *down*ward ones increased throughout.

B. *After making a south pole below.*—Ten alternately reverse currents were transmitted, commencing with an upward one; and the following are the results:—1. ↓ →6·5; 2. ↑ ←8·0; 3. ↓ →6·75; 4. ↑ ←5·33; 5. ↓ →5·25; 6. ↑ ←5·5; 7. ↓ →5·0; 8. ↑ ←6·0; 9. ↓ →5·25; 10. ↑ ←5·5.

The results set down under "A" show that the north polarity was first diminished and reversed, and then the south polarity increased until a downward current produced a movement of 4·9 to the left (and was still increasing); and those under "B" show that the south polarity decreased until a downward current produced a movement of 5·5 to the left—as if in each case the iron was gradually approaching a normal magnetic state by the influence of a disturbing cause and terrestrial magnetism. These results support the hypothesis that the axial current loosens the molecules of the iron, and enables terrestrial magnetic influence to reduce the iron to its normal degree of magnetism with south polarity downwards. But assuming such explanation to be true, it is not complete, because axial currents of opposite direction leave the iron in opposite states (see p. 535). The results generally also indicate that a magnetized soft-iron bar may be rendered normally magnetic by placing it vertical, and passing alternately reversed electric currents axially through it until it ceases to show a changing degree of torsion. I have not examined whether axial-electric currents would entirely remove

the ordinary longitudinal magnetism if the wire was at right angles to the terrestrial magnetic meridian.

As the torsions in these experiments with an iron wire depended upon the influence of both coil and axial currents, it is evident that the magnitude of them was limited by the weakest; and as the residual longitudinal magnetism was feeble and the axial current strong, the magnitudes of the torsions varied with the amount of that residual magnetism. Assuming that the magnitude of the first torsion was to some extent in each case a measure of the residuary magnetism, and no interfering circumstances had occurred, the magnetism left in the wire by the coil-current which produced a *north* pole below was to that left by the same strength of current after producing a *south* pole as 4·0 to 6·5; and this agrees with the fact that terrestrial magnetic influence weakens the magnetism of a vertical soft-iron wire, the *north* pole of which is below. Further investigation might perhaps disclose the exact conditions under which difference of magnitude of the torsions constitutes a true measure of residual magnetism.

Similar series of experiments were made with a steel wire 2·6 m. long and 2·16 mm. diameter; the following are the results:—

A. *After making a north pole below.*—Eighteen currents were passed:—1. $\frac{1}{1}$ +∝ 6·0; 2. $\frac{1}{1}$ ∞+ 5·25; 3. $\frac{1}{1}$ +∝ 4·0; 4. $\frac{1}{1}$ ∞+ 3·75; 5. $\frac{1}{1}$ +∝ 3·0; 6. $\frac{1}{1}$ ∞+ 2·4; 7. $\frac{1}{1}$ +∝ 2·0; 8. $\frac{1}{1}$ ∞+ 2·0; 9. $\frac{1}{1}$ +∝ 1·75; 10. $\frac{1}{1}$ ∞+ 1·75; 11. $\frac{1}{1}$ +∝ 1·5; 12. $\frac{1}{1}$ ∞+ 1·5; 13. $\frac{1}{1}$ +∝ 1·6; 14. $\frac{1}{1}$ ∞+ 1·5; 15. $\frac{1}{1}$ +∝ 1·2; 16. $\frac{1}{1}$ ∞+ 1·33; 17. $\frac{1}{1}$ +∝ 1·33; 18. $\frac{1}{1}$ ∞+ 1·0. No reversion of the direction of torsion or of magnetic polarity took place, and the deflections produced both by the upward and downward currents diminished throughout. The non-reversal of polarity, and the slowness of decrease of magnetism, compared with that which occurred in iron, were probably due to the greater coercive power of steel.

B. *After making a south pole below.*—Sixteen currents were transmitted:—1. $\frac{1}{1}$ ∞+ 5·5; 2. $\frac{1}{1}$ +∝ 6·0; 3. $\frac{1}{1}$ ∞+ 5·25; 4. $\frac{1}{1}$ +∝ 4·25; 5. $\frac{1}{1}$ ∞+ 4·0; 6. $\frac{1}{1}$ +∝ 4·0; 7. $\frac{1}{1}$ ∞+ 3·75; 8. $\frac{1}{1}$ +∝ 3·8; 9. $\frac{1}{1}$ ∞+ 3·75; 10. $\frac{1}{1}$ +∝ 3·5; 11. $\frac{1}{1}$ ∞+ 3·6; 12. $\frac{1}{1}$ +∝ 3·4; 13. $\frac{1}{1}$ ∞+ 3·5; 14. $\frac{1}{1}$ +∝ 3·4; 15. $\frac{1}{1}$ ∞+ 3·5; 16. $\frac{1}{1}$ +∝ 3·4.

On comparing the results of "A" and "B," we find that (as in iron) the magnetism was much more weakened when the *north* pole was below than when the south pole was beneath; this result was of course due to the influence of terrestrial magnetism. With a *south* pole below, both with iron and steel, the polarity was diminished, but more so with steel than with iron.

17. *Effect of direction of axial currents upon the residuary longitudinal magnetism of iron and steel.*

I also examined the effect of a succession of axial currents, all in one direction, using the same battery-arrangement.

A. *In a soft-iron wire* 1·75 *mm. diameter.*—1. With a *south* pole below previously produced by the coil-current, six axial currents in succession were passed *down* it. The first produced a movement of 4·75 mm. to the left hand, the second 1·25 mm., and

the others gradually diminished to 0·5 mm. After restoring the south polarity by a coil-current, ten axial currents were passed *up* it. The first torsion was 6·5 mm. to the right hand, the second 1·5 mm., and the others 1·25 mm. In each of these two series the torsions after the first one were largely due to the renewal of the longitudinal magnetism by terrestrial influence. 2. With a *north* pole below, six *down*ward currents were transmitted: the first produced a movement of 3·75 mm. to the right hand, and made the lower end of the wire neutrally magnetic to a compass-needle, and the following ones yielded no movement. After restoring the same polarity by means of the coil-current, six *up*ward currents were passed: the first produced a deflection of 2·5 mm. to the *left*, and rendered the lower end of the wire a feeble south pole; the second 0·6 to the *right* hand, the 3rd 1·0 to the right, and the remainder the same.

B. *In a steel wire* 2·17 *mm. diameter.*—1. With a *south* pole below, produced by the coil-current, six *down*ward currents were first passed. The first of them produced a movement of 5·5 mm. to the left hand, and the subsequent ones yielded no torsions. The polarity was then restored by means of the coil-current, and six axial currents in succession then passed *up*wards. The first gave a deflection of 3·25 mm. to the right hand, the second ·25, and the others no movement. 2. With a *north* pole below, six *down*ward currents were first transmitted; the first of them gave a torsion of 5·5 mm. to the right hand, and the succeeding ones had no effect. After restoring the polarity as before, six *up*ward currents were passed: the first produced a movement of 6·25 mm. to the left hand, and the others had no visible effect.

The sudden cessation of torsion after the first current in steel, and the much more gradual cessation of it in iron, are quite conspicuous. The fact that the magnitudes of the first torsions, both in iron and steel, are smaller with a north pole below than with a south one, agrees with the view that the magnitude of the movements depends upon that of the residuary longitudinal magnetism.

To ascertain whether in such experiments as these some of the longitudinal magnetism remained after the first axial current had passed, although no torsion was produced by the subsequent axial currents, I transmitted through the coil (containing a steel wire) a current producing a south pole below, then passed 10 axial currents in succession *down* the wire; only the first one produced torsion. On then transmitting a current *up* the wire, a torsion occurred, proving that the magnetic effect of the coil-current still remained.

18. *Will a previous axial current enable a coil-current to produce torsion?*

To ascertain also if the previous passage of an axial current conferred upon the iron the capacity of being twisted by a coil one, I passed a current from the 12 cells in one series *up* the bar of 11 mm. diameter, then through the helix several times, making the lower end of the bar a *south* pole; then up the bar again, and then several times through the coil, producing a *north* pole; and I repeated this series of experiments, employing a current *down* the bar. In each case the *first* application of the coil-current produced

a large torsional movement, the index returning a small distance on stopping the current; and each subsequent contact produced in the same direction the small movement only: the *large* deflections, therefore, could not be produced in the same direction a second time without intervening reversal. I have not examined whether the small detorsions which occur on the cessation of each single coil-current succeeding an axial one are due to the ordinary elasticity of the metal. The directions of the large and small movements are shown by figures 1, 2, 3, 4, Class B, Plate XLII.*

These results confirm the view that to produce torsion freely requires the two currents (see Section 10, p. 535), and that although each current alone will produce its own magnetic effect, neither alone will twist the bar if the iron is free from mechanical strain. They also show that, although a coil-current alone produces no torsion in an annealed iron bar (see p. 533), the previous passage of a current axially through the bar puts the iron into such a magnetic state that it becomes capable of being afterwards freely twisted in opposite directions by opposite coil-currents, and that opposite axial currents cause the iron to assume two opposite directions of such state, because they enable one direction of coil-current to produce opposite directions of torsion. The results further show that the two ends of an iron rod, wire, or tube, through which an electric current has been axially passed, possess opposite properties.

It appears also that each of the opposite longitudinal magnetic states produced by the two directions of coil-current is different from each of the conditions produced by opposite axial currents, because previous magnetization of an iron bar by a coil-current in either direction did not enable the subsequent magnetization of that bar by an opposite coil-current to produce twist, whereas the previous passage, in either direction, of an axial current through it did enable such subsequent treatment of it to produce torsion; and, further, because an axial current does not lengthen an iron bar (see p. 530), but a coil-current does; the acoustic effects of the two directions of current are also different (compare Sections 12 & 23). [All these conclusions agree with the view that the axial current imparts poleless magnetism tangentially to the outer layer of molecules of the iron, and that each of the four different directions of current imparts to the free end of the iron a different property.]

As in each of these experiments with a coil-current the *first* movement of torsion was a *large* one, and the index returned only a small distance back on cessation of the current, the temporary action of a coil-current succeeding an axial one (like that of an axial current succeeding a coil one, see Section 10) leaves an iron bar in a twisted state; and the direction of that twist is opposite with opposite directions of the coil-current. By this method, therefore, as well as by the previous one (see Section 10), detorsion is prevented by some influence within the bar, and mechanical power becomes stored up.

The results of these experiments, and of those previously described (see Section 10), show that the direction of torsion in all of them depends upon that of each of the two currents, and that it was reversed by reversing either the axial or coil-current, but not

* Some cases, apparently, of torsion in reverse directions to those of figs. 1 and 4 are described on p. 552.

by reversing both. They also prove that the directions of torsion produced by a coil-current succeeding an axial one are identical with those produced by an axial current following a coil one.

The general characters of the torsions produced by a coil-current succeeding an axial one are very similar to those produced by the reverse order of currents (see Sections 9 & 10), and indicate that both are produced by the same general cause.

19. *Magnitudes of torsions produced by coil-currents succeeding axial ones.*

The magnitudes of these torsions averaged about two fifths of those produced by axial currents succeeding coil ones. With an iron wire 2·6 m. long and 3 mm. thick, and a current from 12 cells arranged as 3, coil-currents succeeding axial ones produced torsions which varied in magnitude from 1 to 5·25 mm. of movement of the end of the index; and with an iron wire 3·77 mm. diameter, supporting a weight of 5½ kilogrammes, the movements varied from ·5 to 1·25 mm. The magnitudes of the torsions produced by this method also diminished with increase of thickness of the iron.

20. *Influence of the order of succession of the two currents upon the magnitude of the torsions.*

To ascertain this I made four series of experiments with an iron wire 1·75 mm. diameter, supporting no weight; and employed a current from 4 cells arranged as 2 for the coil, and one from 8 cells connected as 4 for the axial wire. Twelve single currents, *i. e.* six of each, were transmitted in each series:—*1st Series.* With coil-currents producing *north* poles below, alternated with *down*ward axial ones, the torsions produced by the coil-currents were all to the right hand, and averaged 1·62 mm.; and those by the axial currents were also in that direction, and averaged 1·64 mm.

2nd Series. The same direction of coil-currents, alternated with *up*ward axial ones. The torsions produced by each kind of currents were all to the left hand; those due to the coil-currents averaged 1·60 mm. in magnitude, and those produced by the axial ones averaged 2·05 mm. The large inelastic torsions produced on changing from one series to another, one of the currents being then reversed, were of course not included in the reckoning.

3rd Series. With coil-currents producing *south* poles below, alternated with *down*ward axial ones. The coil-currents produced torsions to the right hand*, and the axial ones produced torsions to the left. The former averaged ·91 mm. in magnitude, and the latter 3·82 mm.

4th Series. With coil-currents in the same direction, alternated with *up*ward axial ones, the torsions yielded by the coil-currents were to the left hand*, and averaged ·8 mm. in magnitude, whilst those due to the axial ones were to the right hand, and their average range was 5·21 mm. The average magnitude of all the coil-current torsions in the 4 series was 1·23 mm., and of axial-current ones 3·18 mm.:

* The *apparently* abnormal direction of these torsions will be treated of subsequently (see Section 21).

this great difference was no doubt due to the much greater degree of retentive power which iron possesses for longitudinal than for tangential magnetism.

The magnitude of the torsion produced by a given current depends not only upon the kind of current which immediately precedes it, but also upon the description of current which precedes that one. An axial current in a given direction succeeding a coil one nearly always produced a greater torsion if the coil one was preceded by an axial one in the opposite direction than if it followed one in the same direction. A similar but less general result occurs with coil-currents succeeding axial ones. A series of coil-currents producing north poles below, which gave an average deflection of 1·50 mm. when preceded by upward axial currents following coil ones of *similar* direction, yielded an average torsion of 3·25 mm. when the axial currents followed coil ones of *opposite* direction (see Section 30, page 549).

Alternate coil and axial currents therefore produced the largest torsions when *both* kinds of currents were alternately reversed in direction. The torsions in general produced by alternate currents appear to be due, not only to the energy of the acting current, but also to the liberated potential energy of the residual magnetism left by previous currents.

21. *Apparently exceptional cases of torsion.*

On examining the notes of the experiments of the instances of apparently abnormal direction of torsion mentioned in Section 20, it was found that in each of the whole twelve instances detorsion and not opposite torsion alone occurred—as if the current in each case simply liberated the particles of the iron from their state of permanent twist, and allowed the pointer to return to zero. It is singular that the passage of an axial current after a coil one producing a *south* pole below enables a subsequent coil-current of similar direction thus to undo the effect of a former one, whilst no corresponding phenomenon occurs with an axial current following a coil one which produces a *north* pole (see Section 20).

22. *Effect of mechanical pull.*

When making the four series of experiments described in Section 20, I made four precisely similar series whilst a weight of 5½ kilogrammes was suspended from the end of the wire. In the 1*st Series.* With coil-currents producing *north* poles below, alternated with *down*ward axial ones, the torsions produced by coil-currents averaged ·1 mm. less, and those yielded by axial ones 1·48 mm. greater magnitude than without the weight. In the 2*nd Series.* With *up*ward axial currents, those yielded by coil-currents averaged ·15 mm. more and those by axial ones ·84 mm. less than without the weight. In the 3*rd Series.* With coil-currents producing *south* poles below, alternated with *down*ward axial ones, the coil-current torsions averaged ·03 mm. more and the axial-current ones 1·53 mm. more than without the weight. In the 4*th Series.* With *up*ward axial currents, the coil-current torsions averaged ·46 mm. more and the axial-current ones ·12 mm. more than without the weight. No instance of reversal of direction of torsion,

nor even detorsion, by influence of the weight occurred. It is evident from these results that the torsions produced by alternate coil and axial currents are affected by mechanical pull, and are therefore related to the forces of cohesion (compare also Section 42).

23. *Relation of coil-current torsions to electric sounds.*

To ascertain this I employed the same iron wire &c. as before (see Section 12). Coil-currents in each direction were tried, both in succession and alternation, and coil-currents succeeding axial ones were also employed. In every instance a coil-current acting alone produced a sound, both at its commencement and termination, the former being the loudest and the latter more metallic; and by repetition of the current in the same direction the two sounds were more feeble. Coil-currents succeeding axial ones also produced sounds, both at their commencement and cessation, the former being in every case louder and more dull, and the latter feebler and more metallic (see also Section 24); and by repeating the coil-current in the same direction the two sounds were more feeble. In each case, with coil-currents alone, the louder and duller sounds accompanied the large inelastic torsions, and the feebler and more metallic ones occurred with the small elastic movements; but the sounds produced by repeating a coil-current in the same direction did not appear to be weakened in proportion to the degree of diminution of magnitude of the torsions. These results are generally similar to those obtained with axial currents (see Section 12), and similar general conclusions to those stated on page 536 may be drawn from them. (For sounds produced by simultaneous and divided currents, see Section 36, p. 555.)

24. *Does a red heat destroy residual axial-current effect?*

A soft-iron wire 1·55 mm. diameter was placed in the helix, and a series of alternate coil-currents (producing south poles below) and downward axial ones transmitted, in order to ascertain the average amount of movement produced by the coil-currents. The coil-current movements were all detorsions (see Section 21) to the right hand, and averaged 5·1 mm. Ten downward axial currents were then transmitted to impart the axial state. The wire was now heated to redness throughout its length whilst in the magnetic meridian, and allowed to cool in that position. On replacing it in the helix, and passing a single coil-current in the same direction as before, not the slightest torsion occurred, although a strong metallic sound was heard in the wire on making contact. The wire was again similarly imbued with the axial influence, then heated to redness, and cooled whilst in a horizontal position at right angles to the magnetic meridian. On again replacing it in the helix and passing a coil-current as before, a torsional movement, amounting only to ·5 mm., occurred, and a dull thud was heard in the wire on making contact. It is evident that the residuary axial-current state is destroyed by a red heat, and is but little restored by cooling the wire whilst in a direction at right angles to the terrestrial magnetic meridian.

As electrotorsion is closely related to the magnetism, cohesion, and mechanical states

of the iron, and these are all profoundly affected by rise of temperature, and as it does not occur in non-magnetic metals (see Section 4), it is very probable that the torsions would be modified and prevented by heating the iron to redness during the passage of the currents. In a paper "On the Magnetism of Electrodynamic Spirals" (see Phil. Mag., Oct. 1870) I have shown that with the same electric current passing through two successive spirals of iron and copper wire, the tangential or transverse magnetism of the iron is much greater than that of the copper at ordinary temperature; but as the temperature of the two spirals is raised to full redness, the tangential magnetism of the two becomes equal.

25. *Is the axial-current state uniformly distributed?*

To examine this I employed an iron wire 2·6 m. long and 1·75 mm. diameter, and compared the magnitudes of the torsions produced in it by a current in a helix 61 centims. long surrounding the lower end of the wire, with those obtained when the coil surrounded its middle portion. The former averaged 3·1 mm. and the latter 3·2. The magnitudes of the movements, compared with those obtained with longer coils, indicated that the amount of torsion varied directly as the length of the wire within the helix.

I also strewed fine iron filings upon a thin sheet of glass lying upon a horizontal annealed iron wire conveying a strong voltaic current in a direction at right angles to the terrestrial magnetic meridian, and vibrated the glass; no unequal distribution of the filings could be detected.

26. *Retentive power of iron for axial-current influence.*

As an axial current passed through a recently annealed rod of iron or steel leaves it in a different physical state (without twisting it), and one succeeding a coil-current leaves the bar in a twisted condition, it is evident that iron possesses a retentive power, not only for the influence of a coil-current, but also for that of an axial one, and that the residuary axial-current state may exist with or without the condition of twist.

In consequence of this retentive power of iron and steel for those effects, both the direction and the magnitude of the torsions in every case depend not only upon the kind and direction of the currents being applied, and upon the condition of the rod with regard to previous mechanical strain, but also upon the state of it with regard to both those residuary influences.

27. *Will coil-currents remove the residuary effect of axial ones?*

In these experiments a soft iron wire 2·6 m. long and 3 mm. diameter was used, no weight being attached to it, and the battery of twelve cells was arranged for intensity of three.

(1) *After passage of a* downward *current.*—Five coil-currents of alternately opposite direction were passed, the first producing a north pole below. The directions and

magnitudes of movement of the pointer were as follows:—1st, ⇒ 2·25 mm.; 2nd, ⇐ 1·5; 3rd, ⇒ 1·0; 4th, ⇐ 0·5; and 5th, ⇒ 0·5.

(2) *After an upward one.*—Four alternately opposite currents were passed, the first producing a south pole below. The movements were:—1st, ⇒ 2·25; 2nd, ⇐ 1·75; 3rd, ⇒ 0·5; and 4th, ⇐ 0·5. A series of alternately opposite coil-currents, therefore, gradually removes the residual effect in an iron wire of an axial current in either direction.

On comparing these results with those described on page 538 it appears that it required more persistent treatment to remove the residual effect of a coil-current than that of an axial one, probably partly because the influence of terrestrial magnetism assisted in maintaining the former and in removing the latter.

If we assume that the first torsion in each of these two sets of experiments was, to some extent, a measure of the residuary effect of the axial current, we find that in iron the magnitudes of residuary effect of a downward axial current and of an upward one were equal.

As a series of coil-currents only gradually removes the residual effect of axial ones, and a series of axial currents only very gradually changes the ordinary magnetic polarity of iron and steel (see Sections 16 & 28), the degree of persistency of those residuary conditions must be remembered whilst making experiments in electrotorsion.

It is probable that the longitudinally magnetizing power of terrestrial magnetism, like that of a coil-current, would also remove the residual effect of an axial current if the axis of the iron was in the terrestrial magnetic meridian and sufficient time was allowed, especially if vibration was also applied.

Although it requires a *series* of alternately opposite axial currents to remove the residual effect of a coil one, and a *number* of alternately reversed coil-currents to remove that of an axial one, a single opposite coil-current, also a single opposite axial one, each of sufficient power, respectively produce those effects. It would appear from this that the two states produced by opposite coil-currents (or axial ones) are incompatible and cannot coexist, and that a coil-current acts in a more mechanically advantageous way in obliterating the effect of an opposite coil-current than in removing that of an axial one; and an axial current acts more effectively in effacing the influence of an opposite axial one than in reversing or removing that of a coil one. Further investigation of the phenomena in a mechanical aspect, and of the relations of the torsions to mechanical changes, electric sounds, expansion by heat, &c., may possibly disclose a glimpse of the relative directions of the molecular movements produced in iron by the two kinds of currents.

28. *Effect of coil-currents upon residuary axial-current influence in steel.*

These experiments were similar to those in Section 27. The wire was 2·6 m. long and 2·16 mm. diameter, and the same battery-current employed as before.

(1) *After passing a downward axial current.*—Fourteen coil-currents, alternately

opposite in direction, were passed, commencing with one which produced a north pole below. The following represent the results:—1. N. $\rightarrow$ 4·0; 2. S. $\leftarrow$ 4·0; 3. N. $\rightarrow$ 3·5; 4. S. $\leftarrow$ 3·25; 5. N. $\rightarrow$ 2·75; 6. S. $\leftarrow$ 2·5; 7. N. $\rightarrow$ 2·7; 8. S. $\leftarrow$ 2·25; 9. N. $\rightarrow$ 2·7; 10. S. $\leftarrow$ 2·0; 11. N. $\rightarrow$ 2·3; 12. S. $\leftarrow$ 1·75; 13. N. $\rightarrow$ 2·0; 14. S. $\leftarrow$ 1·8.

(2) *After an upward current.*—Sixteen coil-currents, the first producing a south pole below:—1. S. $\rightarrow$ 2·75; 2. N. $\leftarrow$ 5·25; 3. S. $\rightarrow$ 4·25; 4. N. $\leftarrow$ 3·75; 5. S. $\rightarrow$ 3·70; 6. N. $\leftarrow$ 3·25; 7. S. $\rightarrow$ 3·25; 8. N. $\leftarrow$ 3·10; 9. S. $\rightarrow$ 3·20; 10. N. $\leftarrow$ 3·00; 11. S. $\rightarrow$ 3·00; 12. N. $\leftarrow$ 3·00; 13. S. $\rightarrow$ 2·75; 14. N. $\leftarrow$ 3·25; 15. S. $\rightarrow$ 2·83; 16. N. $\leftarrow$ 2·88.

On comparing these results with those obtained with iron (described in Section 27), we find that the residual effect, both of a downward and of an upward axial current, was much more persistent in steel than in iron; and we may conclude that steel possesses, in a greater degree than iron, a coercive power for the influence of an axial electric current.

29. *Influence of* direction *of coil-currents upon residuary effect of axial ones.*

To determine whether coil-currents producing a north pole below had a different effect upon the residuary influence of an axial one from those in an opposite direction, I made the following experiments, using the current from twelve GROVE'S cells arranged as three:—

(A) *With a soft iron wire* 1·75 *mm. diameter.*—1st. After an *up*ward axial current from the same battery, six successive coil-currents, each producing a *north* pole below, were passed: the first produced a deflection of 2·75 mm. to the left hand, and the others no movement. After restoring the axial condition by several upward currents, six coil ones, each producing a *south* pole, were transmitted: the first produced a movement of 1·6 mm. to the right hand, and the others no effect. 2nd. After a *down*ward axial current, six coil-currents, producing north poles below, were passed: the first yielded a movement of 4 mm. to the right hand, and the remainder had no effect. The downward axial-current influence was then restored, and six coil-currents producing south poles below transmitted: the first resulted in a movement of 4·75 mm. to the left hand; the others were without effect.

On comparing these results with those described in Section 17, page 540, it will be seen that the cessation of torsion was less sudden with axial currents succeeding coil ones than with the reverse, probably because the residuary axial condition is less stable than the residuary coil one, and the latter was renewed by the influence of terrestrial magnetism.

(B) *With a steel wire* 2·17 *mm. diameter.*—1st. After an *up*ward axial current, six coil-currents, producing north poles below, were transmitted: the first produced a movement of 0·5 mm. to the left, and the others no effect. The smallness of the torsion was partly because the lower end of the wire was previously a north pole. After restoring the upward axial current several times, six coil ones, producing south poles below, were passed: the first developed a movement of 6·75 mm. to the right hand,

and the others gave only traces of effect. 2nd. After a *down*ward current, six coil ones, producing *north* poles below, were transmitted: the first gave a movement of 5·25 mm. to the right hand, and the others scarcely any effect. The downward current being restored several times, six coil ones, producing south poles below, were passed: the first produced a movement of 5·5 mm. to the left hand, and the others scarcely any effect.

These results may be compared with those obtained with the same wire and battery-current, and described in Section 17, page 540.

To ascertain whether in these cases a portion of the residual axial-current influence still remained in the steel, I passed an axial current down a steel wire in the helix, and then a series of ten currents through the coil, each producing a north pole below. The first coil one produced a movement of 3·5 mm. to the right hand, and the others only minute torsions. On now passing an opposite current through the coil, a movement of 4 mm. to the left hand took place, proving that the axial influence still remained.

30. *Relative values of residuary effects of opposite coil-currents and of axial ones in producing torsion.*

Several of the series of experiments already described have shown that the magnitude of the first torsion produced by a current is specially liable to be affected by various mechanical and magnetic conditions of the axial wire or rod, which may be very readily overlooked, and is therefore only a very crude measure of the amount of residuary effect of a previous current.

In order, therefore, to obtain more precise knowledge respecting the relative amounts of residuary effect of opposite *coil*-currents I transmitted two series of such currents, and passed, in the interval of time between each current, in the first series a momentary *up*ward axial one, and in the second series a *down*ward one; and to obtain similar information respecting opposite *axial* currents I passed two series also of them, and in the space of time between each current transmitted in the first series a momentary coil one, producing a *north* pole below, and in the second series one producing a *south* pole. In all the experiments the iron wire employed was 1·75 mm. thick, and had no weight attached to it; and the electric current was from twelve cells arranged as three.

The following are the results tabulated, and the determinations in each series numbered in the order in which they were made. The capital letters indicate the kind of polarity of the lower end of the wire, the vertical arrows show the direction of the axial current, the horizontal ones that of the movements of the index, and the decimal numbers their magnitudes; the variations in these numbers in each vertical column arose chiefly from the difficulty of getting the pointer perfectly steady. A rigid apparatus, and a firm foundation for its support, are highly necessary in cases where the torsions require to be accurately measured.

The lower end of the wire was a weak south pole at the commencement of the expe-

riments; the pointer also was at zero, and the same zero was maintained throughout the whole of the experiments.

(A) *Residuary effect of coil-currents.—First Series.* With *up*ward axial currents, the first being preceded by a coil one, producing a north pole below:—

↑	S.	↑	N.
1. 2·0 ←■	2. 1·25 ■→	3. 5·5 ■→	4. 2·75 ←■
5. 1·5 ←■	6. 1·25 ■→	7. 5·25 ■→	8. 3·25 ←■
9. 1·75 ←■	10. 1·25 ■→	11. 4·75 ■→	12. 3·50 ←■
13. 1·25 ←■	14. ·75 ■→	15. 5·00 ■→	16. 3·25 ←■
17. 1·25 ←■	18. 1·00 ■→	19. 4·75 ■→	20. 3·50 ←■
5)7·75	5)5·50	5)25·25	5)16·25
Averages 1·55	1·1	5·05	3·25

The largest torsions were produced by axial currents, and the smallest by coil ones; therefore the former left the smallest residual effect. Upward axial currents succeeding coil ones which produced a south pole below, gave larger torsions than those succeeding coil ones which yielded a north pole, in the proportion of 5·05 to 1·55; therefore coil-currents producing a south pole below left a larger residual effect than those producing a north pole. These two numbers exhibit a greater difference than those previously found (see Section 16, p. 539).

Second Series. With *down*ward axial currents, the first being preceded by a coil one producing a south pole below:—

↓	N.	↓	S.
1. 3·8 ←■	2. 3·5 ■→	3. 3·0 ■→	4. ·5 ←■
5. 3·75 ←■	6. 3·3 ■→	7. 2·8 ■→	8. ·5 ←■
9. 4·0 ←■	10. 2·75 ■→	11. 2·25 ■→	12. ·0
13. 4·0 ←■	14. 3·5 ■→	15. 3·10 ■→	16. ·0
17. 4·0 ←■	18. 3·3 ■→	19. 3·75 ■→	20. ·0
5)19·55	5)16·35	5)14·90	5)1·0
Averages 3·91	3·27	2·98	·2

The largest torsions were caused by the axial currents, and the smallest by coil ones. Downward axial currents succeeding coil ones which produced a south pole below gave larger torsions than those succeeding coil ones producing a north pole, in the proportion of 3·91 to 2·98.

(B) *Residuary effect of axial currents.—First Series.* With coil-currents producing a *north* pole below, the first axial one being preceded by a coil one in that direction:—

↓	N.	↓	N.
1. 1·5 ←•	2. 1·25 ←•	3. 3·25 •→	4. 2·0 •→
5. 1·75 ←•	6. 1·50 ←•	7. 3·25 •→	8. 1·5 •→
9. 2·0 ←•	10. 1·50 ←•	11. 3·5 •→	12. 2·25 •→
13. 2·2 ←•	14. 1·50 ←•	15. 3·25 •→	16. 1·0 •→
17. 1·75 ←•	18. 1·75 ←•	19. 3·3 •→	20. 1·1 •→
5)9·20	5)7·50	5)16·55	5)7·85
Averages 1·84	1·50	3·31	1·57

The largest torsions were caused by axial currents, and the smallest by coil ones. Coil-currents producing a north pole below, succeeding *down*ward axial ones, yielded very slightly larger torsions than those succeeding *up*ward ones; therefore the residual effects of downward and upward currents were not widely different: the numbers given in Section 27, page 546, agree with this result.

Second Series. With coil-currents producing a *south* pole below, the first axial one being preceded by a coil one of that direction:—

↓	S.	↓	S.
1. 5·25 •→	2. 1·0 ←•	3. 5·0 ←•	4. 1·0 •→
5. 6·25 •→	6. 1·25 ←•	7. 4·5 ←•	8. 1·75 •→
9. 6·0 •→	10. 1·5 ←•	11. 4·5 ←•	12. 1·5 •→
13. 5·83 •→	14. 1·0 ←•	15. 5·25 ←•	16. 1·5 •→
17. 6·25 •→	18. 1·5 ←•	19. 4·5 ←•	20. 1·25 •→
5)29·58	5)6·25	5)23·75	5)7·00
Averages 5·91	1·25	4·75	1·40

The largest torsions were produced by axial currents, and the smallest by coil ones. Coil-currents producing a south pole below, succeeding *down*ward axial ones, yielded torsions not much larger than those succeeding *up*ward ones; this agrees with the immediately preceding result obtained with coil-currents producing a north pole.

In these four series of experiments, axial currents succeeding coil ones which produced a south pole below, yielded larger torsions than those which succeeded the opposite direction of coil-currents, because the influence of terrestrial magnetism strengthened a residuary south pole and weakened a residuary north one. The average magnitude of all the coil-current torsions was 1·69 mm., and of all the axial-current ones 3·66 mm.

Comparison of the average numbers in these series of experiments confirms the statement already made (Section 20, p. 543), that "the magnitude of the torsion produced by a given current depends not only upon the kind of current which immediately precedes it, but also upon the description of current which precedes that one. An axial current in a given direction succeeding a coil one nearly always produced a greater torsion if the coil one was preceded by an axial one in the opposite direction than if it

followed one in the same direction. A similar but less general result occurs with coil-currents succeeding axial ones." Thus:—

				mm.							mm.	mm.		
↑	N.	↓	gave	1·55,	whereas	↓	N.	↑	gave	1·84 =	·29	increase.		
↓	N.	↓	„	2·98,	„	↑	N.	↓	„	3·31 =	·33	„		
↑	S.	↑	„	5·05,	„	↓	S.	↑	„	5·91 =	·86	„		
↓	S.	↓	„	3·91,	„	↑	S.	↓	„	4·75 =	·84	„		
N.	↓	N.	„	1·57,	„	S.	↓	N.	„	3·27 =	1·70	„		
N.	↑	N.	„	1·50,	„	S.	↑	N.	„	3·25 =	1·75	„		
S.	↓	S.	„	1·40,	„	N.	↓	S.	„	·2 =	1·20	decrease.		
S.	↑	S.	„	1·25,	„	N.	↑	S.	„	1·1 =	·15	„		

The exceptions only occur with south-pole coil-currents succeeding axial ones (see Section 20, p. 542).

As several of the different orders of succession of currents produced torsions in the same directions, attempts were made, but unsuccessfully, to *accumulate* the torsions produced by each.

31. *Which current most determines the direction of torsion?*

The symmetry of all the torsional movements in the four immediately preceding series becomes conspicuous on classing the right-hand ones in the order of their relative magnitudes, and arranging the left-hand ones similarly; thus:—

↑	S.	↓	−4·75	←		↓	S.	↑	−5·91	→
↓	S.	↓	−3·91	←		↑	S.	↑	−5·05	→
S.	↑	N.	−3·25	←		S.	↓	N.	−3·27	→
↓	N.	↑	−1·84	←		↑	N.	↓	−3·31	→
↑	N.	↑	−1·55	←		↓	N.	↓	−2·98	→
N.	↑	N.	−1·50	←		N.	↓	N.	−1·57	→
S.	↓	S.	−1·40	→*		S.	↑	S.	−1·25	←*
N.	↓	S.	− ·20	←		N.	↑	S.	−1·10	→

As in each combination of three currents in the foregoing Table (except the two which produce torsions in an apparently abnormal direction) the first current may be omitted without altering the direction of torsion†, it is evident that the direction of torsion depends upon one or both of the remaining two; and as each combination of currents in the right-hand column would then differ from the corresponding one of the left-hand column only in having its AXIAL current in a reverse direction, it is probably the *axial* current which most determines the direction of torsion. It is, of course, possible so to arrange the different members of the Table that each member of the right-hand column

* Those marked * are apparently abnormal in direction (see Section 32).

† The first current of the three in those cases affects only the *magnitude* of the torsion.

would differ from the corresponding left-hand one only by a reverse direction of its *coil*-current; but in that case the magnitudes would be unsymmetrical.

32. *Apparently exceptional cases of torsion.*

The directions of the torsions given in the two columns headed "S" on page 550 disagree with those shown by the figures in Class B, Plate XLII. (see Section 18), and apparently contradict the law stated in Section 6. On examining the notes of the experiments they were, however, found, like those described in Section 20, page 542 (see also Section 21), to be cases not of torsion, but of detorsion; they coincide also with the instances of decrease of magnitude of movement noticed in Section 30, page 551, and Section 31. Why a coil-current producing a south pole below should behave thus in cases where it followed an axial one which was preceded by a *similar* coil one, and not in those where the axial one was preceded by a *dissimilar* coil one (see Section 30, p. 549), and why such a phenomenon does not occur with a coil-current which produces a *north* pole below (see Section 30, page 550), I have not investigated. These detorsions are, however, probably dependent upon the influence of terrestrial magnetism, because they do not occur when the south pole is *above*.

As real torsion and detorsion can only be detected and measured by the aid of a proper zero-point, it is necessary to have the wire well annealed and free from magnetism and mechanical twist before commencing a series of experiments, and not to disturb the zero-point by mechanical motion of the apparatus. A very effectual way to remove the residual effects of both coil- and axial currents is to heat the rod or wire to redness in a direction at right angles to the magnetic meridian, and allow it to cool in that position without disturbing it (see Section 24). A more convenient but less perfect way is to repeatedly and simultaneously pass a coil-current producing a south pole below, and an axial one of proper relative strength (see Section 37, page 555), and stop the two currents simultaneously; the pointer will then settle very near zero, and the wire will only possess the usual magnetism induced by terrestrial influence (see Section 35).

33. *Coexistence of the coil-current and axial-current states in iron and steel.*

The two conditions, or rather directions, of magnetic condition were observed to coexist in the same wire in many of the experiments. All rods or wires of iron or steel in which there remained the effect of an axial current were at the same time in a more or less longitudinally magnetic state by the influence of terrestrial magnetism, and could then be twisted by the application either of a suitable coil-current or of an axial one. These properties may, however, be also interpreted according to the view already given (see Section 5, p. 532), that in all wires of iron under such circumstances the axes of the molecules lie in a spiral direction with regard to the axis of the wire, and are therefore in a position to be moved by the influence either of a longitudinally or tangentially magnetizing force.

The view that the two conditions are to some extent distinct and independent agrees

with the fact that a suitably powerful coil-current *at once* removes and reverses the residuary longitudinal magnetic polarity in a soft iron wire, but only gradually removes the residual effect of an axial current, and does not at all reverse it (see Section 27). Each axial current also transmits its own characteristic influence through several subsequent coil-current changes, and each coil-current similarly through axial-current changes, in a kind of hereditary manner.

34. *The condition produced in iron and steel by an axial current.*

The experiments generally described in this paper show conclusively that the magnetism produced in iron by an axial-electric current is distributed very differently from that in a cross section of an ordinary electromagnet; it also arises from an influence within the iron, whereas the latter is produced by one from without. They also show that the phenomenon of electrotorsion is essentially magnetic, and strongly support the view that the state existing in iron when an electric current is passing axially through it, upon which the torsion depends, is not the transverse magnetism of the current itself, and which is inseparable from it (see Section 24, page 545), but that induced in the iron by the current, because electrotorsion does not occur in non-magnetic metals (see Section 4), and because the residuary axial-current state is destroyed by a red heat (see Section 24).

I have not made any experiments to ascertain whether the effect of an axial current can be communicated at a distance from one piece of iron to another by mere proximity or contact; nor have I examined whether the same condition can be acquired by vibrating a demagnetized iron rod at right angles to the terrestrial magnetic meridian*.

35. *Torsions produced by simultaneous coil- and axial currents.*

Several modifications of this method were examined:—1st, with undivided currents; 2nd, with divided ones; and 3rd, with one current passed temporarily during the continuance of the other, &c.; and in each case torsions were freely produced. The second and third of these arrangements were the best.

There was a special difference between the torsions produced by alternate currents and those yielded by simultaneous ones. In the former case, on cessation of the current, the pointer only slightly returned towards zero, and the wire remained twisted (except in a limited number of special instances, see Section 20, p. 542, and Section 32); on repeating the current in the same direction, only the small elastic torsions occurred, and the large movements in the same direction could only be again obtained by reversing the current, and then again repeating it in the original direction. But with the two currents flowing simultaneously, on stopping them the index returned nearly to zero,

* About the year 1777 Beccaria noticed "that a needle, through which he had sent an electric shock, had in consequence acquired a curious species of polarity; for, instead of turning as usual to the north and south, it assumed a position at right angles to this, its two ends pointing to the east and west."—Roget's "Treatise on Electromagnetism," page 3, in 'Library of Useful Knowledge,' 1832.

and the wire did not remain twisted; on repeating the currents in the same direction, the very large torsion in the original direction was again produced; and any number of such torsions could be consecutively obtained without any intervening reversal of the currents. It is evident therefore that the coercive force or condition within the bar, which retains the iron in a twisted state after the passage of alternate coil- and axial currents, is either overcome or does not operate when simultaneous currents are employed.

As simultaneous currents produced very much larger torsions than alternate ones, and appeared to aid each other, and notwithstanding coil-currents destroy the effect of axial ones, and *vice versâ*, their influences, although dissimilar, are not contradictory, but appear to act upon the principle of "composition of forces."

The very much greater magnitude of the torsions obtained by this method was probably a consequence of the two magnetic conditions being very much stronger during the continuance of the currents than after their cessation. Not unfrequently, with an iron wire 1·75 mm. thick, the first movement of the index exceeded 25 millimetres. The torsional push is not limited to a small angle, but continues through the entire range of the largest arc through which the pointer can be made to swing, even though that exceeds one third of a circle.

36. *Torsions yielded by simultaneous and divided currents.*

With the two currents commenced together and terminated also simultaneously, the index made a very large movement at the commencement of the currents, remained considerably (though much less) deflected during their continuance, and returned nearly to zero on their cessation, provided the two currents were of proper relative degrees of strength. In all the experiments in which currents simultaneous in their commencement were employed (and they were very many), the direction of torsion agreed with the law (see Section 6), probably because such a method of applying them largely removes at once all interfering residuary influences.

Eight series of experiments, including sixty distinct and different orders of succession of currents, were made—passing simultaneous currents after single ones, both coil and axial, and single currents after simultaneous ones; employing an iron wire 1·75 mm. diameter, and currents from six cells arranged as three upon each circuit. In fifty-four of these cases the directions of the torsions agreed with the law, and the magnitudes of them agreed with the results usually obtained. In the other six, all instances in which single currents following simultaneous ones were employed, and associated only with coil-currents producing a *north* pole below, no movements took place on *making*, but small detorsions occurred on *breaking* the contact.

Although the two currents of each pair of simultaneous ones in these experiments were not of proper relative strength, the coil one being in excess, the average magnitude of all the torsions produced by a single coil-current, after repeated simultaneous ones, was only 1 mm., and by a single axial one after them was 1·5 mm. By comparing these results with those obtained by means of alternate single currents under nearly similar

circumstances (see Section 20, page 542), it will be seen that simultaneous currents leave less residual effect than single ones.

Simultaneous and divided currents, on the first time of passing, produced a loud and dull sound on making contact, and a weaker and more metallic one on breaking contact, but by each repetition in the same direction the reverse, and their first passage produced louder sounds than their repetitions.

37. *Influence of relative strength of the two currents.*

As the magnitude of the torsions produced by simultaneous currents depends upon both currents, and is therefore limited by the weakest, the two currents must be properly proportioned to each other in order to produce the maximum degree of torsion. To effect that object, I have increased the strength of the axial current and decreased that of the coil one (or *vice versâ*), until, after passing and stopping the two simultaneously, a small sign of residual axial-current effect was detected by torsion produced on passing a single current through the coil only. The best proportion of the two forces, with the apparatus and iron wire (2·6 m. long and 1·75 mm. diameter) I usually employed, was the current from four cells arranged as two for the coil, and that from eight arranged as four for the axial wire. By actual measurement it was found that the electric conduction-resistance of the helix in that apparatus (see Section 1) was equal to ·309 ohm, and that of the iron wire 2·6 m. long and 1·75 mm. diameter was equal to ·137 ohm; therefore with two batteries of equal power attached to the coil and iron wire respectively, the quantity of electricity passing through the axial wire was two and a quarter times the amount of that circulating through the coil; and with eight cells attached to the axial wire and four to the coil, the difference would of course be double that amount.

The torsional effects produced by simultaneous and undivided currents passing through the bar and helix in one continuous circuit were similar to those described in Section 36, but were comparatively small, evidently in consequence of the electric power being disadvantageously applied, the axial wire requiring a relatively much greater current.

38. *Influence of metal screens.*

A brass tube 2·6 m. long and 11·5 mm. external diameter was fixed in the helix, with the soft iron wire 1·75 mm. diameter suspended in its axis, and the battery divided into two portions of six cells each. By arranging each six cells as three, and transmitting the two currents simultaneously, deflections varying from 18·5 to 23·5 mm. took place, showing that the brass tube did not materially intercept the torsional influence of the coil-current. In some similar experiments, in which a thin *iron* tube was employed instead of the brass one, diminution of torsion occurred.

39. *Torsions produced by the temporary action of one current during the continuance of another.*

Iron is extremely susceptible of being affected by an electric current, and consequently every different way of applying the two currents produces a difference of effect upon it. With the present method the current which is applied *first*, whether axial or coil, produces little or no torsion according to the residuary magnetic state of the wire; whereas the *second* one produces a very large torsion, nearly or quite as great as would have occurred if the two currents commenced together. And, generally, if one of the two currents is stopped after the other, the discontinuance of the first, whether coil or axial, is attended by a much greater degree of detorsion than that of the second. The effects were modified if a weight was suspended from the wire.

Some experiments were also made of commencing one current (A) soon after the other one (B), and continuing A a short time after B had ceased. Six GROVE'S cells arranged as three were used for one circuit, and six similar ones for the other; and the iron wire employed was 1·75 mm. diameter. Every possible combination and order of succession (eight in number) of the two currents was tried. In each case the first current produced at its commencement only a small movement, varying from 0 to 5·5 mm., and the second a very large one, varying from 20 to 27·25 mm. The current which was first stopped, whether coil or axial, produced a detorsional movement, varying in magnitude from 15 to 19·5 mm.; and on stopping the other current a further detorsion took place, varying in range from 3·5 to 6·25 mm.; the needle then settled either at zero or very near it. According to these results, and allowing a greater value to the degrees most distant from zero, more than three fourths of the torsion ceased when one only of the currents was stopped; probably this was partly a result of momentum of the recoil. The magnitudes of the torsions produced by the axial currents in these experiments varied from 25 to 27 mm., and averaged 25·5 mm.; and of those yielded by the coil-currents ranged from 20 to 27·25 mm., and averaged 23·44 mm., indicating a stronger preparatory condition produced by the coil-current and an excess of that current influence.

In two experiments of these eight, the large torsion was in an opposite direction to that required by the law (see Section 6). In one of them a coil-current producing a *south* pole below was established during the continuance of a *down*ward axial current, which had been preceded by a coil one yielding a *north* pole below in an immediately previous experiment; in the other a coil one producing a *north* pole below was commenced during the flow of a *down*ward axial one, which had followed a coil-current producing a *south* pole below in the previous experiment.

These exceptions were not instances of detorsion, but actual reversals; they were found to depend upon the residual coil-influences mentioned, and upon the successive commencement of the two acting currents, because they did not occur if the residual state was prevented (by terminating the two currents of the immediately preceding

experiment simultaneously) or reversed, nor if the two acting currents were made to commence simultaneously.

The residuary state preceding these instances was not manifested by a conspicuous degree of permanent twist; in the first case that amounted to 1·5 mm., and in the second to only ·25 mm.; this, however, agrees with constant experience in the subject: a non-magnetized iron wire is not visibly twisted by a powerful coil-current (or axial one) alone, but acquires an invisible potent condition which reveals itself by torsion on the subsequent passage of suitable currents. I have not examined why these exceptions to the law occur only in cases where a *down*ward axial current is employed.

These peculiar instances, together with the various other phenomena of electrotorsion and detorsion, support the view that the molecular mechanism of iron is a complex one; they also illustrate the very great influence which the order of succession of the currents exerts in some cases, and to which attention has already been called (see Section 20, page 543).

40. *General influence of the order of succession of the currents.*

A general review of the phenomena described in this paper shows that the hereditary action and order of succession of the various currents affects the torsions in all cases; in all it affects the direction and apparently also the magnitude, in a less number of cases it causes detorsion to occur, and in a very few instances it enables torsion to be produced in the opposite direction to the fullest extent.

41. *Relative magnitudes of torsional effect of electric currents during and after their passage.*

Whilst making the experiments on the magnitudes of the torsions produced by alternate coil- and axial currents (described in Section 20), I made a series of other experiments with the same iron wire (*i. e.* 1·75 mm. diameter and without any weight attached to it) and arrangement of battery, but passing one current during the continuance of the other, for the purpose of obtaining some idea of the relative magnitudes of residuary torsional influence of the two currents to that of their torsional power during their circulation in the coil and axial wire:—

(A) *With the coil-current continuous and the axial one temporary.*—1. With *south* pole below and a *down*ward current and also with an *up*ward one. 2. With a *north* pole below and a *down*ward axial current and with an *up*ward one.

(B) *With the axial current continuous and the coil one temporary.*—1. With a *north* pole below and a *down*ward axial current and with an *up*ward one. 2. With a *south* pole below and an *up*ward axial current and with a *down*ward one.

Each single experiment was repeated. The magnitudes of the whole of the torsions varied from 19 to 24 mm., and averaged 22·4 mm. In the other experiments referred to, the average magnitude of all the torsions produced by coil-currents was

1·23 mm., and of those yielded by an equal number of axial ones 3·18 mm., and of the two collectively 2·20.

It would appear from these results that in iron the residuary torsional influence of the currents generally is about one tenth of that exerted by them during their continuance*. In steel it would be a much greater proportion in consequence of the comparative smallness of the torsions yielded by that substance with simultaneous currents (see Section 44). The general result in these cases would be considerably affected by variation of the relative strengths of the two currents, because, when alternate, the axial currents yielded torsions two and a half times the range of that yielded by the coil ones, and, when simultaneous, if one current was in excess it would only exert a part of its power effectively. To diminish this latter source of error I previously adjusted the currents as accurately as I was able in the way already described in Section 37.

42. *Effect of mechanical pull on torsions produced by simultaneous currents.*

With an iron wire 3·77 mm. thick, a current from six cells arranged as three, producing a north pole below, passed temporarily through the coil during the continuance of an upward current from a similar battery through the iron wire, produced a torsional movement of 3·5 mm. whilst a weight of 5½ kilogrammes was attached to the wire, and of 13·5 mm. in the opposite direction without the weight. As mechanical pull affects the magnitude of the torsions, both with alternate currents (see Section 22) and with simultaneous ones (see also Section 43), it is evident the weight of the pointer and its counterpoise exercised a similar effect in all the experiments.

43. *Relative magnitudes of torsions by different methods.*

The magnitudes of the torsions in all cases depended upon the advantageous application of the forces of the two currents. With an iron wire 3·77 mm. thick, supporting a weight of 5½ kilogrammes, and a current from twelve GROVE's cells arranged as three:—(A) Alternate axial currents in opposite directions gave torsional movements varying in magnitude from ·5 to 3·25 mm. (B) Simultaneous and undivided currents from the same battery-arrangement gave movements varying in extent from 1·4 to 2·25 mm. (C) Simultaneous and divided currents, the coil one being from six cells arranged as three, and the axial one from the other six connected as three, yielded movements varying from 8 to 10 mm. in magnitude. (D) Similar divided currents, the axial ones being passed temporarily during the continuance of the coil ones, the torsional movements varied in extent from 1 to 6 mm. (E) Similar divided currents, with the axial ones continuous and the coil ones temporary, the movements ranged from 0 to 9 mm., and without the weight 13 mm. In these last experiments the coil-currents produced the largest torsions, but in some former experiments (see Section 39, p. 556) temporary axial currents passed during the continuance of coil ones produced the largest. The

* As the amount of residuary axial effect is less than that of coil-influence, the proportion of the former would be less, and of the latter more, than one tenth.

weight appeared to have a strong effect in modifying and diminishing the magnitude of the torsions under the headings D and E.

44. *Comparison of magnitudes of the torsions generally in iron and steel.*

I obtained the torsions with these two substances under nearly similar conditions. The diameter of the iron wire was 3 mm. and of the steel 2·7 mm., and no weight was attached to either. The electric current was from twelve GROVE'S cells, and was applied in each of the following ways:—

1st. *Alternately reversed axial currents only*, and the twelve cells arranged as three.—The torsions obtained with iron averaged 5·4 times the magnitude of those with steel; they varied from 3·25 to 3·5 mm. with iron, and from ·5 to ·75 mm. with steel.

2nd. *Alternately reversed coil-currents.*—The movements were 2·55 times as large with the steel as with the iron; their magnitudes varied from ·5 to 1 mm. with the iron, and from 1·5 to 2·33 mm. with the steel.

3rd. *Alternate axial and coil-currents*, in every possible order.—The magnitude of the movements obtained with iron averaged 1·77 time that of those obtained with steel; those with iron varied from 1 to 5·5 mm., and those with steel from ·0 to 4 mm. Several cases of detorsion occurred in this series.

4th. *Simultaneous coil- and axial currents.*—The current from six cells arranged as three was used with the coil, and a similar current with the axial wire, and the two currents were commenced and stopped simultaneously. Every possible combination of the two currents, and every order of succession (twelve in number) of each pair of them, was tried. The movements obtained with iron averaged 2·42 times the magnitude of those produced with steel. The magnitudes of those with iron varied from 5·75 to 16·25 mm., and with steel from ·25 to 7·75 mm.

The results of these four classes of experiments show that, except with alternately opposite coil-currents succeeding an axial one, iron is much better adapted than steel for producing large electrotorsions.

[It is probable that the generally smaller torsions obtained with steel than with iron was partly due to the greater degree of mechanical resistance which that substance offers to torsion, and partly to its differences of magnetic properties and chemical composition. Electrotorsion therefore affords prospectively a new method of investigating the mechanical and magnetic properties, and the chemical composition, of magnetic metals.]

45. *Is the voltaic coil twisted during experiments of electrotorsion?*

Some experiments were made of passing currents from the twelve cells arranged as six through a loose spiral, 630 mm. long and 16 mm. outer diameter, of thick copper wire, fixed at its upper end and surrounding a fixed cylindrical rod of soft iron 640 mm. long and 10 mm. thick, the lower end of the coil being free and provided with a pointer 380 mm. long. Torsional movements amounting to 7 mm. (as well as the well-known

shortening effect) occurred if the bar was in the coil, but not otherwise, and was lessened to 5 mm. if the undivided current was caused also to traverse the bar. The torsional movement did not change in direction on changing the course of the current either in the coil, the bar, or both, but in each case agreed with a diminution of diameter of the coil; it was therefore of a different kind to the phenomenon of electrotorsion described in this paper.

46. *Electrotorsion of nickel.*

A bar of nickel 60 centims. long and 19 mm. diameter was subjected to the influence of simultaneous and divided axial and coil-currents from twelve GROVE'S cells whilst in the axis of a suitable coil; but only very minute torsional movements occurred, apparently in consequence of unsuitable dimensions of the bar.

47. *Electrotorsion in telegraph-wires.*

[As nearly all the overground telegraph-wires on the surface of the earth are composed of iron, and are more or less magnetized by terrestrial magnetic influence, especially those lying in the direction of the terrestrial magnetic meridian, it is evident that on the passage of every electric current through them electrotorsional movement tends to occur.]

Note on Mr. GORE'S *Paper on Electrotorsion.* *By* Sir WILLIAM THOMSON, *F.R.S.**

In Section 5, "General cause of the torsions," the phenomena are attributed to the combined influence of ordinary magnetic polarity and the magnetic condition of iron at right angles to that. To see precisely how this combined influence produced the results discovered by Mr. GORE, we have only to look to JOULE'S discovery of the effects of magnetism on the dimensions of iron and steel bars, and of the musical sounds consequently produced in an electromagnet every time battery-contact is made or broken. This great discovery was first described in public on the occasion of a *conversazione* held at the Royal Victoria Gallery of Manchester, on February 16th, 1842. A printed account of it is to be found in the eighth volume of STURGEON'S 'Annals of Electricity,' p. 219, and in the 'Philosophical Magazine,' 1847, first half year. The following are the chief results obtained by JOULE:—

1. When a wire or bar of iron (or steel) is alternately subjected to, and left free from, longitudinal magnetizing force, it alternately becomes longer and shorter.

2. In the same circumstances its volume remains sensibly unaltered; and therefore it experiences lateral shrinking to an extent equal to half the extension in length†.

3. JOULE verified the lateral shrinking by passing a current through an insulated

* Not read before the Society, but ordered to be printed.

† It is understood, of course, that the shrinking is reckoned in proportion to the transverse diameter, and the extension in proportion to the length, as is usual in the geometry of strains and in the theory of elasticity.

wire along the axis of a piece of iron gas-pipe*, 1 yard long, $\frac{3}{16}$ of an inch in bore, and $\frac{3}{16}$ of an inch in thickness, and found, as he anticipated, that the *length* of the gas-pipe became *diminished* when the current was instituted, and *increased* when the current was stopped.

4. Residual magnetism leaves residual changes of dimension in iron and steel of the same signs as those exhibited when magnetizing force is first applied or afterwards re-applied.

5. Longitudinal pull†, if sufficiently intense, reduces to zero the magnetic extensions and contractions; and if more intense still, puts the metal into such a state that opposite strains are produced by it. An iron or steel wire stretched vertically by a small weight becomes elongated by magnetization, but if kept stretched by a constant sufficiently heavy weight it is shortened by magnetization‡.

Now the passage of a current along a straight iron or steel wire of circular section gives rise to poleless magnetization in circles perpendicular to the length of the wire and with their centres in its axis. Let γ be the strength of the current through the wire, reckoned of course in absolute units. If the wire be infinitely long, the resulting field of force (whether the wire be of iron or of any other metal) is fully specified by saying that the lines of force are circles in planes perpendicular to the axis and having their centres in this line, and that the intensity of the force is

$$\frac{2\gamma}{d^2}\,r \text{ for points in the substance of the wire,}$$

and

$$\frac{2\gamma}{r} \text{ for external points,}$$

* The bends of the insulated wire outside the gas-pipe in Joule's experiment complicate the circumstances somewhat by superimposing upon the circular poleless magnetization, which a single straight wire along the axis of the pipe would produce, magnetization in which there is northern polarity along one semicylinder, and southern polarity along the other semicylinder of the outer boundary of the iron pipe, and fainter opposite polarities on the inner cylindrical surface. But if the wire had been continued straight for several inches outside the pipe at each end, and then carried away to the battery without ever being brought near the gas-pipe externally, it is clear that effects in the same direction, though of slightly less magnitude (by an almost infinitesimal difference), would have been observed.

† Rankine's nomenclature regarding stresses and strains (which is consistent with Huyghens's celebrated *ut tensio sic vis*) ought to be carefully followed. It is therefore necessary to introduce two nouns, pull and thrust, common enough in familiar language, but not hitherto much used in the theory of elasticity, to express longitudinal forces in the directions which would elongate or shorten the bar or wire. With reference to a stretched wire we ought to talk of the pull along the wire, and ought not to use the word strain or tension to express a stretching force. The only objection to the word pull is that some people might consider it too familiar; but surely it is not a valid objection to the mathematician or philosopher that a word, the use of which enables him to avoid ambiguity in scientific statements, is already understood by non-scientific people. According to Rankine's nomenclature we must confine the word strain to a change of dimension or figure caused by stress; thus the longitudinal strain of a wire or of a beam experiencing a pull or thrust is the (positive or negative) elongation produced by the force.

‡ Hence the "Young's modulus" of iron or steel is increased by longitudinal magnetization.

where a denotes the radius of the wire, and r denotes distance from its axis. (The same description, as is now well known—thanks to the beautiful illustrations and diagrams of iron-filings by which FARADAY showed it in the Royal Institution—is approximately applicable to the field of force in the neighbourhood of a straight portion of wire conveying a current, provided no other part of the wire is near.) Hence the intensity of the magnetization of the substance is equal to

$$\mu \frac{2\gamma}{a^2} r,$$

where μ denotes the "magnetic susceptibility" * of the substance. Let now a uniform magnetizing force X be applied along the whole length of the wire. This, combined with the force due to the current through the wire, gives at any point of the substance a resultant force equal to $\sqrt{\left(\frac{4\gamma^2 r^2}{a^4} + X^2\right)}$ in a direction inclined to the length of the wire at the angle whose tangent is $\frac{2\gamma r}{a^2 X}$. The lines of force in the resultant field are therefore spirals. The wire being supposed infinitely long, the magnetization will still be poleless †, and will be everywhere in the direction of the resultant force; and its intensity will be equal to the resultant force multiplied by the magnetic susceptibility. The extension of the substance along the spiral lines of magnetization, and its shrinkage along the orthogonal spirals, to be anticipated from JOULE'S old results, give rise to GORE'S phenomena of electrotorsion.

Although we thus see GORE'S "electrotorsion" as a geometrical consequence of the earlier discovery of JOULE, we must, nevertheless, regard Mr. GORE'S investigation as having led to an independent discovery of a remarkably interesting character, enhanced by the well-designed and necessarily laborious working out of varied details described in his paper.

It is difficult to conceive any physical investigation, except FARADAY'S magnetic rotation of the plane of polarization of light, more important towards a physical theory of magnetism than JOULE'S result (No. 5) above. It suggests an interesting extension of Mr. GORE'S investigation. Let the wire rod or tube experimented upon be stretched by a heavy weight, and at the same time subjected to a constant twisting-couple of sufficient magnitude, then, no doubt, the torsions as well as the elongations observed under varying magnetic influences will be the reverse of those discovered by Mr. GORE. The investigation ought, of course, to be varied by applying couple alone and longitudinal pull alone.

* THOMSON'S reprint of 'Electrostatics and Magnetism,' § 610. 3.

† The free polarity in the actual experiments due to the finiteness of the iron bar or wire and of the magnetizing helix reduces somewhat the magnitude of the effects, but does not alter their general character.

XVII. *The Winds of Northern India, in relation to the Temperature and Vapour-constituent of the Atmosphere.* By Henry F. Blanford, *F.G.S., Meteorological Reporter to the Government of Bengal. Communicated by Major-General* Strachey, *R.E., C.S.I., F.R.S.*

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Contents.

	Page
Introduction	563
Part I.—Description of the Winds.	
The Punjab	565
The Gangetic Plain	568
Plateau of Rajpootana and Bundelkund	571
Central India	573
Western Bengal and Orissa	574
The Gangetic Delta	576
Assam	578
The Arakan Coast	580
Summary	581
Part II.—Relations of the Winds to other Elements of Climate.	
Temperature. Horizontal and vertical distribution	585
Vapour-tension, Humidity and Rainfall	594
Atmospheric Pressure. Horizontal and vertical distribution	602
Certain effects of the Winds	613
General Summary	617
Appendix.	
On the Cyclones of the Bay of Bengal	623
Tables.	
Wind-directions &c.	629
Temperature	643
Vapour-tension	645
Humidity	646
Rainfall	647
Atmospheric pressure	651
Description of Plates	653

INTRODUCTION.

It is my object in this paper to describe the normal wind-currents of Northern India and their annual variation, and to trace out their origin and causes, in so far as these can be discovered in the local physical changes of the atmosphere. For this kind of inquiry India offers many peculiar advantages. At opposite seasons of the year it exhibits an almost complete reversal of the wind-system and of the meteorological conditions depending on it or on which it depends; its almost complete seclusion, in a meteorological point of view, from the remainder of the Asiatic Continent, by the great mountain-chain along its northern border, simplifies, to a degree almost unexampled elsewhere, the conditions to be contrasted by limiting them to those of the region itself and of the seas around; while it presents in its different parts extreme modifications of climate and geographical feature. In its hill-stations it affords the means of gauging the condition of the atmosphere at permanent observatories up to a height of more than 8000 feet, and in the loftier peaks and ridges of the Himalaya, at temporary

observing-stations, up to the greatest elevations to which man can ascend when unaided by the balloon. The periodical variations of temperature, vapour-tension, pressure, &c., both annual and diurnal, are strongly marked and regular; and their changes proceed so gradually, that the concurrence and interdependence of their several phases can be traced out with much precision, even in the unanalyzed registers.

These numerous and great advantages indicate this country as preeminently a field for the future study of meteorology. Most of the great problems of the science are here presented in the form of *instantiæ ostensivæ*; and comprehensive systematic observation, intelligently conducted, is all that is wanting to place them at the command of European science.

What James Prinsep, Colonel Sykes, Dr. Hooker, and General Strachey have already effected in this field by their own unaided labours is too well known to need more than a passing reference; and it is mainly owing to the exertions of the last, acting both independently and in concert with the Asiatic Society of Bengal, that within the last few years a beginning has been made, at the expense of the Government, to gather its fruits more extensively. In 1865 the first steps were taken by the Governments of Bengal and the North-western Provinces to obtain regular meteorological registers from a number of selected stations, under the charge of a special Government officer for each Government. Up to that date, the only regular observatories had been those of the three Presidency towns, with four other stations in Bombay, that of Trivandrum and Agastyamully in Travancore, and that carried on for three years by Colonel Boileau at Simla. Attempts had indeed been made in previous years to register the rainfall, temperature, and, in some cases, other kinds of observations through the agency of the medical officers of the East-India Company; but, owing mainly to the absence of any organized control, the results were for the most part of little value. Even after a beginning had been made on a better system, owing to various difficulties arising from local causes, a year or two elapsed before the system could be brought into good working order; but there has been a marked improvement year by year, and as the Governments of most of the other provinces have since taken up the scheme, regular observations are now carried on over the greater part of the empire.

The results are not indeed in all cases accessible, and in some of the more remote provinces, where the facilities for scientific work are less than at the older seats of Government, there is still much to be desired. It is impracticable, therefore, at present to extend the discussion of my subject much beyond the geographical limits I have adopted. These include Bengal proper and its dependencies (Orissa, Behar, Assam, and a portion of the Arakan coast), the North-western Provinces with Oude and a part of Rajpootana, the northern part of the Central Provinces, and the Punjab.

The data from all these, except the last and Upper Assam, that have served as the materials for this discussion, consists of registers of the temperature, hygrometry, pressure, wind-direction and velocity, and the rainfall, also in some cases the temperature of radiation. In the case of the Punjab, I have been able to use only the ther-

mometric, hygrometric, wind, and rainfall data, and some of these are less complete and satisfactory than I could have wished. Dr. Murray Thomson's Reports for the North-western Provinces, and Dr. Townshend's for the Central Provinces, have, however, furnished very ample and excellent materials; and these gentlemen, and Mr. Elliott of Roorkee, have taken much pains to ascertain with accuracy the constants to be applied to the observations of atmospheric pressure in their provinces, in order to render them comparable with those of Bengal. But for the cordial cooperation of these gentlemen, it would have been impracticable to collate materials from so large an area for comprehensive discussion.

The leading geographical features of the country are too well known to necessitate any detailed description. In describing the winds, I have adopted the following divisions, most of which correspond to natural orographical divisions:—1st, the Punjab, formed by the lower plain of the five rivers and the smaller plateau above the Salt range that lies along the foot of the Hazarah hills; 2nd, the Gangetic plain, which extends along the foot of the Himalaya from the Indo-Gangetic watershed to the commencement of the delta at Rajmahal; 3rd, the plateau of Rajpootana and Bundelkund, lying to the south of the Gangetic plain and drained by its river; 4th, Central India, which term I have, for the present purpose, restricted to that portion of the flanks of the Sátpoora range which is drained by the upper waters of the Wynegunga and the Nerbudda; 5th, Western Bengal and Orissa—this consists of two distinct parts, viz. the plateau west of the Gangetic delta and the alluvial plain of the Orissa coast; 7th, the Gangetic delta; 8th, the Assam valley; and 9th, the coast of Arakan as far south as Akyab. Some further details of each tract, and the positions of the several stations, the wind-registers of which have furnished the materials of the discussion, will be given in the text.

Part I.—DESCRIPTION OF THE WINDS.

The Punjab.—For illustrating the winds of this region, I have the four stations Rawul Pindee, Lahore, Mooltan, and Dera Ishmail Khan. The first is situated in the north-east corner of the plateau north of the Salt range, and near the foot of the Hazarah hills, at an elevation of 1700 feet. An open plain extends to the south and west, but on the north and east it abuts against the hills of Hazarah and the sub-Himalaya; the latter not very lofty. The second (Lahore) is on the lower plain of the Punjab, about 160 miles south-east of Rawul Pindee and 240 miles west-north-west of Roorkee, at about 700 feet above the sea. The outer range of the sub-Himalaya lies 80 miles off to the north-east, and running from north-west to south-east. The third (Mooltan) is likewise on the lower plain, near the Jhelum river, 190 miles south-west of Lahore, and is about 400 feet above sea-level; and the last (Dera Ishmail Khan) is situated on the main stream of the Indus, 120 miles north by west of Mooltan, and 200 miles west of Lahore. The Sulaiman range, which lies to the west of the Indus, running parallel with the river, is about 50 miles distant from Dera Ishmail Khan; but nearly opposite to the station the valley of the Gomal debouches from the uplands

of Affghanistan, and affords a passage to the winds from the west and north-west. On the north, at a distance of 24 miles, the hills of the Salt range, rising to 4600 feet, advance to the river, which here issues from the range and affords a passage to winds from between north and north-east; while to the east and south the great unbroken plain of the five rivers stretches away in the direction of Lahore and Mooltan.

The wind-registers of these stations extend over a period of a little more than three years. For three months at Lahore and one month at Mooltan only two years' observations are obtainable; so that the data are less complete than those for other parts of our area, and, perhaps in consequence, they exhibit greater irregularities*. The observations are those of the daytime only, viz. at the hours of 10 A.M. and 4 P.M., which are also those of the Gangetic-valley stations and of the Central Provinces.

Beginning with Rawul Pindee, we find a general predominance of west winds, the annual proportion of which is 42 per cent. of the observations. East winds are rather more than half as numerous, and amount to 24 per cent. Winds from the north-east (4 per cent.) are the least frequent, and those from the remaining quarters in no case much exceed 7 per cent. The prevailing directions are undoubtedly mainly determined by the trend of the hills to the north of the plateau, and that of the Peshawur valley beyond the Indus. East winds attain their maximum, and west winds their minimum, in July. The latter are chiefly predominant from October to May, and the former are least frequent in November. In this latter month south-west winds are almost as frequent as those from the west, while in August south-east winds attain their maximum frequency, still subordinate in importance to those from the two dominant quarters. An annual rotation of the winds is slightly, but unmistakably, indicated by the Table. From April to July the veering is by north-west and north to east; and from July to December through south-east, south, and south-west to west.

The geographical position of Dera Ishmail Khan, under the lee of the Sulaiman range (trending from north to south), and at a distance from the Himalaya, determines a very different system of wind-currents. Here north-east winds predominate on the average of the year, amounting to 26 per cent., and west winds (5 per cent.) are least numerous. Winds from the east and south-east amount in each case to 15 per cent., and those from the remaining four points to between 9 and 11 per cent. In the cold-weather months, *i. e.* from November to April, west, north-west, and north winds are at their maximum, always subordinate, however, to those from north-east; while from

* I cannot but think that they are, to a considerable extent, vitiated by another cause, viz. the omission to record calms. Dr. NEIL, who superintends the Punjab registers, assures me, indeed, that absolute calms are of rare occurrence in the Punjab; but I must confess I cannot think that Lahore, for instance, differs in this respect so greatly from Roorkee and Agra as the registers seem to show; and since it is clear that the registration of calms at the Punjab stations has been uniformly neglected, the position of the wind-vane being always recorded as a wind-direction, some doubt must still attach to the probable frequency of calms in this region.

Since writing the above, I have been informed by Dr. CALTHROP that calms are very common in all parts of the Punjab.

May to October winds from between north-east and south-east contribute from 70 to 80 per cent. of the observations. The annual change is rather one of oscillation than rotation. From February to August there appears to be a gradual veering of the mean direction from north by west, through north, north-east, and east to east-south-east; and the change from the summer to the winter monsoon is retrograde, and consists in a gradual strengthening of the northerly current till the latter attains its extreme direction from north by west in January.

At Mooltan the wind-system is again different. Here south-west is the predominant quarter, to the extent of 29 per cent. on the average of the year; north-west stands next in importance, and due east and west winds amount to only 2 and 3 per cent. respectively. It is possible that the low proportion of west winds may be due to some local obstacle, influencing the currents that act on the wind-vane and diverting them either to north or south of their primitive direction. But even if we admit that a portion of the north-west and south-west winds are possibly diverted west winds, the fact remains, that at this station winds from the southerly quarters are equally numerous with those from northern directions, which is not the case at any other of the Punjab stations here noticed. The predominance of westerly over easterly winds, on the other hand, is a condition which also obtains at Rawul Pindee and Lahore, though not at Dera Ishmail Khan. With respect to the annual change of mean direction, the Table shows considerable irregularities, which may be due in part to the cause above suggested, and in part to the inequality of the periods from which the data for the several months have been obtained. In January the resultant appears to be decidedly north-north-west; in February and March less decidedly north by east and north by west; and the wind then appears to back through north-west and west to south-west by south, which is its prevailing mean direction during the summer monsoon, and is most decided in September. In the latter months of the year the direction of the change seems to be reversed, and the winds veer normally and somewhat abruptly through west to north and north by east, which is the mean direction in December.

The wind-system of Lahore resembles to a certain extent that of Rawul Pindee, except that, owing to the more exposed position of the station, the prevalent currents are less exclusively east and west. The most frequent wind is from north-west (25 per cent.), and north-east is second in importance (18 per cent.). South winds are rare, and do not exceed 2 per cent. on the average of the year; and those from the three southerly points are only 22 per cent. against 52 from the opposite directions. As at Rawul Pindee and Mooltan, westerly winds preponderate over easterly, but to a less extent, the proportions being 50 to 39. The prevalent mean direction from October to April is north-west and north-north-west; but north-east winds on the one hand, and west winds on the other, form a not inconsiderable proportion of the whole. From March onward, east and south-east winds become more frequent, and in July and August preponderate. They are not, however, very steady, and north-east and south-west winds are nearly as common as those from south-east. In September westerly winds regain

their ascendency, and veer by west towards the north in October. The annual rotation is therefore normal as at Rawul Pindee.

It results from the above analysis that the winds of the Punjab are far from uniform in different parts of the great plain. Except in the region that lies under the lee of the Sulaiman range, currents from the westward preponderate on the average of the year; and this we shall see is a general rule throughout Northern India. On the plateau north of the Salt range, west winds and their opposites prevail almost to the exclusion of those from other quarters—the former coinciding with the cold and the hot dry seasons, the latter with the summer monsoon. To the south of this, northerly winds preponderate over southerly in the greater part of the Punjab, but at Mooltan the latter are somewhat in excess. In the winter months (December to February) the mean direction is west at Rawul Pindee, nearly north at Dera Ishmail Khan and Mooltan, and between north and north-west at Lahore—diverging, therefore, from the angle formed by the mountain-ranges that bound the plain on the north and west. With the rise of temperature in Northern and Central India in the months of March, April, and May, the wind-currents draw round to the north-east along the foot of the Sulaiman range, to west over the more exposed parts of the plain; but in the latter month, or in June, when the rains are setting in over the greater part of India, reducing its temperature, and thus transferring to the Punjab the locus of greatest heat*, easterly winds begin to increase in frequency, and in July and August preponderate in the central and northern parts of this region. At Mooltan, however, as in Sindh, easterly winds never gain the upper hand, and during the height of the summer monsoon the prevailing winds come from the direction of the Arabian coast. The south-west wind is not, however, here a rain-bearing current. It probably comes as much from the desert as the sea, and passing in its course over the heated arid plains that surround the lower course of the Indus, the increase of its temperature counteracts any tendency to precipitation which may be induced by the upward diffusion of its vapour; so that Mooltan receives on an average only 5 inches of rain during the five months from June to October.

Thus it appears that during the south-west monsoon the winds perform a kind of cyclonic circulation in the Punjab, converging from the plains to the south and east; and this region is the goal of the Indian portion of the monsoon. In Affghanistan, further to the westward, easterly winds are irregular and uncertain, even when the summer monsoon is at its height, and dry west winds predominate throughout this season.

The Gangetic Plain, North-western Provinces, and Behar.—Of the four stations that I have selected for illustrating the winds of this region, the most northerly (Roorkee) is situated 15 miles from the foot of the Sivaliks or sub-Himalayan range, on the *Doab* or fork between the Ganges and Jumna; its elevation is 880 feet above mean sea-level. The second (Agra) lies due south from Roorkee, at a distance of 190 miles, on the south bank of the Jumna, just at the point where a low spur of the great Malwa plateau abuts

* See *post*, Part II., page 588.

on the river; the station is 550 feet above sea-level. The third (Benares), 320 miles east-south-east from Agra and 150 miles from the foot of the Himalaya, is also situated near the southern edge of the plain, the bounding escarpment of the Bundelkund plateau (or eastern extension of that of Malwa) being only about 20 miles to the south of the station. Benares is situated on the north bank of the river, and 255 feet above the sea. Lastly, Patna (or more correctly the civil station of Bankipore), below the confluence of the Sone and Ganges, 180 miles east and a little north of Benares, stands in the midst of the great alluvial plain of Behar, at an elevation of 172 feet.

The wind-registers of the three first-named stations extend over a period of from six to seven years, and comprise day observations only, as in the case of the Punjab stations. That of Patna includes four years and eight months' observations, of which 11 months give day observations only, and the remainder the 10 P.M. and 4 A.M. observations in addition. The Patna observations are unfortunately vitiated to a considerable extent by the observer's omission to record calms, except during the last twelve months of the period. I have therefore, in computing the percentage Table, taken the proportion of calms in these twelve months as an average, and reduced the number of recorded winds in previous years in relative proportion. The result, if not quite satisfactory, is at least less erroneous than it would otherwise have been.

The continuous chain of the Himalaya, skirting the northern edge of the Gangetic plain, determines in a great measure the direction of its prevailing winds. At Roorkee, where this influence is most directly felt, and where the neighbouring hill-chains run almost due north-west and south-east, the winds from these two quarters greatly exceed those from all other directions, and are of nearly equal frequency, amounting together to 38 per cent. of the observations; of the remainder, not less than 41·5 per cent. are calms. At Agra the influence of the Himalayan range is less marked, and the prevailing winds are modified by other causes. Next to calms (25 per cent.), west winds are most numerous (23·3 per cent.), and north-west and east winds stand next in frequency, being respectively 10·3 and 10·5 per cent. North and north-east winds are about equally frequent (between 7 and 8 per cent.), and those from the southerly semicircle are less frequent, amounting altogether to only 15 per cent. of the observations. At this station that predominance of westerly winds which has already been remarked at the Punjab stations is very distinct.

At Benares the proportion of calms becomes reduced to 7 per cent., and the mean movement of the air is nearly half as great again as at Roorkee. The winds from the several quarters have nearly the same relative frequency as at Agra. Thus west winds maintain their preponderance (30·5 per cent.), and east and north-west winds stand next in order. South and south-east winds are of rare occurrence; but there is a slight relative increase of south-west winds, which here form 9·3 per cent. of the observations.

At Patna the proportion of calms to winds, as given in the Table, greatly exceeds that at any other station; but the great excess is in part due to the inclusion of night observations. At both Patna and Roorkee calms are twice as frequent at night as in

the daytime, and at Benares (judging at least from one year's register) between nine and ten times as frequent*. Nevertheless, after making due allowance for this difference in the comprehensiveness of the Tables, Patna still *appears* to equal Roorkee and surpass all other stations in the stillness of its atmosphere; but the anemometric records of the mean diurnal movement of the air do not confirm this conclusion, and they are probably a safer guide than the column of recorded calms. The former show the average movement of the wind to be considerably greater than at Benares, and still more in excess of Roorkee.

Winds from the north-west quarter are most frequent, being half as numerous again as those from west. East and north-east winds count second and third in relative frequency, and those from the eastern semicircle are equal to, or rather in excess of, those from westerly quarters. Winds from the south are very rare, and are almost restricted to the month of August; but those from south-east are relatively more numerous, and those from south-west less numerous than at Benares. Northerly winds are twice as common as southerly.

At all the stations of the Gangetic plain the winter season is that in which calms are most prevalent and the average movement of the winds is at a minimum, November being the month of greatest stillness at the highest stations. At Roorkee, in this season, the average direction of the wind is north-west, at Agra west-north-west or a little more northerly, at Benares west by north, and at Patna west-north-west. There is throughout this season a secondary maximum of winds from the opposite quarters,—from south-east at Roorkee, and east at Agra and Benares; and these winds, though quite subordinate to the principal currents from the westward, are of much importance to agriculture, since on them depends the occurrence of the winter rains and the fortune of the *rubbee* or winter crops. With the approach of the hot weather, the winds blow with greater force and steadiness from the westward; calms become less frequent, and attain their annual minimum in April at Agra, in April and May at Patna, and in May and June at Benares and Roorkee. The wind blows from about the same direction as in the cold season; but the westerly winds are now hot and exceedingly dry, and blow with great force during the heat of the day and the fall of the barometric tide. As the hot season advances, easterly winds gradually increase: at Patna and Roorkee this increase is very distinct as early as April, chiefly from north-east at the former and south-east at the latter station; at Benares it occurs a month later from east; and at Agra it proceeds gradually from April onwards, accompanied, however, by an incursion of south-west winds in April. This last phenomenon will be again met with, more distinctly developed, at Ajmere. At Patna easterly winds preponderate as early as May;

* Night observations of the winds were not recorded in the North-western Provinces until lately. The proportions of calms in the day and night hours are as follows:—

Patna	276 day,	559 night	. . . 1 year.
Benares.	37 „	358 „	. . . 1 „
Roorkee	156 „	408 „	. . . 8 months.

but at Benares and the higher stations westerly winds maintain their preeminence till the latter part of June or the beginning of July.

In the North-western Provinces easterly winds attain their maximum in July, the first month in which, as a rule, the rains become general. North-east winds are at their annual maximum in May at Patna, between May and June at Benares, and at Agra in June and July. Thus, as the Tables show, along the southern edge of the Gangetic plain north-east winds are more than twice as frequent in the so-called south-west monsoon as at the opposite season, when the north-east monsoon prevails at sea.

At Roorkee, from July onwards, south-east winds gradually give place to those from north-west, up to the end of the rains in the beginning of October; but at Agra, Benares, and Patna the month of August is marked by a temporary check of the easterly winds and an incursion of winds from another quarter. This is between south-east and south-west at Agra, west at Benares, and south and south-west at Patna. The latter decline in September, easterly winds resuming their sway. This feature is not peculiar to these stations, but is equally well marked in Orissa, and is perceptible even in Lower Bengal. It appears to be due to an incursion of the monsoon current from the Arabian Sea.

At stations on the southern border of the plain, the winds veer normally from west or north-west round through north and east to their extreme south-easterly direction in the summer monsoon; the opposite change is more abrupt. At Roorkee any rotation that can be detected is retrograde.

Plateau of Rajpootana and Bundelkund.—Dr. Murray Thomson's reports for the years 1863–69 give meteorological registers* of the stations Beawur, Ajmere, and Jhansi. The two former stations are situated within a few miles of each other and under similar geographical conditions; and since their registers refer to different years, I treat the whole as those of one station. Ajmere and Beawur are situated at an elevation of between 1500 and 1800 feet, near the western edge of the plateau, where it declines to the desert plains of Bikaneer. A few miles west and north-west from the stations are some low hills, spurs of the Aravulli range, and an outlying spur of the same range to the southward forms the watershed between the Gangetic basin and the streams draining towards the Gulf of Cambay.

On the average of the year, winds from the west and south-west greatly exceed those from any other quarters, and together amount to 52 per cent. of the observations; of the remainder, 10 per cent. are calms. Winds from other quarters are about equally frequent, with the exception of north-east winds (8 per cent.), which are slightly in excess of others (5 to 6 per cent.). Calms are most prevalent in the winter months (October to February), during which they contribute from 15 to 20 per cent. of the observations; and they are least so in May, when they amount to only 2 per cent. North, north-east, and east winds are at their maximum in the winter season, and December and January are the only months in which easterly components preponderate

* Consisting of day observations only. The winds are recorded at 10 A.M. and 4 P.M.

over westerly, or, with the addition of November, northerly over southerly: the distinction between Ajmere and Agra in this and other respects is very striking. In February west and south-west winds begin to set in with increased frequency, and in April blow with considerable steadiness—the latter attaining their annual maximum in May, the former in September, while the mean direction throughout the summer monsoon is west-south-west. Up to October this wind-current scarcely veers or slackens, but in November the wind comes more from north, and eventually north by east. It is to be noticed that at Ajmere the two monsoons prevail alternately in what may be termed their normal directions, the south-west, however, greatly preponderating in duration and steadiness, while the north-east monsoon is weak and unsteady and much interrupted by calms. In many respects the wind-system resembles that of Mooltan.

Jhansi is situated on the plateau of Bundelkund, 250 miles east by south of Ajmere, at an elevation of 836 feet above the sea. From Agra it lies south by east, at a distance of 130 miles, while Benares lies 320 miles to the eastward. There are a few scattered hills about the station, "one very close to the city and overhanging it, another a little way off to the south-east"*. The registers of this station, given in Dr. THOMSON'S Reports, cover a period of six years; but some are imperfect, specially in the earlier months of the year.

The winds at Jhansi are chiefly west or north-west and east (30, 14, and 16 per cent. respectively on the average of the year). Calms appear to be comparatively rare, amounting to only 3 per cent., and winds from south-east are only 5 per cent. of the observations. Westerly elements predominate greatly over easterly, and northerly to some extent over southerly. In this latter respect Jhansi affords a marked contrast with Ajmere. It is probably owing to its situation on or near the boundary between the very diverse wind-systems of the Gangetic plain and Central India that at all times of the year the winds are more or less conflicting, and that the figures that express the percentage of the resultants in the Table are consequently lower than at any other station except Roorkee.

In the cold-weather months north and north-east winds are at their maximum, but even then they are quite subordinate to those from west, north-west, and east; and in December and in January the resultant direction is from 1 to 2 points west of north. In February and March northerly elements decrease rapidly, and in the latter month south winds attain their maximum. In April and May hot westerly winds set in, and the resultants veer back to west and west-north-west. South-west winds are frequent in April, and after a temporary suppression in May they recur, and are at their maximum during the three months June to August. In this latter month there appears to be a sudden incursion of east winds—a somewhat remarkable irregularity; for at Agra, Benares, and stations further eastward, at which the rainy season is characterized by easterly winds, the reverse phenomenon takes place. In September north winds undergo a sudden increase and reach their annual maximum; and in this and the two following months the mean direction is pretty steadily north-west by west.

* Dr. THOMSON'S Report for 1863. The direction of the first-mentioned hill is not stated.

Central India.—Dr. Townshend's Reports, published in the 'Central Provinces Gazette' and in the Sanitary Commissioner's Reports for those provinces, give the anemometric results of Jubbulpore and Nagpore for the three years 1869–71, together with those of other stations for shorter periods. I select these two as the most complete and trustworthy.

Jubbulpore is situated on the north, Nagpore to the south, of the belt of hills which stretches across Central India, and is now generally known as the Satpoora range. At its northern foot this range or series of ranges dips to the alluvial valley of the Nerbudda, lying between two escarpments, and allowing an uninterrupted passage to the prevalent westerly winds. Jubbulpore is situated at the upper end of this valley, 1350 feet above the sea, not far from the low watershed that separates the drainage of the Nerbudda and the Ganges. North of the station the escarpment that borders the valley on the north retreats, and a gently ascending plain (over which is carried the railway to Allahabad) forms a break in the tableland, and affords a channel to northerly winds.

Nagpore lies 150 miles south-south-west of Jubbulpore, on the plain that extends along the southern foot of the Satpooras, in a kind of bay between two prominent offshoots of the hill-country, which is drained by the Wynegunga and its tributaries. The station is about 1000 feet above the sea.

At both stations westerly winds preponderate on the whole, the dominant direction being west at Jubbulpore, north-west at Nagpore. East and south-east are the directions of least frequency at the former, and south-east and south at the latter station. Thus the wind veers on an average four points between Jubbulpore and Nagpore. Calms are not very frequent at either station. At Jubbulpore the mean velocity of the wind up to May is rather below that at Patna, but during the rains it is much greater. At Nagpore it is greater than at Jubbulpore in all months of the year. The velocity is highest in June, lowest in November or December.

At Jubbulpore north and north-east winds blow pretty steadily in November and December, but south and south-east winds are not infrequent, being about half as numerous as the former. These are probably in great part local mountain winds. In January westerly elements begin to preponderate over easterly, and southerly to gain on northerly winds. South winds attain their maximum frequency in March, in which month winds from between south and west amount to 54 per cent. of the observations, while those from the opposite quadrant have diminished to 28 per cent. In April and May west and north-west winds gain the ascendant; but on the setting in of the rains the wind backs towards south-west and blows very steadily during the three months June to August from a mean direction about a half a point south of west. In September the monsoon slackens; north and north-west winds begin to preponderate, veering towards north and north-east; and in November the north-east monsoon is reestablished.

On comparing this series of changes with that shown by the Jhansi registers, it appears that in the cold season the prevailing winds of Jubbulpore are from 1 to 8 points more easterly than those of the latter station; that at both stations southerly winds increase from January to March, and attain their maximum in this latter month; that west and north-west winds become ascendant in April and May, and then back towards south-west up to the setting in of the rains. At Jhansi, however, easterly winds are more frequent at all times of the year than at Jubbulpore, and especially so in August.

These two stations exhibit a sort of graduated passage from the wind-system of the Ganges valley to that of the peninsula south of the Satpoora range. This last is illustrated in Nagpore.

At this latter station the average direction of the wind in the winter months is steadily east-north-east, while winds from north-west and west are extremely rare. In February, and still more in March, the currents become unsteady and conflicting, though the movement of the air is on the whole increasing. South winds are at their maximum from February to April, and south-west winds are not infrequent. In April north-west winds gain the ascendant, and blow with increasing steadiness in the following months, backing, however, to west, which is their mean direction in July. North winds are at their maximum in May, north-west winds in June, and west and south-west winds in July. After July, northerly and easterly winds begin to increase; the mean direction veers to north-west in September, to north-east by east in October, and finally attains its extreme easting in December.

Thus, then, in the cold-weather months and those of the rains, that is to say when the north-east and south-west monsoons are at their height on the seas around India, the wind-currents south of the Satpooras have a direction almost diametrically opposite to those of the Gangetic plain. The former blow to and from the Arabian Sea, the latter to and from the Bay of Bengal; only in the hot season do the two wind-systems approximately coincide, and then they are, in both regions, from between west and north-west, blowing from the comparatively dry region lying to the north-west towards the thermal focus of Central India and Western Bengal. The evidence of this will be given in another place.

Western Bengal and Orissa.—For illustrating the winds of this region I have the three stations Hazareebagh, Cuttack, and False Point. The first is situated 2000 feet above the sea, on one of the culminating points of the plateau that lies between the Sone, the Ganges, and the Gangetic delta. This plateau forms the eastern termination of the elevated range that, beginning with the Rajpipla hills at the Gulf of Cambay, is continued by the Gawilgurh, Mahadeva hills, and other divisions of the Satpooras to the umbilical plateau of Umarkuntuk; and, after a short break, by the not less elevated tablelands of Jameera Pat, Main Pat, Chota Nagpore, and Hazareebagh, up to the angle of the Ganges at Rajmahal. To the east of Jubbulpore it separates the Gangetic drainage from that of the peninsula, and its influence on the wind-currents is scarcely less important.

Cuttack and False Point are both situated to the south of this range, on the alluvial plain of Orissa. The former, 80 feet above the sea, lies close to the low hills from which the Mahanuddy debouches on its delta. The latter is one of the more prominent points of the same delta, 50 miles east from Cuttack, and, jutting beyond the general outline of the coast, is fully exposed to the winds that at different seasons of the year sweep up and down the Bay of Bengal.

At all these stations wind observations have been recorded night and day, at intervals of six hours.

The Tables are drawn up from the registers of three years; but it is to be regretted that, in the case of Hazareebagh and Cuttack, more especially the latter, they are vitiated by the omission of calms. I infer from the more recent registers that at Cuttack in the cold-weather months the atmosphere is as calm as in Behar.

At Hazareebagh, as in the Ganges valley, the prevailing winds are from north-west and west; these amount together to 46 per cent. on the average of the year. North-east is the quarter of least frequency (4 per cent.), and winds from north and east do not exceed 7 and 8 per cent. respectively. During the cold-weather months the wind blows pretty steadily from between west and north-west. As the temperature increases on the approach of the hot weather, this current tends to back towards south-west and south; and in June south and south-east winds preponderate, bringing the rains from the Bay of Bengal. In July there is a further backing towards south-east and east; but in August a sudden increase of west and south-west winds implies an incursion of the monsoon current from the Arabian Sea. In September, however, the easterly backing is resumed, and east and south-east winds attain their annual maximum, amounting together to 45 per cent. of the observations. In October winds from the opposite quarters regain the ascendant, and the winter monsoon sets in from north-west by west.

It is to be observed that the annual partial rotation of the winds is here chiefly retrograde, viz. from north-west through south-west and south to south-east, and that when a semirevolution has been completed, and the rainy monsoon has attained its extreme easting in September, it is followed and supplanted by land-winds setting in from the opposite quarter of the compass. We shall presently see that the winds of the Gangetic delta follow a similar course. The incursion of the south-west current in August has been already noticed in the case of Agra, Benares, and Patna. It is much more decided at Hazareebagh, and equally so, but more prolonged and regular, at Cuttack and False Point. The mean velocity of the wind at Hazareebagh is more than twice as great as at Benares, and nearly twice as great as at Patna. At the time of its maximum in May and June it is twice as great as when at its minimum in November and December. In point of steadiness, however, the periods of maximum and minimum are reversed. The north-west winds of November and December, if gentle, blow very steadily, and the resultants show an excess of 62 per cent. In May, on the other hand, there is an excess of 17 per cent. only in the direction of the resultant.

At Cuttack and False Point we meet with a wind-system very different in its more

striking features from any yet described. The land-winds from north and north-west are here quite of subordinate importance; and a great predominance of those from south-west and south blowing along the shore, or obliquely from the Bay towards the hilly country of the interior, characterizes Orissa and the Northern Circars. At both the above stations, winds from between north and north-east set in as early as October, and with the increasing cold of the interior and strengthening of the land-winds they become more northerly in the two following months. At False Point, indeed, they maintain a marked ascendency till the end of January, veering back, however, towards north-east and east; but at Cuttack the veering proceeds further, and sea-winds from east and south-east predominate in this month. In February south-west winds gain the ascendant at False Point, and south winds at Cuttack, a difference of about four points between the two stations being maintained throughout the hot weather and rainy months. With the increase of temperature in the interior, south winds at Cuttack and south-west winds at False Point increase in steadiness, backing through one or two points up to May. In June, with the setting in of the rains in Bengal, the veering again becomes normal, *i. e.* the westing increases, and this tendency is maintained till the month of August. In September the south-west monsoon slackens, and the wind once more backs rapidly through south-east and east to north-east in the month of October.

At False Point, as at most coast-stations, calms are of rare occurrence. In October, the month of their greatest frequency, they amount to only 11 per cent. of the observations, while in May none are recorded. They are somewhat more frequent in the cold weather than the rains, and least so in the hot-weather months. This appears to be a universal rule in Northern India. At Cuttack, to judge from one year's register, they are common in the cold weather, and in accordance therewith the mean velocity of the wind is low. The summarized register for a part of the two years 1871–72 will serve to correct the deficiencies of those of the longer period.

Gangetic Delta.—The delta of the Ganges may be regarded as the northern end of the great wedge-shaped depression occupied by the Bay of Bengal. This arm of the ocean affords an unobstructed channel to the interchanging air-currents between the equatorial seas and the plains of Upper India. The delta lies, so to speak, in the neck of the funnel formed by the converging coasts of the two peninsulas—that on the east being bordered by a continuous mountain-range, the Arakan Yoma, not less than 4000 or 5000 feet in height; that on the west bordered also by hills, more broken indeed, and of much less mean elevation, but still opposing a certain barrier to the free passage of the winds to and from the interior of the peninsula. At the upper end of the Bay these hill-tracts advance to within 200 miles of each other, enclosing between them the united deltas of the Ganges and Brahmapootra; and further inland the advanced plateau of the Garo hills constricts the plain to a width of 150 miles, allowing free access to the plains of the Upper Provinces, but almost entirely obstructing the entrance of the narrow valley of Assam.

Of the four stations that I select for illustrating the wind-system of the delta, three (viz. Saugor Point, Calcutta, and Berhampore) are situated along a line from south to north parallel to its western border and at no great distance from it; the fourth (Dacca) lies 120 miles to the eastward of the last-named station, about equidistant from the sea and the eastern hill boundary of Tipperah. Berhampore, 160 miles from the coast, is 64 feet above sea-level, the other stations at various less altitudes. Saugor Point, at the entrance of the Hooghly estuary, is a mere marsh, protected by embankments from submergence at high water.

As a consequence of the physical configuration of the country, nearly the whole of the lower stratum of air that sweeps over the delta is *in transitu* between the sea and the Gangetic plains to the westward. The Assam valley, owing to its narrowness and the abruptness of its junction with the broader Ganges valley, affords but an obstructed and tortuous passage to this stratum, and it is chiefly an upper current that, passing over the low hills of Tipperah and the Khasi and Jynteah hills to the north of these, makes its way to or from Upper Assam as the monsoon wind.

The wind Table for Calcutta has been drawn up from the hourly observations of ten years, and, notwithstanding the roughness of the observations, represents the wind-system probably with as much accuracy as is attainable in the absence of self-registering instruments. The velocities are obtained from four years' observations. For other stations, three years' registers of observations recorded at six-hour intervals have been used. In two cases, viz. Berhampore and Saugor Island, the registration of calms has been neglected during the greater part of the period: the Saugor-Island Table is probably scarcely affected by this omission; but such is not the case at Berhampore, and I give therefore the register of one year in addition in which calms have been recorded.

At Saugor Island south and south-west winds predominate greatly over those from any other quarter, amounting together to 52 per cent. on the average of the year. North-west and west winds and their opposites are least common, amounting to from 5 to 8 per cent. respectively. At Calcutta south winds form 31 per cent. of the annual average, and there is a slight but decided preponderance of westerly over easterly components. At Berhampore the case is very different. The excess of south winds over those from any other quarter is but small; and there is a difference of only 5 per cent. between them and south-west winds, which are here the least frequent. At Dacca, again, south winds preponderate, and south-east winds stand next in order, while easterly components slightly exceed the westerly.

At all these stations the annual rotation of the wind is incompletely retrograde, and such as has already been described at Hazareebagh. The winter monsoon or land-wind sets in in October and becomes well established in November, with a mean direction which is nearly north at Dacca and Saugor Island, north-north-west at Calcutta, and north-west by north at Berhampore. At Saugor Island it blows pretty steadily during the three months November to January from a direction a few degrees east of north, and at Berhampore it is almost equally steady; but at Calcutta and Dacca it backs

gradually towards the west, and by February blows from west by south. At Saugor Island the sea-wind sets in in February, somewhat suddenly from south-west by south, and during the hot-weather months backs gradually through four points of the compass, increasing in steadiness and mean velocity till the setting in of the rains in June. At Calcutta, Berhampore, and Dacca, a similar change in direction occurs, but through a greater range. Thus at Calcutta the wind backs through 8 points, at Dacca through 9, and at Berhampore through 10 points, between February and May.

On the setting in of the rains the wind veers normally (to the westward) about half a point or a point—a change small in amount, but equally distinct in all parts of the delta, in the 10-year Table of Calcutta and the 3-year Tables of the other stations. In the following months, however, the winds again acquire more easting, until in September, the last month of the rains, the mean directions are S. 4 E. at Saugor Point, S. 30 E. at Calcutta, and S. 70 E. at Berhampore. At Dacca and Saugor Island that incursion of westerly winds in August which is so marked at Hazareebagh and other stations already noticed is distinct, though less striking, and it is traceable even in the Calcutta Table. At the two first-named stations there is a temporary increase of south, south-west, and west winds in August, and a corresponding decrease of south-east winds, such as to cause a normal veering of the resultant through nearly two points at Dacca and one at Saugor Island. At Calcutta and Berhampore this does not take place; but the backing of the wind is somewhat less between July and August than either in the preceding or following months. In October the winds are chiefly from the east, but unsteady and stormy, alternating with calms in the earlier part of the month, and northerly or north-westerly in the latter part.

In all parts of the delta the velocity of the wind is lowest in November and highest in May and June. This difference is greatest at the inland stations Dacca and Berhampore. At the former the mean movement of the air in June is six times as great as in November, and at the latter more than four times as great. At all times of the year it decreases rapidly from the coast inland. Thus at Saugor Island, Calcutta, and Berhampore the mean diurnal movement of the wind in May and November is as follows:—

	May.	November.
Saugor Island	345 miles	111 miles.
Calcutta	209 „	82 „
Berhampore	100 „	29 „

Assam.—The Assam valley extends in an east by north direction along the foot of the Eastern Himalaya, from the extremity of the Garo hills in east longitude 90° to the point where the Brahmapootra issues from the Brahmakoond gorge in about longitude 96°. Its length is thus about 420 miles, while its width nowhere much exceeds 60 miles. On the south it is enclosed by a tableland formed by the Garo, Khasi, Jynteah, and Naga hills, of which the second are the most elevated, the highest ridges being between 5000 and 6000 feet. The Jynteah hills next to the eastward are 1500 or

2000 feet lower, and the Naga hills still less elevated, with the exception of the Burrail range, on the south-east and south, which rises to about 5000 feet. Between the western extremity of this range and the Khasi hills is left a comparatively free passage, at a lower level, for the monsoon winds blowing to or from the upper part of Assam. On the north the Bhotan Himalaya runs parallel with the valley up to the gorge by which the great Dihong river breaks through the mountains, and affords an open passage to the monsoons to penetrate to the region north of the Himalaya. Seebsaugor lies to the south and a little to the west of this break in the mountain-chain, full in the course of the alternating currents, but sheltered somewhat by its depressed position on the alluvial plain between the two hill-ranges. Goalpara, on the other hand, is situated at the lower end of the valley, due north of the Garo hills, and completely shielded on the north by the Bhotan Himalaya.

The Tables for both these stations are drawn up from the observations of three years (recorded four times daily). The Seebsaugor observations are evidently somewhat rough; but there seems no reason to doubt their general trustworthiness, except in so far as they may be vitiated by the usual omission of calms. Both Tables show a preponderance of easterly winds, as decided as is that of westerly winds in the peninsula of India. At Goalpara this excess of easterly winds holds good in every month of the year, and the only change in the mean direction of the wind is an oscillation through about 7 points of the compass, from east by north in March to south-south-east in July, and back again during the remainder of the year. North-east winds are most frequent in the winter months, and attain their maximum in March. West winds are also at their maximum in the same months, in which respect Goalpara resembles Berhampore. In May east winds attain their annual maximum, amounting to 59 per cent. of the observations; but in the following months, during the prevalence of the south-west monsoon, winds from south, south-west, and west are frequent. In respect of mean velocity, there are two epochs of maximum and two of minimum annually. The former occur in April and September, the latter in July and December; of these, the April maximum is the absolute maximum of the year, and the December minimum is generally below that of July.

At Seebsaugor, winds from the north and north-east amount together to 54 per cent. on the average of the year, while those from south and south-west do not exceed 29 per cent. The air-current from the direction of the great Dihong valley (possibly derived from Tibet) preponderates considerably over that from the sea. Nevertheless the climate of Seebsaugor, though cool, is not so dry as this fact might lead us to anticipate; the rainfall is not much less than at Goalpara, and occurs at all seasons of the year: this fact leads me to doubt whether in reality any considerable body of air is drawn from the trans-Himalayan region. The winter monsoon from north and north-east sets in in October and blows with great steadiness through November and December. In January westerly winds begin to be felt, and gradually increase during the following months, chiefly from south-west, until in June they predominate over those

from between north and east. Through July and August the south-west wind, the undiverted monsoon, maintains a very decided mastery, but slackens in September and ceases in October, when the winter monsoon regains its supremacy.

Arakan Coast.—Although geographically a part of the Burmese peninsula, this coast may fitly be treated as belonging to the Indian area in respect of the winds, since those of the lower stratum of its atmosphere are more influenced by the heat and cold of Bengal than by any alternations of temperature &c. in the interior of the eastern peninsula. This is owing to the mountain-chain of the Arakan Yoma, which runs from the eastern end of Cachar, parallel with the coast, at an average distance of 40 or 50 miles, down to Cape Negrais, and is of an average height of 4000 or 5000 feet. It would seem, from the wind Tables here given, that the greater part of the air-currents at Chittagong and Akyab are *in transitu* to or from the Gangetic delta; but it is probable that a certain portion of them make their way up or down the river-valleys that drain the western slopes and spurs of the Yoma, and, crossing the low ranges of the Chittagong and Cachar Hill tracts, form part of the Assam branch of the monsoons.

Chittagong is situated at the south-eastern extremity of the Gangetic delta, about the junction of the great Megna estuary with the sea. To the east and north-east extends the tract of low hills above referred to, and on the very margin of which the station is built, a strip of alluvium about three miles broad intervening between the hills and the sea. Chittagong is 140 miles south-east from Dacca, and 240 east by north from Saugor Point.

Akyab is 150 miles further down the coast, at the point where the Koladyne river enters the sea. The station is built on a low point of land between the estuary and the Bay of Bengal, and the harbour formed by the estuary is further enclosed by a group of rocky islands to the southward.

At Chittagong there is no very decided preponderance of any one wind-direction on the average of the year; but north-west winds are less common than others, and on the whole southerly winds are in excess of northerly, in the proportion of 42·5 to 30 per cent. As in the Gangetic delta, the annual rotation is retrograde from November to the following September (in this case through more than two thirds of the compass), and the change from the characteristic direction of the summer to that of the winter monsoon takes place somewhat abruptly in the month of October. In the cold-weather months the average direction is from between north and north-west, but less westerly by about a point than at Dacca; northerly elements preponderate till the end of February, that is to say a month later than either at Dacca or Saugor Island. Between February and June the wind works round gradually to the southward, and in the latter month it backs further to about south-east by south, which is its average direction as long as the south-west monsoon prevails over the Bay. At all times of the year the mean direction is modified by land- and sea-breezes. The average velocity is more uniform than at inland stations. When greatest, viz. in June, it is less than twice as great

as at its minimum in October or November. At the former period it is less than at Dacca, at the latter nearly three times as great. At these two stations the periods of maximum and minimum are approximately the same, and about a month later than at most stations to the westward. Calms are not very common at any time of the year, and occur chiefly at the close of the south-west monsoon.

The wind-system of Akyab differs from that of Chittagong in much the same way as this latter differs from that of Dacca, *i. e.* the corresponding phases of the wind's rotation occur about six weeks or two months later at the more southerly station; and while the average direction of the summer monsoon is less easterly, that of the winter monsoon is less westerly. Thus at Akyab northerly elements preponderate over southerly in April, and southerly over northerly in October, the reverse of the case at Chittagong; in other words, both monsoons continue to be felt on the coast of Akyab one or two months after the change has occurred at Chittagong. This accords completely with the results of Captain MAURY'S discussion of ship observations in the Bay of Bengal*, and is also what might be anticipated from the character of the barometric changes presently to be discussed†. The result is of high importance to the theory of the formation of cyclonic storms, respecting which I shall say a few words in the sequel. One other point of interest to be noticed in the Akyab wind Table is the interruption of the otherwise regular retrograde movement of the winds by a sudden westing in August through one point of the compass, followed by a return to eastward in September. This reminds us of the similar phenomenon already noticed in the wind Tables of the North-western Provinces and Bengal.

Summary.—From the foregoing discussion of wind-registers it appears that the wind-system of Northern India is very different from that of the adjacent seas. Instead of two monsoons from north-east and south-west prevailing alternately during about equal periods of the year, we find a great diversity of prevalent wind-currents, depending on the directions of the mountain-ranges and great valley plains; and, with respect to period, to be classified under three rather than two distinct seasons—excepting, indeed, in Upper Assam, where the normal monsoons prevail. Thus in the cold-weather months (November to January) the winds are light or completely calm, and the air flows in a gentle current from the plains of Upper India and the Punjab down the valleys of the Indus and the Ganges, or across the hilly watershed of Central India to join the north-east or east monsoon of the peninsula proper. This appears to have a distinct source in the hill-region of Chota Nagpore and the country south of the Satpooras. It is possible that in Upper Assam and in the Upper Punjab, at the two extremities of the Himalaya, some portion of the cool air from the transmontane region may find egress to the southward; but if so, the amount must be small, and the mountain-barrier of the Himalaya, which throughout the interval of 1500 miles extends unbroken along the northern margin of our area, completely secludes India from the influence of more

* Physical Geography of the Sea, 12th edition, p. 368.

† See *post*, Part II., pp. 603, 604.

northern regions*. I have not included in the foregoing discussion the wind-registers of any Himalayan stations: a detailed analysis, such as I have attempted for those of the low-country stations, would in their case only mislead, since at all such stations local influences are so powerful as in a great measure to obscure the effects of such as are more general, and with which alone I am now dealing; but I may observe that at Darjeeling (7000 feet), even in the cold-weather months, southerly elements of wind-direction predominate over northerly†, and according to Dr. HOOKER's observations, at great heights in the interior of Sikkim, a southerly current prevails throughout the year. At the stations of Chuckrata and Nynee Tal in the north-west Himalaya, at about the same elevation as Darjeeling, Dr. MURRAY THOMSON's registers show that the winds are almost exclusively from the south or some southerly quarter at all seasons of the year. In Lower Bengal, south and south-east winds are not very common from November to January; but their representatives (viz. easterly winds) are more common in Upper India, specially in proportion to winds from the opposite quarters, calms being most frequent of all. I have already mentioned that these easterly winds bring the winter rains; and it will presently be shown that the latter are more regular and copious in the Punjab and upper part of the Ganges valley than in Lower Bengal. From these facts we must, I think, conclude that a portion of the upper or anti-monsoon current, following the same course in the upper atmosphere as the summer monsoon does in the lower, descends on the plains of Upper India, while another portion impinges on the southern slopes of the Himalaya or even crosses them into Tibet. In the narrower valley of Upper Assam, at least at Seebsaugor, this current appears to be less felt, and winds from north and east blow steadily and persistently; but the ômission of the observer at this station to record calms, and the frequent occurrence of rain in the winter months, lead me to doubt whether the effects of the return current are not felt to a great extent in that region also.

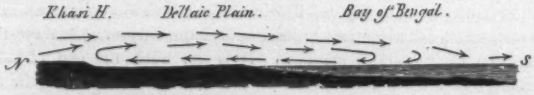
With the advent of the hot weather the winds of Northern India draw round to the westward, and dry currents (partly perhaps derived from the mountainous and desert country lying to the west of the Indus) radiate out over the whole region as far eastward as the eastern limits of the Gangetic delta, and, becoming heated in their passage over the plains, form the well-known hot winds of April and May. These winds, however, are not steady, continuous currents: they are, as Dr. HOOKER has described them, essentially *diurnal* winds, due to the local heating of the soil; they set in about 10 o'clock in the day and blow sometimes in gusts, but in general with tolerable steadiness till sun-down, when they are followed by a calm. On the coast of Orissa, and in the

* In the Report on the Meteorology for Bengal for 1871, page 122, I wrote otherwise, having been misled by the apparent steadiness of the winds in the Punjab and Upper Assam. Further information on this point, and a consideration of the rainfall distribution, which in the cold-weather months seems incompatible with the existence of dry currents from the Tibetan regions, have induced me to modify the views then expressed.

† See the Reports of the Meteorological Department of Bengal, 1869–71. The prevailing directions are east and west—that is, parallel to the great Rungeet valley below the observatory ridge.

delta of the Ganges, sea-winds begin to prevail, however, as early as February—first on the coast-line and in the immediate neighbourhood of the hills, and afterwards more inland. Thus they gradually encroach upon and eventually displace the land-winds* near the ground surface, so that in April and May the opposing currents meet obliquely in the hilly region that lies to the west of the delta and of the plains of Orissa. Here at least is their average line of meeting; in March the winds of the delta are nearly as much from west as south, while in May westerly elements preponderate but slightly over easterly at Hazareebagh and even at Benares. The gradual retrograde rotation exhibited by the wind-resultants of the Lower Bengal stations in the spring months is due to the increasing displacement of north-west or land-winds by those from the sea; and the latter advance further and further inland, coalescing with the southerly and easterly currents of the Himalayan slopes, and in Behar curving round and blowing as north-easterly winds at Patna and Benares. In Eastern Bengal in like manner they blow up to the Garo and Khasi hills, where they meet the north-east current from Upper Assam; while in Lower Assam a portion of the latter turns with the valley and, blowing steadily from the eastward, probably coalesces with the south-easterly current from the delta. At sea, as Captain MAURY has shown, and as further appears from the registers of Chittagong and Akyab, southerly winds gain possession of the Bay by a like gradual extension southwards from the coast-line of the delta. They back down, as Captain MAURY expresses it, about 5° or 6° of latitude on an average between February and March, but less rapidly on the Arakan coast than on that of India. Thus at Madras, in latitude 12°, southerly winds predominate in March; whereas at Akyab, in latitude 20°, they do not gain the upper hand until the month of May. A section of the atmosphere from the Khasi hills to the Bay near the Arakan shore in the latitude of Akyab would probably, in the month of March, present a system of currents somewhat resembling the accompanying diagram (fig. 1).

Fig. 1.



In Western India, in Sind, and the desert of Bikaneer southerly winds prevail in like manner in the spring months, and penetrate inland as far as Mooltan and Ajmere. At the latter station, situated at nearly 2000 feet above the sea, they predominate as early as February, while at the former they do not exceed northerly winds until May. At Agra they are felt in April, but do not preponderate.

In June the south-west monsoon sets in on both coasts of the peninsula. The current from the Arabian Sea, sweeping across the Sahyadree mountains and up the valleys of

* In this place and throughout this paper I use the terms "land-winds" and "sea-winds" to designate winds which originate respectively in the interior or at sea, without regard to their periodicity, whether it be annual or diurnal.

the Taptee and Nerbudda, blows as a west or west-south-west current over Central India up to the very confines of the Gangetic plains, and across the peninsula south of the Satpooras up to the coasts of Orissa. That from the Bay of Bengal pours into the funnel-shaped opening occupied by the delta, and then, turning westward, passes up the Ganges valley towards the Punjab; while upper currents sweep over the Hazareebagh plateau in the same direction, and across the hills of Eastern Bengal towards the valley of Assam and the river-gorges that afford them an entrance to Tibet. It is not improbable that a portion of the current that traverses the Bundelkund plateau from the Arabian Sea may, on reaching the Gangetic plain, curve round towards the north-west as an upper current, coalescing with that from the Bay of Bengal. But no such passage can be detected in the registers of the surface-winds. Indeed along the southern margin of the plain the winds are on the whole rather from north-east, blowing towards the Malwa and Bundelkund plateau during the height of the monsoon. In any case the Punjab is the limit of these winds. On reaching the plain of the five rivers they perform a kind of cyclonic circulation around it; and in Affghanistan, although easterly winds are occasionally felt at this season, the dominant current is from the westward.

Up to the close of the south-west monsoon in October, the Coromandel coast and the plains of the Carnatic have received but little rain, and remain at a higher temperature than any other part of the peninsula. Unsteady north-east winds, alternating with calms and variable winds from other quarters, then set in on the Orissa coast and over the north-west part of the Bay; while the southerly current recurves from south-east and blows towards this heated region, bringing the late autumn rains, which some writers have erroneously attributed to the north-east monsoon*. At the same time the winds of Lower Bengal are conflicting and variable, and calms (alternating with storms) are at their maximum frequency. But in the North-western Provinces, where the rains cease somewhat earlier and the temperature falls more rapidly, there is, during the greater part of October, a decided movement of the air from the west and north-west. In like manner in the Central Provinces light land-winds have set in from east and north-east, which latter (or east to the south of the Satpooras) is there the characteristic direction of the winter monsoon. It is not until the end of this month, or in November, that north-west winds blow over the delta, connecting the north-west current of Upper India with the north-east winds of the northern part of the Bay in a continuous stream, and that it can therefore be said with strictness that the north-east monsoon has set in.

The manner in which the change of monsoons takes place in the Bay of Bengal has been already described by Captain Maury in the following terms:—"In October the north-east trades lead off the attack, and the two combatants have force enough about the parallel of 15° N. to blow during this month nine days each. The conflict, instead of being back to back, is now face to face; instead of blowing away from the medial

* This error was distinctly pointed out by Professor Dove in his Treatise, 'Ueber die Vertheilung des Regens auf der Oberfläche der Erde,' page 98.

line (as in March), they now blow towards it; instead of being a place of high, the medial line is now a place of low barometer. By November the north-east trade has pushed the place of equal contest as far down as the parallel of 5° N." With some demur to certain expressions, which might be taken to imply that the winds are impelled towards their place of meeting against mutual resistence by a *vis a tergo*, the above represents in a general way the character of the north-east monsoon's advance. But, as in the case of the opposite change already described, this advance is more rapid on the Indian than the Burmese coast of the Bay. At Akyab southerly winds still preponderate in October, while at False Point north and north-east winds are in considerable excess; and even at Madras northerly winds are quite as frequent as those from southerly quarters in that month.

Such being the general scheme of the wind-currents of Northern India, it remains now to trace out their relations to temperature, humidity, and atmospheric pressure at the different seasons of the year. With this view I proceed to give a sketch (imperfect indeed in many respects, but still not without value) of the distribution and annual changes of these important elements of climate.

Part II.—Relations of the Winds to other Elements of Climate.

Temperature.—The general distribution of temperature, which I have deduced from the registers of the last few years*, agrees generally with that represented on the Messrs.

* I give the results in two Tables—the first of which shows the mean temperature of each place in each month of the year, as deduced from the observations, the second the equivalents of these at sea-level. The means have been obtained in various ways. For the Lower Provinces and Assam, as well as for Roorkee and (in part) Benares, I have taken the arithmetical means of observations recorded at 4 and 10 A.M. and P.M. In the case of Seebsaugor alone, sunrise observations are substituted for those at 4 A.M. At stations in the Central and North-western Provinces (other than Roorkee and Benares), the mean of the minimum and 4 P.M. temperatures is taken as the mean of the day; and in the case of the Punjab stations, I have applied to the means of the mean maxima and minima a correction proportional to the mean diurnal range, derived from the Roorkee registers. The sea-level values in Table III. are obtained by adding to the figures in Table II. 1° for every 350 feet of elevation.

Owing to the shortness of the periods from which, in many cases, the mean temperatures of Tables II. and III. have been obtained, the diversity of the methods of reduction, and, above all, my uncertainty how far the instruments employed in the Central Provinces, the North-western Provinces, and the Punjab can be accepted as accurate and comparable, I hesitate to base any conclusions on small differences of temperature, even when the registers of several stations yield concordant evidence of their reality. For these reasons I am far from confident that the peculiar loop which the isotherm of 60° makes in the Gangetic plain in the January chart will be substantiated by more complete and trustworthy data. Its existence rests on the evidence of Benares and Lucknow given in the Tables, with that of Bareilly, Goruckpore, and Fyzabad (each for four years). The January temperatures of these latter stations are respectively 57°·4, 60°·1, and 60°·5, which, reduced to sea-level values, are 59°, 60°·8, and 61°·5. There are, however, independent grounds for regarding as probabl the distribution of temperature thus indicated. There is an evident relation between the loop of abnormally low temperature and that of high pressure shown in the same chart; and the fact already noticed, that southerly winds prevail on the southern slopes of the Himalaya throughout the cold-weather months, might lead us to

Schlagintweit's charts in the 153rd volume of the Philosophical Transactions. But the Messrs. Schlagintweit's division of the year into four equal periods does not admit of the phenomena being represented in sufficient detail for the present purpose; nor does it accord with the natural arrangement of the seasons, which in Northern India present only three distinct phases. These are:—(1) the *cold season*, lasting from the breaking up of the rains in October to February or March; (2) the *hot season*, characterized in general by a dry atmosphere and a great diurnal range of temperature; and (3) *the rainy season*, in which the temperature is moderately high and equable and the air very humid. The beginning and ending of these seasons differ in period somewhat in different parts of the area, and the gradations of temperature which accompany them are very different both in period and amount. In describing these latter it will be convenient to take the month of October as our starting-point. Except where otherwise specified, the figures quoted in the following description of the horizontal distribution of temperature are those of Table III., viz. the *sea-level equivalents* of the observed mean temperatures.

At the close of the rains in the early part of October (Plate XLVII.), the temperature of Northern India is nearly uniform at about 81° or 82°. But changes soon set in: evaporation and radiation to a cloudless sky speedily reduce the temperature of the more elevated plains of the interior below that of the maritime belt; and thus the average temperatures of the whole month, given in Table III., show an extreme variation of 8°. In the two following months the inequalities thus arising become greatly intensified; and they culminate in January, when there is a difference of nearly 20° between Mooltan and Bombay. The distribution of temperature in this month is shown in the chart (Plate XLIII.), on which the isotherms are interpolated from the figures given in Table III., with some additions in particular cases. Without insisting on small differences, which may be subject to correction on the review of a longer period and with more accurate coefficients of the temperature correction for altitude, it is clear that the Punjab is in January the seat of the greatest cold, while Rajpootana on the one hand, and the Gangetic delta and Lower Assam on the other, are warmer than the regions immediately adjacent under the same latitudes. The difference of the mean temperature of the Punjab stations (55°) and that of Calcutta, Berhampore, and Goalpara (66°) is not less than 11°. The great Gangetic plain, lying between them, enjoys a nearly equable temperature throughout of 60° or a little under. Benares in this month appears to have a lower temperature (for its elevation) than Roorkee, and very much lower than Goalpara, both in higher latitudes. Opposite to the Gulf of Cambay and the Bay of Bengal, especially the latter, the isotherms advance northwards, retreating

anticipate a somewhat higher temperature in the neighbourhood of the hills, where, as I suppose, the anti-monsoon current descends to a lower level than further south. It is true that the isotherm of 65°, as laid down on the chart, shows no corresponding loop where it might be expected to occur; but I have no observations for determining this part of its course nearer than Hazareebagh, which is at an altitude of 2000 feet.

again to the south in a festoon-shaped curve in the interval, so that Benares and neighbouring stations in the Gangetic plain enjoy a lower temperature than any other part of India under the same latitude. In the peninsula south of the Satpooras the isotherms appear to be more regular, but the data available for this region are as yet insufficient to show their course with any pretence to accuracy.

In February and March (Plates XLIII. and XLIV.) a general rise of temperature is accompanied by important modifications in its relative distribution. The isotherm of 70°, which in January ran across the peninsula south of the Satpooras, is now (in March) pushed up to the Punjab, indicating a general average rise of 10°. The greatest rise is at Benares and Patna (16°), the least at Bombay (4°·3), and two thermal foci begin to make their appearance in the Central Provinces and the hilly country of Western Bengal. The temperature-difference of Rawul Pindee (the coldest station of the Punjab) and Calcutta is 15°; that of Rawul Pindee and Nagpore 19°.

In April (Plate XLIV.) the thermal focus of Central India is well developed. The mean temperature of Nagpore in this month is 7° above that of Bombay (allowing for altitude), 13½° above that of Rawul Pindee, and 6° above that of False Point. The region of greatest heat is indicated on the chart (Plate XLIV.) by the isotherm of 90°, which includes the stations of Nagpore and Hoshungabad. The whole of Central India with Rajpootana, Bundelkund, and Behar, Western Bengal south of the Ganges, with Chota Nagpore and Orissa, have a mean temperature between 85° and 90°. The Upper Punjab (including Rawul Pindee) and Upper Assam are the coolest parts of our region, having a mean temperature of 75° to 77°.

In May (Plate XLV.) the thermal focus moves up somewhat to the north-west, Ajmere and Jhansi being now the hottest stations in the list, while the isotherm of 95° embraces nearly the whole of the Malwa plateau together with the region lying around Nagpore. But the rise of temperature in the Punjab is greater than in Rajpootana. At Ajmere and Jhansi the mean May temperature is 8° and 9° higher than that of April; but at Mooltan it is 10°, at Lahore 11°, and at Rawul Pindee, the most northerly station, nearly 14°. In Upper Assam there is a rise of 6° or 7°; but Lower Assam remains cool, the May temperature of Goalpara being only 1° above that of April. The mean temperature of the Assam valley in this month is about 80°, that of the Punjab 91°.

In June (Plate XLV.) the focus of heat has travelled up to the Punjab, the temperature of which has risen from 3° to 7° above that of May, while at Nagpore it has fallen 8° in consequence of the cooling effect of the summer rains. The rains do not set in on an average till the middle of the month, so that the decrease of the June mean temperature from that of May is not more than about half the whole reduction produced by the monsoon rains. To the north of the Satpooras and to the west of the Hazareebagh plateau, where the rains do not set in till the end of June or the beginning of July, the mean temperature of June is equal, or nearly so, to that of May; while in the Punjab (as above mentioned) there is a rise of 3° to 7°, and in Upper Assam, where heavy rain falls from March or April, a rise of nearly 4°.

In Upper Assam, indeed, all the seasons are less strongly characterized than in Northern India. Here, as in the temperate zone, the heat increases gradually and uniformly up to July, and from June to September remains about 4° above that of the delta. Even at Goalpara, at the mouth of the valley, no fall of temperature results from the burst of the monsoon rains; but a steady rise of 1° per month after April brings the mean temperature up to between 82° and 83° in July and August, and is followed by an equally steady fall till October. In Assam the hot season, as understood in Northern India, is unknown.

The progressive fall of temperature in Northern India, as the monsoon rains advance, on the one hand from the Arabian Sea, and on the other from the Bay of Bengal, is very clearly shown in the chart for July (Plate XLVI.). The greater part of Central India and the whole of the Lower Provinces, together with Lower Assam and Cachar, have now a mean temperature below 85°. But the Punjab and the Bikaneer desert still range above 90°. The highest temperature is that of Rawul Pindee, where the fall from June amounts only to 2½°.

From July to October (Plates XLVI. & XLVII.) the temperature gradually declines, in such measure that by the end of September or the beginning of October it is nearly equalized over Northern India. In the Central Provinces, indeed, and in Rajpootana, also at Calcutta and some other stations in Bengal, there is a slight rise of temperature in September just before the rapid fall sets in.

To sum up the above facts briefly. The distribution of temperature in Northern India presents three very distinct phases, corresponding to the three seasons already defined. In the cold weather two loci of minimum temperature are situated in the Punjab and Upper Assam, and there is a secondary locus of abnormally low temperature, extending apparently from Bareilly to Benares. With this exception the general course of the isothermals conforms more nearly to the parallels of latitude than at any other season.

In the hot weather a temperature focus is found in Central India, and the uplands and plateaux south of the Ganges and eastward from the Sahyadree mountains have a temperature considerably higher than that of the Gangetic plain, the maritime belt, or the surrounding seas. The Upper Punjab and Upper Assam are still the coolest parts of our area.

Finally, in the rains, the Punjab is the seat of the highest temperature, and Upper Assam, though much lower, nevertheless ranges above Bengal. The coolest regions are those where the rains are most copious, and consist of two tracts extending inland from the coasts of Bombay and Bengal respectively, in the course of the monsoon currents.

To complete this discussion, it remains to consider the distribution of temperature in a vertical direction at different seasons of the year. The only available evidence bearing on this subject is that afforded by the hill-stations, of which the best are Darjeeling, at 6941 feet, with Goalpara 386 feet, as a reference station, and Chuckrata at 6884 feet, which may be compared with Roorkee at 880 feet—the former illustrating the damp

climate of Bengal, the latter the drier and more continental climate of Upper India. Shillong, being situated on a plateau, is less favourable for the purpose, but may be adduced as an additional example of a Bengal station at a lower altitude (4792 feet), and can be compared with Goalpara on the north-west and Silchar (75 feet) on the south-east. Lastly, I shall give a comparison between Nynee Tal at 6400 feet and Roorkee, observing, however, that the situation of the former station, on the banks of a mountain lake in a hollow valley, is such as to impair its character as a representative station. These data are given in the following Table:—

Table of comparative temperatures at four hill-stations and three stations on the plains, showing the variations of the temperature decrement with altitude in each month.

	Darjeeling, 6941 ft.	Goalpara, 360 ft.	Difference, G – D.	Elevation = 1° Fahr.	Shillong, 4792 ft.	Goalpara, 360 ft.	Difference, G – S.	Elevation = 1° Fahr.	Silchar, 75 feet.	Difference, Sil. – Shill.	Elevation = 1° Fahr.	Elevation mean of cols. 8 and 11.
	°	°	°	ft.	°	°	°	ft.	°	°	ft.	ft.
January	42·9	64·3	21·4	301	51·3	64·3	13	339	64·4	13·1	360	350
February	44·5	67·4	22·9	286	54·2	67·4	13·2	333	68·2	14	337	335
March	50·4	73	22·6	290	61·4	73	11·6	280	74·5	13·1	360	370
April	56·1	77·1	21	312	64·5	77·1	12·6	350	78·2	13·7	344	347
May	60·2	78·2	18	363	68·4	78·2	11·8	373	81·1	12·7	371	372
June	63·3	79·9	16·6	395	69·4	79·9	10·5	420	81·9	12·5	377	399
July	63·9	81	17·1	383	69·6	81	11·4	386	82·1	12·5	377	382
August	64	81·3	17·3	379	69·2	81·3	12·1	364	81·6	12·4	380	372
September	62·1	80	17·9	366	67·3	80	12·7	347	81·4	14·1	334	341
October	58	78·3	20·3	322	63·5	78·3	14·8	300	79·8	16·3	289	295
November	50·2	71·1	20·9	313	56	71·1	15·1	292	72·1	16·1	292	292
December	44	64·8	20·8	315	50·1	64·8	14·7	300	65	14·9	317	309
Year	55	74·7	19·7	335	62·1	74·7	12·8	349	75·9	13·8	345	347
Range	21·1	17·0	6·3	109	19·5	17	4·6	128	17·7	3·9	91	107

	Chuckrata, 6884 ft.	Roorkee, 886 ft.	Difference, R. – C.	Elevation = 1° Fahr.	Nynee Tal, 6400 ft.	Roorkee, 886 ft.	Difference, R. – N.	Elevation = 1° Fahr.
	°	°	°	ft.	°	°	°	ft.
January	42·6	57·6	15	400	43·8	57·6	13·8	400
February	44·5	62·3	17·8	337	43·3	62·3	19	291
March	50·2	69·6	19·4	309	51	69·6	18·6	297
April	59·5	80·4	20·9	287	60	80·4	20·4	271
May	68·7	88·6	19·9	302	66	88·6	22·6	244
June	69·5	89	19·5	308	69	89	20	276
July	65·7	84·5	18·8	319	68·7	84·5	15·8	349
August	65·1	84·2	19·1	314	68	84·2	16·2	341
September	62·2	83	20·8	288	66·3	83	16·7	330
October	59·2	76	26·8	224	58·7	76	17·3	319
November	53·5	63·7	20·2	297	50	63·7	13·7	403
December	44·2	57·5	13·3	451	45·7	57·5	11·8	468
Year	57·1	74·7	19·3	320	57·5	74·7	17·2	332
Range	26·9	31·5	13·5	227	25·7	31·5	10·8	224

The average of the three Tables of the Bengal stations gives a mean increment of 343 feet elevation for each temperature decrement of 1° Fahr. In the North-west Himalaya the temperature decrease would appear to be on the whole more rapid, the average of the two Tables being 326 feet per 1°. But these are only the mean results of the year, and the Tables show a wide departure from this average in different months of the year, which is least at Darjeeling, greatest at Chuckrata. The difference of temperature between this latter station and Roorkee in October is more than twice as great as in December.

At Darjeeling the decrease of temperature with elevation is most rapid in February. This is not, indeed, the coldest month at Darjeeling; but in the first two months of the year the temperature rises less than at Goalpara. Between February and June, on the other hand, the rise is greater at the hill-station, especially in the month of May; and about the time when the rains set in generally over Bengal, the difference of temperature between Darjeeling and Goalpara is less than at any other time of the year. The difference increases during the rains, but only to the extent of 1°·1. On their cessation in October a sudden increase of 2°·4 takes place, and the difference is further augmented by 2°·6 up to the month of February.

At Chuckrata the difference from Roorkee is greatest in October and least in December. From this month it increases rapidly till April, after which there is a slight fall till July, followed by a rise which is extremely rapid at the close of the rains up to the maximum in October. At Nynee Tal the variations are similar, except that the first and absolute maximum difference occurs in May, and the increase at the close of the rains is comparatively small in amount.

It would lead me into a lengthy digression, unnecessary to my present purpose, to enter on a detailed discussion of the extremely complicated causes which give rise to these irregularities in the vertical distribution of temperature in the lower atmosphere; and, indeed, I am doubtful whether with our present data a very satisfactory result could be looked for. An investigation of similar phenomena in the case of Hoch Obir compared with Klagenfurt and other stations in Carinthia by M. J. Hann has led him to some general conclusions, which I quote below*, and which seem to help to an expla-

* The problem which M. Hann desired to solve in this inquiry was the relation of the temperature decrement with altitude to the wind-direction; and he expresses his results in the following empirical laws:—

1. "Die Temperaturabnahme nach oben ist bei südlichen und südwestlichen Winden langsamer als bei nördlichen und nordöstlichen.

* * * * * * * *

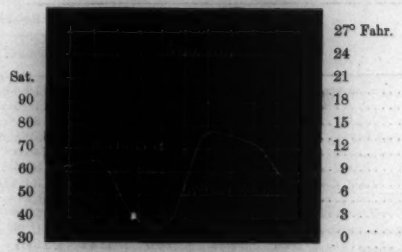
2. "Die Temperaturabnahme mit der Höhe zeigt eine grosse Abhängigkeit von der Windstärke—sie ist stets grösser bei Stürmen, aus welcher Richtung sie auch kommen mögen; aber auch der Unterschied zwischen nördlichen und südlichen Strömen spricht sich dann noch schärfer aus. Die Ursache davon liegt, wie noch gezeigt werden soll, zumeist in dem raschen erzwungenen Emporsteigen der Luft.

3. "Bei schwachen Winden und heiterer Witterung ist die Temperaturabnahme in den unteren Schichten sehr verzögert; sie wächst aber dann rascher in den höheren Luftschichten. Die Temperatur-Verminderung mit der Höhe ist am langsamsten bei heiterer Witterung und schwachen westlichen Strömungen in die Höhe."

nation of the present case; but the climatal conditions of a European hill-tract are so different from those of the Gangetic plain and the Himalayan boundary chains, that conformity of the local empirical laws of the two regions is not to be expected. M. HANN has not suggested any physical cause for the difference he has observed in the temperature decrements with south-west and north-east winds, and I may therefore be permitted to suggest one in the difference of their humidity; since the continual upward diffusion and condensation of water-vapour must tend to equalize the temperatures of the lower and higher strata, and this tendency will be the greater the higher the humidity of the air—that is, the nearer it is to saturation. In the case of the Himalayan stations there appears to be a certain inverse ratio between the relative humidity of the atmosphere and the difference of temperature at the upper and lower stations—not, indeed, such as to explain the whole of the variation, but such as to indicate that the condensation of vapour exercises an important influence on the phenomenon. This is shown in the accompanying diagrams (figs. 2 & 3), which give the curves of mean

Fig. 2.

Fig. 3.



humidity at two of the higher and two lower stations, together with those of the temperature differences between the pair of stations contrasted.

It will be observed that in the case of the damp climate of the Sikkim Himalaya the temperature-difference curve is considerably less irregular than in the dry climate of the North-western Provinces, and that the absolute minimum difference at Darjeeling and the secondary minimum at Chuckrata coincide with the high humidity of the rains. Further, that the rise of temperature over the plains in the early months of the year is felt at Darjeeling two months earlier than at Chuckrata; and the rise of temperature at the former between March and June, which much exceeds that at the lower station of Goalpara (reducing their difference), and which I suppose to be due to the condensation of moisture as cloud and rain in those months, is but faintly indicated, and at a later season at Chuckrata, where the rainfall is small at that period of the year. (See the Rainfall Table at the end*.) One very important datum is yet wanting for the north-western hill-stations, viz. the proportion of cloud-obscuration; and the radiation observations are imperfect. At Roorkee the sky is more free from cloud in October than in any other month of the year, and the same is probably the case at Chuckrata. If so, the great difference of temperature in this month may be due to the increased radiation at the upper station. It is to be hoped that in the course of a few years data will be collected that will admit of a more satisfactory investigation of the subject†.

* The rainfall Tables for Nynee Tal and Simla may be taken as illustrations of the rainfall of the North-west Himalaya in lieu of that of Chuckrata, of which I have the record for only two years.

† The registers of the temperature of nocturnal radiation at the hill-stations of the North-western Provinces are at present imperfect, and there appears to be a peculiar difficulty in obtaining observations at such stations that shall be comparable with those on plains' stations, owing to the convection of the cooled air on sloping ground. It may be therefore, and probably is the case, that the recorded temperatures of radiation at night are much above those that would be given by an instrument placed in a hollow. I find in Dr. Thomson's Reports one year's observations at Nynee Tal, 10 months' at Raneekhet near Nynee Tal, and 7 months' at Chuckrata. The following Table gives in one column the means of all these (without distinction of station), in another those of the same years at Roorkee, and in a third the difference of the two. The remaining columns show the mean maximum temperatures of solar radiation for the same years treated in like manner, and their differences.

	Grass Nocturnal Radiation.			Solar Radiation.		
	Hill-stations.	Roorkee.	Difference, R.—H.	Hill-stations.	Roorkee.	Difference, R.—H.
January	27·3	40	12·7	85·2	110	24·8
February	28·6	41·5	12·9	90·3	114·5	24·2
March	38·5	51·5	13	113	128	15
April	43·6	60	16·4	119·3	129	9·7
May	50·3	67	16·7	121·7	143·5	21·8
June	53	74·5	21·5	121·3	139·5	18·2
July	52·5	76	13·5	114·7	133	18·3
August	54	75	21	115·3	127	11·7
September	51	72	21	123·4	127·9	4·5
October	38·8	56·8	16	123·3	127·8	4·5
November	34·5	43·1	18·6	114·5	121·1	6·6
December	28·9	38	9·1	97·5	109·7	21·2

From this rough comparison it would seem that the excess of nocturnal radiation at the hill-stations in October is not so great as in the rainy months; and so far the fall of temperature in that and the preceding

On the low plateaux to the south of the Ganges, and in the higher parts of the Gangetic plains, the distribution of temperature follows laws very different to the above. In their case a moderate elevation appears to determine an increase of the summer temperature and a decrease only in the winter season, or, in certain cases, in the rains—consequently an increased annual range. Whether the mean temperature of the year shows a diminution corresponding to the altitude above sea-level, cannot very well be ascertained in the case of stations on the Gangetic plain, because their distance from stations of reference at or near the sea-level is so great, that it is impossible to eliminate the effects of other geographical differences, such as the slope of the ground, proximity of hills or water, &c.; and I have already remarked that many of the registers are not, in my opinion, strictly comparable for small differences. But in the case of the somewhat more elevated plateaux, we have more trustworthy means of comparison in the stations of Hazareebagh and Berhampore, situated under nearly the same latitude, about equally distant from the sea, and affording observations with corrected thermometers, observed four times a day at equal intervals of six hours. Moreover their registers extend over the same period of the four years 1868–71. Finally, their geographical *entourage* is such as to offer no striking contrasts, except *that* the effect of which is in question—since Berhampore is on a comparatively dry part of the delta and at no great distance from its margin. Hazareebagh is situated at 2014 and Berhampore at 65 feet above the sea-level. The following Table gives the mean temperature of each station and their difference in each month:—

TABLE of comparative temperatures at stations on a plateau and in the plains, showing the temperature-difference in each month.

	Hazareebagh, 2014 feet.	Berhampore, 65 feet.	Difference, B.–H.
January	61·7	65·3	3·6
February	66·2	70·7	4·5
March	74·3	78·2	3·9
April	82·6	85·5	2·9
May	85·9	86·3	0·4
June	81·4	84·6	3·2
July	79·2	84	4·8
August	77·9	84·1	6·2
September	77·3	83·4	6·1
October	74·5	81·7	7·2
November	68·9	73·5	4·6
December	61·6	66·2	4·6
Year	74·3	78·6	4·3
Range	24·3	21	6·8

month, as compared with Roorkee, remains unexplained. But it also appears that the solar heat is at all times (on an average) less intense than on the plains, and that this difference is least in September and October. It would appear, then, that, on an average, less solar heat reaches the hill-surface at these stations, 7000 or 8000 feet above sea-level, than the surface of the plain immediately below the hills. The figures are far from accounting for the observed variation of temperature-difference.

On the average of the whole year, Hazareebagh is then 4°·3 cooler than Berhampore, which, for a difference of 1949 feet, gives a mean of 1° Fahr. for 453 feet. The temperature-difference is least in May, when it amounts to only 0°·4; but Hazareebagh is still the cooler, and this station is therefore above the range at which elevation produces an actual increase of summer temperature. The difference is greatest in October; but from July to December it remains above the average of the year, and thus (in respect of the rainy season) contrasts strongly with the corresponding variation at hill-stations. The variation may, I think, be traced partly to the character of the winds, but chiefly to the local absorption and radiation of the solar heat. In May there is at Berhampore, as at all stations on the delta, a very large excess of south and east, that is to say sea-winds, more or less deflected; while at Hazareebagh there is still an excess of west and a large proportion of north-west, that is hot land-winds. In October the preponderance of sea- and land-winds respectively at the two stations is similar, but the land-winds are at this season of the year the cooler of the two. But a further and more important cause tending to produce the observed variation is the difference of nocturnal radiation and diurnal absorption of solar heat in those months. This difference is at its minimum in May and its maximum in October in the former case, and *vice versâ* in the latter. The mean of three years' observations of a RUTHERFORD'S minimum thermometer (placed above the ends of the grass on forked sticks, and radiating its heat freely during the night time) is as follows:—

	Hazareebagh.	Berhampore.	Diff. B.—H.
May	72°·2	73°·5	1°·3
October . . .	59°·8	71°·8	12°·0

On the other hand, the mean of three years' observations of a maximum blackened bulb-thermometer enclosed in a vacuum-tube and freely exposed to the sun, the observations having been recorded on all days, whether clear or overcast, is as follows:—

	Hazareebagh.	Berhampore.	Diff. H.—B.
May	160°·5	150°·8	9°·7
October . . .	140°·3	142°·7	−2°·4

A similar excess of nocturnal radiation and decrease of insolation, but less in amount, characterizes the rainy months at Hazareebagh, and serves to explain the lower temperature of that station. I need not pursue this subject further at present. The important point which is so strongly indicated by the above figures is that the temperature of the atmosphere over the plateau is largely affected by the absorption and radiation of the ground, and therefore does not represent that of the free atmosphere at an equal height over the plains. Any difference of temperature thus arising between two masses of air in the same horizontal plane must tend to produce convection-currents; and of such currents tending to or from the plateaux of Central and Upper India and Western Bengal we have many instances.

Vapour-tension, Humidity, and Rainfall.—The existing records of the hygrometric state of the atmosphere, from which I have constructed Tables IV. and V., are less

complete than could be desired. With the exception of the Lower Provinces and Bombay (Colaba Observatory) and two stations in the North-western Provinces, the available humidity-registers give observations of the day hours only; and in order to obtain comparable values, I have multiplied these by factors obtained empirically from the registers of Benares and Roorkee (see note to Table V.). I have rejected the data of several stations which it would have been desirable to add, since they give results so much above other stations similarly situated, as to leave me in little doubt of their untrustworthiness. The Table of vapour elasticities is computed directly from those of the mean humidity and the mean temperature, except in the cases of Calcutta and Bombay, where the figures have been obtained from the psychrometer observations taken hourly and reduced for each observation. With these two exceptions the values are therefore in all cases somewhat too low.

Since, as is well known, Northern India with Eastern Bengal presents in different parts the extreme modifications of continental and maritime climate, it might be expected that the range of vapour-tensions shown by a comparison of the registers would be very great; and such is, indeed, the case. The extreme amounts shown in the present Table are furnished by Mooltan and False Point—the annual mean of the former being 0·382 inch, while that of the latter is 0·862 inch*.

But this, the geographical range, is in certain cases surpassed by the annual range at one and the same station. Of this the stations in the Gangetic plain offer the most striking examples. Thus at Patna the mean vapour-tension rises from 0·328 in January to 0·969 in July, at Benares from 0·325 in December to 0·977 in July, and at Roorkee from 0·293 in December to 0·911 in July, the ranges being therefore 0·641, 0·652, and 0·618 respectively. On the other hand, Bombay and Akyab, the climates of which are more equable than that of any other station, show a range of 0·335 and 0·374 only.

The lowest vapour-tension occurs in January at most of the stations, coinciding with the minimum of temperature. At one or two only it appears to be slightly lower in December. Such is the case at Cuttack, False Point, and Saugor Island, where sea-winds set in very early in the year (see *ante*, Part I. pp. 576, 578), and at Benares. Generally in the upper part of the Gangetic plain, in Central India, and the Punjab the means of December and January are almost equal. In the Gangetic delta and Orissa, on the coast of Arakan, and in Eastern Bengal and Lower Assam† the elasticity rises regularly and rapidly up to the setting in of the rains, indicating a steady increase in the supply of vapour as well as a rising temperature. But in the dry regions of the interior, where land-winds prevail throughout the spring months, the rise of vapour-tension is very slow, not much greater probably than would be produced by the actual rise of temperature on the local supply of vapour. The elevation, when the summer monsoon begins to be felt in June or July, is then very sudden, and the fall at the close of the monsoon between September and November equally strongly marked.

* The January mean at the former station is 0·208 inch; the May mean at the latter 1·079 inch.

† Probably also in Upper Assam; but I have no register for that region.

As regards the decrease of vapour-tension with elevation above sea-level, a comparison of its mean values at Hazareebagh, Shillong, and Darjeeling with those of neighbouring stations near the sea-level affords additional confirmation of the conclusions drawn by General STRACHEY* from the observations of Dr. HOOKER in Sikkim and those of Mr. WELSH in England. Comparing, in the first place, the Darjeeling values with those of Goalpara, it appears that in most months the former are rather more than half as great as the latter, and that the proportions are relatively greater in the months of the rains than in those of the cold or hot seasons. The total atmospheric pressure, however, at the former station is at all seasons more than three fourths of that at the latter, so that the diminution of atmospheric density over Bengal from January to June, in so far as it is due to the admixture of water-vapour, chiefly affects the lower fourth of the atmosphere. Of this additional proofs will appear presently. At Shillong the proportion is somewhat higher, but more constant as compared with the means of Goalpara and Silchar, the same reference-stations that have been selected for a comparison of temperatures. But at Hazareebagh, while the mean proportion is higher than either of the above, nearly three fourths of that on the plains, the range is also greater than either, varying from 59 to 90 per cent. of that at the lower stations. For this station I have taken as a standard of reference the means of Patna and Calcutta conjointly, these stations being alternately to windward and leeward of Hazareebagh at opposite seasons of the year. The results of these several comparisons are shown in the following Table:—

Table of comparative vapour-tensions at stations at different elevations, showing the ratios of decrement in each month.

	Darjeeling, 6941 ft.	Goalpara, 386 ft.	Ratio, D : G.	Shillong, 4792 ft.	Mean of Goalpara and Silchar.	Ratio, $S : \frac{G+S}{2}$.	Hazaree-bagh, 2014 ft.	Mean of Calcutta and Patna.	Ratio, $H : \frac{C+P}{2}$
	inch.	inch.		inch.	inch.		inch.	inch.	
January	0·212	0·439	0·48 : 1	0·265	0·459	0·58 : 1	0·280	0·407	0·68 : 1
February ...	0·238	0·435	0·55 : 1	0·273	0·486	0·56 : 1	0·264	0·447	0·59 : 1
March.....	0·252	0·487	0·52 : 1	0·321	0·562	0·57 : 1	0·330	0·548	0·60 : 1
April	0·338	0·632	0·53 : 1	0·394	0·690	0·57 : 1	0·423	0·644	0·65 : 1
May	0·427	0·790	0·54 : 1	0·534	0·851	0·63 : 1	0·557	0·776	0·72 : 1
June	0·529	0·897	0·59 : 1	0·598	0·923	0·64 : 1	0·796	0·899	0·88 : 1
July	0·546	0·919	0·59 : 1	0·629	0·946	0·66 : 1	0·867	0·961	0·90 : 1
August	0·548	0·916	0·60 : 1	0·627	0·942	0·66 : 1	0·812	0·949	0·85 : 1
September ...	0·501	0·900	0·55 : 1	0·588	0·926	0·63 : 1	0·786	0·932	0·84 : 1
October	0·381	0·782	0·49 : 1	0·503	0·828	0·63 : 1	0·563	0·778	0·72 : 1
November ...	0·276	0·593	0·48 : 1	0·341	0·623	0·55 : 1	0·360	0·539	0·66 : 1
December ...	0·213	0·459	0·46 : 1	0·271	0·480	0·56 : 1	0·279	0·415	0·67 : 1
Year	0·388	0·687	0·56 : 1	0·445	0·726	0·61 : 1	0·526	0·691	0·76 : 1
Range.....	0·336	0·484	.....	0·364	0·487	.....	0·603	0·554	

Turning now to the Table of relative humidity, we find that while stations on the coast-line have at all times of the year a higher degree of humidity than those on the

* Proceedings of the Royal Society, vol. xi. p. 182.

plains of the interior, the rate of decrease is very different in different seasons, and that in the first three months of the year the rule of increasing dryness with increase of distance from the coast-line holds good only as far inland as Behar in the Gangetic plains. From Patna to Lahore the humidity of the atmosphere steadily increases to such an extent, that in March the latter station exceeds the former by 19 per cent. of saturation. I have little doubt that the Central Provinces are in like manner more humid in this season than the strip of country lying between them and the Sahyadree range, to judge from the rainfall Tables: humidity observations for the latter, though existing, are unfortunately not accessible and cannot be appealed to in verification; but during a part of the time Nagpore and Jubbulpore show a higher humidity than Patna, and even than Benares.

At the hill-stations the rule is also modified, and at Darjeeling from June to September the humidity of the atmosphere exceeds that of any other station either on the coast or elsewhere. I shall presently consider the effect of elevation on this element.

The coast-stations show some variations which evidently have reference to the prevailing winds. Thus in the earlier months of the year (from January to May) False Point has a higher degree of humidity than Akyab, though situated under nearly the same latitude; and during the remainder of the year this relation is reversed. A similar relation exists between False Point and Bombay, except that Bombay falls below False Point a month earlier; and a similar tendency, though less marked, is observable in the registers of Chittagong and Saugor Island. Thus, then, it appears that the west coasts of both peninsulas have a lower degree of humidity than the east coast of India during the drier half of the year, while they range higher as soon as the south-west monsoon sets in.

At Saugor Island and Chittagong January is the driest month of the year, and August the most humid, the annual range being 13 per cent. at the former station, and 18 per cent. at the latter. At Akyab the range is only 11 per cent., and March is the driest and July the most humid month. At False Point the humidity is lowest in November, that is, during the height of the rains lower down on the Madras coast; and from April to August the proportion of moisture scarcely varies, and is about 15 per cent. of saturation higher than in November.

Proceeding inland, a distance of 50 or 60 miles suffices to produce a marked decrease of humidity, especially during the prevalence of the land-winds. Thus in the month of March Calcutta stands 13 per cent. of saturation below Saugor Island, and Cuttack 16 per cent. below False Point; and in the month of February Dacca ranges 13 per cent. below Chittagong. In Cachar, where the winds are chiefly from the south and the country around is either marsh or clothed with dense forest, the humidity of the air remains high and equable throughout the year; only in two months, viz. in March and April, does it fall below that of Chittagong. But in Behar and in the North-western Provinces after March the ordinary rule holds good. Thus in April Patna ranges 39 and Benares 43 per cent. of saturation below Saugor Island, while in May

Jubbulpore and Nagpore are respectively 53 and 50 per cent. below False Point, and 42 and 39 per cent. below Bombay. The driest climate is that of Mooltan, where in May and June the humidity is 45 and 52 per cent. of saturation below that of Bombay.

In these instances it will have been noticed that the period of greatest siccity falls the later in the season the greater the distance from the sea, measured along the course of the prevailing wind-current. Thus while at Saugor Island and Chittagong the minimum of humidity falls in January, at Dacca in February, at Calcutta between February and March, and at Cuttack, Berhampore, Cachar, and Goalpara in this latter month, it occurs at Hazareebagh between March and April, at Patna and Benares decidedly in April, at Jubbulpore, Nagpore, and Roorkee in May, and in the Punjab between that month and June. This is clearly dependent on the advance of the sea-winds, as already described in the first part of this paper.

In the North-western Provinces and the Punjab a secondary minimum occurs at the beginning of the cold weather. This minimum falls *the earlier* the greater the distance from the sea, in the sense above defined. At Benares it falls in November, and from that month to January the air approaches more nearly to saturation as the temperature falls. At Lahore and Rawul Pindee it is in October, and at Mooltan as early as September: at this last station the normal variation of the Indian climate is reversed, and the absolute maximum of the year falls in December. At Rawul Pindee and Roorkee, both situated close to the northern hills, the winter (here the secondary maximum) falls in February, or a month later than at other stations. This winter maximum is evidently related to the winter rains of the Upper Provinces, and, like the corresponding winter maximum and rains of Europe, is traceable to the descent of the equatorial (here the anti-monsoon) current and the low winter temperature. But for this it may be inferred that the Punjab winter would be much more rigorous. The increased humidity of the winter does not affect places much below Benares. At Patna the last vestige of it is perceptible in the fact that the mean humidity of January does not fall below that of December, and at Hazareebagh the proportion remains constant from November to January. At Cuttack, however, the phenomenon reappears, and the humidity of January appears to be higher than that either of the preceding or following months. This may possibly be due to the descent of an anti-monsoon current from the Arabian Sea, a conclusion which is favoured by several circumstances, which I shall refer to later on.

It was observed by Dr. Hooker, in the meteorological appendix of his 'Himalayan Journals,' that the relative humidity of the atmosphere remains pretty constant throughout all elevations in the Himalaya, except in a Tibetan climate*. The data for the hill-stations in the present Table generally confirm this observation, but they also show that the law is subject to considerable exceptions at certain seasons of the year; moreover, that the local law of variation is by no means the same at all the hill-stations, but varies with the character of the wind-currents prevailing at each station. At

* Bessel assumed a similar law of distribution in computing his barometric formula.

Chuckrata, in the North-west Himalaya, the humidity of the air is from 10 to 12 per cent. (of saturation) lower than at Roorkee in the months of the cold weather (November to February), and from 3 to 16 per cent. higher from March to September. The general form of the curve of annual variation is the same at both stations (see fig. 3, p. 591), but both the winter and summer maxima occur a month later at the hill-station, and the spring minimum a month earlier. Next, comparing Darjeeling with Goalpara, it appears that it is only in the last three months of the year that the humidity of the hill-station ranges below that of Goalpara, and then only to the extent of from 4 to 1 per cent. of saturation. In May they are equal, and in all the other months Darjeeling is the higher. This is especially the case in February, when there is a difference of 16 per cent. of saturation between the two stations. Darjeeling exhibits in a very marked degree the subordinate winter maximum already described in the Upper Provinces, but it falls later, the secondary minimum being in December and the maximum in February. The cause is doubtless the same in both cases, Darjeeling, as we have seen, being brought by its elevation within the influence of the anti-monsoon current. At Shillong, however, no such phenomenon is to be observed, and the cause is again obvious. The north-east current from Assam, flowing down the valley and across the Khasia plateau, must drive the compensating equatorial current into a higher region of the atmosphere, and the registers of the station show that in the cold-weather months the preponderating winds at this station are from the north.

The Shillong registers show further that, as compared with Goalpara, the humidity of the station is low. Except in February and December and from July to October, when it is equal to that of Goalpara (in the last month somewhat higher), the mean humidity of the hill-station is from 1 to 5 per cent. less than that of the Assam valley. A similar relation is shown, but far more decidedly, by Hazareebagh, which from October to May has a drier atmosphere, not only than Calcutta and Cuttack, but even than Patna, 120 miles further inland: from October to April its humidity ranges below that of Jubbulpore, although it is less than 700 feet higher. I should not indeed venture to infer from this fact that in a vertical column of the atmosphere over the plains the relative humidity of the atmosphere would be found in the cold and early hot-weather months lower than near the land-surface, to any thing like the extent shown by the Hazareebagh registers. The situation of this station on the highest part of a dry plateau, freely absorbing and emitting the solar heat, while it affords little evaporation, presents conditions very different from those of the free atmosphere; and probably by raising the temperature raises the tension of saturation, while it raises that of the vapour actually in the air only in about the same ratio as that of dry air. But I have already noticed this subject in a previous part of this paper.

In a paper published in 1870 in the 39th volume of the Journal of the Asiatic Society of Bengal, I gave Tables of the average monthly rainfall at 47 stations in Bengal, Behar, Orissa, Assam, and on the Arakan coast. In Table VI. (at end) these have been corrected and supplemented by the registers of subsequent years and data drawn from other sources; and I have added similar Tables for 41 stations in the Central Provinces, the

North-western Provinces, the Punjab, and Bombay, chiefly taken from recent registers. The whole, numbering 92 stations, give a very fair conspectus of the distribution of rainfall in Northern India, with the exception, indeed, of the Bombay Presidency, for which I have been able to obtain, in addition to that of the Colaba Observatory, only a few old and published Tables. The rainfall map (Plate XLIX.) has been drawn up from these data. It shows the distribution of the total annual rainfall by lines of equal precipitation, which represent increments or decrements of 10 inches. The chart cannot of course pretend to accuracy in detail, since rainfall, more than any other climatic element, is subject to local variation, being affected by local geographical conditions, such as the proximity of small hills, &c.; but the lines show with tolerable faithfulness the broader features of its distribution.

I have indicated by inscriptions on the chart the tracts in which rain is received at each of the three principal seasons. Of these latter, the summer and early autumn rain, that of the *rainy season* emphatically so called, is by far the most copious and extensive; so that, beyond a slight recession towards the coast of the ishyetic lines of the Punjab, Upper Provinces, and Assam by the omission of the winter rains, and of those of Orissa, Bengal, and the eastern districts generally by that of the spring or hot-weather rains, the map would require little modification to represent the rainfall of the first-named period only.

The cold-weather rains are received most regularly and in the largest quantity in the North-western Provinces and the Punjab, in Upper Assam and Cachar. In Behar and the Gangetic delta they are less regular and lighter. They usually begin in December and continue till March in the North-western Provinces, till April in the Punjab. In Bengal, as we have seen, sea-winds begin to be felt in February and March, and there is no period of demarcation between winter and spring rains. The same is the case in Assam and Cachar. The mean rainfall of eight stations in the Gangetic plain from Goruckpore and Benares upwards, and of eight stations in the Punjab from November to April, is as follows:—

	Gangetic Plain.	Punjab.		Gangetic Plain.	Punjab.
November ...	0·04 inch.	0·00 inch.	February....	1·05 inch.	1·02 inch.
December ...	0·29 „	0·53 „	March.....	0·91 „	1·29 „
January.....	1·02 „	0·53 „	April.....	0·54 „	0·81 „

These rains, therefore, reach their maximum a month later in the Punjab than in the North-western Provinces. This does not coincide with the period of greatest cold, nor even with the winter maximum of humidity on the plains; but it appears to coincide rather with the latter at an elevation of 6000 or 7000 feet, to judge from the register of the single station Chuckrata. In any case it is determined by some cause other than mere cold, and this I take to be the humidity of the anti-monsoon current. In both cases the winter rains are followed by a break of about two months, during which a scanty and uncertain rainfall only is received from occasional thunder-storms. Stations situated near the northern hill-range receive more rain than those lying towards the

borders of Rajpootana, in the cold weather as at other seasons; and on the lower parts of the hills (Dehra, Kangra) they are very copious. On the south of the Gangetic plain they are felt at Ajmere, Jhansi, and in the Central Provinces, but not further westward. Even at Mahableshwar the mean fall from December to March does not amount to half an inch, and at Bombay from December to April to less than a quarter of an inch on the mean of twenty-three years.

The spring or hot-weather rains prevail over all that region over which sea-winds set in from the Bay of Bengal at an early period of the year. In Assam and Eastern Bengal showers are pretty frequent in March or even February, and in April the rainfall is pretty general and copious. In Cachar and Sylhet in this latter month it amounts to 12 or 14 inches, and at Seebsaugor and Nazeerah, in Upper Assam, it is between 9 and 10 inches on an average. Lower down the Assam valley it is lighter, viz. 6 or 7 inches between Tezpore and Gowhatty, and at Goalpara but little more than 5 inches. Similarly on the eastern margin of the Gangetic delta it is less than in Cachar and Sylhet, viz. 8·3 inches at Comillah (Tipperah), 4 inches at Noakhally, and 5 at Chittagong. In the western part of the delta it amounts to between 2 and 3 inches (except on the coast-line, where it is heavier). These rains are felt as occasional thunder-storms, known as *north-westers*, as far inland as Nagpore; and also in Behar and in the western half of the delta much is received in this form.

The spring rains have no very definite termination, even in Western Bengal; but generally a fortnight or three weeks of hot dry weather precedes the setting in of the monsoon rains. The break of the monsoon is further marked by a change in the general direction of the wind, and seems therefore to be a phenomenon of a distinct character—something more than a mere increase in the force and regularity of the sea-winds which bring the spring rains above described. In Lower Bengal, Orissa, and the Central Provinces the change consists in a shifting of the wind to the westward in the Gangetic plains to the setting in of east or south-east winds. In Assam and Cachar, however, no change of the kind takes place; from February or March the rainfall increases rapidly and steadily up to June, and then decreases gradually to the end of the south-west monsoon. But on the Arakan coast, and on the Sikkim Himalaya, the commencement of the rains is more definite, and occurs three weeks or a month earlier than on the western part of the delta.

In Lower Bengal and Orissa the rains begin on an average in the first or second week in June, and the fall averages from 9 to 15 inches in that month: in the neighbourhood of the hills, both on the east and north, it is much heavier. They reach the North-western Provinces later, and at Agra and Delhi the mean fall in this month does not exceed 1 or 2 inches. In Bombay they set in about the same time as in the Gangetic delta, and in the Central Provinces a week or so later; but in Rajpootana there is little rain till the following month. In the Punjab the rains are much lighter than elsewhere (except in Sind and the Thaneer desert, which latter is rainless, or nearly so). They

are heaviest in July; but even at Rawul Pindee, in the immediate neighbourhood of the hills, they amount to only 17·21 inches between June and September. At Lahore the total average fall is 7·71 inches, at Peshawur 4·48, and at Mooltan 3·61 only; but in the cool Himalayan valleys of the outer ranges the rains begin in June, and are far more abundant. In Hazarah from June to September the fall amounts to 22·08 inches, and in Kangra to not less than 87·08 inches according to Dr. NEIL's Reports.

On the Himalaya the heaviest rainfall is unquestionably on the lower and outer slopes, and it diminishes from Bhotan to the westward, but in what ratio the present data are insufficient to show. Buxa (said to be situated at 1800 feet above the sea), on one of the outer spurs of the Bhotan Dooars, has an average annual fall (to judge from three years' registers) second only to that of Cherra Poonji, viz. 280 inches; Rungbee, at 4000 feet in outer Sikkim, somewhat less exposed, has 175 inches; and Dehra, in the Doon or low valley between the Sivaliks and the Himalaya, 72·29 inches. The stations at elevations of 7000 feet or 8000 feet receive less, and show a decrease towards the north-west like those at the lower levels. Thus Darjeeling has 127 inches, Nynee Tal 86·58, and Simla 56·20 inches.

Generally the quantity of rainfall diminishes, *ceteris paribus*, with the increase of distance from the coast-line, especially within the first few miles: compare, for example, Saugor Island and Calcutta, False Point and Cuttack. But it increases rapidly on approaching a hill-range on the windward side, whenever the latter presents a steep face in that direction. Instances of this are afforded by Rungpore and Dinagepore as compared with Maldah, by Mymensing and Sylhet as compared with Dacca, by Goruckpore as compared with Benares, by Roorkee as compared with Agra, and by Rawul Pindee compared with Lahore. To leeward of a range, on the other hand, the decrease at the foot and gradual increase beyond is very marked: instances are afforded by Nowgong, Tezpore, and Gowhatty on the north and north-east of the Khasi hills, and even by Shillong on their northern slope, at all of which the rainfall is much below that of Seebsaugor and Nazeerah, higher up the valley in the direction followed by the rain-bearing south-west wind; and especially by Poonah and other stations under the lee of the Sahyadree range, as contrasted with Nagpore and others of the Central Provinces. Of the influence of mere elevation, apart from exposure and slope of ground, no sufficient evidence is yielded by any registers available to me.

Atmospheric Pressure.—Our knowledge of the distribution and changes of pressure is much less complete than that of temperature, or even of vapour-pressure and humidity. Although barometric registers are obtainable for the whole of the area here treated of, many of them cannot be utilized in this discussion owing to the elevation of the Observatories not being known with sufficient accuracy, or to the readings being uncorrected for the error of the instrument or even for temperature. The Punjab registers are defective in all these respects, and those of Ajmere and Dholebagan (a station in Upper Assam), which are otherwise good, cannot be used in the absence of

any trustworthy determinations of level*. The reduced readings of the Jhansi registers, here given, must be regarded as somewhat doubtful from the same cause. There remain those of stations on the Gangetic plain, in Central India, in Bengal, and on the coasts of the Bay; and for these I have data for periods of from three to five years. These are given in Tables VII. and VIII., the former being the mean of the observed pressures reduced to 32° FAHR., the latter the sea-level values corresponding thereto.

For the majority of the stations the mean pressure is obtained by taking the arithmetical mean of observations recorded daily at intervals of six hours, viz. at 4 and 10 A.M. and P.M. The exceptions are Port Blair, Madras, Nagpore, Jubbulpore, and Hoshungabad, for which I have taken the means of the 10 A.M. and 4 P.M. observations, the corresponding night observations being wanting. Excepting Madras, all the means are corrected to those of the Calcutta standard barometer, which instrument reads 0·011 inch higher than the Kew standard.

In October, on the mean of the month, the pressure is nearly uniform in Bengal, and on both coasts of the Bay, in the Central Provinces, and the Gangetic valley. Such inequalities as are shown in the Table and Chart are small and irregularly distributed. Cuttack and False Point show the highest pressure, and Goalpara the lowest; but their difference is less than 0·1 inch, and generally over the whole area a mean pressure of 29·85 or 29·86 prevails. In the North-western Provinces and Behar on one side, and on the Arakan coast on the other, the pressure ranges slightly above that of Bengal, and the difference, though small, finds† its expression in a slight converging tendency of the winds from both quarters. In like manner the small difference between Cuttack and Nagpore causes easterly winds in the direction of the latter, and that between Cuttack and the lower part of the coast a small excess of north-east winds in the north-west of the Bay.

In the following months the pressure rises over the whole area, but chiefly in the North-western Provinces, Chota Nagpore, and Orissa. In December an axis of maximum pressure lies over Cuttack, Benares, Lucknow, and Roorkee; and Agra, Jhansi, and Jubbulpore on one side, and Calcutta, Hazareebagh, and Patna on the other, all

* This cannot be ascertained with sufficient approximation from the barometric readings. Barometric differences in India are in general so small, that an inconsiderable error in the assigned elevation may lead to very deceptive results when the readings are reduced to sea-level; and, on the other hand, owing to the persistence of these differences of pressure, small though they be, the sea-level equivalents of the mean annual pressure at stations one or two hundred miles apart differ sufficiently to vitiate any reasoning based on their assumed equality, and thus the fundamental assumption in the determination of heights by the barometer, viz. that when reduced to sea-level the compared pressures of the two stations are equal, is invalid. An example which strikingly illustrates this is given in the Bengal Meteorological Report for 1869. Cuttack and Saugor Island are about 160 miles apart, the former 80, the latter 6 feet, above mean sea-level; yet, owing to a persistently low pressure during many months of 1868 in the neighbourhood of Cuttack, a comparison of the mean readings of the barometers of the two stations (recorded four times daily during the whole year), indicates a difference of 205 instead of 74 feet.

† See *ante*, Part I. pp. 583, 584.

range lower. In January and February the distribution remains much the same in its general character, except that in the latter month it falls less over the Bundelkund plateau than elsewhere, and the ridge of high pressure is pushed somewhat southward and westward. Jhansi, Benares, Jubbulpore, and Agra now range highest in the Table.

In March the pressure falls rapidly over the whole of Northern India, as represented in the Table, and most so in the delta, on the Hazareebagh plateau, and in Central India south of the Satpooras. These two regions are still separated by a ridge of somewhat higher pressure, which further extends across the Bay of Bengal between the low-pressure area of the delta and that of Ceylon and the south of the Bay. In the existence of this ridge we have doubtless the immediate cause of the back-to-back winds described in the summary of Part I. Its existence is alluded to by Captain MAURY*, though whether as an observed phenomenon or an inference from the observed course of the winds, does not clearly appear in the description. In either case its existence is now placed beyond doubt.

In April, with a further and more rapid fall of pressure in Northern and Central India, the areas of minimum pressure in Central India and Western Bengal coalesce, and form a trough of minimum pressure, which extends from Berhampore to beyond Nagpore. To the north and north-west of this the pressure is but little higher, at least as far as Agra and Roorkee. In Southern India, and over the Bay, on the Arakan coast, and in Eastern Bengal the pressure exceeds that of any part of Northern India (except possibly the Punjab). In May the trough of low pressure formed in Western Bengal and Nagpore moves up somewhat to the north, and now runs east and west, from Hazareebagh along the Sone valley. The pressure of the North-western Provinces falls below that of the delta, and a still lower pressure is established in Rajpootana.

In June the direction of the baric gradients is much the same as in May, except that, as appears from the Roorkee and Agra registers, the fall of pressure in the Punjab is much greater than elsewhere; so that the seat of minimum pressure is probably transferred to the upper part of that province. The mean difference between Port Blair at the Andamans and Roorkee in June amounts to nearly 0·3 inch, and that between Calcutta and Port Blair to more than 0·2 inch. Since Calcutta is situated about midway between Port Blair and Roorkee in the course followed by the monsoon current, it follows that the baric gradient over the Bay of Bengal is about twice as great as up the axis of the Ganges valley. The former is about one tenth of an inch in 400 miles, the latter one tenth in 800 miles. Such, and no greater, are the gradients that sustain the steady current of the south-west monsoon.

In July no further change of importance occurs. The pressure has reached its annual minimum. In August a general rise of ·04 or ·05 inch takes place over the whole of Northern India. At Madras and in Arakan the rise is about half as much, and at Port Blair *nil*. The trough of low pressure that was formed in May south of the Ganges

* *Op. cit.* p. 366.

lasts throughout the rains, and has a marked influence on the winds. The seat of *lowest* pressure lies, however, in the direction of the Punjab and the desert of Bikaneer.

Finally, in September and October the pressure increases more rapidly, and in such measure as to become nearly equalized, prior to the very different distribution which ushers in the cold season and its northerly winds.

It is evident, on inspection of the charts, that the distribution of pressure, to a certain extent, follows that of temperature near the ground surface, in an inverse ratio of intensity. Thus (omitting from consideration the Punjab and Upper Assam, for which barometric data are wanting) Benares is in the cold weather the seat of highest pressure and also of lowest abnormal temperature*. The ridge of high pressure of which this is the culminating point, and which extends in a curve from Roorkee to Cuttack†, is in like manner evidently coincident with the southward bend of the isothermals shown in the cold-weather charts from November to February, especially the first of these months; and, on the other hand, the relatively low pressure of the delta coincides with the northward bend of the isothermals. In these cases, then, there is an apparent coincidence of the isobaric with the isabnormal rather than the isothermal curves. But in the hot-weather months, although a similar coincidence is still traceable, the infusion of water-vapour, which tends to equalize the temperature at higher levels with that near the ground‡, is evidently influential. The trough of low pressure in April between Berhampore and Nagpore has along its axis a temperature not much below 90° when reduced to sea-level, and *probably* a vapour-tension higher than any part of the region to the north-west. That of Nagpore, *e. g.*, is 0·504 inch, and that of Jubbulpore only 0·398; that of Hazareebagh does not indeed seem so high as that of Patna and Benares; but it would appear from the Table at page 596 that the vapour-tension in the hot-weather months diminishes very rapidly in the first few thousand feet of the atmosphere; and although I think it probable that, owing to the high temperature of the ground surface of the plateau, the humidity of Hazareebagh is lower than that of the free atmosphere at the same elevation, there is no reason to believe that the vapour-tension is very much influenced by this cause, since there is no evaporating surface. If the vapour-tension of Hazareebagh and Patna in April be compared with that of the same stations in January, the rise at Hazareebagh is seen to be proportionally greater than at Patna; and at all stations to the south and east of Hazareebagh the relative rise very greatly exceeds that of stations to the north and west.

I infer, therefore, that while the temperature of Hazareebagh near the ground level in April is but little lower than that of the delta (nearly 2000 feet nearer the sea-level) and of Patna (which is more than 1700 feet lower), the admixture of water-vapour, though less than over the former, is probably greater than at the same elevation over the latter and all places to the west and north-west; that a higher temperature is thus imparted to the still more elevated strata of the atmosphere, the result being to render their mean density somewhat lower than over either. The temperatures

* The term *abnormal* is used in the sense given to it by M. Dove.

† See preceding page. ‡ See *ante*, page 591.

of radiation quoted at page 594 show that while more solar heat reaches the ground of the plateau in May (and it is the same in April), the nocturnal radiation is scarcely greater than on the plains of the delta, if the extreme temperatures of radiation may be accepted as a criterion of the *quantities* of heat received and emitted. And since the column of the atmosphere over the former is 2000 feet shorter than over the latter, it may be inferred that more heat is retained in the atmosphere over the plateau than in that above the same level over the plains. In any case, these low plateaux appear to have the effect of locally reducing the pressure of a humid atmosphere, since the Table of S. L. pressures and the charts for the months May to September show that a trough of pressure exists to the south of the Gangetic plain throughout the rains.

Throughout the rains the seat of lowest pressure appears, both from the direct evidence of the Agra and Roorkee barometers and the indirect evidence of the winds, to be in the Punjab; and this is also the seat of the highest temperature at that season. But it is also to be noticed that the commencement of the rains, between May and July, is marked by a fall of temperature of not less than 15° at Nagpore and nearly 12° at Jubbulpore, while the atmospheric pressure also falls by ·087 inch at the former and ·128 inch at the latter. Here the explanation is probably to be found in the diffusion of heat from the lower to the higher strata by the influx of water-vapour, the tension of which increases from 0·539 inch to 0·776 inch at Nagpore, and from 0·439 to 0·654 at Jubbulpore. It has been shown in the Table at page 596, that after March the tension of water-vapour at all elevated stations up to 7000 feet rises more rapidly than near the ground surface, and also in the Table at page 589 that the temperature becomes relatively higher—owing, as has also been shown with much probability, to the diffusion and condensation of water-vapour. I shall presently prove, I think irrefragably, that high temperature is the most influential cause of the fall of pressure; and if sufficient time be allowed for the communication of heat from lower to higher levels, a low pressure may be produced without any very copious accession of vapour, since the Punjab, which is the region of least rainfall and highest temperature, is that towards which the winds tend throughout the rains.

There is yet another cause which may affect the pressure to an appreciable extent, and to which I have not yet referred. It is one not usually considered by meteorologists, although Mr. Espy has resorted to it in explanation of the diurnal tides, and more recently Mr. Laughton has assigned to it more importance than I should be disposed to concede to it in atmospheric physics*. This is the dynamic pressure of the atmosphere in motion. It follows from elementary mechanical laws that wherever the motion of a current of air is checked or diverted, pressure must be produced; and the only question in the present connexion is whether, and to what extent, it will be appreciable. A case in which, if in any, it would, I think, be sensible is that in which the lower winds diverge from a circumscribed area or ridge of high pressure, the supply being necessarily drawn from above, and maintained by currents flowing in, in the upper

* Phys. Geog. p. 329.

atmosphere, from opposite quarters, and subject at the same time to cooling, by radiation, to an extent greater than is compensated by the heat set free from their arrested motion. The dynamic pressure which would arise from the contraction and sinking of the cooled air may, I think, be sensible to the barometer. In no other way am I able to account for the ridge of high pressure in the cold-weather months between Benares and Cuttack, where the temperature of the air, though lower than on either side, is several degrees higher than over the Gangetic plain, while the pressure is also higher. I shall presently collate the evidence, already given in these pages, of the probable existence of those upper currents which I suppose to produce this phenomenon.

I have now to consider a very important part of the subject of pressure, viz. its changes at Shillong and the Himalayan stations Darjeeling and Simla, and the evidence they afford of the height to which the density of the atmosphere is affected by the heat of the spring and summer months and the accession of vapour. Goalpara, situated about midway between Shillong and Darjeeling, affords a convenient standard of reference for these two stations; and Roorkee, at the foot of the hills, 100 miles from Simla, may be compared with that station.

It is apparent, on a glance, that the annual range of the mean monthly pressures at all the hill-stations bears a much smaller ratio to that of stations in the plains, than do the total pressures at any season of the year; in other words, that the change of mean pressure over the plains between January and July, and *vice versâ*, is chiefly due to a change in the density of the atmosphere below the elevations of these stations. The amount of this change in each month is shown in the following Table. In the cases of Darjeeling and Shillong the density is computed in two different ways:—first, (d_1) from the difference of pressures at the higher and lower stations; and secondly, (d_2) from the mean pressure, temperature, and vapour-elasticity of the intervening atmospheric column*. In both columns, dry air at 32° FAHRENHEIT and 29·921 inches pressure is taken as the standard $=1$.

* By the formulæ

$$d_1=\frac{(b_1-b_2)}{12(h_2-h_1)}\,10516,$$

$$d_2=\frac{\frac{b_1+b_2}{2}-\frac{3(e_1+e_2)}{16}}{29{\cdot}921}\cdot\frac{1}{1+{\cdot}002036\left(\frac{(t_1+t_2)}{2}-32\right)},$$

where b_1, b_2, t_1, t_2, e_1, e_2 are respectively the reduced barometric readings, the temperatures, and vapour-tensions at the lower and higher stations, and h_1 and h_2 their elevations in feet.

It may be objected to the above method of computing the mean density of a column of the atmosphere from the observations of pressure, temperature, and vapour-tension of two stations only, that it makes assumptions as to the distribution (more especially) of vapour and temperature, the validity of which is extremely doubtful, and which, indeed, can be only approximately true, when minor and temporary irregularities have been eliminated by taking for the values of t_1, t_2, e_1, and e_2 the averages of a very large number of observations. It may further be objected that the composition and condition of a very oblique column of the atmosphere (one of 100 miles in horizontal length) cannot legitimately be assumed to represent those of a vertical column, and that

TABLE showing the mean density of the air-columns between three hill-stations and two reference-stations on the plains in each month of the year.

1. DARJEELING AND GOALPARA.

	Goalpara, b_1.	Darjeeling, b_2.	Difference, b_1-b_2.	Mean density of air, d_1.	$\frac{b_1+b_2}{2}$.	$\frac{t_1+t_2}{2}$.	$\frac{e_1+e_2}{2}$.	Mean density of air, d_2.
January	29·603	23·344	6·259	0·837	26·473	53·6	0·325	0·844
February	·522	·322	·200	·829	·422	55·9	·336	·838
March	·442	·311	·131	·820	·376	61·7	·369	·827
April	·380	·326	·054	·809	·353	66·6	·485	·817
May	·286	·273	·013	·804	·279	69·2	·608	·809
June	·206	·228	5·978	·799	·217	71	·713	·803
July	·195	·224	·971	·798	·209	72·4	·732	·799
August	·247	·261	·986	·800	·254	72·6	·749	·801
September	·328	·322	6·006	·803	·325	71	·700	·807
October	·426	·391	·035	·807	·408	68·1	·582	·815
November	·567	·424	·143	·821	·495	60·6	·615	·829
December	·621	·409	·212	·830	·515	54·4	·336	·843
Year	29·402	23·320	6·082	0·813	26·360	64·6	·546	·819
Range	0·426	0·202	0·288	0·039	0·306	19·0	·424	·045

the conclusions subsequently drawn from the physical analysis of such a column, on this assumption, are therefore invalid. Some weight must doubtless be given to both these objections; but I believe nevertheless that the conclusions I have drawn are substantially trustworthy. Leaving for a moment the question of the obliquity of the column, a question which affects both computations of the density, I would point out that the errors of the values of d_2, in so far as they are due to an irregular distribution of temperature and vapour, are shown by their difference from those of d_1, since these latter are computed from independent data; and although this difference is considerable in some months, it is least so in January and July (or June at Shillong), when the exchange of air between the upper and lower strata of the atmosphere is most active, and when therefore both temperature and vapour must be least irregularly distributed. The question remains how far can d_1 be accepted as representing the mean density of a vertical column. (d_1) gives the mean density of a column of air (h_2-h_1) feet in length, which is equilibrated by a column of mercury at 32° FAHR., represented by (b_1-b_2). The error, if any, must lie in the assumption here made, that b_2, the pressure at a somewhat distant hill-station, really represents the pressure at height (h_2-h_1) vertically over the station when the pressure is b_1, and such error would arise were there a persistent difference of pressures between the two places in the same horizontal plane. Now any such difference would tend to produce wind-currents from one place towards the other, and this tendency should be shown by the wind-registers. *There are* such currents in July, from Goalpara towards Darjeeling, and from Roorkee towards Simla, indicating therefore the existence of a somewhat greater pressure over the former stations than at the latter in the same horizontal plane. But from the annual range of pressure at the hill and plains' stations given in the Table above, it appears that this difference of pressure at the height of 7000 feet is probably not more than half as great as at the ground surface, where the barometric gradient does not exceed 0·1 inch in 800 miles, or 0·012 in 100 miles (see page 604). Take the probable error of b_2, then, as ·006. Then increasing the value of b_2 by that amount, the value of d_1 will be diminished by 0·001, and that of d_2 increased by half as much. The error of the results in the month of July cannot therefore much exceed this. In January the mean movement of the air is so low that the errors of the assumption similarly estimated must be quite insignificant.

I should expect that the differences of d_2 and d_1 are principally due to the real mean temperature of the air-column in most months being somewhat higher than the assumed temperature $\frac{t_1+t_2}{2}$. A certain amount of error must arise also from the fact that the adopted values of vapour-tension are probably in all cases lower than the real tensions; but the effect of this cannot be very great.

2. SHILLONG AND GOALPARA.

	Goalpara, b_1.	Shillong, b_2.	Difference, b_1-b_2.	Mean density of air, d_1.	$\frac{b_1+b_2}{2}$.	$\frac{t_1+t_2}{2}$.	$\frac{e_1+e_2}{2}$.	Mean density of air, d_2.
January	29·603	25·262	4·341	0·863	27·432	57·8	0·352	0·863
February	·522	·238	·284	·852	·380	60·6	·354	·854
March	·442	·211	·231	·842	·326	67·2	·404	·846
April	·380	·195	·185	·832	·287	70·8	·513	·839
May	·286	·098	·188	·833	·192	73·3	·662	·830
June	·206	·060	·146	·824	·133	74·6	·747	·823
July	·195	·058	·137	·823	·126	75·2	·774	·817
August	·247	·100	·147	·825	·173	75·2	·789	·825
September	·328	·172	·156	·826	·250	73·6	·744	·831
October	·426	·247	·173	·830	·333	70·9	·642	·838
November	·567	·320	·247	·844	·443	63·5	·467	·856
December	·621	·324	·297	·854	·472	57·4	·365	·868
Year	29·402	25·190	4·211	0·837	27·296	68·3	·568	·841
Range	0·426	0·266	0·204	0·040	0·346	17·8	·437	·051

3. SIMLA AND ROORKEE.

	Roorkee, b_1.	Simla, b_2.	Difference, b_1-b_2.	Mean density of air, d_1.		Roorkee, b_1.	Simla, b_2.	Difference, b_1-b_2.	Mean density of air, d_1.
January	29·126	23·225	5·900	0·835	September	·792	·194	·598	·793
February	·048	·198	·850	·828	October	·960	·277	·683	·804
March	28·985	·197	·788	·819	November	29·104	·293	·811	·823
April	·878	·207	·671	·803	December	·154	·254	·900	·835
May	·747	·142	·605	·793					
June	·632	·061	·571	·789	Year	28·895	23·185	5·712	0·808
July	·638	·069	·569	·788	Range	0·522	0·232	0·331	0·047
August	·695	·101	·594	·792					

From these Tables we learn a fact of high importance, viz. that of the whole annual oscillation of the atmospheric pressure over the Gangetic plain, which may be taken as the measure of the forces that produce the alternating monsoons of India (up to the equator, where the pressure is nearly invariable), nearly one half is due to an alteration of the density of that stratum of the atmosphere which lies between 400 and 5000 feet (Goalpara and Shillong), more than two thirds to that of the stratum between 400 and 7000 feet (Goalpara and Darjeeling), and nearly two thirds to that between 800 and 7000 feet in the North-western Provinces (Roorkee and Simla). If for the actual annual barometric range at Goalpara and Roorkee we substitute their equivalents at sea-level, as given in Table VIII., viz. 0·445 inch at Goalpara and 0·595 at Roorkee, and increase the range of the values of b_1-b_2 by a corresponding amount, then the oscillation below Shillong becomes more than one half, and that below Darjeeling and Simla nearly seven tenths of the whole.

Further, it is to be noticed that the change of density in the North-western Provinces considerably exceeds that in Bengal, and affects the atmosphere to a greater height,

since the annual oscillation of pressure is absolutely greater at Simla at an elevation of 7071 feet than at Darjeeling at 6941 feet. Calling the annual mean density of the atmospheric column below the hill-station in each case =1, the range between January and July is 0·048 in Bengal, and 0·058 in the North-western Provinces.

If the method of computation which gives the values d_s be accepted as legitimate, by varying successively the values of each of the terms (b_1+b_2), (t_1+t_2), and (e_1+e_2) in the formula for d_s while the others remain constant, we may ascertain another fact of high importance, viz. the degree in which the density of this column is affected respectively by the variation of the top pressure, the rise of temperature, and the introduction of additional vapour in the place of dry air of the same pressure. Between January and July the mean density of the air below Darjeeling falls from 0·837 to 0·798. This total difference is made up in the manner following*:—

(1) By decrease of the top pressure from 23·344 inches to 23·224 inches, the mean density of 0·837 would be reduced to 0·832.

(2) By increase of the mean temperature from 53°·6 to 72°·4, the mean density of 0·837 would be reduced to 0·807.

(3) By the infusion of water-vapour, which, in conjunction with the rise of temperature, raises the mean vapour-tension from 0·325 inch to 0·732 inch, the mean density of 0·837 would be reduced to 0·832, supposing the temperature to remain constant.

And summing up these differences, we find—

Density of air in January	0·837
Reduction due to decreased pressure above	0·005
Reduction due to increased temperature of column . .	0·030
Reduction due to introduction of water-vapour . . .	0·005
Total reduction	0·040
Density of air in July	0·797

which accords almost exactly with that obtained from the difference of observed pressures, viz. 0·798.

* These values are obtained by the following formulæ, by which, from the figures in the Table on pages 608, 609, the change of density arising from the variation of any element between two given months may be found. Calling B_1 and B_2 the initial pressures at the lower and higher stations, b_2 the final pressure at the latter T and t, E and e the initial and final *mean* temperatures and vapour-tensions respectively, and D, d_b, d_t, and d_e the inital observed and final computed densities respectively, we have

$$d_b = D\,\frac{\left(b_2\left(1+\frac{B_1}{B_2}\right)\right)}{B_1+B_2},$$

$$d_t = D\,\frac{1+\cdot 002036(T-32)}{1+\cdot 002036(t-32)},$$

$$d_e = D\,\frac{(B_1+B_2)-\frac{3}{8}e}{(B_1+B_2)-\frac{3}{8}E}\cdot\frac{1+\cdot 002036(T-32)}{1+\cdot 002036(t-32)}.$$

Thus, then, it would appear that the increased temperature of the column itself reduces its density to an extent six times as great as is effected either by the decrease of pressure above or by the large increase of its vapour constituent; and if we may further assume that the second of these elements results from the variation of the other two in the like proportion, and that the condition of the oblique column of air between Darjeeling and Goalpara fairly represents, for our present purpose, the general state of the atmosphere over Lower Bengal, we must conclude that of *the reduction of the atmospheric pressure between January and July on the plains of Lower Bengal, six sevenths are due to the increased temperature of the atmosphere, and only one seventh to the displacement of dry air by aqueous vapour.*

I have not at present the data for a similar analysis of the density of the atmospheric column between Simla and Roorkee, or indeed any pair of stations in the drier climate of the North-western Provinces; but it can hardly be doubted that such an analysis would show results more or less similar to the above; and it may be expected that the lower mean pressure of the atmosphere from May to September would be found to depend on its higher mean temperature, up to a height of about 9000 or 10,000 feet. That the mean temperature of the column below 7000 feet is actually higher in the neighbourhood of the hills of the North-western Provinces than that of the similar column below the Sikkim Himalaya, is shown by the following comparison of mean temperatures of the atmosphere below Chuckrata and Darjeeling, situated at nearly equal elevations above sea-level.

TABLE of the mean temperature of the air-columns below Darjeeling (6941 feet) and Chuckrata (6884 feet).

	Darjeeling.	Chuckrata.		Darjeeling.	Chuckrata.
January	53·6	50·1	July	72·4	75·1
February	55·9	53·4	August	72·6	74·6
March	61·7	59·9	September	71	72·6
April	66·6	69·9	October	68·1	67·6
May	69·2	78·6	November	60·6	58·6
June	71	79·2	December	54·4	50·8
Year	64·6	65·9	Range.....	19	29·1

Changes of temperature are then the principal cause of the variations in the weight of the atmosphere; but the part played by vapour is not the less important, though its action is chiefly indirect. This action is evidently to equalize the temperature of the air-column, to carry heat from the lower and more highly heated strata to those at greater elevations, and also, as Dr. TYNDALL has shown, to arrest and absorb both solar and terrestrial radiated heat in its passage through the atmosphere. Indirectly, therefore, water-vapour greatly influences the pressure, though the change of density that

arises from its displacement of the heavier constituents of the atmosphere is relatively small, and in some cases unimportant*.

With respect to the interior of India, the course of events appears to be as follows:—From March onwards the air immediately over the elevated plains is gradually raised to a high temperature, but at first only in the lowest stratum. By degrees, convection-currents from the sea, of no great vertical thickness, are drawn into this lower stratum, introducing vapour, which carries the heat by diffusion and condensation to a greater and greater height. Up to the end of May or the beginning of June this diffusion of heat, while reducing the pressure, does not lower the temperature of the surface; and it is only when in June a strong steady current of nearly saturated air is drawn from the equatorial seas, that the precipitated vapour, partly as cloud and partly as rain re-evaporating, absorbs the excess of solar heat and reduces the temperature of the lower atmosphere to the extent shown in the temperature Table.

There is yet another point of importance to be noticed in the Tables on pages 608, 609. While the stations at the lower levels, Goalpara and Roorkee, as well as all others on the plains, show but one annual maximum and one minimum of pressure, the former in December, the latter in June or July, the two hill-stations (Darjeeling and Simla) have two maxima and two minima, like places on the Atlantic coast of Europe. The epochs of the maxima and minima are, however, very different in the two cases. The absolute minimum of the year at these hill-stations coincides with that on the plains, and is doubtless due to the same cause; but the absolute maximum falls in November at the former, and is followed by a fall, at first rapid, and gradually decreasing till March. A small rise then takes place, which brings the mean pressure of April above that of February; it is, however, only temporary, and in May and June the pressure falls rapidly to its minimum.

* The facts thus indicated relative to the effects of temperature and vapour in affecting pressure seem to explain the apparent anomalies that have led some authors, especially Mr. Laughton (Phys. Geog. pp. 120, 123), to question the soundness of Hadley's theory of the trade-winds. These are, that winds do not blow centripetally toward the Sahara, the Arabian Desert, the interior of Australia, &c., all of them dry regions with a day temperature very much above that of the neighbouring seas. In the first place Mr. Laughton has, I think, insufficiently considered the fact that a high day temperature in these dry regions generally alternates with a low night temperature, the nocturnal radiation being intensified by the same conditions which increase the incident solar heat; so that the mean temperature of the 24 hours, one element of importance in determining the general system of the winds in such regions, is frequently below that of places with a much lower day temperature. *Ex. gr.*, Lahore in April has a mean maximum diurnal temperature of 98°·4, Calcutta one of 93°·5; but the mean temperature of the 24 hours is only 79°·1 at Lahore, while at Calcutta it is 84°·5. Further, the facts discussed in the text and tabulated at pages 589, 608, 609, show that in a dry atmosphere the high temperature prevails only in its lowest stratum, and that the mean temperature of a column of comparatively dry air, say 7000 feet in height, may have an average temperature 16° below that of the surface, while another equal column of moist air averages only 9° or 10° less than near the ground. Surface-temperature alone, especially that of the daytime, is a very unsafe criterion of the average temperature of the atmosphere above the place of observation; but in the equatorial calm belt, to which region Mr. Laughton applies conclusions drawn from Arabia, Australia, &c., the atmosphere is highly charged with vapour, and the decrease of temperature with elevation therefore probably slow.

The fall of pressure in December and January is, I think, evidently due to the rapid condensation of the lower stratum of the atmosphere by the radiation of its heat, and, while this cooled air flows away to the south as the source of the north-east monsoon, by the consequent setting in of a compensating anti-monsoon current of higher humidity and comparatively equable temperature at (or, in the case of Darjeeling, chiefly *above*) the level of the hill-stations. From January to March, as well as from April to October, the pressure at Darjeeling and Simla must, on this assumption, be lower than at equal elevations to the southward in the course of this upper current.

The temporary rise in April is, I think, to be attributed to the expansion of the lower atmosphere, by which, for a time, a larger proportion of the atmosphere is lifted above the level of 7000 and 8000 feet. This requires further investigation.

Certain effects of the Winds.—I have now discussed most of the more important facts relating to temperature, vapour-diffusion, and atmospheric pressure, to be gathered from the registers of the past few years, in so far as they bear on the causes of the winds. It remains to notice certain effects of the winds, especially on temperature and rainfall; and I shall then briefly sum up the results of the whole discussion, and add a few remarks, in an Appendix, on the storms of the Bay of Bengal, to the explanation of which a knowledge of the normal wind-system is indispensable.

In respect of temperature, it is obvious that, except in the case of dynamic heating and cooling, a wind cannot *per se* raise the temperature of a place above, nor depress it below, that of the region from which it immediately comes. Any change of temperature that it may undergo along its course must therefore be due to local causes, such as evaporation, radiation, or the absorption of solar or terrestrial heat *in transitu*. The effect of a wind is to tend to equalize the temperature of places along its path. Before applying this postulate to the meteorology of the region in question, it is, then, necessary to consider those changes of temperature that may arise from dynamic causes; and of these one class of cases only need be noticed, viz. that of a current which is cooled by a rapid ascent to a higher level, or heated by descent to a lower level. The latter, if recognizable, will chiefly affect the temperature of stations on the plains, the former that of hill-stations.

Both these actions doubtless take place on a great scale over the plains of Northern India, since we have seen that at one season this region is the *terminus a quo*, at another the *terminus ad quem* air-currents are set in motion, and during the continuance of these currents there must be a constant passage of air from the higher to the lower strata, or *vice versâ*. If, then, dynamic heating be appreciable, it should be detected in a relatively higher temperature of the air on the plains of Upper India in the cold-weather months; and for evidence of cooling by the ascent of the air, we should look for a relatively lower temperature of the hill-stations in the months of the rains—the effect in either case being shown by a difference of temperature between hill- and plain-stations greater than at those times when the interchange between the different strata is at a minimum. But the evidence tabulated at page 589 shows that it is precisely at these former seasons

that the temperature-difference is *least*, and that it is *greatest* at the change of the monsoons. We have also seen that where the anti-monsoon current descends in greatest volume, viz. the plains of Upper India, is the coldest part of India at that time of year; and the temperature-difference between Chuckrata and Roorkee is then not only at its annual minimum, but it is also less than between Darjeeling and Goalpara, where the descent of the anti-monsoon is but little felt. The conclusion is obvious. Any change of temperature that may arise from dynamic causes is completely neutralized by other causes operating in a reverse direction, and the residual excess of the opposite effects are at their maximum when these dynamic causes are most active. This is in entire accordance with the conclusions of *à priori* reasoning, and consistent also with the view that differences of temperature are the principal cause of wind.

Dr. MÜHRY, in a recent work on the winds (Untersuchungen über die Theorie und allgemeine Geographische System der Winde, page 99), attributes the heat and dryness of the westerly winds of the Gangetic plains between March and June in a great measure to dynamic heating; but his view of the origin of these winds is certainly erroneous. He adduces them as an example, on a great scale, of an air-cascade flowing from Tibet over the ridge of the Himalaya, and a retroversion of the current in the hollow of the fall (*Windschatten*). I transcribe the passage in a footnote*. Of the existence of a north-east current across the crest of the Himalaya, I can find no evidence in the account of any traveller in high Himalayan regions. From the accounts of Dr. HOOKER, General STRACHEY, M. SCHLAGINTWEIT, Mr. SHAW, and others, it appears that amid all the local irregularities due to the varying direction of the valleys, the prevalent winds are southerly up to the principal range, and that day winds of great force, attributed by General STRACHEY to the heating of the elevated plains, blow in the same direction through the passes. Only in Turkistan does the prevalent wind appear to be northerly up to the Mustakh or Kàrakoràm range. The hot westerly winds of the Gangetic plain and an extensive region to the south, in April and May, are, as Dr. HOOKER long ago pointed out, essentially day winds, and due to the heating of the soil†. Were

* "In Asien gibt zunächst der Himalaya, Gelegenheit zu Entstehung eines grossen Beispiels unserer Erscheinung auf dem Continente; so scheint wenigstens ein gewisser endemischer Wind im nördlichen Ostindien seine Erklärung zu finden. Längs der südlichen Seite dieses mächtigsten Gebirges, welches ja ebenfalls nach Nordwest hin streicht, etwa von 27° bis 35° N., ist wohl bekannt, dass in der heissesten Zeit, im trockenen Frühjahr, von März bis Mai, zur Zeit, wenn in übrigen Ostindien, und man kann sagen in übrigen Süd-Asien, der Nordostmonsun herrscht, das ist der Polarstrom, noch ungestört in seiner untersten Schicht durch den sommerlichen Seewind, den S.W. Monsun, dass dann das ganze Gangesthal hinunter ein N.W. anhaltend weht, warm und von excessiver Trockenheit. Es darf uns kaum Zweifelhaft erscheinen, dass dieser Wind gleichfalls einen Windfall und Retroversion des N.O. Passats darstellt." It is doubtless owing to the dearth of information hitherto accessible on the subject of the normal winds of India, that Dr. MÜHRY has been misled into the belief that N.E. winds are generally prevalent in India between March and May. The idea that a polar stream flows from Central Asia across the Himalaya is an error of old standing.

† These winds form a very interesting subject for investigation, but I cannot attempt it at present. They must be considered in connexion with the diurnal variation of temperature and pressure. In connexion with the latter, I may mention that they generally set in at the time of the morning maximum, and I expect their explanation is to be found in a study of the barometric tides.

they what Dr. Mühry supposes, they would probably be felt by night as well as by day; but we have seen that in the hot weather in Upper India the nights are generally calm, and the temperature falls lower than in Bengal.

The effect of the winds in tending to equalize the temperature along their path is very distinctly exhibited in the charts for the cold-weather months, more especially that of November. The cool current from the North-western Provinces flows most steadily down the Gangetic valley, across Western Bengal and the tract intervening between this province and Nagpore; and it is here that the isothermals make their great southerly bend. Their northerly curvature opposite to the Gulf of Cambay and in Eastern Bengal, shown more or less distinctly in all the cold-weather months, I can attribute only to the influence of the anti-monsoon currents flowing at lower levels in those regions. The evidence on this head afforded by the tabulated observations of the surface-winds is, of course, not very distinct, but is not altogether wanting, and, as far as it goes, lends support to this view. I have noticed, in Part I. (pages 581 & 585) of this paper, that southerly winds blow on the Arakan coast a full month after they have ceased on the opposite coast of India, and at Dacca calms are very common in the cold-weather months, which is not the case normally at Calcutta*. Moreover the diurnal movement of the wind in the four months of the cold weather averages 46·6 miles only at Dacca, while at Calcutta it is 98·1 miles per day. At Ajmere, again, southerly winds are in excess both in October and February, and they are very common both in December and January; and here, too, as in the North-western Provinces (but not in the Central Provinces), calms are very common throughout the cold weather.

The isobars afford evidence to a similar effect. In all the cold-weather months there is a lower mean pressure over Eastern Bengal, and, as far as evidence goes, apparently opposite the Gulf of Cambay (certainly on the Bombay coast), than in the region between Nagpore and the Gangetic delta. I conclude, then, that while the anti-monsoon currents exist probably over the whole of Northern India, they flow in greater volume and at lower levels in Eastern Bengal and opposite the Gulf of Cambay than elsewhere.

In the hot weather the course of the isothermals is evidently determined chiefly by the form of the land; but in July the cooling influence of the monsoon currents, setting in from both coasts, is very distinctly shown in the chart; and up to September the hottest region is that most remote from the coast, as measured along the course of the rain-bearing winds.

With respect to the influence of the winds on rainfall, little is to be added to what has already been said. The winter rains are, I conclude, dependent upon the descent of the anti-monsoon current, and to the cooling which it undergoes by radiation either of the air directly, or of the land-surface with which it comes in contact. In December and January the isotherm of 63° or 64° coincides approximately with the limit of the

* At both of these stations calms have been recorded regularly throughout the period represented in the Tables.

region in which the rains are pretty regular, but they are felt occasionally up to the isotherm of 70°, and even beyond.

When the sea-winds set in on the coasts of Bengal and Orissa in the early spring months—winds which I have said are at first restricted to the lowest stratum of the atmosphere—they bring the vapour, which is precipitated as the spring rains, chiefly in the form of storms. It is at the beginning of this season that hail-storms are most frequent. In Lower Bengal two or three such storms generally occur in the month of March, and may be traced to the meeting of the land- and sea-winds, and probably to the dynamic cooling of ascending currents*. The heavier rains of Eastern Bengal are determined probably by the character of the country—hills for the most part covered with forest, and with marshes extending up to their foot. The evaporation is such as to keep the ground and superincumbent air lower in temperature and more humid than anywhere in Western Bengal.

In the rains as well as in the hot weather the winds are checked on reaching the coast-line, as is evident from the great difference in the mean diurnal movement of the air at Saugor Island and Calcutta; and there must accordingly be an ascending stream or eddy, which causes a large precipitation of the vapour within the first few miles bordering the coast. Of this the rainfall Table and chart bear evidence.

The strip of elevated country south of the Ganges, in which (it would appear from the few registers that have been kept in this tract) the rainfall averages more than 50 inches, coincides with the trough of low pressure described at page 604 and indicated on the charts for the summer months, especially May and August. It coincides also approximately with the line separating the westerly monsoon current of Central India from the easterly monsoon of the Ganges valley.

The Central Provinces to the south of the Satpooras, and even for some distance to the north of that chain, receive their rains wholly from the west coast. Much of this country is very hilly, but it is not all of this character; and the numerous feeders of the Godavery drain a system of plains, which have a mean elevation of less than 1000 feet above the sea, and fall away to the south-east. Yet the rainfall is here heavier than on the Deccan plateau nearer the Ghâts, which must of course be crossed by the rain-bearing winds, excepting such portions as may pass up the valleys of the Taptee and Nerbudda. This appears anomalous; but I think the explanation may be found partly in the facts to which General Strachey drew attention in the paper I have before referred to (page 596). He there showed that the distribution of vapour vertically in the atmosphere, on the hypothesis of independent atmospheres of dry air and water-vapour, is inconsistent with the known ratio of temperature decrement with elevation, inasmuch as such a distribution would require the existence at comparatively moderate heights of

* The storms of this time of year are well known as nor'-westers. They are generally accompanied by violent electrical discharges, and the barometric column invariably rises rapidly on their approach, sometimes to the extent of 0·1 inch; so that, on looking back through a series of diurnal barometric curves, it is easy to detect these days on which storms have occurred. They occur most frequently about 5 or 6 in the afternoon.

a vapour-tension above that of saturation. But vapour must always *tend* to assume such a distribution; and thus a current which has already been robbed of a large part of its vapour by passing over the Sahyadree ridge, even though rendered hygrometrically dry on redescending to lower levels, may gradually become resaturated in its higher strata by the upward diffusion of its residual moisture. After the first falls, moreover, if the current is uninterrupted, a portion of the precipitated rain, which may be roughly estimated between one fourth and one third, will be again taken up by evaporation and carried further inland, again to be precipitated. In this way probably also is to be explained the gradual advance of the rains up the Ganges valley to the Punjab, a progress which occupies three or four weeks from the setting in of the heavy rainfall in Bengal.

The rainless, or nearly rainless, climate of Sind and the plains of Bikaneer owes its character doubtless to that of the arid countries to the west and north-west. In the absence of any wind-observations in this region, this cannot be verified; but a very probable explanation is furnished by the following considerations. This region, including the Punjab, appears to be the seat of the lowest pressure in the rainy months; and it has been shown, from the wind-registers of the last-named province*, that during this period of the year there is a kind of cyclonic circulation of the winds around it. Accordingly (and this the wind- and rain-registers show) on its northern and eastern borders it receives a moderate rainfall from winds that reach it from the east and south-east, having travelled up the Ganges valley or across the Satpooras and the Malwa and Bundelkund plateau; but on the south and west, if the rule there holds good, they must come from Baloochistan, Arabia, and Persia, all exceedingly dry countries. As far as I can speak from my recollection of a register kept at Khelat some years since, such rain as falls on the hills to the west of the Indus comes from the eastward.

GENERAL SUMMARY.

The north-east monsoon of Indian seas is produced by the cooling and condensation of a comparatively calm atmosphere over the land-surface of India. It has its origin in the plains of the Punjab, Upper and Central India, and Assam; probably also on the southern slopes of the Himalaya, where the air, cooled by radiation and contact with the surface of the hills, flows down the large valleys to mingle with the similarly cooled air of the plains. These currents are fed by an upper current, which I have termed the *anti-monsoon*. This is felt as a southerly wind on and over the south face of the Himalaya, and descends on the plains of Upper and Central India, bringing the winter rains. There would appear to be two principal branches of this current, the course in each case being indicated by a higher temperature and lower pressure at the surface of the ground, as well as by the longer duration of southerly winds. One of these flows at a lower level opposite to the Gulf of Cambay, over a part of Rajpootana, and a portion at

* *Ante*, Part I., pages 565 *seqq.*

higher level flows eastward above the Ghâts, and descends on the hilly country east and north of Nagpore. The other branch, coming from the Bay of Bengal, holds its course at a lower level over Eastern Bengal, and then the major portion, curving to the west and north-west, blows on the south face of the Himalaya, and, flowing off sideways, probably meets the Arabian-Sea current along a line indicated by a ridge of high mean pressure at the ground surface. This line passes through Roorkee, Lucknow, Benares, and Cuttack, and coincides with the axis of the stream of cold air from Upper India. The east or north-east monsoon of the Wynegunga plain and the north-west monsoon of Lower Bengal radiate out from this ridge of pressure and flow away as very gentle currents, in the one case towards the Arabian Sea, in the other to the Bay of Bengal. A portion of the Bengal anti-monsoon current probably flows north-eastwards to Assam, where it brings rain, and feeds the Assam and East Himalayan branch of the north-east monsoon; but further evidence is required to establish the existence of this current.

This double system of upper and lower currents will be rendered more comprehensible by the annexed woodcut (fig. 4), on which the upper currents are shown by dotted, the lower by continuous lines.

Fig. 4. Lower and Upper winds N.E. monsoon.

The evidence on which the above description is based may be briefly summed up as follows:—First, the course of the winds in the lower atmosphere as shown by the wind-registers. The cold dry north-west current, which begins in Upper India, increases in steadiness and strength, probably therefore in volume, as it moves towards Western Bengal. It increases also as the season advances. This is shown by the following velocities registered at stations on or near the central course of the current, and extracted from the wind Tables given at the end of this paper: Calcutta, it should be remarked, lies at a lower level than Hazareebagh; and it is probable that after passing the plateau of Western Bengal, the current does not descend completely to the low level of the delta, but more frequently holds on its course at the elevation it has already attained.

TABLE showing the increasing mean movement of the North-west Current during the Cold Weather.

	November.	December.	January.	February.	
Roorkee	19·2	25·4	40·2	47	Miles per day.
Benares	55	34·2	55·6	73·3	"
Patna	51·7	34·4	64·6	72·8	"
Hazareebagh	91·1	92·7	98	131·4	"
Calcutta	82·1	91·2	104·5	114·8	"

Secondly, on the Himalaya, especially the north-west portion, southerly winds prevail throughout the cold weather; and at stations 7000 or 8000 feet above the sea the atmospheric pressure falls in December and January, while on the plains it rises in December, and does not begin to fall till the end of the latter month. Thirdly, in Upper India and the Punjab, easterly winds, bringing rain, are much more common than in Bengal, and the atmosphere is characteristically calm. Winter rains also occur in Central India, where the lower current is from east and north-east. Fourthly, there is a ridge of high mean pressure running from Roorkee through Benares to Cuttack, which I can account for only on the supposition that it coincides with a trough of low pressure in the upper atmosphere where the currents from north-east and south-west meet and descend. Fifthly, there is in Eastern Bengal a region of low pressure where the northerly wind is unsteady and much interrupted by calms. It is in the prolongation of the line of the Arakan coast where the southerly monsoon blows a month longer than on the Indian coast. This I suppose to indicate the course of a main stream of the anti-monsoon, which is here lower than over Western Bengal, but does not, at least in general, descend to the land-level*. Sixthly, the isothermal lines bend northward (indicating a relatively high temperature) opposite the Gulf of Cambay and in Eastern Bengal, and southwards between Western Bengal and Central India. The former I suppose to indicate the course of the two principal branches of the anti-monsoon flowing northwards, the latter the place of their meeting, descent, and return as the beginning of the northerly monsoon.

The south-west monsoon is produced by the heating of the land-surface of the peninsula and the superincumbent air to a temperature much above that of the sea to the southward. Six weeks before the vernal equinox, sea-winds begin to set in in the lowest stratum of the atmosphere, on the maritime belt of Lower Bengal and Orissa, and gradually advance further and further inland. At the same time over the whole of Northern India the winds continue to blow from the westward, rising gradually in temperature, and at length blowing only or chiefly in the daytime as the hot winds of April and May. This state of things depends probably on the high temperature being restricted to the stratum of air immediately over the ground. But with the advance of the sea-winds

* When rain does occur, or the sky is cloudy, at Calcutta in the cold weather there is generally an easterly, south-easterly, or southerly current above and calm below.

and the upward diffusion and condensation of their vapour, the heat also is diffused to higher levels. In May the rise of temperature at 7000 and 8000 feet proceeds as rapidly as on the plains, or even somewhat more so. By this diffusion of heat and the increasing temperature of the ground surface and the lower strata of the air under a nearly vertical sun, the pressure falls steadily, and the sea-winds are drawn from a greater distance south. At length, as seems probable, in June, the ridge of high pressure over the sea, which has steadily receded southwards since February, is obliterated, and the south-east trade, or perhaps only a portion of it, crossing the line, brings the monsoon rains to Bengal and the west coast of India*. The two principal divisions of the monsoon, advancing from opposite sides of the peninsula, appear to follow a course very similar to that of the anti-monsoon currents of the winter season, but of course more influenced by the irregularities of the land-surface. The Arabian-Sea branch blows right across India, and is felt as a westerly wind in Orissa, and even beyond to the eastward, being probably influenced by the direction of the Satpooras and their virtual continuation in the plateaux of Sirgoojah, Chutia Nagpore, and Hazareebagh; while the Bengal current is restricted to Assam and Cachar, Bengal and the Gangetic plain. The moist south winds blow up to the Himalayan snows and even beyond into Tibet (but this is probably their limit in that direction), and westward up to the further limits of the Punjab plains. The greater part must form an ascending current over the plains, and return southwards at such a height that it is doubtful if any direct evidence of it is forthcoming. I am unable to agree with Dr. Mühry's conclusions on this head; but of these I will speak presently. That the mean movement of the air decreases from the coast-line inland is shown by the following Table:—

Table showing the decreasing velocity of the South-east Current up the Gangetic valley during the South-west Monsoon.

	June.	July.	August.	September.	
Saugor Island ...	201	255	262	242	Miles per day.
Calcutta	198	157	139	132	"
Hazareebagh	185	152	132	129	"
Patna	90	81	81	84	"
Benares	95	73	70	58	"
Roorkee	73	53	38	27	"

The accompanying chart shows the average course of the wind-currents during the height of the south-west monsoon. To what height they extend there is no evidence to show, but it is probably much greater than that of the north-east monsoon currents in Northern India.

* See *ante*, Part I., page 583. Also Maury's Physical Geography, edit. 12th, page 367; and Meldrum, British Association Report, 1867, Trans. of Sect. p. 21.

Fig. 5. Lower Winds S.W. monsoon.

The conclusion is, I think, in accordance with the evidence adduced in the foregoing pages, which at least establishes the superior steadiness, velocity, and extension of the south-west monsoon current. It is, however, at variance with that of Dr. MÜHRY; and it is but due to the recognized eminence of that writer that I should specify why and wherein I differ from him. Dr. MÜHRY considers that the south-west monsoon is a phenomenon of less magnitude than the north-east monsoon, which he regards as identical in character with the north-east trade of the North Atlantic and of equally remote origin. The passage has been quoted on a preceding page, in which he speaks of it as a current proceeding from Central Asia. He regards both the north-west monsoon of Australia and the south-west monsoon of India as a deflection or retroversion of the lowest stratum of a perennial trade-wind; and infers from the perennial northerly flow of the smoke of the Merapi volcano in Java, and the prevalence of winds from between north and west at Dodabetta during the south-west monsoon, described by Colonel SYKES, that the former current is not more than 6000 feet, the latter 9000 or 10,000 feet in vertical thickness. In regard to the latter, I will observe, in the first place, that it has already been shown that the north-east monsoon of Indian seas has its origin in Northern India, and is there, at all events, a current of less depth and magnitude than Dr. MÜHRY supposes, since the winds on the North-west Himalaya at 7000 or 8000 feet are throughout southerly. There is, then, no reason to infer that the south-west monsoon is merely a deflected current of a trade-wind, the very existence of which over India is negatived by the evidence. Secondly, Dr. MÜHRY has omitted to notice certain observations of Colonel SYKES, which at least imply doubt of the north-west winds of Dodabetta being the return current of the south-west monsoon. He says that "it very frequently blows from only one or two points to the northward of west, and may belong to the monsoon of Western India, local physical circumstances having given it a slant"*. There is nothing improbable in this explanation, as the broad valley in which lies the station of Ootacamund runs up to the north-west of the peak, and in mountain-tracts

* Philosophical Transactions (1850), vol. cxi. p. 373.

the winds on peaks and ridges are always much influenced by the direction of the valleys. I am not prepared to deny the possibility of a return current being occasionally felt at 8600 feet (the height of Dodabetta); but the evidence is at least inconclusive, and the superior velocity and steadiness of the south-west monsoon render it probable that it is a current of greater depth and volume than the north-east monsoon. As to the north-west monsoon of Australia, and the evidence of the Merapi volcano, I do not think the case has much bearing on the question of the monsoons of India. It is clear that the monsoon is not a single great current, proceeding to or from Central Asia, but consists of several currents, to some extent independent of each other, flowing to or from more than one centre on the Asiatic continent; and there may be a deep north-easterly current flowing from India to the south and south-west of that region, and a very shallow one opposite the Malay peninsula. I certainly could not accept, without much stronger evidence than has yet been adduced, the complicated system of winds during the south-west monsoon, supposed by Dr. Mühry, viz. a south-west current up to about 9000 or 10,000 feet, which is a retroversion of the lowest stratum of the trade-wind; above this "the remainder of the trade-wind, moving undisturbed from the north-east; and over this the returning anti-trade moving from south-west at a still greater elevation." With much of what Dr. Mühry has written on the subject I should, however, add that I cordially agree.

The annual range of atmospheric pressure, which at Roorkee, when reduced to sea-level, amounts to 0·6 inch on the means of the months, is due principally to a change of density in the lower 7000 or 8000 feet. The proportion below Simla and Darjeeling is nearly seven tenths of the whole. Six sevenths of this in the case of the latter are probably due to the expansion of the atmosphere by increase of temperature, and only one seventh to the substitution of water-vapour for dry air. But water-vapour, though of subordinate importance in this respect, plays an important part in communicating heat to the higher strata of the atmosphere, carrying it upwards by its diffusion in the form of latent heat which is emitted by its condensation, and also by arresting and absorbing the solar and terrestrial radiation. The temperature-difference of Darjeeling and Goalpara varies nearly inversely as the humidity of the lower station.

To conclude: I have in this paper sketched out the general system of normal wind-currents of Northern India, and have shown in general terms their relations to heat and moisture, as far as they are to be gathered from existing observations. This work is almost an essential preliminary to any more detailed inquiry; but it is to be regarded as only a first rough sketch of a very important and characteristic wind-system. In many, I may say in *all*, respects it requires detailed verification. In the first place, the relations of the winds to temperature, moisture, and pressure have to be verified by the reduction of the original data in the form of wind-roses of these several elements; the progressive diffusion of heat and vapour vertically in the hot weather has to be followed out in detail, if possible, at a well-selected series of stations at heights intermediate between the present observatories and the plains; and the physical analysis which I have

attempted for a vertical column of the atmosphere over Lower Bengal has to be executed for similar atmospheric columns in the drier climate of Upper India. The peculiar variations in the absorption and radiation of heat at hill-stations and on the great plateaux which have been adverted to in the foregoing pages have to be worked out and explained; as a preliminary to which, the present methods of observation must be retested and probably modified, in order to exclude the disturbing effect of convection-currents. These convection-currents, both on the hill-sides and over the open plains, are in themselves an important object of inquiry, especially those that must be formed between the land- and sea-winds: the upper currents, the existence of which I have inferred from various indications, await verification, wherever possible, by regular observations of the drifting of the higher clouds; and the whole question of the diurnal variation of the winds and other meteorological elements, such as the barometric tides, remains yet for investigation. Furthermore, the causes of those persistent irregularities that characterize the monsoons of different years, to which I drew attention in the 39th volume of the Journal of the Asiatic Society of Bengal, demand special inquiry, both on account of their scientific and their economic importance. And, lastly, the data from other parts of India not yet treated of, or of which the treatment has been especially deficient, have to be brought together and collated with those now given. When this shall have been done, it is not improbable that the conclusions here drawn may be found in some respects to require modification. But I think that in all their leading features they are justified by the evidence adduced; and even were it otherwise, since truth may be educed from error, but never from confusion, I might still hope that this discussion may serve a useful purpose in urging forward the work yet to be done in India, and that it may contribute something of value to the general progress of the science.

APPENDIX.

Note on the Cyclones of the Bay of Bengal.—Of seventy-three storms, notices of which I have met with in old records and in Mr. PIDDINGTON'S works, or which I have myself recorded in recent years, the distribution in the several months is as follows:—

January	2	May	17	September	3
February	0	June	4	October	20
March	1	July	2	November	14
April	5	August	2	December	3

All that have occurred between November and the end of April have been restricted to the south of the Bay; and the same is to be said of the greater part of the November storms. May and the first half of June and October with the first week of November are the only periods in which cyclones can be said to be prevalent in the north of the Bay, though they occur occasionally in the intervening months, that is, during the south-west monsoon. It will be sufficient therefore for my present purpose to consider the

normal state of the winds and atmospheric pressure at these two periods of the year, and how far they help to throw light on the conditions which favour the formation of these storms.

It has been shown, in the summary of Part I. of the foregoing discussion, that southerly winds set in on the coast of Bengal in February, at first as mere local sea-breezes, and that as the sun ascends in declination they come from further out at sea and extend further inland. The change from north to south takes place on the Indian coast earlier than on that of Arakan; but in May southerly winds prevail over the whole of the Bay*, and indeed, according to Maury and Cornelissen, down to the line. But generally the winds are light except near the coast, and the south-west monsoon has not yet set in. The mean pressure at Port Blair, reduced to sea-level, is 29·788 inches; at Akyab 29·774; at Madras 29·747; at Calcutta 29·685; so that the mean barometric gradient is about 0·1 inch in 800 miles, or half as great as in July. There can then be no question of conflicting currents being the cause of cyclones in this month, unless it be assumed that (as Sir John Herschel has suggested in the case of the West-Indian storms) an upper return current strikes down with a high velocity into the southerly surface current. But there is no evidence of this in any case that I have investigated; and were this the determining cause, it would remain unexplained why cyclones are not more frequent during the south-west monsoon, when the lower current evidently at sea, and the upper therefore inferentially, have a higher velocity than in May. It would still remain to ascertain what brings the upper current down.

In October the circumstances are very different. Unsteady north-east winds do then blow on the coast of Orissa, while south-west or more frequently south-east winds prevail

* The Port Blair observations of four years give the following result in May:—

North.	North-east.	East.	South-east.	South.	South-west.	West.	North-west.
8	13	4	26	27	146	7	17

The following Table is taken from a notice of Cornelissen's work, "Route voor Stoomschepen &c.," in the Zeitschrift d. österr. met. Gesellschaft; I have not seen the original work:—

Winds in the Indian Ocean, north of 10° south latitude, in the month of May.

80° to 90° East Longitude.

	15°-20° N.	10°-15° N.	5°-10° N.	0-5° N.	0-5° S.	5-10° S.
North-east	5	7	4	3	12	10
East-south	20	20	14	16	33	72
South-west	68	59	64	49	26	8
West-north	5	9	16	25	16	5
Calms	1	5	2	7	13	5

90° to 100° East Longitude.

North-east	1	7	4	5	19	10
East-south	10	6	12	18	29	68
South-west	65	63	67	51	21	10
West-north	17	16	11	20	16	7
Calms	7	8	6	6	15	5

over the east and south of the Bay. The mean (sea-level) pressure at Port Blair is then 29·861, at Akyab 29·867, at Madras 29·855, and at Calcutta 28·859. Hence the pressure is nearly uniform around the coasts of the Bay, and, as might be expected, calms are very frequent. Here, again, although less decidedly than in May, there can hardly be any question of conflicting currents other than such as are temporary and due to local irregularities. The north-east monsoon has not yet set in; and it is to be remarked that at False Point, where north-east winds are most frequent, calms are also more frequent than in any other month in the year.

The atmosphere over the Bay is, then, calmer in October than in May; but storms are most frequent in the former month, and indeed, if we regard only those which disturb the northern part of the Bay, one and a half times as frequent in October as in May*. Consequently a calm atmosphere or variable winds would appear to be a condition favouring the formation of cyclones; and this is verified by the facts of the few storms of which I have been able to trace the antecedent conditions.

I. The Calcutta storm of the 5th October, 1864, appears to have originated on or about the 2nd of the month, to the west of the Northern Andaman. For several days previously the winds in the north of the Bay were variable, but on the whole southerly; at Madras they were from east-south-east; at Port Blair from south-west, then south-east, and finally west-south-west; and the pressure was lower at Port Blair than at any other station. It was not until the 2nd (that is, the day on which the storm formed) that a north-east wind set in down the Madras coast. From the readings of an uncompared barometer at Port Blair, it would appear that the pressure at the Andamans had been lower than in Ceylon or at Madras or Calcutta, at all events since the 26th of September, perhaps earlier, and that it fell not less than 0·12 from noon of the 30th September to noon of the 1st October.

II. The Calcutta cyclone of the 2nd November, 1867, was formed on the 27th October, in latitude 10° to the west of the Nicobar Islands†. For at least four days previously the barometric pressure in this region was lower than elsewhere in or around the Bay. "It was also lower (on the 24th October certainly, and probably on the previous day also) than on the open sea to the southward. The depression was gradually intensified up to the 27th, when it began to blow a hurricane on the northern limit of the area." It appeared that over the greater part of the Bay the pressure was nearly equable, and that the depression was local and bounded by a high barometric gradient. For many days prior to the 24th, light south-easterly winds had prevailed on the Indian coast, while in Bengal the wind was variable with a predominance of easting. To the south, between the equator and north latitude 5°, a squally, damp, west-north-west wind blew continuously, having prevailed at least from the 11th of the month. On the 27th it became west-south-west, drawing round towards the area of depression. On the 24th

* See Report on the Calcutta Cyclone of October 1865, p. 101.

† The origin of this storm was described at length in the Proc. Roy. Soc. vol. xvii. p. 472. I quote above some passages from this paper.

and 25th a north-east wind set in in Bengal and down the west of the Bay, displacing the south-east wind, which, however, continued to be felt in the immediate neighbourhood of the Nicobars; and the cyclone-vortex was formed by the indraught of these three currents to the preexisting area of barometric depression.

III. "The storm of the 13th to the 17th* May, 1869, originated near Cape Negrais on the first of these days. A south-west breeze had been blowing pretty steadily over the greater part of the Bay for some days before, the mean direction being south-south-west in the north of the Bay, south-west in the middle, and west-south-west in the south." Only "in the neighbourhood of the Andamans, Cape Negrais, and Rangoon a northerly or north-westerly wind was felt occasionally during the ten days previous; but it was unaccompanied by any fall of temperature, and would seem to have been a local deflection of the south-west current." "Some observations of pressure in the neighbourhood of Cape Negrais would seem to indicate the existence of a slight barometric depression in the Gulf of Martaban and to the west and north of Cape Negrais, but the instruments were not sufficiently trustworthy to establish the fact in a satisfactory manner."

IV. The storm of the 5th to the 10th June† of the same year was apparently formed in the middle of the Bay, in about 16° north latitude. At Port Blair the wind had been steady from south and south-west for two or three days; at Chittagong and Akyab southerly, with alternations of land- and sea-breezes; at Saugor Island south; at False Point south-east; and a steamer crossing the head of the Bay from Calcutta to Akyab had a steady south-east breeze. Around the coasts, the pressure was lowest at Saugor Island on the 4th, when it was 0·1 below that at Port Blair, *i. e.* rather less than the normal difference in this month. On the 5th the difference was reduced one half of this; but in the middle of the Bay, when the storm originated, the barometer stood at more than 0·2 inch lower than at Saugor Island, and there is evidence of the winds beginning on that day to curve in around the local depression.

V. The cyclone of the 7th and 8th October of the same year‡ commenced in the northern part of the Bay on the morning of the former day. At Port Blair the wind had been steady from the south-west for several days, and the barometer had not varied more than ·03 inch on the mean of the day from the 2nd to the 6th. At Akyab the barometric change had been but little greater and the wind had been light from south-south-east, with one or two changes to north and east, for a few hours, which were probably due to local causes. On the 6th it was steady from between south-south-east and south-west, blowing moderate to fresh. At Saugor Island the wind was light and from the south up to midnight of the 5th, after which a light wind set in from east-north-east and continued without much change till sunset of the 6th. At False Point the barometer had been steady and the wind south-east or east. In the south of the Bay the wind had been from west or west-south-west for several days and the weather squally; this appears to be always the case before a cyclone.

VI. The Vizagapatam storm of the 5th November, 1870§, probably commenced on

* Meteorological Report for Bengal, 1869, p. 102. † Ib., p. 103. ‡ Ib., p. 107. § Ib., 1870, p. 116.

the 1st of the month to the west of the Andamans. During the last four days of October a considerable change had taken place in the distribution of atmospheric pressure over the Bay. On the 28th it had been lowest at Vizagapatam and Port Blair, and at Madras to the southward, and at all the coast stations around the north of the Bay, only about 0·05 in. higher than at Vizagapatam. On the three following days the Madras pressure remained almost unchanged; but a general rise took place to the north-west, which brought the pressure at Vizagapatam up to an equality with that of Madras, and that of Saugor Island and False Point above all other stations. At the same time a fall of 0·1 inch took place at Port Blair, which reduced the pressure at that station to about 0·15 inch below that in the north-west of the Bay. Over the north of the Bay, as far down as 18° or 19° on the Indian coast, and down to Port Blair on the other side, the wind was from north and north-east. At Vizagapatam it was variable, and at Madras chiefly from the westward. On the open sea to the southward, the westerly monsoon, with its characteristic squally weather, prevailed; but over a triangular area, extending across the Bay between Coconada, Madras, and Rangoon, calms and variable winds had prevailed for two weeks previously, and it was along the middle of this calm belt that the cyclone subsequently took its course.

These instances show at least that the existence of opposite currents antecedently to the formation of a cyclone is by no means an essential condition. In the case of storms I., IV., and V. no northerly or north-easterly wind was felt in the north-west of the Bay until the day that the storm-vortex was found, and the barometric depression which, in the first case certainly, and probably in the others pre-existed, must have been the cause both of the north-east wind and the cyclone; and in the other cases mentioned, the north-east wind in no case blew further south than the place of the storm's origin.

It is generally difficult to ascertain the exact barometric pressure at the place at which a storm originates for a week or ten days before the cyclone is formed, since such data can be obtained only from such ships as may subsequently arrive in Calcutta, if even from them; and in most cases the instruments in use on board ships are not read with that accuracy that is requisite to establish the existence of a depression of less than about 0·1 inch (a large amount in tropical seas), and their inherent error is generally unknown; but such data as I have obtained have led me to infer that, as in the case of storm II., an area of barometric depression is, as a rule, formed several days before the cyclone is generated; and *à priori* considerations would lead us to expect that such must be the case, producing a convergence of the air around. When such a convergence has once been established, the formation of a cyclone is easily explained from known physical laws.

I must therefore conclude that the views put forward by Mr. Meldrum with respect to the formation of storms in the South Indian Ocean*, viz. that they are produced between parallel currents flowing in opposite directions, do not hold good in the case of the Bay of Bengal. I infer that a calm state of the atmosphere, or one in which the

* Proceedings of the Meteorological Society of London, 1869, vol. iv. p. 283.

winds are light and variable over the open sea, is a condition favourable to the formation of these storms, and that a second condition is a high or moderately high temperature. The consequence of this collocation will be the production and ascent of a large quantity of vapour, which will be condensed with the liberation of its latent heat over the place of its production, instead of being carried away to some distant region. If this state of things last for some days, the atmospheric pressure will be locally lowered, causing, or tending to cause, an indraught of air towards the place of minimum pressure. In order that a cyclone may ensue, one further condition appears to be essential. It was pointed out in a paper on the origin of the November cyclone, 1867, published in the Proceedings of the Royal Society*, that most of the storms of the Bay of Bengal originate along a line running from south to north by the Nicobars, Andamans, and the islands of the Arakan coast. Some, such as those of the 5th and 10th June and the 7th and 8th October, 1869 (*supra* IV., V.), also a storm that occurred during the present year (1872) on the 30th June and 1st July†, originate about the middle of the Bay. But I do not know of a single case in which a cyclone has been actually formed in the north-west of the Bay, or under the lee of the Madras coast, although more than one case has been recorded in which most of the requisite conditions were fulfilled, and in which for some days there has seemed reason for apprehending one‡. These facts, I think, find their explanation in another fact, surmised by Colonel GASTRELL and myself as a probable local law in 1865§, and verified in the case of every storm I have yet investigated (where sufficient evidence has been forthcoming), viz. that the formation of a cyclone is determined by an inrush of a saturated stormy current from the south-west or west-south-west. Now, under the lee of the Madras coast or in the north-west of the Bay, any wind coming from this direction must pass over the peninsula, a course which would drain it to a great extent of its vapour, while its free passage would be enormously impeded by friction and the irregularities of the land-surface; but in the eastern half of the Bay no such impediments present themselves to its free access, and its high velocity and abundant vapour seem to be the determining conditions of the formation of cyclones.

I would especially guard myself against being supposed to extend the above views, *mutatis mutandis*, to the case of the South Indian Ocean, or, indeed, any area other than that I am specially dealing with. Doubtless it will be found that in each region subject to cyclones the determining conditions present local modifications; and the best way to arrive at a general theory of the formation of cyclonic storms will be to ascertain the local conditions as accurately as possible in each case by independent study, as Mr. MELDRUM has done in the South Indian Ocean, and as I have here attempted for the Bay of Bengal. It will then be easy to eliminate all that is merely local, and to establish general laws by a comparison of the results so obtained.

* Proceedings of the Royal Society, vol. xvii. p. 479.

† Described by Mr. W. G. WILLSON in a Report to the Government of Bengal.

‡ See the Bengal Meteorological Reports for 1870, p. 115, and 1871, p. 124. Another case occurred during the present year on the 26th October.

§ Report on the Calcutta Cyclone of 1864, p. 105, &c.

TABLE I. WIND DIRECTIONS AND VELOCITIES.

PUNJAB.

I. Rawul Pindee.

	Observed.										Per cent.								Resultant.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.
January	3	13	8	39	7	8	5	94	12	...	7	4	21	4	4	3	51	6	...	31	N. 79 W.
February	3	8	6	26	8	8	8	98	6	...	5	3	16	5	5	5	58	3	...	41	S. 88 W.
March	3	11	9	51	2	13	7	94	8	...	6	5	28	1	7	4	45	4	...	19	N. 84 W.
April	3	7	7	33	8	11	3	101	9	...	4	4	18	5	6	2	56	5	...	37	W.
May	3	25	3	33	2	9	10	85	19	...	13	2	18	1	5	5	46	10	...	39	N. 72 W.
June	3	17	5	64	8	12	6	61	7	...	9	3	36	4	7	3	34	4	...	3	N. 33 W.
July	3	12	6	110	6	6	7	34	1	...	7	3	60	3	3	4	19	1	...	43	N. 89 E.
August	4	18	11	64	49	19	15	66	3	...	7	5	26	20	8	6	27	1	...	19	S. 37 E.
September	4	15	15	66	25	24	21	66	8	...	6	6	28	10	10	9	28	3	...	11	S. 17 E.
October	4	9	6	29	17	27	24	118	17	...	3	2	12	7	11	10	48	7	...	43	S. 73 W.
November	4	15	25	22	14	17	68	73	6	...	6	10	9	6	7	28	31	3	...	35	S. 63 W.
December	3	11	12	26	2	3	10	103	17	...	6	7	14	1	2	5	56	9	...	31	N. 69 W.
Year	...	...	...	...	...	...	...	...	...	...	6·5	4	24	5·5	6	7	42	5	...	...	.....

II. Dera Ishmail Khan.

January	3	33	43	10	11	18	9	25	41	...	17	23	5	6	9	5	13	22	...	26	N. 10 W.
February	3	26	47	8	9	7	10	19	42	...	16	28	5	5	4	6	11	25	...	41	N. 7 W.
March	3	32	40	24	19	11	6	16	37	...	17	22	13	10	6	3	9	20	...	33	N. 18 E.
April	3	21	37	19	26	18	27	6	25	...	12	21	11	14	10	15	3	14	...	13	N. 66 E.
May	3	15	54	42	25	12	25	5	7	...	8	29	23	13	7	13	3	4	...	38	N. 81 E.
June	3	9	54	36	28	21	14	3	9	...	5	31	21	16	12	8	2	5	...	43	N. 88 E.
July	3	3	57	37	33	25	24	...	4	...	2	31	20	18	14	13	...	2	...	45	S. 77 E.
August	4	6	54	52	95	19	29	...	3	...	2	21	20	37	7	12	...	1	...	57	S. 66 E.
September	4	6	61	64	68	27	20	1	1	...	2	24	26	27	11	8	1	1	...	58	S. 74 E.
October	4	31	85	31	43	25	20	9	13	...	12	33	12	17	10	8	3	5	...	37	N. 71 E.
November	4	31	53	22	34	22	25	21	28	...	13	23	9	14	9	11	9	12	...	15	N. 46 E.
December	4	31	57	27	20	28	11	29	48	...	12	23	11	8	11	4	12	19	...	22	N. 11 E.
Year	...	...	...	...	...	...	...	...	...	...	10	26	15	15	9	9	5	11	...	...	.....

TABLE I. (continued).

PUNJAB.

III. Mooltan.

	Observed.										Per cent.									Resultant.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.
January	3	46	17	6	11	7	20	5	55	...	27	10	4	7	4	12	3	33	...	45	N. 25 W.
February	3	42	29	7	20	17	12	4	30	...	26	18	4	13	11	7	2	19	...	18	N. 10 E.
March	2	12	34	2	12	7	15	2	38	...	10	28	1·5	10	6	12	1·5	31	...	30	N. 7 W.
April	3	17	27	7	23	12	36	4	48	...	10	15	4	13	7	21	2	28	...	14	N. 49 W.
May	3	15	21	5	14	13	63	13	35	...	8	12	3	8	7	35	7	20	...	30	S. 76 W.
June	3	5	24	4	18	26	81	4	16	...	3	13	2	10	15	46	2	9	...	41	S. 31 W.
July	3	9	21	...	20	16	100	1	10	...	5	12	...	11	9	56	1	6	...	49	S. 35 W.
August	3	19	28	5	22	30	53	12	14	...	10	15	3	12	16	29	7	8	...	21	S. 29 W.
September	3	8	5	2	10	48	101	1	8	...	4	3	1	6	26	55	1	4	...	70	S. 31 W.
October	4	27	31	1	16	34	83	18	46	...	11	12	...	6	13	33	7	18	...	31	S. 73 W.
November	3	17	31	1	20	17	40	11	43	...	9·5	17	1	11	9·5	22	6	24	...	19	N. 72 W.
December	4	31	60	10	24	14	34	5	49	...	14	26	4	11	6	15	2	21	...	23	N. 6 E.
Year	...	...	...	...	...	...	...	...	...	...	12	15	2	10	11	29	3	18	...	...	.....

IV. Lahore.

January	3	23	23	11	10	4	13	37	62	...	13	13	6	5	2	7	20	34	...	46	N. 41 W.
February	3	20	26	23	5	1	5	30	60	...	12	15	13	3	1	3	18	35	...	47	N. 23 W.
March	3	33	39	12	9	4	6	35	49	...	18	21	6	5	2	3	19	26	...	43	N. 20 W.
April	3	15	25	18	4	...	9	48	60	...	8	14	10	2	...	5	27	34	...	49	N. 41 W.
May	2	10	27	12	5	3	11	10	33	...	9	24	11	4	3	10	9	30	...	35	N. 9 W.
June	2	8	17	18	13	1	27	12	24	...	7	14	15	11	1	22	10	20	...	10	N. 48 W.
July	2	2	27	20	30	4	26	4	10	...	2	22	16	25	3	21	3	8	...	28	S. 63 E.
August	3	5	40	38	45	7	32	5	12	...	3	22	21	24	4	17	3	6	...	35	S. 72 E.
September	3	9	34	15	15	4	32	47	22	...	5	19	9	9	2	18	26	12	...	21	N. 72 W.
October	4	24	35	22	25	7	15	32	74	...	10	15	9	11	3	6	14	32	...	31	N. 24 W.
November	4	19	38	20	25	4	11	40	82	...	8	16	8	10	2	5	17	34	...	36	N. 29 W.
December	4	42	42	28	12	...	8	29	90	...	17	17	11	5	...	3	11	36	...	50	N. 14 W.
Year	...	...	...	...	...	...	...	...	...	...	9	18	11	10	2	10	15	25	...	...	.....

Gangetic Valley.

I. Roorkee.

	Observed.										Per cent.									Resultant.		Mean daily movement.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.	Years.	Miles.
January	7	1	10	8	32	7	34	9	125	209	...	2	2	8	2	7	2	29	48	18	N. 63 W.	4	40·2
February ...	7	7	24	17	26	5	14	27	115	161	2	6	4	7	1	4	7	29	40	25	N. 22 W.	4	47·0
March	7	7	21	9	19	17	21	41	124	184	2	5	2	4	4	4	9	28	42	27	N. 59 W.	3	51·5
April	7	5	25	8	59	12	29	42	85	145	1	6	2	15	3	7	10	21	35	14	N. 81 W.	3	64·7
May	7	7	9	8	115	24	29	31	98	116	2	2	2	26	6	7	7	22	27	11	S. 30 W.	3	73·8
June	7	7	19	20	93	24	21	25	92	119	2	4	5	22	6	5	6	22	28	5	S. 17 W.	4	72·7
July	7	3	16	22	167	16	19	15	32	144	1	4	5	38	4	4	3	7	34	34	S. 43 E.	4	53·4
August	7	4	14	14	152	7	18	17	52	156	1	3	3	35	2	4	4	12	36	23	S. 41 E.	4	38·1
September ...	7	6	16	12	84	6	21	15	92	168	1	4	3	20	1	5	4	22	40	3	N. 80 W.	4	27·3
October	7	1	18	10	85	18	12	12	64	219	...	4	2	20	4	3	3	14	50	7	S. 32 E.	4	18·2
November ...	7	3	7	2	63	8	8	8	48	273	1	2	...	15	2	2	2	11	65	4	S. 13 E.	5	19·2
December ...	7	4	13	3	49	...	15	28	91	231	1	3	1	11	...	4	6	21	53	15	N. 60 W.	6	25·4
Year	...	...	...	...	...	...	...	...	...	...	1·1	3·7	2·6	18·4	3·0	4·7	5·2	19·8	41·5	...	.....	...	.....

II. Agra.

January	6	36	14	24	5	10	8	80	24	90	12	5	8	2	4	3	27	8	31	27	N. 56 W.	...	...
February ...	6	34	21	20	4	11	8	106	33	74	11	7	7	1	3	2	34	10	24	36	N. 62 W.	...	...
March	7	59	33	10	18	18	13	111	48	75	15	6	3	5	5	3	29	12	20	37	N. 55 W.	...	...
April	6	22	22	31	24	20	39	83	64	20	7	7	9	7	6	12	25	20	6	29	N. 79 W.	...	...
May	6	29	31	35	21	24	15	110	45	31	8	9	10	6	7	4	32	13	9	26	N. 68 W.	...	...
June	6	27	37	47	16	9	26	85	43	40	8	11	14	5	3	8	26	13	12	21	N. 48 W.	...	...
July	7	33	52	89	21	12	19	46	33	90	8	13	23	5	3	5	12	8	23	20	N. 48 E.	...	...
August	7	10	24	68	43	24	42	45	18	129	2	6	17	11	6	10	11	4	32	14	S. 37 E.	...	...
September ...	7	24	30	66	12	4	18	65	53	116	6	8	18	3	1	5	17	14	30	17	N. 24 W.	...	...
October	7	24	19	12	13	21	22	109	27	156	6	5	3	3	5	5	27	6	39	27	N. 85 W.	...	...
November ...	7	21	9	18	18	26	35	73	26	164	5	2	5	5	7	9	19	7	42	21	S. 78 W.	...	...
December ...	7	22	29	39	23	15	19	91	41	146	5	7	9	5	4	4	21	9	34	20	N. 65 W.	...	...
Year	...	...	...	...	...	...	...	...	...	...	7·7	7·2	10·5	4·8	4·5	5·8	23·3	10·3	25·1	...	.....	...	...

TABLE I. (continued).

GANGETIC VALLEY.

III. Benares.

	Observed.										Per cent.									Resultant.		Mean daily movement.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.	Years.	Miles.
January	6	12	25	35	22	4	37	112	52	40	4	7	10	7	1	11	33	15	12	33	N. 75 W.	3	55·6
February ...	6	8	19	43	18	...	40	136	36	30	2	6	13	5	...	12	41	11	10	37	N. 87 W.	3	73·3
March	6	14	14	48	15	7	31	132	65	13	4	4	14	5	2	9	39	19	4	38	N. 76 W.	3	66·2
April	6	21	22	29	34	3	30	112	75	16	6	6	8	10	1	9	33	22	5	37	N. 70 W.	3	62·9
May	7	30	61	85	13	1	33	85	76	13	8	15	22	3	...	8	22	19	3	24	N. 14 W.	3	87·3
June	7	15	54	88	20	9	36	81	65	12	4	14	23	6	2	10	21	17	3	14	N. 13 W.	3	95·2
July	7	9	39	114	26	15	58	91	23	14	2	10	29	7	4	15	23	6	4	6	S. 28 E.	3	73·4
August	7	9	33	99	32	10	47	101	38	36	2	8	24	8	2	11	27	9	9	4	S. 67 W.	3	69·7
September ...	7	14	48	103	27	14	24	74	27	22	4	13	29	8	4	7	21	8	6	5	N. 69 E.	3	58·3
October	7	13	28	60	19	7	31	120	68	35	3	7	16	6	2	8	31	18	9	27	N. 68 W.	3	60·9
November ...	7	26	28	30	13	12	33	123	70	45	7	7	8	3	3	9	32	19	12	38	N. 69 W.	3	25·0
December ...	6	18	19	44	4	4	8	136	69	15	6	6	14	1	1	2	43	22	5	16	N. 62 W.	3	34·2
Year	...	...	...	...	...	...	...	...	...	...	4·3	8·6	17·5	5·8	1·8	9·3	30·5	15·4	6·8	...	.....	...	...

IV. Patna.

	A.—Observed. Calms not recorded.										B.—Observed. Calms recorded.									
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Year.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.
January	4	24	26	23	16	13	65	144	121	?	1	1	5	2	1	...	2	12	11	90
February ...	4	32	34	32	14	11	49	96	124	?	1	1	6	7	5	1	4	12	10	70
March.....	4	40	29	25	15	7	57	94	171	?	1	1	...	...	...	...	3	27	29	64
April	4	44	74	57	28	2	20	42	148	?	1	1	15	13	5	...	4	19	19	44
May	4	71	119	82	37	1	18	26	80	?	1	11	24	22	5	...	2	5	10	45
June	4	61	105	96	54	2	19	28	54	?	1	1	19	19	6	1	...	...	...	74
July	4	41	72	130	65	3	37	28	58	?	1	1	13	13	2	1	2	2	2	88
August	3	5	38	55	59	38	47	38	30	?	1	3	9	20	25	8	8	6	4	41
September ...	4	14	62	111	116	4	26	20	16	?	1	2	11	22	18	...	...	...	2	65
October	4	20	61	56	60	19	42	50	68	?	1	6	10	5	2	3	7	3	9	79
November ...	4	50	63	30	13	9	20	60	115	?	1	2	6	4	4	1	7	11	8	77
December ..	4	23	29	34	20	7	39	111	148	?	1	...	4	3	1	...	3	8	7	98

IV. Patna.

	Per cent.									Resultant.		Mean daily movement.	
	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.	Years.	Miles.
January	1	2	1	1	1	4	9	8	73	15	N. 71 W.	2	64·6
February	2	4	4	2	1	4	10	12	61	16	N. 60 W.	2	72·8
March	4	2	2	1	1	5	13	20	52	30	N. 61 W.	2	96·2
April	5	12	9	4	...	3	9	21	37	24	N. 12 W.	2	144·2
May	10	18	13	5	...	2	4	11	37	30	N. 33 E.	2	138·7
June	5	11	10	5	...	1	2	4	62	19	N. 57 E.	2	90·3
July	2	6	9	4	...	2	2	4	71	11	N. 67 E.	2	81·2
August	1	8	13	15	8	9	7	6	33	14	S. 41 E.	2	81·1
September	2	8	15	15	...	2	2	2	54	26	S. 82 E.	2	84·4
October	3	6	5	5	2	4	4	7	64	4	N. 13 E.	2	50·1
November	4	6	3	2	1	3	7	10	64	12	N. 37 W.	2	51·7
December	1	2	2	1	...	2	6	7	79	9	N. 69 W.	2	34·4
Year	3·3	7·1	7·2	5	1·2	3·4	6·2	9·3	57·3	.....	.....	.....	.....

TABLE I. (continued).

RAJPOOTANA.

I. Beawur and Ajmere.

	Observed.										Per cent.									Resultant.		Mean diurnal movement.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.	Years.	Miles.
January	7	36	59	34	52	23	41	31	28	54	10	16	10	15	6	11	9	8	15	10	N. 62 E.	...	...
February ...	7	31	22	37	24	27	46	62	26	47	10	7	12	7	8	14	19	8	15	14	S. 75 W.	...	...
March	7	29	30	11	41	29	72	53	16	20	9	10	4	14	9	24	17	6	7	21	S. 46 W.	...	...
April	6	22	7	6	21	29	113	87	44	28	6	2	2	6	8	32	24	12	8	52	S. 69 W.	...	...
May	7	12	6	5	13	16	200	101	14	9	3	2	1	4	4	53	27	4	2	73	S. 59 W.	...	...
June	7	7	18	7	10	18	188	111	9	20	2	5	2	2	5	48	29	2	5	67	S. 60 W.	...	...
July	7	1	14	8	14	15	168	142	14	23	...	3	2	3	4	42	36	3	6	69	S. 63 W.	...	...
August	7	8	32	16	13	7	168	115	29	33	2	8	4	3	1	40	27	7	8	53	S. 68 W.	...	...
September ...	7	5	26	5	15	10	111	193	20	30	1	6	1	4	2	27	47	5	7	84	S. 68 W.	...	...
October	7	22	47	5	16	8	79	156	15	65	5	11	1	4	2	19	38	4	16	48	S. 87 W.	...	...
November ...	6	40	44	35	14	21	31	65	15	63	12	13	11	4	6	9	20	5	20	11	N. 37 W.	...	...
December ...	6	48	44	40	26	50	26	30	22	51	14	13	12	8	15	8	9	6	15	8	N. 73 E.	...	...
Year	...	...	...	...	...	...	...	...	...	...	6	8	5	6	6	27	25	6	10	...	.....	...	...
II. Jhansi.																							
January	3	14	26	34	9	12	7	39	38	7	8	14	18	5	6	4	21	20	4	20	N. 19 W.	...	...
February ...	3	11	15	20	11	16	8	51	23	1	7	9	13	7	10	5	33	15	1	20	N. 70 W.	...	...
March	4	21	14	42	18	37	13	62	13	3	9	6	19	8	17	6	28	6	1	12	S. 40 W.	...	...
April	5	22	13	40	16	24	39	95	32	21	7	4	13	5	8	13	32	11	7	28	S. 84 W.	...	...
May	5	8	30	47	16	11	26	92	86	4	3	9	15	5	4	8	28	27	1	33	N. 62 W.	...	...
June	6	25	30	57	32	8	44	98	51	23	7	8	15	9	2	12	27	14	6	18	N. 72 W.	...	...
July	5	8	28	41	14	11	57	112	26	13	3	9	13	5	4	18	36	8	4	33	S. 81 W.	...	...
August	6	18	19	86	14	26	46	103	53	5	5	5	23	4	7	13	28	14	1	17	W.	...	...
September ...	5	57	23	25	12	17	5	106	40	15	19	8	8	4	6	2	35	13	5	38	N. 51 W.	...	...
October	5	31	37	33	11	23	13	105	37	17	10	12	11	4	8	4	34	11	6	28	N. 59 W.	...	...
November ...	6	44	18	58	8	26	7	107	40	16	14	6	18	2	8	2	33	12	5	25	N. 53 W.	...	...
December ...	5	35	43	64	22	15	10	78	48	3	11	13	20	7	5	3	25	15	1	20	N. 8 W.	...	...
Year	...	...	...	...	...	...	...	...	...	...	9	9	16	5	7	7	30	14	3	...	.....	...	...

Central India.

I. Jubbulpore.

	Observed.										Per cent.									Resultant.		Mean diurnal movement.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.	Years.	Miles.
January	3	49	39	3	14	26	13	12	25	5	26	21	2	8	14	7	6	13	3	27	N. 6 W.	2	61·3
February ...	3	34	18	4	17	26	19	16	20	9	21	11	2	10	16	12	10	12	6	11	N. 58 W.	2	50·7
March.....	3	26	18	7	14	40	16	43	21	...	14	10	4	8	22	9	23	11	...	22	S. 79 W.	2	83·3
April	3	28	16	8	5	22	14	51	27	8	16	9	4	3	12	8	28	15	4	34	N. 68 W.	3	93·1
May	3	24	11	2	7	13	25	63	39	2	13	6	1	4	7	13	24	21	1	41	N. 76 W.	3	123·1
June	3	5	4	5	1	5	40	97	22	1	3	2	3	1	3	22	54	12	1	74	S. 85 W.	3	145·7
July	3	4	4	1	...	9	32	115	21	1	2	2	1	...	5	17	61	12	1	80	S. 86 W.	3	132·0
August	3	3	3	1	7	12	26	116	17	1	2	2	1	4	6	14	62	9	1	75	S. 83 W.	3	123·0
September ...	3	32	15	6	11	12	13	62	24	5	18	8	3	6	7	7	35	13	3	39	N. 64 W.	3	92·0
October	3	39	31	16	4	21	23	30	18	4	21	17	8	2	11	12	17	10	2	16	N. 29 W.	3	52·7
November ...	3	49	41	14	17	24	15	5	8	7	27	23	8	10	13	8	3	4	4	28	N. 42 E.	2	47·1
December ...	3	38	50	6	26	26	16	10	11	3	20	27	3	14	14	9	5	6	2	21	N. 50 E.	3	55·8
Year	...	...	...	...	...	...	...	...	...	...	15	12	3	6	11	12	27	12	2	...	.....	...	.....

II. Nagpore.

January	3	15	52	51	16	13	13	8	6	12	8	28	27	9	7·5	7·5	4	3	6	43	N. 74 E.	2	70·1
February ...	3	18	24	30	17	21	12	10	22	9	11	15	18	10	13	7	6	14	6	16	N. 70 E.	2	85·8
March.....	3	24	16	32	12	34	27	20	22	2	13	8	17	6	18	14	11	12	1	6	S. 15 W.	2	99·9
April	3	31	11	17	11	23	18	29	37	...	18	6	10	6	13	10	16	21	...	24	N. 59 W.	3	116·4
May	3	55	7	9	8	12	13	32	49	1	29	4	5	4	6	7	17	27	1	47	N. 40 W.	2	162·1
June	3	17	5	4	2	14	27	48	60	4	9	3	2	1	8	15	26	33	2	58	N. 74 W.	3	163·4
July	3	7	4	3	...	3	50	82	44	2	4	2	1	...	1	26	42	23	1	73	N. 89 W.	3	156·5
August	3	9	4	5	3	12	25	66	56	6	5	2	3	2	6	13	36	30	3	62	N. 80 W.	3	122·9
September ...	3	24	10	14	10	11	20	30	58	3	13	5·5	8	5·5	6	11	17	32	2	42	N. 49 W.	3	120·4
October	3	35	69	28	13	3	8	2	22	6	19	37	15	7	2	4	1	12	3	56	N. 38 E.	3	88·7
November ...	3	22	55	63	19	5	3	3	5	6	12	30	34	10	3	3	2	3	3	62	N. 68 E.	3	71·4
December ...	3	19	55	54	28	10	7	2	4	7	10	30	29	15	5	4	1	2	4	57	N. 76 E.	3	59·7
Year	...	...	...	...	...	...	...	...	...	...	12	14	14	6	7	10	15	18	3	...	.....	...	...

TABLE I. (continued).

WESTERN BENGAL AND ORISSA.

I. Hazareebagh.

	Observed.										Per cent.									Resultant.		Mean diurnal movement.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.	Years.	Miles.
January	3	32	12	8	18	33	26	113	139	...	8	3	2	5	9	7	30	36	...	60	N. 70 W.	3	98·0
February ...	3	27	5	5	14	25	38	111	110	...	8	2	2	4	7	11	33	33	...	58	N. 77 W.	3	131·4
March.....	3	26	3	7	12	32	68	113	111	...	7	1	2	3	9	18	30	30	...	61	N. 85 W.	3	147·6
April	3	24	18	26	35	41	52	70	92	...	7	5	7	10	11	14	20	26	...	32	S. 89 W.	3	168·4
May	3	40	14	27	59	77	36	42	62	...	11	4	8	16	22	10	12	17	...	17	S. 36 W.	3	187·3
June	3	15	21	31	72	94	49	49	34	...	4	6	9	20	26	13	13	9	...	35	S. 6 W.	3	184·6
July	3	12	15	63	71	86	36	46	38	4	3	4	17	19	23	10	13	10	1	31	S. 13 E.	3	152·3
August	3	16	15	36	46	54	48	113	44	...	4	4	10	12	15	13	30	12	...	32	S. 58 W.	3	131·9
September ...	3	32	25	79	90	61	29	25	22	...	8	7	22	25	17	8	7	6	...	36	S. 50 E.	3	129·2
October	3	48	19	18	38	36	35	72	108	...	13	5	5	10	10	9	19	29	...	34	N. 66 W.	3	103·5
November ...	3	59	25	10	2	22	26	85	130	...	16	7	3	1	6	7	24	36	...	62	N. 50 W.	3	91·1
December ...	3	37	13	6	14	19	38	104	141	...	10	3	2	4	5	10	28	38	...	62	N. 66 W.	3	92·7
Year	...	...	...	...	...	...	...	...	...	...	8	4	7	11	13	11	22	24	...	...	.....	...	.....

II. Cuttack.

January	3	33	51	94	49	62	21	22	47	...	9	13	25	13	16	6	6	12	...	25	S. 68 E.	1	38·6
February ...	3	23	34	57	35	88	39	38	25	...	7	10	17	10	26	12	11	7	...	23	S. 18 E.	1	66·7
March.....	3	12	15	31	61	149	38	52	23	...	3	4	8	16	39	10	14	6	...	47	S. 2 W.	1	85·1
April	3	7	22	18	34	187	50	25	11	...	2	6	5	10	53	14	7	3	...	63	S. 3 W.	2	123·3
May	3	9	13	26	34	257	25	6	5	...	2	3	7	9	69	7	2	1	...	75	S. 7 E.	2	137
June	3	6	8	30	38	151	70	43	16	...	2	2	8	10	42	20	12	4	...	56	S. 8 W.	2	134·8
July	3	15	5	7	19	129	85	76	34	...	4	1	2	5	35	23	21	9	...	52	S. 39 W.	2	94·2
August	3	9	9	20	34	92	73	81	52	...	2	2	5	10	25	20	22	14	...	54	S. 44 W.	2	78·2
September ...	3	20	44	61	29	106	36	33	26	...	6	12	17	8	30	10	9	8	...	25	S. 24 E.	2	78
October	3	56	87	55	42	39	19	21	46	...	15	24	15	11	11	5	6	13	...	30	N. 51 E.	2	56
November ...	3	108	56	23	9	15	17	49	82	...	30	16	6	2	4	5	14	23	...	51	N. 16 W.	2	35·9
December ...	3	56	46	39	24	14	12	46	65	...	19	15	13	8	5	4	15	21	...	27	N. 1 W.	2	34·8
Year	...	...	...	...	...	...	...	...	...	...	8	9	11	9	30	11	12	10	...	...	.....	...	.....

Cattack, 1 year (calms recorded).

	Observed.										Per cent.									Resultant.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.
January	1	7	19	19	20	34	11	8	1	5	6	15	15	16	28	9	6	1	4	37	S. 40 E.
February	1	5	16	5	13	35	19	6	5	12	4	14	4	11	30	17	5	4	11	33	S. 3 E.
March	1	3	11	10	11	34	33	7	4	11	2	9	8	9	27	27	6	3	9	42	S. 8 W.
April	1	3	5	3	7	30	25	3	44	...	2·3	4	2·3	6	25	21	2·3	37	...	50	N. 41 W.
May	1	...	7	8	15	53	36	3	2	...	...	6	6	12	43	29	2	2	...	67	S. 4 W.
June	1	5	4	1	6	27	27	18	2	...	6	4	1	7	30	30	20	2	...	57	S. 36 W.
July	1	1	2	1	22	26	34	21	8	9	1	2	1	18	21	27	17	6	7	55	S. 29 W.
August	1	1	2	5	6	17	49	35	8	1	1	2	4	5	14	39	28	6	1	68	S. 53 W.
September	1	7	10	3	9	35	27	4	2	23	6	8	2	8	29	23	3	2	19	40	S. 10 W.
October	1	14	12	...	2	7	14	6	9	59	11	10	...	2	6	11	5	7	48	15	N. 46 W.
November	1	18	17	8	8	9	6	8	12	34	15	14	7	7	7	5	7	10	28	18	N. 15 E.
December	1	23	36	19	14	3	7	4	6	12	19	29	15	11	2	6	3	5	10	45	N. 49 E.
Year	...	...	...	...	...	...	...	...	...	...	6	10	5	9	22	20	9	7	12	...	.....

III. False Point.

January	3	44	140	47	29	25	27	11	13	21	12	39	13	8	7	8	3	4	6	44	N. 55 E.
February	3	19	51	31	34	28	92	35	32	12	6	15	9	10	8	28	10	10	4	16	S. 37 W.
March	3	16	27	13	22	28	162	60	21	4	4	8	4	6	8	46	17	6	1	51	S. 52 W.
April	3	9	9	3	13	47	228	23	11	2	2	2	1	4	14	66	7	3	1	77	S. 42 W.
May	3	4	5	5	15	69	247	12	8	...	1	1	1	4	19	68	3	2	...	83	S. 35 W.
June	3	...	3	6	38	37	173	57	16	13	...	1	2	10	10	51	7	5	4	69	S. 42 W.
July	3	...	2	1	17	33	140	123	32	2	...	1	...	5	9	40	35	9	1	76	S. 63 W.
August	3	9	15	7	22	25	123	85	63	15	3	4	2	6	7	34	23	17	4	54	S. 72 W.
September	3	18	43	16	41	29	132	38	23	9	5	12	5	12	8	38	11	7	2	34	S. 40 W.
October	3	35	102	29	42	28	32	23	27	39	10	28	8	12	8	9	6	7	11	21	N. 53 E.
November	3	85	142	23	21	11	15	10	37	4	24	40	7	6	3	4	3	11	1	58	N. 28 E.
December	3	76	139	30	18	11	7	9	47	24	21	39	8	5	3	2	2	13	7	57	N. 28 E.
Year	3	...	...	...	...	...	...	...	...	...	7	16	5	7	9	33	11	8	4	...	.....

TABLE I. (continued).

GANGETIC DELTA.

I. Saugor Island.

	Observed.										Per cent.									Resultant.		Mean diurnal movement.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.	Years.	Miles.
January	3	97	89	16	15	36	51	24	30	...	27	25	5	4	10	14	7	8	...	30	N. 4 E.	2	126
February ...	3	44	22	13	7	78	104	30	37	...	13	7	4	2	23	31	9	11	...	38	S. 54 W.	3	192·5
March	3	18	7	12	9	99	156	41	29	...	5	2	3	2	27	42	11	8	...	62	S. 40 W.	3	231·4
April	3	3	9	14	19	139	148	19	9	...	1	2	4	5	39	41	5	2	...	76	S. 22 W.	3	394·1
May	3	3	5	7	61	176	110	4	6	...	1	1	2	16	47	30	1	2	...	82	S. 6 W.	3	344·8
June	3	4	8	10	47	142	114	24	10	...	1	2	3	13	39	31	8	3	...	73	S. 15 W.	2	204·1
July	3	4	6	13	49	122	132	35	11	...	1	2	3	13	33	36	9	3	...	71	S. 20 W.	2	254·8
August	3	12	14	22	34	91	140	49	10	1	3	4	6	9	24	38	13	3	...	60	S. 28 W.	2	261·8
September ...	3	11	23	33	57	122	88	9	7	...	3	7	9	16	35	25	3	2	...	58	S. 4 E.	2	242·3
October	3	68	50	21	34	67	56	26	44	6	18	13	6	9	18	15	7	12	2	5	N. 81 W.	2	135·5
November ...	3	158	80	14	8	13	27	17	42	...	44	22	4	2	4	7	5	12	...	60	N. 3 E.	2	103
December ...	3	124	102	22	2	17	45	22	38	...	33	27	6	1	5	12	6	10	...	49	N. 5 E.	...	...
Year.....	...	...	...	...	...	...	...	...	...	...	12·5	9·5	5	8	25	27	7	6	...	...	.....	...	.....
II. Calcutta.																							
January	10	2003	293	356	159	651	517	984	1253	121	32	5	6	2	10	8	15	20	2	40	N. 38 W.	3	104·5
February ...	10	846	334	453	265	1215	775	1069	842	39	15	6	8	4	21	13	18	14	1	23	S. 79 W.	3	114·8
March	10	466	151	233	277	2479	1198	1006	515	30	7	2	4	4	39	19	16	8	1	47	S. 33 W.	4	133·1
April	10	116	102	280	677	3684	846	295	147	31	2	2	4	11	60	14	5	2	...	74	S. 3 W.	4	203·9
May	10	162	194	621	1144	3211	607	154	112	95	3	3	10	18	51	10	2	2	1	65	S. 12 E.	4	209·7
June	10	185	288	550	879	2841	840	369	159	141	3	5	9	14	45	13	6	3	2	58	S. 5 E.	4	198·3
July	10	110	240	713	1159	3056	812	253	61	77	2	4	11	18	47	12	4	1	1	65	S. 12 E.	4	157·5
August	10	137	340	974	1149	2470	735	342	96	121	2	5	15	18	39	12	5	2	2	55	S. 18 E.	4	138·9
September ...	10	340	524	1028	1049	1965	479	407	224	152	5	8	17	17	32	8	7	4	2	40	S. 30 E.	4	131·7
October	10	1292	440	582	509	1039	487	941	928	173	20	7	9	8	16	8	15	14	3	13	N. 53 W.	4	87·0
November ...	10	2499	521	336	126	155	283	736	1421	51	41	9	5	2	2	5	12	23	1	58	N. 19 W.	4	82·1
December ...	10	2380	399	173	58	299	262	846	1658	53	39	6	3	1	5	4	14	27	1	60	N. 27 W.	4	91·2
Year.....	...	...	...	...	...	...	...	...	...	...	14	5	8	10	31	11	10	10	1	...	.....	...	.....

III. Berhampore.

	Observed.										Per cent.									Resultant.		Mean diurnal movement.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.	Years.	Miles.
January	3	73	47	11	11	5	28	83	114	...	20	13	3	3	1	7	22	41	...	57	N. 40 W.	2	37·2
February ...	3	46	30	14	17	29	45	81	71	...	14	9	4	5	9	14	24	21	...	38	N. 68 W.	2	41·4
March	3	24	19	10	24	39	97	116	43	...	6	5	3	6	11	26	31	12	...	51	S. 72 W.	2	56·5
April	3	14	33	43	38	100	64	47	21	...	4	9	12	10	28	18	13	6	...	32	S. 7 W.	2	88·7
May	3	11	44	80	71	88	28	36	14	...	3	12	21	19	24	7	10	4	...	40	S. 43 E.	2	99·7
June	3	12	41	63	68	118	24	23	11	...	3	11	18	19	33	7	6	3	...	47	S. 35 E.	2	128·9
July	3	6	51	90	86	96	28	6	2	6	2	14	24	23	26	7	2	...	2	56	S. 49 E.	2	99·4
August	3	11	84	65	73	66	40	15	4	14	3	22	17	20	18	11	4	1	4	41	S. 59 E.	2	78·1
September ...	3	24	77	71	84	58	20	4	5	21	7	21	20	23	16	5	1	1	6	49	S. 76 E.	2	75·6
October	3	50	68	39	39	29	34	49	37	27	13	18	11	11	8	9	13	10	7	13	N. 19 E.	2	47·7
November ...	3	106	21	16	18	8	25	59	108	...	29	6	4	5	2	7	16	30	...	56	N. 34 W.	2	29·0
December ...	3	121	32	10	2	2	17	57	131	...	32	9	3	0·5	0·5	5	15	35	...	71	N. 30 W.	2	28·1
Year	...	...	...	...	...	...	...	...	...	...	11	12	12	12	15	10	13	14	1	...	.....	...	...
Berhampore, 1 year (calms recorded).																							
January	...	42	21	4	1	6	5	17	20	8	34	17	3	1	5	4	14	16	6	50	N. 14 W.	...	...
February ...	...	16	11	6	3	3	15	27	11	24	14	9	5	3	3	13	23	9	21	29	N. 61 W.	...	...
March	...	10	5	1	3	6	23	34	16	26	8	4	1	2	5	19	27	53	21	44	W.	...	...
April	...	8	8	14	10	11	25	14	13	17	6·5	6·5	12	8	9	21	12	11	14	16	S. 47 W.	...	...
May	...	2	8	42	27	17	15	5	1	7	1	6	34	22	14	12	4	1	6	51	S. 52 E.	...	...
June	...	1	14	34	33	23	14	...	...	...	1	12	28	28	19	12	...	...	...	61	S. 51 E.	...	...
July	...	3	42	12	15	30	16	...	...	6	2	34	10	12	24	13	...	...	5	37	S. 65 E.	...	...
August	...	7	63	10	5	6	10	8	1	14	6	51	8	4	5	8	6	1	11	45	N. 49 E.	...	...
September ...	...	17	60	5	5	6	3	...	3	21	14	50	4	4	5	2·5	...	2·5	18	59	N. 42 E.	...	...
October	...	26	47	3	...	...	7	6	8	27	21	38	2	...	...	6	5	6	22	53	N. 18 E.	...	...
November ...	...	17	5	3	...	3	9	26	18	39	14	4	2	...	2	8	22	15	33	39	N. 56 W.	...	...
December ...	...	27	10	2	1	2	5	19	29	29	22	8	2	1	2	4	15	23	23	49	N. 34 W.	...	...
Year	...	...	...	...	...	...	...	...	...	...	12	20	9	7	8	10	11	8	15	...	.....	...	...

TABLE I. (continued).

GANGETIC DELTA.

IV. Dacca.

	Observed.										Per cent.									Resultant.		Mean diurnal movement.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.	Years.	Miles.
January	3	59	39	10	12	34	17	63	100	34	16	11	3	3	9	5	17	27	9	40	N. 45 W.	3	46·2
February ...	3	32	20	11	21	53	59	73	62	8	10	6	3	6	16	17	22	18	2	37	S. 80 W.	3	60·2
March.....	3	12	16	18	36	97	101	49	38	2	3	4	5	10	26	28	13	10	1	48	S. 32 W.	3	92·8
April	3	26	17	26	57	146	50	13	24	...	7	5	7	16	40	14	4	7	...	48	S. 5 E.	3	149·2
May	3	10	22	38	113	135	36	9	5	1	3	6	10	31	37	10	2	1	...	65	S. 24 E.	3	167·6
June	3	1	8	19	143	136	42	7	3	...	...	2	5	40	38	12	2	1	...	78	S. 18 E.	3	228·6
July	3	...	...	20	166	159	20	2	3	...	...	...	5	45	43	5	1	1	...	88	S. 23 E.	3	213·7
August	3	1	7	11	96	174	55	11	8	7	...	2	3	26	47	15	3	2	2	75	S. 6 E.	3	159·0
September ...	3	3	12	38	114	104	40	7	9	18	1	3	11	33	30	12	2	3	5	64	S. 23 E.	3	129·0
October	3	32	30	26	50	54	34	6	69	62	9	8	7	14	15	9	2	19	17	3	S. 23 E.	3	62·2
November ...	3	76	67	18	22	13	21	23	63	51	21	19	5	6	4	6	7	18	14	36	N. 1 W.	3	37·4
December ...	3	77	45	7	12	13	16	44	103	51	21	12	2	3	4	4	12	28	14	47	N. 28 W.	3	42·7
Year	...	...	...	...	...	...	...	...	...	...	8	7	6	19	26	11	7	11	5	...	.....	...	...

ASSAM.

I. Goalpara.

January	3	10	36	159	27	13	30	65	16	...	3	10	45	8	4	8	18	4	...	30	S. 86 E.	3	90·2
February ...	3	12	43	138	30	9	34	58	12	...	3·5	13	41	9	3	10	17	3·5	...	30	S. 88 E.	3	129·0
March.....	3	14	47	129	13	8	22	67	10	...	5	15	42	4	2	7	22	3	...	27	N. 75 E.	3	167·9
April	3	8	34	156	22	9	20	36	15	...	3	11	52	7	3	7	12	5	...	45	N. 88 E.	3	196·2
May	3	10	29	221	31	21	18	25	17	...	3	8	59	8	6	5	7	4	...	58	S. 86 E.	3	173·0
June	3	18	34	132	25	33	37	55	28	...	5	9	37	7	9	10	15	8	...	21	S. 78 E.	3	127·1
July	3	11	20	115	29	55	46	74	20	...	3	5	31	8	15	13	20	5	...	20	S. 23 E.	3	91·7
August	3	13	22	107	40	57	48	60	23	...	4	6	29	11	15	13	16	6	...	23	S. 29 E.	3	106·1
September ...	3	18	32	130	30	34	40	50	23	...	5	9	36	8	10	11	14	7	...	23	S. 71 E.	3	126·7
October	3	5	34	174	40	33	31	41	14	...	1	9	47	11	9	8	11	4	...	43	S. 74 E.	3	97·7
November ...	3	8	35	190	25	17	21	34	14	17	2	10	52	7	5	6	9	4	5	63	S. 88 E.	3	102·1
December ...	3	14	42	191	51	13	21	24	10	...	4	11	52	14	3	6	7	3	...	58	S. 86 E.	3	84·2
Year	...	...	...	...	...	...	...	...	...	...	3	10	44	8	7	9	14	5	...	...	.....	...	...

II. Seebsaugor.

	Observed.										Per cent.									Resultant.		Mean diurnal movement.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.	Years.	Miles.
January	3	122	110	91	...	12	6	31	...	...	33	30	24	...	3	2	8	...	...	61	N. 36 E.	...	...
February ...	3	125	27	69	...	8	16	29	...	...	45	10	25	...	3	6	11	...	...	40	N. 33 E.	...	...
March.....	3	126	103	48	...	13	54	18	...	...	35	28	13	...	4	15	5	...	...	44	N. 27 E.	...	...
April	3	116	94	55	...	17	67	11	...	...	32	26	15	...	5	19	3	...	...	37	N. 28 E.	...	...
May	3	138	88	13	...	13	104	16	...	...	37	24	3·5	...	3·5	28	4	...	...	45	N. 8 W.	...	...
June	3	99	53	4	...	36	150	18	...	...	27	15	1	...	10	42	5	...	...	23	S. 86 W.	...	...
July	3	37	27	1	...	88	179	40	...	...	10	7	...	...	24	48	11	...	...	58	S. 43 W.	...	...
August	3	35	21	5	...	106	167	38	...	...	9	6	1	...	29	45	10	...	...	59	S. 38 W.	...	...
September ...	3	68	95	5	...	51	110	31	...	...	19	26	1	...	14	31	9	...	...	10	N. 80 W.	...	...
October	3	124	145	33	...	35	21	14	...	...	33	39	9	...	9	6	4	...	...	56	N. 31 E.	...	...
November ...	3	131	160	55	...	12	2	...	...	...	36	45	15	...	3	1	...	...	...	80	N. 36 E.	...	...
December ...	3	122	180	62	...	4	4	...	...	...	33	48	17	...	1	1	...	...	...	82	N. 38 E.	...	...
Year	...	...	...	...	...	...	...	...	...	...	29	25	11	...	9	20	6	...	...	...	.....	...	...

Arakan Coast.

I. Chittagong.

January	3	159	76	49	8	4	29	92	70	...	33	15	10	1	1	6	20	14	...	19·4	N. 34 W.	3	134·4
February ...	3	81	53	50	17	36	62	79	65	...	18	12	11	4	8	14	18	15	...	22·6	N. 43 W.	3	131·8
March.....	3	70	49	28	23	64	116	95	47	...	14	10	6	5	13	24	19	9	...	31·0	S. 79 W.	3	143·6
April	3	33	27	40	51	120	122	63	21	...	7	6	8	11	25	26	13	4	...	39·5	S. 21 W.	3	189·4
May	3	25	23	60	78	105	131	56	15	1	5	5	12	16	21	27	11	3	...	40·9	S. 6 W.	3	188·9
June	3	11	19	96	157	120	57	15	3	1	2	4	20	33	25	12	3	1	...	61·4	S. 33 E.	3	199·7
July	3	1	13	131	167	125	42	8	...	6	0	3	27	34	25	8	2	...	1	69·6	S. 40 E.	3	195·2
August	3	8	30	118	141	128	80	27	4	7	1	5	22	26	24	15	5	1	1	54·1	S. 31 E.	3	160·7
September ...	3	15	41	121	110	96	94	26	11	24	3	8	23	20	18	17	5	2	4	42·3	S. 34 E.	3	132·8
October	3	67	82	85	58	41	61	91	51	20	12	15	15	10	7	12	16	9	4	6·9	N. 21 E.	3	102·6
November ...	3	118	135	58	18	9	34	83	69	15	22	25	11	3	2	6	15	13	3	40·2	N. 2 E.	3	106·2
December ...	3	201	63	43	8	9	34	99	96	3	36	11	8	1	2	6	18	17	1	52·4	N. 19 W.	3	120·2
Year	...	...	...	...	...	...	...	...	...	...	13	10	14·5	14	14	14·5	12	7	1	...	.....	...	...

TABLE I. (continued.)

ARAKAN COAST.

II. Akyab.

	Observed.										Per cent.									Resultant.	
	Years.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm.	Per cent.	Direction.
January	3	45	92	26	21	12	17	71	78	6	12	25	7	6	3	4	19	23	1	44·7	N. 14 W.
February	3	40	94	25	4	2	8	43	113	19	11	27	7	1	1	2	12	33	6	51·5	N. 11 W.
March	3	39	81	48	24	7	16	75	145	46	8	17	10	5	1	3	16	30	10	36·7	N. 22 W.
April	3	29	36	34	28	29	54	86	100	6	7	9	9	7	7	13	21	25	2	30·3	N. 72 W.
May	3	19	27	42	55	49	89	63	63	22	4	6	10	13	11	21	15	15	5	22·8	S. 46 W.
June	3	6	15	33	74	96	126	10	22	16	1	4	8	19	24	32	2	6	4	51·7	S. 5 W.
July	3	1	6	40	166	169	97	6	7	2	0	1	8	33	34	20	1	2	1	71·7	S. 13 E.
August	3	11	13	48	119	119	138	26	16	4	2	3	10	24	24	28	5	3	1	54·5	S. 1 E.
September	3	17	28	69	105	140	78	37	31	8	3	5	14	21	27	15	7	6	2	42·2	S. 13 E.
October	3	24	78	87	62	62	71	68	49	17	5	15	17	12	12	14	13	9	3	10·4	S. 38 E.
November	3	88	110	37	37	14	42	84	91	3	17	22	7	7	3	8	17	18	1	32·5	N. 13 W.
December	3	91	142	24	19	2	23	81	82	88	17	26	4	3	0	4	15	15	16	30·4	N. 6 W.
Year	...	...	...	...	...	...	...	...	...	...	7	13	9	13	12	14	12	16	4	.....	.....

Table II. MONTHLY MEANS OF OBSERVED TEMPERATURES.

	Elevation in feet.	Years.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Method of reduction.
Rawulpindee ...	1700?	3	50·9	54·9	60·4	71·9	85·6	92·1	89·5	86	84·1	72·8	61·2	52·7	Mean of max. and min. corr. by Roorkee.
Mooltan	400?	7	52·9	59·4	68·8	80·3	89·9	92	92·6	89·5	85·7	75	64·6	55·5	„ ditto.
Lahore	700?	3	53·5	58·6	66·3	79·7	90·6	93·4	89·5	88	85·6	78·1	65·7	55·7	„ ditto.
Roorkee.....	880	5	57·6	62·3	69·6	80·4	88·8	89	84·5	84·2	83	76	63·7	57·5	„ 4^h, 10^h, 16^h, 22^h.
Agra	550	4	58·7	63·9	73·3	84·1	93·6	92·9	87·5	84·7	83·9	79·1	69·3	61·8	„ min. and 16^h.
Benares	263	3–5	58·5	67·8	75·2	86·2	91·2	90·2	85·9	85·2	84	79·2	68·5	59·1	„ 4^h, 10^h, 16^h, 22^h (part min. and 16^h.)
Patna	179	4	60·7	66·2	76·6	83·7	87	87·2	84·1	83·4	82·3	78·9	68·9	61·8	„ ditto.
Ajmere	1800	4	58·3	65	72·9	84·4	92·4	91·5	86	83·3	84·9	79·1	70·3	63·8	„ min. and 16^h.
Jhansi.....	900	4	60·8	68·3	75·4	85·6	95·1	90·9	83·6	82·3	82	77·9	69·6	63	„ ditto.
Jubbulpore.....	1353	3	62·9	67·2	73·5	84·2	90·2	87·4	78·6	77·9	78·7	75	67·6	62·7	„ ditto.
Nagpore	1025	3	70·5	75·1	81·1	87·3	93·8	86	78·4	78·9	79	77·5	72·3	68·7	„ ditto.
Hoshungabad...	1030	2–3	65·9	72·1	77·5	88·5	93·4	95·2	79·2	78·9	79·8	78·7	73·1	69·8	„ ditto.
Bombay	20	18	73·7	75·8	77·4	83	85·5	83·2	81·3	80·4	80·3	81·6	79·4	76	„ hourly observations.
Monghyr	160	4	62·8	68·5	77·1	84·7	86·8	86·2	84·1	83·8	82·8	80·4	71·4	63·5	„ 4^h, 10^h, 16^h, 22^h.
Hazareebagh ...	2014	4	61·7	66·2	74·3	82·6	85·9	81·4	79·2	77·9	77·3	74·5	68·9	61·6	„ ditto.
Cuttack	80	4	70·2	74·4	80·1	86	89	85·8	84	84	83·2	81·5	74·6	69·6	„ ditto.
False Point.....	19	4	71·2	75·2	80·1	84	87·1	86·3	85·3	85·1	85·2	84·1	77·4	70·9	„ ditto.
Saugor Island...	6	4	69	74	79·9	83·9	85·6	85·1	84·1	83·5	83·1	81·2	75	68·2	„ ditto.
Calcutta	18	16	68·8	73·3	79·6	83·6	85	83·7	83·1	83·2	82·6	81·7	75·1	68·5	„ hourly observations.
Berhampore ...	65	4	65·3	70·7	78·2	85·5	86·3	84·6	84	84·1	83·4	81·7	73·5	66·2	„ 4^h, 10^h, 16^h, 22^h.
Dacca.....	20	4	67·5	72·8	79·8	82	83·6	83·5	83·2	83·8	83·2	81·6	74·9	68·5	„ ditto.
Cachar	89	2–3	64·4	68·2	74·5	78·2	81·1	81·9	82·1	81·6	81·4	79·8	72·1	65	„ ditto.
Chittagong.....	90	4	67·6	71·7	77·9	81·2	83	81·6	80·9	81·3	81·5	80·5	74·2	67·9	„ ditto.
Akyab	21	2–4	70·5	74·6	78·9	83·6	84·4	82·3	81	81·6	82	81·5	77·8	72	„ ditto.
Darjeeling	6941	4	42·9	44·5	50·4	56·1	60·2	63·3	63·9	64	62·1	58	50·2	44	„ 4^h, 10^h, 16^h, 22^h.
Goalpara	386	3	64·3	67·4	73	77·1	78·2	79·9	81	81·3	80	78·3	71·1	64·8	„ ditto.
Shillong	4792	3	51·3	54·2	61·4	64·5	68·4	69·4	69·4	69·2	67·3	63·5	56	50·1	„ ditto.
Seebsaugor.....	500?	3	58·6	62·6	66·8	73·4	80	83·8	84·8	84·6	83·2	79·8	69·9	61·3	„ sunrise, 10^h, 16^h, 22^h.
Chuckrata	6884	2–3	42·6	44·5	50·2	59·5	68·7	69·5	65·7	65·1	62·2	59·2	53·5	44·2	„ 4^h, 10^h, 16^h, 22^h.

TABLE III. MEAN TEMPERATURES AT SEA-LEVEL.

(Reduced from TABLE II.)

	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
Rawulpindee	55·7	59·7	65·2	76·7	90·2	96·9	94·3	90·8	88·9	77·6	66	57·5
Mooltan	54	60·5	69·9	81·4	91	95·1	93·7	90·6	86·8	76·1	65·7	56·6
Lahore	55·5	60·6	68·3	81·1	92·6	95·4	91·5	90	87·6	80·1	67·7	57·7
Roorkee	60·1	64·8	72·1	82·9	91·3	91·5	87	86·7	85·5	78·5	66·2	60
Agra	60·2	65·4	74·8	85·6	95·1	94·4	89	86·2	85·4	80·6	70·8	63·3
Benares	59·2	68·5	75·9	86·9	91·9	90·9	86·6	85·9	84·7	79·9	69·2	59·8
Patna	61·2	66·7	77·1	84·2	87·5	87·7	84·6	83·9	82·8	79·4	69·4	62·3
Ajmere	63·4	70·1	78	89·5	97·5	96·6	91·1	88·4	90	84·2	75·4	68·9
Jhansi	63·4	70·9	78	88·2	97·3	93·5	86·2	84·9	84·6	80·5	72·2	65·6
Jubbulpore	66·8	71·1	77·4	88·1	94·1	91·3	82·5	81·8	82·6	78·9	71·5	66·6
Nagpore	73·4	78	84	90·2	96·7	88·9	81·3	81·8	81·9	80·4	75·2	71·6
Hoshungabad	68·9	75·1	80·5	91·5	96·4	98·2	82·2	81·9	82·8	81·7	76·1	72·8
Bombay	73·7	75·8	77·4	83	85·5	83·2	81·3	80·4	80·3	81·6	79·4	76
Monghyr	63·2	68·9	77·5	85·1	87·2	86·6	84·5	84·2	83·2	80·8	71·8	63·9
Hazarecbagh	67·4	71·9	80	88·3	91·6	87·1	84·9	83·6	83	80·2	74·6	67·3
Cuttack	70·4	74·6	80·3	86·2	89·2	86	84·2	84·2	83·4	81·7	74·8	69·8
False Point	71·2	75·2	80·1	84	87·1	86·3	85·3	85·1	85·2	84·1	77·4	70·9
Saugor Island	69	74	79·9	83·9	85·6	85·1	84·1	83·5	83·1	81·2	75	68·2
Calcutta	67·6	72·8	80·5	84·5	86	84·9	83·5	83	83·2	81·4	74·7	67·8
Berhampore	65·5	70·9	78·4	85·7	86·5	84·8	84·2	84·3	83·6	81·9	73·7	66·4
Dacca	67·5	72·8	79·8	82	83·6	83·5	83·2	83·8	83·2	81·6	74·9	68·5
Cachar	64·7	69·6	74	78·9	82·1	81·9	82·3	81·8	81·6	80	72·3	65·2
Chittagong	67·9	71·9	78·2	81·5	83·3	81·9	81·2	81·6	81·8	80·8	74·5	68·2
Goalpara	65·4	68·5	74·1	78·2	79·3	81	82·1	82·4	81·1	79·4	72·2	65·9
Seebsaugor	60	64	68·2	74·8	81·4	85·2	86·2	86	84·6	81·2	71·3	62·7

TABLE IV. MONTHLY MEAN VAPOUR-TENSIONS.

	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
Rawulpindee	0·227	0·271	0·304	0·406	0·392	0·511	0·651	0·658	0·537	0·370	0·324	0·231
Mooltan	0·208	0·248	0·330	0·371	0·293	0·479	0·611	0·638	0·504	0·373	0·292	0·237
Lahore	0·283	0·310	0·406	0·448	0·488	0·579	0·777	0·754	0·686	0·430	0·341	0·280
Roorkee	0·299	0·365	0·390	0·404	0·488	0·737	0·911	0·891	0·823	0·537	0·336	0·293
Benares	0·339	0·373	0·393	0·475	0·585	0·864	0·977	0·943	0·943	0·657	0·438	0·325
Patna	0·328	0·347	0·402	0·484	0·639	0·851	0·969	0·948	0·914	0·729	0·466	0·342
Jubbulpore	0·326	0·359	0·412	0·398	0·439	0·714	0·654	0·687	0·725	0·633	0·466	0·341
Nagpore	0·447	0·478	0·583	0·504	0·539	0·769	0·776	0·749	0·782	0·631	0·594	0·462
Bombay	0·586	0·631	0·727	0·825	0·890	0·933	0·921	0·884	0·869	0·855	0·709	0·623
Hazareebagh	0·280	0·264	0·330	0·423	0·557	0·796	0·867	0·812	0·786	0·563	0·360	0·279
Cuttack	0·509	0·569	0·676	0·831	0·941	0·949	0·954	0·954	0·941	0·827	0·581	0·484
False Point	0·603	0·698	0·831	0·978	1·079	1·040	1·020	1·013	0·972	0·911	0·647	0·551
Saugor Island	0·531	0·662	0·813	0·939	1·005	1·025	1·016	1·008	0·995	0·893	0·659	0·523
Calcutta	0·487	0·548	0·695	0·805	0·893	0·947	0·954	0·950	0·950	0·828	0·613	0·489
Berhampore	0·417	0·442	0·511	0·769	0·877	0·985	1·001	0·993	0·970	0·832	0·585	0·450
Dacca	0·437	0·483	0·660	0·774	0·908	0·974	0·976	0·995	0·965	0·851	0·596	0·459
Cachar	0·479	0·538	0·637	0·749	0·913	0·950	0·974	0·969	0·952	0·874	0·652	0·500
Chittagong	0·479	0·566	0·715	0·861	0·924	0·937	0·926	0·949	0·934	0·863	0·650	0·497
Akyab	0·581	0·650	0·739	0·885	0·955	0·947	0·929	0·926	0·938	0·902	0·761	0·604
Darjeeling	0·212	0·238	0·252	0·338	0·427	0·529	0·546	0·548	0·501	0·381	0·276	0·213
Goalpara	0·439	0·435	0·487	0·632	0·790	0·897	0·919	0·916	0·900	0·782	0·593	0·459
Shillong	0·265	0·273	0·321	0·394	0·534	0·598	0·629	0·627	0·588	0·503	0·341	0·271

N.B.—The values for Calcutta and Bombay are obtained from hourly observations with the psychrometer, each observation being reduced separately, and are therefore as accurate as are obtainable with that instrument. The remainder are computed directly from the figures in Tables II. and V., and represent only a rough approximation to the real values, which will be somewhat higher.

TABLE V. MONTHLY MEAN HUMIDITY.

(Saturation =100.)

	Years.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
Rawulpindee	5	61	63	58	52	32	34	47	53	46	46	60	58
Mooltan	5	52	49	47	36	28	30	40	46	41	43	48	54
Lahore	5	69	63	63	46	34	37	56	57	56	46	54	63
Roorkee	4	63	65	54	39	36	54	77	76	73	60	57	62
Benares	4–5	69	55	45	38	40	61	79	80	81	66	63	65
Patna	3–4	62	54	44	42	53	66	83	83	83	74	66	62
Jubbulpore	3	57	54	50	34	31	55	67	72	74	73	69	60
Nagpore	3	60	55	55	39	34	62	80	76	79	67	75	66
Bombay	18	70	72	77	73	73	82	86	85	84	79	70	69
Hazareebagh	3–4	51	41	39	38	45	72	87	85	84	66	51	51
Cuttack	4	69	67	65	67	69	77	82	82	83	77	68	67
False Point	4	79	80	81	84	84	83	84	84	82	78	69	73
Saugor Island	3–4	75	79	80	81	82	85	87	88	88	84	76	76
Calcutta	16	71	68	67	69	73	81	85	86	85	78	73	72
Berhampore	3–4	67	59	53	63	70	83	86	85	85	77	71	70
Dacca	4	65	60	65	71	79	85	86	86	85	79	79	66
Cachar	2–3	79	75	74	76	84	88	89	90	89	86	83	81
Chittagong	3–4	71	73	75	81	82	87	88	89	87	83	77	73
Akyab	3–4	78	76	75	77	81	86	88	86	86	84	80	77
Darjeeling	4	77	81	69	75	82	91	92	92	90	79	76	74
Goalpara	3	73	65	60	68	82	88	87	86	88	83	78	75
Shillong	3	70	65	59	65	77	84	87	88	88	86	76	75
Chuckrata	3–4	51	55	60	42	44	67	90	92	86	58	44	50

N.B.—The mean humidities of the three Punjab stations are obtained from observations of the psychrometer at 10 A.M. and 4 P.M. multiplied by factors deduced from the Roorkee registers. Those of Nagpore and Jubbulpore from similar data, multiplied by factors deduced from the Benares registers. Those of Bombay and Calcutta are the means of hourly observations, and all others are the means of observations recorded daily at 4^h, 10^h, 16^h, and 22^h.

TABLE VI. AVERAGE MONTHLY AND ANNUAL RAINFALL IN NORTHERN INDIA &c.

Arakan, Eastern Bengal, and Assam.

	Years.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	Total.
I. Sandoway	3–5	0·14	.....	0·24	1·35	14·01	55·32	64·50	51·36	30·91	17·09	1·15	...	236·07
II. Akyab	12–14	0·35	0·28	0·53	1·30	10·50	50·91	53·81	36·93	25·67	13·45	5·91	·25	199·89
III. Chittagong	13–15	0·29	1·30	1·39	5·08	8·76	22·68	21·95	22·50	12·99	7·17	1·93	·46	106·50
IV. Noakhally	13–15	·36	·72	1·75	4·05	9·09	21·92	17·09	20·12	13·77	8·71	1·54	·03	101·15
V. Tipperah	11–12	·64	·89	2·29	8·34	11·92	20·34	16·81	13·98	10·69	6·76	1·77	·04	94·47
VI. Cachar	11–13	·42	3·26	5·67	12·09	15·01	19·01	23·71	16·79	14·03	7·88	·88	·66	119·41
VII. Silhet	12–15	·13	1·45	4·56	13·93	23·68	30·42	25·56	26·23	4·99	9·87	·71	·14	141·67
VIII. Cherra Punji	5–12	·52	3·34	4·03	34·67	64·34	115·19	150·38	75·93	58·23	18·05	2·58	...	527·26
IX. Shillong	5–6	·14	·59	1·51	3·66	9·31	17·57	13·68	9·84	17·29	6·07	1·63	·16	81·45
X. Seebsaugor	12–14	1·21	2·39	3·85	9·78	11·20	15·41	16·36	15·48	11·17	4·64	1·47	·65	93·61
XI. Nazeerah	9–10	1·51	2·33	2·83	9·18	7·77	14·22	13·56	15·75	10·27	4·67	·72	·75	83·56
XII. Tezpore	10–12	·56	1·00	1·10	7·17	10·28	14·90	15·41	13·26	8·64	3·10	·94	·75	77·11
XIII. Nowgong	10–12	1·14	1·29	2·52	6·04	10·97	12·58	15·43	15·44	12·28	4·64	·61	·31	83·25
XIV. Gowhatty	10–12	·56	1·37	1·43	7·04	10·52	13·73	12·06	11·10	7·87	3·06	·38	·11	69·23
XV. Goalpara	7–8	·31	·64	1·83	5·15	13·09	25·70	21·19	12·46	11·46	5·43	·37	·15	97·78

Gangetic Delta and Northern Bengal.

XVI. Mymensing	7–8	·26	1·27	1·37	8·33	14·26	24·13	20·77	14·99	13·62	5·97	·13	...	105·10
XVII. Dacca	11–12	·62	·76	1·46	7·00	9·94	14·12	13·15	12·43	8·89	5·97	·89	·06	75·29
XVIII. Bogra	8–11	·32	·90	1·00	5·05	9·33	17·62	20·25	12·72	15·71	5·90	1·50	·09	90·39
XIX. Rungpore	10–12	·13	·28	1·00	2·78	10·01	24·33	19·07	13·58	12·07	4·71	·36	·13	88·45
XX. Dinajpore	10–12	·12	·66	·87	2·90	8·64	20·50	17·47	14·19	14·02	5·94	·21	·02	85·54
XXI. Maldah	14–16	·80	1·52	1·63	1·93	3·42	8·80	10·19	9·73	10·67	4·60	·24	·51	54·04
XXII. Rampore Beauleah	10–12	·13	1·14	1·41	2·25	5·79	11·76	13·36	9·93	10·63	5·39	·34	·06	62·19
XXIII. Berhampore	14–16	·33	·81	1·15	2·37	4·22	9·65	10·53	9·87	9·31	5·75	·21	·10	54·30
XXIV. Jessore	10–14	·24	·41	1·96	3·94	7·33	13·52	11·09	11·09	9·25	6·36	·86	·01	66·06
XXV. Kishnagur	8–12	·52	·82	·90	4·51	7·71	11·57	9·56	9·78	7·61	4·90	·35	·18	58·41
XXVI. Burdwan	12–15	·57	1·03	1·47	2·02	4·20	10·67	12·93	11·98	8·98	5·46	·51	·49	60·31
XXVII. Calcutta	32	·53	·72	1·22	2·25	5·43	11·80	13·33	14·18	10·36	5·31	·67	·24	66·04
XXVIII. Saugor Island	4–5	·05	·82	1·73	2·26	5·77	15·09	15·21	16·85	17·43	11·46	·94	...	87·61

TABLE VI. (continued).

Orissa and Plateau of Western Bengal.

	Years.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	Total.
XXIX. Pooree	14–15	·09	1·37	·71	1·81	2·37	7·79	9·29	12·01	9·58	7·26	1·22	·79	54·29
XXX. False Point	6	·02	·77	1·77	2·72	2·70	14·08	13·67	11·22	11·92	12·06	3·47	...	74·45
XXXI. Cuttack	11–14	·49	·48	1·09	1·74	1·63	9·24	11·09	10·63	9·03	5·48	·97	·65	52·52
XXXII. Balasore	10–12	1·09	1·15	1·97	3·15	4·50	12·08	9·06	12·12	13·74	7·25	·89	...	67·00
XXXIII. Midnapore	6–8	·89	·39	2·20	1·72	5·99	14·02	11·62	11·40	9·79	6·27	·48	...	64·77
XXXIV. Bancoorah	14–16	·40	·86	1·49	1·97	3·09	10·09	12·44	10·17	8·13	4·18	·19	·08	53·09
XXXV. Raneegunj	5	·22	·39	1·25	1·05	2·73	10·70	14·56	11·98	9·83	2·51	·14	...	55·36
XXXVI. Soory	9	·47	·53	·80	·81	2·39	8·81	12·41	12·15	8·97	3·81	·13	·18	51·46
XXXVII. Purulia	7–10	·31	·55	·79	·78	1·13	9·22	8·75	11·17	6·64	4·59	·05	·23	44·21
XXXVIII. Hazareebagh	6–7	·30	·41	·80	·46	1·38	9·61	14·68	13·16	8·36	3·54	·13	·08	52·91
XXXIX. Ranchee	14–16	·94	·91	1·63	·56	1·47	6·52	10·37	9·76	6·65	3·22	·12	·10	42·25
Gangetic Plains (Behar and North-western Provinces).														
XL. Bhagulpore	15–16	·50	·74	·44	1·02	2·10	8·44	11·60	10·72	8·28	4·97	·03	·08	48·92
XLI. Monghyr	15–16	·37	·61	·54	·44	1·61	6·21	10·82	8·16	7·73	3·78	·05	·11	40·43
XLII. Gya	8–10	·87	·50	·63	·58	·94	7·01	12·51	9·18	7·35	3·44	·05	·08	43·14
XLIII. Patna	9–10	·49	·79	·27	·49	1·12	6·78	10·61	7·44	6·35	2·66	·10	·06	37·16
XLIV. Arrah	13–16	·88	·54	·65	·89	1·35	7·64	13·90	10·77	10·53	3·05	·24	·07	50·51
XLV. Chuprah	13–16	·68	·50	·47	·56	1·29	6·76	9·14	8·45	7·14	2·87	·02	...	37·88
XLVI. Mozufferpore	11–13	·59	·51	·58	·46	1·89	6·85	10·54	9·98	8·66	3·70	·03	...	43·79
XLVII. Chumparun	8–10	·24	·31	·90	·35	1·80	8·04	10·61	11·37	7·66	3·66	...	·11	45·05
XLVIII. Benares	8–9	·74	·42	·33	·17	·57	4·24	12·43	5·99	6·99	2·13	·02	...	34·03
XLIX. Goruckpore	5	·72	·48	·48	·26	·98	8·19	15·12	9·74	7·60	5·64	...	·04	49·25
L. Lucknow	5	1·56	·05	·39	·30	·62	4·34	16·55	7·85	9·95	2·55	...	·29	44·45
LI. Agra	8–9	·48	·29	·50	·17	·30	1·81	10·02	7·76	3·17	·60	...	·20	25·30
LII. Delhi	6	·87	·40	1·57	·63	·76	2·63	6·85	6·14	1·93	·28	...	·58	22·64
LIII. Bareilly	4–5	·92	1·41	1·00	·46	·29	5·55	13·61	8·44	8·64	1·47	...	·35	42·14
LIV. Roorkee	8	1·62	2·39	1·59	·54	·74	4·06	9·91	9·86	4·30	·75	·34	·78	36·88
LV. Umbala	6	1·27	2·97	1·42	1·59	1·00	4·05	9·30	6·62	3·10	·23	...	·10	31·55

Rajpootana, Bundelkund, and Nerbudda Valley.														
LVI. Nimar	5	0·12	.....	0·07	0·04	·24	9·39	13·75	9·04	6·18	·96	·02	·07	39·88
LVII. Hoshungabad	10	·35	·21	·20	·05	·60	4·50	15·19	13·89	9·31	·77	·71	·28	46·06
LVIII. Nursingpore	10	·03	·16	·21	·15	·52	6·80	14·08	10·93	7·58	1·19	·25	·28	42·18
LIX. Jubbulpore	10	·54	·54	·57	·09	·08	8·96	18·96	12·45	7·86	1·35	·13	·26	51·79
LX. Saugor	14	·27	·78	·30	·11	·50	9·20	15·54	12·32	8·24	1·12	·32	·86	49·56
LXI. Jhansi	4–7	2·04	·24	·47	·02	·25	4·60	14·13	6·17	4·12	1·77	...	·38	34·19
LXII. Ajmere	6–7	1·35	·21	·48	·26	·19	1·13	5·74	9·50	2·82	·19	...	·39	22·26
Central India (South of Satpooras).														
LXIII. Wardha	5	·98	.....	·28	·46	·30	5·72	10·31	11·32	6·58	1·72	·12	·07	37·86
LXIV. Nagpore	13	1·01	·49	·94	·44	·50	8·73	11·33	9·50	8·32	2·75	·60	·11	44·72
LXV. Chindwara	5	1·03	.....	1·33	·18	·35	8·98	10·64	8·86	6·68	2·37	...	·17	40·59
LXVI. Seoni	9	·45	1·02	·72	·49	·48	8·14	14·93	10·65	6·26	1·73	·41	·61	45·89
LXVII. Chanda	5	·05	·21	1·81	·37	·31	8·16	15·96	11·18	8·86	2·45	·30	·16	49·82
LXVIII. Raipore	5	·91	.....	1·39	·70	·22	11·41	14·66	11·94	8·42	4·77	2·74	...	57·16
LXIX. Belaspore	5	·95	·17	1·56	·87	·33	9·72	10·87	10·46	4·59	1·93	·29	·18	41·92
LXX. Sumbulpore	7–10	·56	·61	·59	·28	1·04	10·04	13·05	10·63	7·44	4·38	·06	·12	48·80
Western India (South of Satpooras).														
LXXI. Bombay	23	·05	·01	.....	·05	·56	20·84	24·26	13·16	9·64	2·20	·47	·05	71·30
LXXII. Mahableshwar	15	·05	·25	·15	1·31	3·31	46·53	92·10	72·33	31·32	4·58	2·07	·05	254·05
LXXIII. Poonah	7	·32	.....	·14	·25	1·36	5·65	6·50	3·13	4·34	4·70	·68	·23	27·30
LXXIV. Sattara	4	·14	·04	·04	2·90	1·71	5·46	13·29	8·06	3·38	4·11	2·40	·69	42·15
LXXV. Sholapore	10	·11	·06	·25	1·26	1·78	4·95	3·26	4·57	6·65	3·64	1·25	·12	27·90

TABLE VI. (continued).

Punjab.

	Years.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Total.
LXXVI. Lahore	5–6	·75	1·06	1·16	·57	1·16	·99	4·20	1·18	1·34	·72	...	·44	13·57
LXXVII. Shahpoor	5	·08	·88	·86	·32	·38	1·46	1 70	2·60	·58	·02	...	·44	9·32
LXXVIII. Rawulpindee	5–6	1·57	2·72	2·89	1·85	·87	2·49	6·66	5·40	2·66	·24	...	·94	28·29
LXXIX. Peshawar	5	·64	1·36	1·30	1·80	·24	·28	·72	1·52	1·76	·32	...	·96	10·90
LXXX. Dera Ishmail Khan	5	·20	·64	1·44	·50	·10	1·48	1·04	·85	·44	·02	...	·40	7·11
LXXXI. Mooltan	9	·42	·35	·78	·95	·39	·31	1·62	1·13	·55	·29	·03	·37	7·19
LXXXII. Hissar	6	·34	·62	·61	·24	·18	2·68	4·56	2·68	1·16	...	..	·38	13·45
LXXXIII. Sirsa	6	·23	·52	·74	·26	·34	3·07	3·90	2·12	1·57	...	...	·30	13·05
Himalaya.														
LXXXIV. Buxa Fort	3	·37	1·23	1·63	8·03	27·37	47·37	81·62	58·01	36·10	17·73	·91	·85	280·22
LXXXV. Rungbee	6	·94	1·51	2·56	6·36	10·15	31·78	51·41	34·56	26·82	8·44	·49	·23	175·25
LXXXVI. Darjeeling	9–12	·61	1·48	1·74	3·95	7·22	28·41	29·76	27·57	17·93	7·92	·26	·11	126·96
LXXXVII. Khatmandu	4	0·44	1·79	0·17	2·61	3·11	7·25	12·05	11·86	6·27	4·96	0·47	1·21	52·09
LXXXVIII. Naini Tal	7–8	6·39	4·09	4·47	2·08	2·82	13·20	19·26	20·18	7·88	3·51	·09	2·61	86·58
LXXXIX. Dehra	5	1·58	2·33	2·90	·66	·65	6·44	26·70	20·03	9·83	·81	...	·36	72·29
XC. Simla	6	1·40	·80	3·83	2·35	2·87	9·93	17·72	13·89	4·66	·33	...	·42	58·20
XCI. Kangra	5–6	3·13	6·13	4·78	1·31	1·78	11·80	38·30	28·04	8·94	·48	·32	1·07	106·08
XCII. Hazara	5	2·22	4·90	6·54	2·68	2·10	5·14	8·06	5·90	2·98	1·08	...	2·04	43·64

TABLE VII. MONTHLY MEANS OF OBSERVED ATMOSPHERIC PRESSURES DEDUCED FROM THE REGISTERS OF THE 5 YEARS 1867–71.

	Year.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	Year.
Simla	3	23·225	23·198	23·197	23·207	23·142	23·061	23·069	23·101	23·194	23·227	23·293	23·254	23·185
Roorkee	5	29·125	29·048	28·985	28·878	28·747	28·632	28·638	28·695	28·792	28·960	29·104	29·154	28·896
Agra	4	·509	·437	29·358	29·243	29·098	·982	·931	29·035	29·133	29·321	·477	·522	29·250
Lucknow	3	·694	·608	·527	·403	·276	29·161	29·166	·233	·326	·499	·656	·711	·439
Benares	5	·813	·727	·644	·528	·388	·273	·273	·347	·413	·606	·764	·820	·550
Patna	4–5	·849	·781	·687	·578	·476	·349	·361	·431	·511	·694	·857	·886	·622
Jhansi	3–4	·188	·147	·028	28·927	28·795	28·662	28·667	28·753	28·837	·019	·174	·206	28·950
Jubbulpore	3	28·611	28·603	28·525	·429	·324	·205	·206	·280	·309	28·481	28·622	28·646	·437
Nagpore	3	·947	·896	·808	·702	·625	·528	·538	·616	·642	·794	·939	·983	·751
Monghyr	3–4	29·860	29·775	29·686	29·587	29·493	29·367	29·376	29·442	29·516	29·686	29·832	29·900	29·627
Hazareebagh	4–5	27·966	27·906	27·851	27·765	27·672	27·558	27·553	27·617	27·677	27·848	27·963	28·005	27·782
Cuttack	4–5	29·985	29·897	29·822	29·708	29·620	29·549	29·547	29·594	29·662	29·821	29·960	30·014	29·765
Saugor Island	5	30·021	·942	·879	·784	·680	·557	·547	·590	·666	·827	·976	·041	·792
Calcutta	5	·021	·945	·869	·774	·667	·552	·551	·610	·679	·840	·982	·042	·793
Berhampore	4	29·948	·876	·790	·676	·568	·442	·467	·541	·624	·769	·908	29·975	·715
Jessore	4–5	·994	·922	·850	·753	·666	·537	·534	·599	·674	·832	·967	30·026	·779
Dacca	4–5	·979	·921	·845	·777	·678	·559	·558	·596	·678	·817	·952	·010	·781
Cachar	2–3	·902	·862	·798	·730	·636	·545	·549	·596	·680	·773	·911	29·939	29·745
Chittagong	5	·908	·851	·788	·726	·642	·537	·537	·572	·634	·755	·879	·933	29·730
Akyab	4–5	·995	·929	·896	·840	·753	·689	·693	·711	·760	·845	·951	30·015	·840
Darjeeling	3–5	23·344	23·322	23·311	23·326	23·273	23·228	23·224	23·261	23·322	23·391	23·424	23·409	23·320
Goalpara	3	29·603	29·522	29·442	29·380	29·286	29·206	29 195	29·247	29·328	29·426	29·567	29·621	29·402
Shillong	3	25·262	25·238	25·211	25·195	25·098	25·060	25·058	25·100	25·172	25·247	25·320	25·324	25·190
Port Blair	2–3	29·719	29·715	29·674	29·630	29·568	29·563	29·574	29·574	29·606	29·639	29·689	29·720	29·639
Madras	4	·967	·943	·898	·808	·720	·671	·706	·734	·754	·827	·925	·978	·828

TABLE VIII. MONTHLY MEANS OF ATMOSPHERIC PRESSURES REDUCED TO SEA-LEVEL FROM THE VALUES IN TABLE VII.

	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
Roorkee	30·060	29·972	29·893	29·762	29·613	29·494	29·508	29·567	29·670	29·855	30·027	30·091
Agra	·078	·998	·907	·779	·621	·503	·508	·565	·666	·863	·033	·087
Lucknow	·084	·989	·900	·766	·632	·516	·525	·593	·689	·868	·036	·098
Benares	·094	30·002	·914	·791	·647	·532	·534	·610	·676	·874	·038	·101
Patna	·040	29·971	·871	·759	·655	·527	·540	·612	·692	·877	·040	·077
Jhansi	·072	30·017	·882	·760	·603	·479	·496	·586	·674	·868	·042	·087
Jubbulpore	·018	29·997	·896	·766	·640	·523	·548	·628	·655	·846	·015	·055
Nagpore	·004	·941	·837	·715	·622	·537	·563	·643	·670	·830	29·992	·045
Monghyr	·030	·944	·850	·748	·653	·526	·536	·603	·678	·849	·999	·070
Hazareebagh	·033	·948	·855	·728	·616	·512	·516	·590	·657	·851	·999	·075
Cuttack	·069	·980	·905	·790	·700	·630	·628	·675	·704	·904	30·044	·098
Saugor Island	·027	·948	·885	·790	·686	·563	·553	·596	·672	·833	29·982	·047
Calcutta	·040	·964	·888	·792	·685	·570	·569	·629	·697	·859	30·001	·061
Berhampore	·021	·948	·861	·746	·637	·511	·536	·610	·694	·839	29·979	·047
Jessore	·015	·943	·871	·774	·686	·557	·554	·619	·694	·853	·988	·047
Dacca	·016	·958	·881	·813	·714	·595	·594	·632	·714	·853	·988	·047
Cachar	29·979	·939	·874	·815	·710	·619	·623	·670	·754	·848	·987	·036
Chittagong	30·003	·946	·881	·819	·733	·629	·629	·664	·726	·848	·973	·028
Akyab	·017	·951	·918	·861	·774	·710	·715	·732	·782	·867	·973	·037
Goalpara	·007	·923	·837	·771	·675	·592	·580	·633	·716	·817	·966	·025
Port Blair	29·943	·939	·896	·850	·788	·783	·794	·795	·828	·861	·912	29·944
Madras	·994	·970	·925	·835	·747	·698	·733	·761	·782	·855	·953	30·005

Description of the Plates.

Plates XLIII. to XLVIII. show the mean distribution of temperature and pressure in Northern India in each month of the year. The isobars are represented by thick lines, corresponding to even twentieths of an inch. The value of each is noted against the line, and the sign + and — indicates the directions in which the baric gradient rises and falls. The isotherms are represented by thin lines, corresponding to equal increments of 5° Fahr.

The charts are based on the data given in Tables III. and VIII., with some additions in certain cases (see note, p. 585). Owing to the absence of barometric data for the Punjab, Assam, and Bombay, the isobars cannot be traced in those provinces.

Plate XLIX. shows the average distribution of the total annual rainfall in Northern India, by means of ishyetic lines, each of which corresponds to an increment or decrement of 10 inches. The chart is drawn up from the data given in Table VI. It does not pretend to show minor local variations, nor to give the details of distribution where the annual rainfall exceeds one hundred inches. The stations enumerated in Table VI. are indicated on the chart by Roman figures corresponding to those of the Table.

The principal characteristics of distribution according to season are indicated by inscriptions on the chart.

XVIII. *On a Self-recording Method of Measuring the Intensity of the Chemical Action of Total Daylight.* By Henry E. Roscoe, *F.R.S.*

Received November 27, 1873.—Read February 5, 1874.

Although the method of measuring the varying intensity of the chemically active rays, as affecting chloride-of-silver paper of constant sensitiveness described in the Bakerian Lecture for 1865, has been the means of pointing out many important facts * concerning the distribution of the sun's chemical activity through the atmosphere as well as in different situations on the earth's surface, it has not as yet been introduced as a regular portion of the work of meteorological observatories. Until this is done, and the measurements are regularly continued and made in many situations, we cannot hope to obtain any thing like a knowledge of the laws of distribution of these rays over the earth's surface, or any information as to the yearly variation of the solar chemical activity. This non-adoption of the method has to be explained, not in any want of reliance in the process or in the results, but in the fact that, in order to obtain a satisfactory curve of daily chemical intensity, at least hourly observations need to be made; this involves, however, the expenditure of so much time and labour that the permanent observatories, already too heavily weighted, have found it impossible to undertake the necessary work. In the present communication I have to describe a modification of the above-mentioned method, which, whilst preserving untouched the principles upon which it is based and the amount of exactitude of which it is susceptible, reduces the personal attention needed for carrying out the measurements to a minimum, and thus renders its adoption in observatories possible.

According to this plan, the constant sensitive paper is exposed to the action of total daylight at given intervals, say at every hour, during the day, by a self-acting arrangement, for accurately known times. The insolation-apparatus, stocked with sensitive paper, is placed in position either early in the morning of the day during which the measurements have to be made, or on the previous night; and by means of electric communication with a properly arranged clock, the sensitive paper is exposed every hour during the day, so that, in the evening, the observer has only to read off in the

* (1) Phil. Trans. 1867, p. 555, "On the Chemical Intensity of Total Daylight at Kew and Paris, 1865, 1866, 1867," by H. E. Roscoe, F.R.S.; (2) Phil. Trans. 1870, p. 309, "On the Relation between the Sun's Altitude and the Chemical Intensity of Total Daylight in a Cloudless Sky," by H. E. Roscoe, F.R.S., and T. E. Thorpe, Ph.D., F.R.S.E.; (3) Phil. Trans. 1871, p. 467, "On the Measurement of the Chemical Intensity of Total Daylight made at Catania during the Total Eclipse of December 22, 1870," by H. E. Roscoe, F.R.S., and T. E. Thorpe, F.R.S.E.

ordinary manner the hourly intensities which have been recorded on the paper during the day.

This self-recording arrangement, though at first sight simple enough, involves points which have rendered its successful completion a somewhat lengthy and difficult matter. Thanks, however, to the skill of Mr. CHARLES JORDAN, of Manchester, these mechanical difficulties have now been overcome, and the instrument perfectly answers the desired end.

Owing, in the first place, to the great variations which occur in the chemical intensity of total daylight in different places, at different times of the day, and in different periods of the year, and, secondly, owing to the fact that, in order to be able accurately to estimate chemical intensity, the coloration acquired by the paper must reach, but not much exceed, a given tint, it becomes necessary, on each occasion when an observation is needed, that the sensitive paper should be exposed mechanically, not once, but for several known but varying intervals of time, quickly succeeding each other; so that whatever may be the intensity of the total daylight (supposed during those intervals to remain constant), some one at least of the several exposed papers will possess the requisite shade. This is accomplished by a duplicate arrangement of a clock and insolation-apparatus. The clock has connected with its minute-wheel (Plate L. fig. 1, A) a train of three wheels (B, C, D, fig. 1), so arranged for speed that the last wheel (D) revolves once every two minutes. On the periphery of this metal wheel are fixed eleven stout platinum pins (marked 1 to 11, fig. 1), each projecting about 3 millims. from the face of the wheel. As this wheel revolves, each one of the pins is in turn brought for an instant into metallic contact with the platinum face of the elastic metallic arm (E, fig. 1). As the wheel passes on, metallic contact between the wheel and the arm is interrupted until the next pin comes into position. These platinum pins are so placed on the wheel that by its uniform rotation contact is instantly made, broken, and, after a given interval, again made and broken, in all eleven times in succession. The intervals during which contact is broken are of different lengths, dependent upon the positions of the pins. For use in this country the intervals are so arranged as to break the contact for times approximating to the following number of seconds:—

Interval ..	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.	No. 10.
Seconds ..	2	3	4	5	7	10	12	17	20	30

When the instrument is to be used in situations where the chemical intensity is much greater or much less than in our latitudes, different intervals must be adopted. Whilst the wheel is in metallic contact with the elastic arm (E), a current from four cells of a BUNSEN'S battery (or if a slow and constant current is required twelve to twenty cells of a LE CLENCH) passes by means of a second elastic arm (F) through wires connecting the clockwork with the insolation-apparatus; but this current ceases to pass as soon as the circuit at E is interrupted. The paper of constant sensitiveness, cut into long

narrow strips like those used in MORSE'S telegraph, and of sufficient length (about 3 metres) to last for one day's observations, is wound on to a bobbin (B, fig. 2), from which it passes over the light metallic wheel (W, fig. 2) about 15 centims. in diameter, to the circumference of which one end of the strip is made fast. The escapement (E, fig. 2) is connected with the keeper (K) of a small electromagnet (M), round which the current from the battery passes whenever contact is made by the clock at E, fig. 1. The escapement-wheel (F, fig. 2), worked by a spring (S), is fixed on to the axis of the wheel (W) carrying the paper, so that when a current passes through the electromagnet the keeper (K) is attracted, the escapement (E) released, and the paper carried forward on the wheel to a distance regulated by the number of the teeth of the escapement-wheel (F). As soon as the current ceases, the keeper is brought back into the position shown in the drawing by means of the spiral spring at its extremity.

When the minute-wheel (A, fig. 1) of the clock comes round to a given point, a fixed pin (p) on this wheel presses against the long end of a lever (L), and this pushes down the elastic arm (E) (which is insulated from the rest of the clockwork) on to the thick platinum pins fixed on to the metal wheel (D). By the passage of the pins in front of the end of the elastic arm contact is made, and then a current passes from the clock to the electromagnet on the insolation-apparatus. The paper on the wheel is then suddenly moved forward such a distance as is necessary in order to expose a fresh portion under the circular opening (O, fig. 3) on the upper side of the metal cover of the insolation-apparatus. This small disk of paper (4 millims. in diameter) remains exposed to the daylight until removed by the movement of the keeper caused by the contact of the second pin at E, fig. 1; a fresh disk of paper then instantly becomes visible, in its turn to be removed after having been exposed for a known length of time. This successive exposure of disks of sensitive paper for increasing periods of time up to 30 seconds goes on until the wheel carrying the platinum pins has completed one revolution. The fixed pin (p) on the minute-wheel has during that time passed so far round, that it has now ceased to press upon the curved and thickened end of the lever (L), and the tip of the elastic arm (E) is now drawn back by a small insulated spring (S); so that it remains in this position until an hour has elapsed, when it is again pressed on to the platinum pins by the point on the minute-wheel, and the exposure of another set of disks of sensitive paper occurs again. During the space of nearly an hour, during which the wheel (W, fig. 2) remains stationary, a very dark disk is obtained; and if one of these dark disks be marked during the day with the hour of exposure, the times of all the different exposed disks can be ascertained.

When in use the insolation-apparatus is covered with a light tight blackened metal cover (C, fig. 3), having a thin piece of metal at the top, which, when the instrument is placed in position, lies horizontally. In the centre of this plate a circular opening 4 millims. in diameter is bored, the edges of which are carefully bevelled off. A steel spring (A, B, fig. 2), over which the prepared paper passes, presses the strip against the lower surface of the horizontal metallic plate, so that the disk of paper is seen to lie

close to the plate below the circular opening. In order to keep the paper and apparatus dry during rain, a glass shade is placed in wet weather over the insolation-apparatus, and the loss of light thus occasioned by reflection and absorption experimentally ascertained for each instrument.

On unrolling, at the end of the day, the strip of insolated paper from the wheel (W) in a room illuminated by the monochromatic light of a soda-flame, the black disks of the hours are seen, and between each of these are found ten circles variously tinted, from that (probably scarcely visible) which was exposed for 2 seconds, to that (perhaps too dark to read off) which was insolated for 30 seconds. Amongst these some one at least will be found to be of such a tint as to enable it to be read off on a graduated fixed strip, as described in my former communication*.

In order to be able to read off each hourly observation quickly, the half of each of the tinted disks which is of about the right shade is punched out of the paper by a solid semicircular punch, which is worked by the foot, whilst the long strip of paper is held in both hands. One end of the long paper band is then placed in one of the spring clamps of the reading-drum†, and the band brought through the other clamp, so that the remaining halves of the tinted disks are held against the graduated fixed strip which is placed on the drum. By moving the drum on its horizontal axis, the various shades of the fixed strip are made to pass and repass each of the semicircular holes on the band; and thus the point on the strip identical in tint with the remaining half of the disk can be easily ascertained, the reading being made in a darkened room by the light of a soda-flame. Each tint is thus read off ten times, and the mean taken as the result.

II. *On the Calibration of the fixed Strips and Standard Tints.*

The calibration of the fixed strips‡ can be advantageously effected, independently of the pendulum photometer, as follows:—The strip to be calibrated is gummed on to the reading-drum, and the points on this strip, of equal intensity to papers tinted by simultaneous exposure to zenith-light under vertical cylinders closed at the top with differently sized diaphragms, are ascertained. For a calibration thus made six cylinders were employed, each being 6 decims. long and 1 decim. in diameter, and blackened inside. On the top of each was fitted a metal plate perforated by a circular opening. These openings were of varying size, such that the relative intensities of the diffused zenith-light falling on the sensitive paper at the bottom of each cylinder were as follows:—

	Relative intensity.
Cylinder 1	1·00
„ 2	2·32
„ 3	4·00
„ 4	6·13
„ 5	8·72
„ 6	11·75

* Phil. Trans. 1865, vol. clv. p. 610. † Ibid. p. 615, fig. 6. ‡ Ibid. p. 610.

Standard sensitive papers thus exposed under the several cylinders were then taken into the dark room, and the points of identical tint on the fixed strip then read off. In this way a number of points are found upon the fixed strip, the relative intensities of which are known. The normal tint† (of which the intensity =1) is next read off on the strip; and if its reading corresponds to any one of the tints previously read off, the true value of that tint is known to be that of unity, and those of the other tints can readily be ascertained. Several experiments with the cylinders are made for each strip to be calibrated, and in one or more of these a tint is sure to be found equal to the normal tint. As an example I give the following data obtained in the calibration of a fixed strip marked C, on which the mean reading of the normal tint I=1 was found to lie at 132 millims.

Column I. contains the numbers of the cylinders under which the papers were exposed, column II. the mean reading of these tints on the fixed strip, column III. the corresponding relative intensities, and column IV. the true intensities when the tint of reading, 132 millims., is taken equal to the normal.

Experiment A.

I.	II.	III.	IV.
2.	170	2·32	0·27
3.	163	4·00	0·46
4.	151	6·13	0·70
5.	131*	8·72	1·00
6.	101	11·75	1·35

Experiment B.

2.	168	2·32	0·29
3.	158	4·00	0·52
4.	146*	6·13	0·79
6.	82	11·75	1·51

Experiment C.

1.	166	1·00	0·31
2.	150*	2·32	0·72
3.	106	4·00	1·24

Experiment D.

I.	II.	III.	IV.
1.	173	1·00	0·28
2.	157	2·32	0·65
3.	119*	4·00	1·12
4.	59	6·13	1·72
5.	21	8·72	2·44

Experiment E.

1.	161	1·00	0·44
2.	130*	2·32	1·02
3.	70	4·00	1·76
4.	15	6·13	2·69

Experiment F.

4.	147*	6·13	0·77
5.	123	8·72	1·09

The reading marked with an asterisk in each case is the one used to connect that experiment with the others. In cases in which no reading occurs identical with the one in previous experiments where the true intensities have been found, the true intensity of the nearest reading is obtained by interpolation from those of two previous observations. Thus the true intensity of the mean reading 146 millims. in Experiment B is found by interpolating from the true intensities of the mean readings 151 and 131 found in Experiment A.

From these numbers the following intensities are obtained for the undermentioned point on the strip in question:—

† Philosophical Transactions, 1863, vol. cliii. p. 157.

Millims.	I.	Millims.	I.	Millims.	I.	Millims.	I.
173	0·28	151	0·70	130	1·02	70	1·76
170	0·27	150	0·72	125	1·09	59	1·72
168	0·29	147	0·77	119	1·12	21	2·44
166	0·31	146	0·79	106	1·24	15	2·69
163	0·46	132	1·00	101	1·35		
158	0·52	131	1·01	82	1·51		

Hence the following Calibration Table is obtained for the same strip by graphical interpolation:—

I.	II.	I.	II.	I.	II.	I.	II.	I.	II.
10	2·89	45	2·03	80	1·54	115	1·15	150	0·67
1	2·85	6	2·02	1	1·53	6	1·15	1	0·65
2	2·81	7	2·00	2	1·51	7	1·14	2	0·63
3	2·77	8	1·99	3	1·51	8	1·13	3	0·61
4	2·73	9	1·97	4	1·50	9	1·12	4	0·59
5	2·69	50	1·96	5	1·49	120	1·12	5	0·58
6	2·64	1	1·94	6	1·48	1	1·11	6	0·56
7	2·60	2	1·93	7	1·48	2	1·10	7	0·54
8	2·56	3	1·91	8	1·47	3	1·09	8	0·52
9	2·52	4	1·90	9	1·47	4	1·07	9	0·51
20	2·48	5	1·88	90	1·46	5	1·06	160	0·50
1	2·44	6	1·86	1	1·45	6	1·05	1	0·49
2	2·42	7	1·84	2	1·44	7	1·04	2	0·48
3	2·41	8	1·82	3	1·43	8	1·03	3	0·46
4	2·39	9	1·81	4	1·42	9	1·02	4	0·41
5	2·37	60	1·80	5	1·41	130	1·01	5	0·36
6	2·35	1	1·79	6	1·40	1	1·00	6	0·31
7	2·34	2	1·78	7	1·39	2	0·99	7	0·30
8	2·32	3	1·77	8	1·38	3	0·97	8	0·29
9	2·31	4	1·76	9	1·37	4	0·95	9	0·28
30	2·29	5	1·75	100	1·36	5	0·94	170	0·27
1	2·28	6	1·74	1	1·35	6	0·92	1	0·26
2	2·26	7	1·73	2	1·32	7	0·90	2	0·25
3	2·24	8	1·72	3	1·30	8	0·88	3	0·24
4	2·23	9	1·71	4	1·28	9	0·86	4	0·22
5	2·21	70	1·69	5	1·26	140	0·85	5	0·20
6	2·19	1	1·68	6	1·24	1	0·83	6	0·18
7	2·18	2	1·66	7	1·23	2	0·81	7	0·16
8	2·16	3	1·65	8	1·22	3	0·79	8	0·15
9	2·15	4	1·63	9	1·21	4	0·77	9	0·13
40	2·13	5	1·62	110	1·20	5	0·76	180	0·10
1	2·11	6	1·60	1	1·19	6	0·74		
2	2·09	7	1·59	2	1·18	7	0·72		
3	2·07	8	1·57	3	1·17	8	0·70		
4	2·05	9	1·56	4	1·16	9	0·68		

As a check upon this method of calibrating the fixed strip, a series of tints, obtained by exposure in quick succession in the hand-insolator * for known but varying times to the diffused light of a cloudless sky, were read off on the same strip. A Calibration Table was then constructed for the strip, and the values therein contained were found to coincide (within the limits of observational error) with those obtained by the former method.

* Philosophical Transactions, 1865, vol. clv. p. 612.

In order to avoid the necessity of making the foregoing experiments for the calibration of every new graduated strip, a series of standard tints * was prepared by reading off the points of identical tint on the graduated strip C. The following numbers were obtained :—

Standard tint.	Mean of ten readings.	Intensity.
No. I.	18 millims.	2·56
II.	54 „	1·90
III.	74 „	1·63
IV.	117 „	1·14
V.	132 „	1·00
VI.	146 „	0·74
VII.	157 „	0·54
VIII.	160 „	0·50
IX.	164 „	0·41

By means of these standard tints thus calibrated the intensities of any graduated strip can be read off.

III. *On the preparation of constant Sensitive Paper in long Strips.*

In the paper already referred to †, it has been shown that sheets of salted paper, each having an area of 0·3 square metre, can be silvered, so that each portion of the sheet, after drying, possesses exactly the same degree of sensitiveness. For the purposes of the present method the paper has to be used in the form of long thin strips; and if these strips can be silvered in lengths from 1 to 2 metres, and still retain a uniform degree of sensitiveness, much labour in cutting up the silvered sheets will be saved. That this uniformity can be obtained is seen from the following experiments.

Salted paper was silvered by laying it, in the form of sheets 2 decims. square, on to the surface of a 12 per cent. nitrate-of-silver solution. After lying on the bath for two minutes it was hung up to dry. Some of the same salted paper was next cut into long strips about 10 to 15 millims. in width, and these were silvered by floating on the silver solution contained in a narrow wooden trough 1·5 metre in length; the strips were then hung up to dry. Small pieces were next cut out of both the sheet and strip silvered paper, pasted on the back of the ordinary reading strips, and exposed to sunlight in two different hand-insolators at the same moment for identical periods of time. The tints obtained were read off on a calibrated strip, when it was found that the intensities obtained for the tints on the strip and sheet silvered paper were identical, as is shown by the following Tables :—

* Philosophical Transactions, 1865, vol. clv. p. 615, fig. 3.

† Ibid. 1863, cliii. p. 155.

Experiment A.									
Paper in sheet.					Paper in strip.				
No.	n.	Mean read g.	I.	$\frac{I}{n}$.	No.	n.	Mean reading.	I.	$\frac{I}{n}$.
1.	20	76	0·99	0·049	1.	20	73	1·03	0·051
2.	15	117	0·76	0·051	2.	15	115	0·83	0·055
3.	20	72	1·04	0·052	3.	20	56	1·25	0·062
4.	15	78	0·98	0·065	4.	15	78	0·98	0·065
5.	10	79	0·97	0·097	5.	10	74	1·01	0·101
6.	10	131	0·66	0·066	6.	10	115	0·77	0·077
7.	15	76	0·99	0·066	7.	15	61	1·20	0·080
8.	10	114	0·78	0·078	8.	10	113	0·78	0·078
			Mean	0·066				Mean	0·071
Experiment B.									
1.	10	128	0·69	0·069	1.	10	126	0·70	0·070
2.	10	120	0·75	0·075	2.	10	122	0·73	0·073
3.	15	97	0·87	0·058	3.	15	114	0·78	0·052
4.	5	153	0·47	0·094	4.	5	154	0·45	0·090
5.	5	117	0·76	0·152	5.	5	131	0·66	0·132
6.	10	123	0·73	0·073	6.	10	125	0·71	0·071
7.	5	140	0·60	0·120	7.	5	139	0·61	0·122
8.	5	135	0·63	0·126	8.	5	140	0·60	0·120
9.	5	141	0·59	0·118	9.	5	142	0·58	0·116
			Mean	0·098				Mean	0·094

The mean intensity obtained by insolation of paper in sheet from the two experiments is therefore $\frac{0·066+0·098}{2}=0·082$; that obtained by the insolation of paper cut into strips and silvered is $\frac{0·071+0·094}{2}=0·082$. Hence we may conclude that the method of silvering the paper in strips can be relied upon.

IV. *Determination of the Times of Exposure of the constant Sensitive Paper in the Insolator.*

The time during which each one of the disks of sensitive paper is exposed to the total daylight depends upon the length which elapses between the contact of the elastic arm (E, fig. 1) with the two consecutive platinum pins on the wheel D. A certain time, however, elapses after the passage of the current before the escapement-wheel (F, fig. 2) on the insolator is brought into motion and the paper thus moved. It therefore becomes necessary to ascertain the times of exposure for each particular insolator, by observing the moment at which the paper is released, and noting the space of time which elapses until the paper disk disappears. The estimation of these intervals, some of them of only short duration, was made with a chronograph kindly lent by Mr. J. B. Dancer, by which intervals of time could be accurately measured to within

one fifth of a second. A large number of observations were made with this instrument, and the mean reading of the interval taken as the duration of exposure for each one (see Table *infra*). In order to check these times, the duration of the longer intervals was also ascertained by counting with a watch whose seconds' hand indicated quarter seconds; and these coincided with the times observed with the chronograph.

Interval..	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.	No. 10.
	2·2	2·8	3·8	5·2	6·7	9·5	10·8	17·3	20·2	31·4
	2·0	3·0	2·8	5·3	7·0	8·7	10·3	16·2	20·6	32·0
	1·7	3·8	3·0	6·0	7·0	9·3	11·0	17·8	19·7	32·4
	2·9	3·1	4·3	5·3	6·8	9·9	10·0	16·9	19·4	31·1
	1·7	2·0	3·1	4·8	6·7	9·7	9·7	17·2	20·4	30·7
	2·7	2·6	3·9	5·2	6·7	9·0	10·2	18·1	20·5	31·7
	2·9	1·9	3·6	6·1	6·8	9·5	10·0	16·6	20·2	31·4
		3·0	4·0	5·2	7·0	9·2	10·3	17·0	20·5	31·1
			4·1	5·2	6·3	10·0	9·9	16·7	20·2	31·6
				4·5	6·9	9·2	10·1	17·9	20·5	32·0
				5·3	7·1	9·7	10·6	16·6	19·7	32·6
				5·3	7·0	9·4	10·1	17·0	20·1	32·4
					6·9	9·4	10·3	16·9		
					6·9	10·1	10·0			
					7·0	10·4	10·1			
					7·0	10·1	10·1			
					6·9	9·7	10·3			
						10·2				
Mean....	2·3	2·8	3·6	5·3	6·9	9·6	10·2	17·1	20·2	31·7

V. *Determination of the Coefficient for the Reflection and Absorption due to glass cover.*

In order to keep the disk of sensitive paper dry and to preserve the insolation-apparatus from damage by rain, it is covered during wet weather with a glass shade, chosen of such dimensions that the exposed disk of paper lies as nearly as possible in the centre of the hemisperical top of the shade. For the purpose of determining the value of the coefficient representing the loss of light caused by this glass covering, simultaneous observations of the intensity of total daylight were made with the hand-insolator and with the self-registering instrument covered by the glass shade. From the equation $I_h = I_r \times \text{const}$ (where I_h signifies the intensity obtained by the hand-instrument, and I_r that obtained by the self-recording instrument covered with the shade) the value of the coefficient can be ascertained. The following experiments give the value of the coefficient for the glass shade used in the determinations which follow. A similar series of observations must be made for every shade employed.

Self-registering instrument.						Hand instrument.				
Experiment.	Time.	n.	Mean reading.	I.	$\frac{I}{n}$	Time.	n.	Mean reading.	I.	$\frac{I}{n}$
I.	h m 11 0 A.M.	5·3	169	0·26	0·049	h m 11 0 A.M.	5	154	0·45	0·090
		10·2	154	0·45	0·044		10	133	0·65	0·065
		17·1	144	0·56	0·033				Mean	0·078
				Mean	0·042					
II.	2 0 P.M.	9·6	145	0·55	0·057	2 0 P.M...	10	57	1·24	0·124
		10·2	138	0·61	0·059				Mean	0·124
		17·1	60	1·21	0·070					
				Mean	0·062					
III.	2 30 P.M.	5·3	155	0·44	0·083	2 30 P.M.	5	140	0·60	0·120
		6·9	150	0·50	0·072		5	137	0·62	0·124
		9·6	143	0·57	0·059				Mean	0·122
				Mean	0·071					
IV.	3 0 P.M.	3·6	168	0·28	0·077	3 0 P.M.	5	137	0·62	0·124
		5·3	162	0·36	0·068		5	120	0·75	0·150
		6·9	150	0·50	0·072				Mean	0·137
				Mean	0·072					
V.	3 30 P.M.	5·3	160	0·39	0·073	3 30 P.M.	5	109	0·81	0·162
		6·9	139	0·61	0·088		5	122	0·73	0·146
		9·6	121	0·74	0·077				Mean	0·154
				Mean	0·079					
VI.	4 0 P.M.	9·6	166	0·31	0·032	4 0 P.M.	10	131	0·66	0·066
		10·2	161	0·38	0·037		15	117	0·76	0·051
		17·1	145	0·55	0·032		15	65	1·14	0·076
		20·2	130	0·67	0·033				Mean	0·064
				Mean	0·034					
VII.	4 30 P.M.	20·2	116	0·77	0·038	4 30 P.M...	15	59	1·22	0·081
		31·7	68	1·10	0·035				Mean	0·081
				Mean	0·037					
VIII.	5 0 P.M.	17·1	137	0·62	0·036	5 0 P.M...	15	76	0·99	0·066
		20·2	133	0·65	0·032				Mean	0·066
				Mean	0·034					

The following Table shows that the values of the coefficient obtained at the undermentioned varying hours of the day are sensibly constant. The mean value 1·94 is therefore taken as representing the coefficient for the shade used in the determinations which follow:—

Time.	Coefficient for glass shade.	Deviation from mean.
h m		
11 0 A.M.	1·857	−0·080
2 0 P.M.	2·000	+0·063
2 30 „	1·950	+0·013
3 0 „	1·902	−0·035
3 30 „	1·718	−0·219
4 0 „	1·939	+0·002
4 30 „	2·189	+0·252
5 0 „	1·941	+0·004
Mean	1·937	0·083

VI. *Comparison of Curves of Daily Chemical Intensity obtained* (1) *with the Hand-insolator and* (2) *with the Self-recording Instrument.*

During the months of May, June, and July 1873 simultaneous hourly determinations of the chemical intensity of total daylight were made in Victoria Park, Manchester, with the hand-insolator and the self-recording arrangement described above on twelve separate days, whilst on ten other days the intensity was ascertained by the automatic apparatus alone. Of these I deem it unnecessary to give the details of more than six full days' observations with both instruments for the purpose of showing that the chemical intensities, as measured by the two methods, closely agree, and that therefore the self-recording instrument gives reliable and accurate results.

The daily observations selected for this purpose were made on May 17th and 19th, June 9th, 17th, and 18th, and on July 1st. The following Tables and accompanying curves (fig. 4, Plate L.) show the close correspondence of both sets of observations. The Curves marked in thicker lines give the results of the hand-insolator measurements, those in thinner lines the results of the measurements recorded by clockwork.

The integrals of total chemical intensity * for six of the days on which simultaneous observations were made with the two instruments agree as closely as can be expected from the nature of the case.

Daily Integrals of Chemical Intensity determined

	(1) By clock instrument.	(2) By hand instrument.
May 17th	82·8	81·6
„ 19th	49·5	47·2
June 9th	87·1	96·7
„ 17th	51·0	52·2
„ 18th	52·2	47·4
July 1st	111·2	117·3

I have to thank Mr. Thomas Carnelley, B.Sc., for the able assistance which he has given me in carrying out the above determinations.

* Philosophical Transactions, "Bakerian Lecture," 1865, vol. clv. p. 621.

May 17th, 1873.

Clock.					Hand.				
Time.	*n*.	Mean reading.	I.	$\frac{I}{n}$.	Time.	*n*.	Mean reading.	I.	$\frac{I}{n}$.
h m	0				h m	0			
11 15 A.M.	9·6	41	1·87	0·195	11 15 A.M.	10	56	1·74	0·174
	6·9	90	1·43	0·207		5	120	1·06	0·212
	5·3	148	0·84	0·154				Mean	0·193
			Mean	0·187				Diff.	−0·006
	9·6	47	1·82	0·190		10	56	1·74	0·174
12 15 P.M.	6·9	75	1·57	0·227	12 15 P.M.	5	119	1·10	0·220
	5·3	136	0·86	0·162		10	55	1·75	0·175
			Mean	0·193				Mean	0·190
								Diff.	+0·003
	10·2	63	1·68	0·163		5	130	0·92	0·184
1 15 „	6·9	97	1·37	0·198	1 15 „	10	56	1·74	0·174
			Mean	0·182		5	122	1·00	0·200
								Mean	0·186
								Diff.	−0·004
	3·6	144	0·78	0·216		5	127	0·95	0·190
2 15 „	5·3	140	0·82	0·155	2 15 „	10	57	1·73	0·173
			Mean	0·185		5	126	0·96	0·192
								Mean	0·185
								Diff.	0·000
	5·3	145	0·77	0·145		10	78	1·54	0·154
3 15 „	6·9	92	1·42	0·205	3 15 „	5	139	0·83	0·166
	10·2	58	1·72	0·170		10	85	1·48	0·148
			Mean	0·173				Mean	0·156
								Diff.	+0·017
	6·9	143	0·79	0·114		15	97	1·37	0·091
4 15 „	17·1	71	1·60	0·094	4 15 „	10	122	1·00	0·100
	9·6	135	0·87	0·091		15	99	1·35	0·090
			Mean	0·099				Mean	0·094
								Diff.	+0·005
	10·2	150	0·70	0·068		20	125	0·97	0·049
5 15 „	17·1	124	0·98	0·057	5 15 „	15	128	0·94	0·062
	20·2	114	1·22	0·060		15	129	0·93	0·062
			Mean	0·062		15	135	0·87	0·058
								Mean	0·056
								Diff.	+0·006
	20·2	154	0·64	0·032		20	158	0·58	0·029
6 15 „	31·7	152	1·00	0·032	6 15 „	25	148	0·74	0·030
			Mean	0·032				Mean	0·029
								Diff.	+0·003

May 19th, 1873.

Clock.					Hand.				
Time.	n.	Mean reading.	I.	$\frac{I}{n}$.	Time.	n.	Mean reading.	I.	$\frac{I}{n}$.
h m	6″·9	160	0·54	0·078	h m	25″	73	1·59	0·064
10 30 A.M.	9·6	151	0·69	0·072	10 30 A.M.	15	61	1·69	0·113
	17·1	100	1·34	0·078		10	149	0·72	0·072
	20·2	84	1·49	0·074		15	106	1·29	0·086
			Mean	0·076				Mean	0·084
								Diff.	−0·008
11 30 „	5·3	127	0·95	0·177	11 30 „	5	148	0·74	0·148
	6·9	52	1·78	0·257		10	34	1·94	0·194
	9·6	48	1·81	0·188		10	41	1·87	0·187
			Mean	0·207				Mean	0·176
								Diff.	+0·031
12 30 P.M.	20·2	138	0·84	0·042	12 30 P.M.	20	142	0·80	0·040
	17·1	155	0·62	0·036		15	151	0·69	0·046
	9·6	168	0·37	0·039					
			Mean	0·039				Mean	0·043
								Diff.	−0·004
1 30 „	17·1	98	1·36	0·079	1 30 „	10	141	0·81	0·081
	10·2	145	0·77	0·075		10	145	0·77	0·077
	9·6	152	0·67	0·070		7	146	0·76	0·108
			Mean	0·075				Mean	0·089
								Diff.	−0·014
2 30 „	No observations.				2 30 „	No observations.			
3 30 „	17·1	92	1·42	0·083	3 30 „	10	135	0·87	0·087
	10·2	145	0·77	0·075		15	126	0·96	0·064
	9·6	135	0·87	0·090		10	150	0·70	0·070
	6·9	152	0·67	0·097					
			Mean	0·083				Mean	0·074
								Diff.	+0·009
4 30 „	9·6	162	0·49	0·051	4 30 „	15	156	0·61	0·041
	17·1	152	0·67	0·039		20	146	0·76	0·038
	20·2	140	0·82	0·040		25	128	0·94	0·038
			Mean	0·043				Mean	0·039
								Diff.	+0·004
5 30 „	31·7	126	0·96	0·030	5 30 „	30	146	0·76	0·025
	20·2	155	0·62	0·030		20	159	0·56	0·028
						30	131	0·91	0·030
			Mean	0·030				Mean	0·028
								Diff.	+0·002

June 9th, 1873.

Clock.					Hand.				
Time.	n.	Mean reading.	I.	$\frac{I}{n}$.	Time.	n.	Mean reading.	I.	$\frac{I}{n}$.
h m	17·1	132	0·66	0·039	h m	15	136	0·63	0·042
7 30 A.M.	20·2	120	0·75	0·037	7 30 A.M.	20	115	0·77	0·039
	31·7	56	1·25	0·039		30	56	1·25	0·042
			Mean	0·039				Mean	0·041
								Diff.	−0·002
	6·9	114	0·78	0·113		10	84	0·95	0·095
8 0 „	9·6	85	0·94	0·098	8 0 „	15	17	1·61	0·107
	10·2	55	1·26	0·124		10	50	1·31	0·131
			Mean	0·112				Mean	0·111
								Diff.	+0·001
	5·3	139	0·61	0·115					
9 0 „	6·9	77	0·99	0·143	9 0 „	No observations.			
	9·6	47	1·34	0·140					
			Mean	0·133					
	10·2	140	0·60	0·059		10	137	0·62	0·062
10 0 „	17·1	110	0·80	0·047	10 0 „	15	116	0·77	0·051
	20·2	70	1·07	0·053		20	62	1·19	0·060
			Mean	0·053				Mean	0·058
								Diff.	−0·005
	5·3	138	0·61	0·115		10	30	1·49	0·149
11 0 „	9·6	20	1·58	0·165	11 0 „	10	18	1·60	0·160
	10·2	17	1·61	0·158		15			
			Mean	0·146				Mean	0·154
								Diff.	−0·008
	9·6	117	0·76	0·079		10	70	1·07	0·107
12 0	10·2	105	0·83	0·081	12 0 „	10	73	1·03	0·103
	17·1	11	1·67	0·098		15	17	1·61	0·107
			Mean	0·086				Mean	0·106
								Diff.	−0·020
	3·6	132	0·66	0·183					
1 0 P.M.	5·3	121	0·74	0·140	1 0 P.M.	No observations.			
	6·9	25	1·54	0·223					
			Mean	0·182					
	5·3	114	0·78	0·147		5	113	0·78	0·156
2 0 „	6·9	112	0·79	0·114	2 0 „	10	18	1·60	0·160
	9·6	72	1·04	0·108		10	15	1·63	0·163
			Mean	0·123				Mean	0·160
								Diff.	−0·037

June 9th, 1873 (continued).

Clock.					Hand.				
Time.	n.	Mean reading.	I.	$\frac{I}{n}$.	Time.	n.	Mean reading.	I.	$\frac{I}{n}$.
h m	9·6	113	0·78	0·081	h m	10	67	1·11	0·111
3 0 P.M.	10·2	108	0·81	0·080	3 0 P.M.	10	56	1·25	0·125
	17·1	20	1·58	0·092		15	15	1·63	0·109
			Mean	0·084				Mean	0·115
								Diff.	−0·031
	9·6	132	0·66	0·069					
4 0 „	10·2	129	0·68	0·067	4 0 „	10	83	0·95	0·095
	17·1	55	1·26	0·074		15	57	1·24	0·083
			Mean	0·070				Mean	0·089
								Diff.	−0·019
	9·6	163	0·35	0·036					
5 0 „	10·2	161	0·38	0·037	5 0 „	10	140	0·60	0·060
	17·1	137	0·62	0·036		10	140	0·60	0·060
						15	124	0·71	0·047
			Mean	0·036				Mean	0·056
								Diff.	−0·020
	10·2	159	0·40	0·039					
6 0 „	17·1	143	0·57	0·033	6 0 „	10	152	0·48	0·048
	20·2	123	0·72	0·036		15	131	0·66	0·044
						20	109	0·81	0·041
			Mean	0·036				Mean	0·044
								Diff.	−0·008

June 17th, 1873.

Clock.					Hand.				
Time.	n.	Mean reading.	I.	$\frac{I}{n}$.	Time.	n.	Mean reading.	I.	$\frac{I}{n}$.
h m	20·2	149	0·51	0·025					
5 20 A.M.	31·7	117	0·76	0·024					
			Mean	0·025					
	17·1	116	0·77	0·045					
6 20 „	20·2	76	0·99	0·049					
			Mean	0·047					
	5·3	161	0·38	0·072	No observations.				
7 20 „	6·9	147	0·53	0·077					
			Mean	0·075					
	5·3	153	0·47	0·089					
8 20 „	6·9	139	0·61	0·089					
			Mean	0·089					

June 17th, 1873 (continued).

Clock.					Hand.				
Time.	n.	Mean reading.	I.	$\frac{I}{n}$.	Time.	n.	Mean reading.	I.	$\frac{I}{n}$.
h m	"				h m	"			
9 20 A.M.	5·3	145	0·55	0·104	9 20 A.M.	No observations.			
	6·9	138	0·61	0·088					
			Mean	0·096					
10 20 „	6·9	60	1·21	0·175	10 20 „	5	119	0·75	0·150
	9·6	10	1·68	0·175		5	118	0·76	0·152
						10	20	1·58	0·158
			Mean	0·175				Mean	0·153
								Diff.	+0·022
11 20 „	5·3	153	0·47	0·088	11 20 „	5	150	0·50	0·100
	6·9	145	0·55	0·080		5	153	0·47	0·094
	9·6	129	0·68	0·071		10	117	0·76	0·076
			Mean	0·080				Mean	0·087
								Diff.	−0·007
12 20 P.M.	10·2	154	0·45	0·044	12 20 P.M.	10	156	0·43	0·043
	17·1	125	0·71	0·042		15	137	0·62	0·041
			Mean	0·043				Mean	0·042
								Diff.	+0·001
1 20 „	5·3	124	0·71	0·134	1 20 „	No observations.			
	6·9	95	0·89	0·129					
			Mean	0·132					
2 20 „	2·8	151	0·49	0·175	2 20 „	3	145	0·55	0·183
	3·6	131	0·66	0·183		5	123	0·72	0·144
	5·3	113	0·78	0·147		5	132	0·66	0·132
			Mean	0·168				Mean	0·153
								Diff.	+0·015
3 20 „	10·2	155	0·44	0·043	3 20 „	10	153	0·47	0·047
	17·1	142	0·58	0·034		15	146	0·54	0·036
			Mean	0·039				Mean	0·041
								Diff.	−0·002
4 20 „	10·2	134	0·64	0·063	4 20 „	10	138	0·61	0·061
	17·1	90	0·91	0·053		15	127	0·69	0·046
	20·2	63	1·17	0·058		20	69	1·09	0·055
			Mean	0·058				Mean	0·054
								Diff.	+0·004
5 20 „	10·2	158	0·41	0·040	5 20 „	10	158	0·41	0·041
	17·1	146	0·54	0·032		15	153	0·47	0·031
	20·2	123	0·72	0·036		20	120	0·75	0·035
	31·7	65	1·14	0·036		30	73	1·03	0·034
			Mean	0·036				Mean	0·035
								Diff.	+0·001

June 18th, 1873. (Protector for first four hours.)

Clock.					Hand.				
Time.	n.	Mean reading.	I.	$\frac{I}{n}$.	Time.	n.	Mean reading.	I.	$\frac{I}{n}$
h m	"				h m	"			
4 30 A.M.	31·7	168	0·28	0·017					
5 30 „	20·2	151	0·49	0·024					
	31·7	118	0·76	0·024					
			1·94 × mean	= 0·046					
6 30 „	10·2	146	0·54	0·053					
	17·1	117	0·76	0·044					
			1·94 × mean	= 0·093					
7 30 „	6·9	152	0·48	0·070	No observations.				
	17·1	61	1·20	0·070					
			1·94 × mean	= 0·136					
8 30 „	3·6	147	0·53	0·147					
	5·3	122	0·73	0·138					
			1·94 × mean	= 0·123					
9 30 „	3·6	111	0·80	0·222					
	5·3	57	1·24	0·234					
			Mean	0·228					
10 30 „	2·3	133	0·65	0·282	10 30 A.M.	3	116	0·77	0·257
	2·8	118	0·76	0·271		5	22	1·57	0·314
	6·9	10	1·68	0·243		5	42	1·39	0·278
			Mean	0·265				Mean	0·283
								Diff.	−0·018
11 30 „	3·6	139	0·61	0·169	11 30 „	5	144	0·56	0·112
	6·9	107	0·82	0·111		5	143	0·57	0·114
	9·6	83	0·95	0·099		10	68	1·10	0·110
			Mean	0·129				Mean	0·112
								Diff.	+0·017
12 30 P.M.	3·6	119	0·75	0·208	12 30 P.M.	5	71	1·06	0·212
	5·3	62	1·19	0·225		5	71	1·06	0·212
	6·9	30	1·49	0·216		7	25	1·54	0·220
			Mean	0·216				Mean	0·215
								Diff.	+0·001
1 30 „	10·2	133	0·65	0·064	1 30 „	No observations			
	17·1	73	1·03	0·060					
			Mean	0·062					
2 30 „	17·1	147	0·53	0·031	2 30 „	10	166	0·31	0·031
						15	157	0·42	0·028
								Mean	0·030
								Diff.	+0·001

June 18th, 1873 (continued).

Clock.					Hand.				
Time.	n.	Mean reading.	I.	$\frac{I}{n}$.	Time.	n.	Mean reading.	I.	$\frac{I}{n}$.
h m					h m				
3 30 P.M.	"	No observations.			3 30 P.M.	"	No observations.		
4 30 „	31·7	167	0·30	0·010	4 30 „				
5 30 „	17·1	149	0·51	0·030	5 30 „	15	121	0·74	0·049
	20·2	112	0·79	0·039		20	71	1·06	0·053
	31·7					30	10	1·68	0·056
			Mean	0·035				Mean	0·053
								Diff.	−0·018

July 1st, 1873. (Protector for first two hours.)

Clock.					Hand.				
Time.	n.	Mean reading.	I.	$\frac{I}{n}$.	Time.	n.	Mean reading.	I.	$\frac{I}{n}$.
h m					h m				
5 0 A.M.	" 20·2	160	0·39	0·027	5 0 A.M.	"	No observations.		
6 0 „	17·1	143	0·57	0·064	6 0 „				
(Henceforward fixed strip A used.)									
7 0 „	9·6	157	0·59	0·061	7 0 A.M.	10	152	0·67	0·067
	10·2	155	0·62	0·062		10	148	0·74	0·074
	17·1	118	1·14	0·067		15	128	0·94	0·063
			Mean	0·063				Mean	0·068
								Diff.	−0·005
8 0 „	10·2	172	0·31	0·030	8 0 „	10	169	0·36	0·036
	17·1	165	0·43	0·025		15	167	0·39	0·026
	20·2	165	0·43	0·023		20	158	0·58	0·029
			Mean	0·026				Mean	0·030
								Diff.	−0·004
9 0 „	17·1	160	0·54	0·032	9 0 „	10	155	0·62	0·062
	20·2	146	0·76	0·038		15	143	0·79	0·053
						20	124	0·98	0·049
			Mean	0·035				Mean	0·055
								Diff.	−0·020
10 0 „	5·3	162	0·49	0·092	10 0 „	5	154	0·64	0·128
	9·6	103	1·32	0·138		10	90	1·43	0·143
	17·1	10	2·18	0·127		15	29	1·98	0·132
			Mean	0·119				Mean	0·134
								Diff.	−0·015

July 1st, 1873 (continued).

Clock.					Hand.				
Time.	n.	Mean reading.	I.	$\frac{I}{n}$.	Time.	n.	Mean reading.	I.	$\frac{I}{n}$.
h m	"				h m	"			
11 0 A.M.	5·3	127	0·95	0·179	11 0 A.M.	5	165	0·43	0·086
	6·9	157	0·59	0·085		5	164	0·45	0·090
	9·6	152	0·67	0·070		10	142	0·80	0·080
			Mean	0·111				Mean	0·085
								Diff.	+0·026
12 0	2·8	148	0·74	0·264	12 0	3	150	0·70	0·233
	3·6	127	0·95	0·264		3	145	0·77	0·257
	5·3	119	1·10	0·207		5	119	1·10	0·220
			Mean	0·245				Mean	0·234
								Diff.	+0·011
1 0 P.M.	2·8	147	0·75	0·268	1 0 P.M.	3	145	0·77	0·257
	3·6	140	0·82	0·228		3	144	0·78	0·260
	5·3	122	1·00	0·190		5	86	1·47	0·294
			Mean	0·230				Mean	0·270
								Diff.	−0·040
2 0 „	3·6	122	1·00	0·278	2 0 „	No observations.			
3 0 „	2·8	163	0·47	0·168	3 0 „	3	159	0·56	0·187
	3·6	156	0·61	0·170		3	162	0·49	0·163
			Mean	0·169				Mean	0·175
								Diff.	−0·006
4 0 „	9·6	165	0·43	0·045	4 0 „	10	149	0·72	0·072
	17·1	127	0·95	0·056		15	143	0·79	0·053
			Mean	0·051				Mean	0·062
								Diff.	−0·011
5 0 „	6·9	162	0·49	0·071	5 0 „	15	120	1·06	0·071
	9·6	159	0·56	0·058		10	147	0·75	0·075
			Mean	0·065				Mean	0·073
								Diff.	−0·008

XIX. *On the Organization of the Fossil Plants of the Coal-measures.*—Part VI. Ferns.

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Received March 18,—Read March 26, 1874.

In no part of the investigation upon which I have been so long engaged have I encountered so many real difficulties as in that of which I am about to lay the results before the Society. Amongst the earliest sections which I made I found anomalous structures, generally consisting of a single bundle of vessels, such bundles varying much in size and form, enclosed in a parenchymatous or prosenchymatous bark. Few of these examples exhibited any clear indication of the group of plants to which they had respectively belonged. Many of them might have been either Lycopods or Ferns, so far as structure was concerned. But in addition to the difficulty of assigning them to their respective groups, was the further one of determining which were independent plants or which merely varying portions of the same plant. This latter difficulty is more real in the case of Ferns than of Lycopods, because nothing is more common amongst the recent examples of the ferns than to find a rhizome possessing one structure, its primary petiole another, and its secondary and tertiary petioles yet different structures*; consequently it became exceedingly probable that similar variations would be found in fossil types. This possibility was converted into a certainty by the researches of Cotta, Corda, and Renault, all of whom obtained stems with petioles attached, and which exhibited differences such as I have referred to. But supposing all these difficulties to have been overcome, supposing the *disjecta membra* of each plant to have been properly collocated, and each species to have been correctly referred to its natural order, a new difficulty arose from the plans of procedure adopted by previous writers on this subject. In 1832 C. Bernard Cotta published his 'Dendrolithen,' giving descriptions and somewhat defective figures of a number of specimens to which he assigned new generic and specific names. He threw these forms into the three families of Rhizomata, Stipites, and Radiati, the latter family being defined as "Caules ad tertiam familiam pertinentes strias radiales continent, quæ horizontaliter perscissæ vel inter se separatos concentricos annulos formant, vel inde ab axe incipientes usque ad peripheriam exeunt"

* I may observe that, before completing this portion of my inquiries, I made an extensive series of observations upon the structure of the rhizomes and petioles of all the recent ferns which I could obtain, especially in reference to the way in which the bundles supplying the secondary petioles and nervures originated in the primary one. Having thus made myself practically master of a mass of detailed information on these subjects, I was enabled to study with the greatest advantage the admirable memoir of M. Trécul ("Remarques sur la position des Trachées dans les Fougères," Annales des Sciences Naturelles, tom. xii., 5ème série) and to enter upon the present inquiry.

(*loc. cit.* p. 58). The plants thus defined are obviously such as I should have considered to possess some modification of exogenous growth, though COTTA points out certain features in which he considers that they differ from the stems of true Dicotyledons.

The above publication was followed in 1845 by CORDA's noble work entitled "Beiträge zur Flora der Vorwelt." In this admirably illustrated volume he described and figured a large number of hitherto unknown forms, and included in his respective genera all those previously described by COTTA, the whole being thrown into two primary groups. One of these groups was composed of what he regarded as Monocotyledonous and Dicotyledonous plants, the other of Ferns. These groups he further divided into secondary natural families. The only one of these latter belonging to his first division which concerns me now is that of the Palmæ. The Ferns he divided into seven families and more than forty genera, the latter being too often based upon the most insufficient characters. The undue multiplication of genera by this distinguished botanist was very properly objected to by M. BRONGNIART, who says, "M. CORDA, dans son essai sur la flore de l'ancien monde, me paraît avoir trop multiplié, pour l'état actuel de nos connaissances, les genres fondés sur les tiges des Fougères, dont nous ne connaissons généralement la structure que d'une manière trop imparfaite pour établir des divisions bien définies" ('Tableau des genres de Végétaux Fossiles,' p. 84, 1849). He then proceeds to throw many of CORDA's genera into more comprehensive generic groups, still retaining sixteen genera, nine of which, however, he regards as merely provisional ones.

The second great difficulty with which I have had to contend springs out of CORDA's multiplied genera even as reduced in number by BRONGNIART. They are all based chiefly upon variations in the form of the vascular bundles, a character which a study of living forms demonstrates to have no generic value whatever; and his definitions further take no cognizance of the immense differences which generally subsist between the form of those bundles in the primary and in the secondary petioles. Thus, on the one hand, there is a close resemblance between the transverse sections of the bundles in the petioles of *Dicksonia rubiginosa*, *Pteris umbrosa*, *Adiantum trapeziforme* and *macrophyllum*, and *Aspidium molle*, whilst, on the other, nothing can be more distinct than the vascular arrangements in *Pteris umbrosa* and *P. aquilina*, in *Adiantum trapeziforme* and *A. cuneatum*, and in numerous other examples of a similar kind which might be quoted. Such being the case, it is obvious that no guides can well be less trustworthy than transverse sections of these vascular bundles in establishing generic distinction amongst the fossil ferns.

These difficulties are not diminished by the fact that in the fossil petioles the tissues forming the sheath of each bundle, and the elongated cells which are enclosed along with the vessels, are rarely, if ever, so distinctly marked as in the living one. Indeed in many of them the cortical cells graduate uninterruptedly up to the scalariform vessels, which latter constitute the only tissue that is sharply differentiated from the cortical ones; in other words, such bundles appear to be *vascular* rather than *fibro-vascular*.

Under these circumstances the question which is forced upon our consideration is a practical one. I find amongst my sections several well-marked structures which cannot be located in any of Corda's genera. Hence, if I act upon Corda's method, I must not only adopt all his generic distinctions, but even add to the evil by creating several new genera.

I was at first disposed to adopt the latter plan, and had proposed the names of *Edraxylon* and *Arpexylon** for two undescribed types, and also intended adopting Mr. Binney's name of *Stauropteris*† for a third one. The reasons already assigned make the absurdity of this plan obvious. I have therefore determined to reduce the number of these genera, and to adopt one common term for the entire series of what appear to be fern-petioles, trusting to time and diligent research to enable us to identify them with the several fronds and stems to which they belong, all of which have doubtless already received generic names. Of course I exclude from my common genus such names as *Caulopteris* and *Psaronius*, which represent arborescent stems whose external aspects are more or less known, and which consequently are not genera exclusively based upon the form of the vascular bundle of the petiole. Nothing better indicates the necessity for some such course as that which I propose to adopt than the way in which even Brongniart has classed these petioles or isolated rachis. He divides them into two groups:—"*Pétioles a faisceau vasculaire unique" and "**Pétioles a faisceaux vasculaires multiples." I shall have to describe examples in which the primary petiole would have to be located in one of these divisions and its secondary branches in the other—a condition sufficiently common amongst living ferns.

Corda has embraced a number of these petioles in one family under the name of Rachiopterideæ. In order to avoid the introduction of new names, I propose to adopt the term *Rachiopteris*, and to include in that genus all the several condensed genera adopted by M. Brongniart from Corda, under the names of *Zygopteris*, *Selenochlæna*, *Selenopteris*, *Gyropteris*, *Anachoropteris*, *Ptilorachis*, *Diplophacelus*, *Calopteris*, and *Tempskia*, as well as my own proposed genera of *Edraxylon* and *Arpexylon* and Mr. Binney's *Stauropteris*. I am fully alive to the inconveniences to which this plan exposes us; but it will be easy to indicate the several forms by provisional specific names. It appears almost certain that the stems and petioles which are now represented by the names just quoted will be ultimately found to belong to various species of *Pecopteris*, *Neuropteris*, *Sphenopteris*, and their allies. Consequently all other names can but be provisional and temporary ones. By adopting the plan proposed we avoid burdening science with a number of meaningless genera, based upon characters which have little, if any, generic value, and which only possess even specific ones under peculiar limitations, such limitations arising from the variations which a single petiole exhibits according to the portion of it from which a section is made.

Transverse sections of the petioles of recent ferns exhibit four common types of fibro-vascular bundles:—(1) that in which the transverse section of the bundle represents

* See Proc. Roy. Soc., No. 136. † Proceedings of the Lit. and Phil. Soc. of Manchester, vol. xi. p. 69.

various modifications of the letter H; (2) similar modifications of T; (3) modifications of U; and (4) of X. In many instances these primary bundles have secondary ones superadded. The arrangement of the fibro-vascular bundles in the *stem* or *rhizome* is that of a network constituting a hollow cylinder, prolonged through the entire stem. From each mesh of this cylinder there arise one, two, or more fibro vascular bundles, which pass outwards to form the petiolar bundles; but, as M. Trécul has pointed out, many of these are abortive, because, generally speaking, some or even many of the meshes, especially in creeping rhizomatous stems, fail to develop corresponding leaves. Where leaves are developed we find great variations in the arrangements of the petiolar and radicular bundles. Sometimes both these are given off from the primary cylinder of the stem; at others one or more bundles are thus given off to the petiole, and the radicular bundles spring from them as they pass through the cortical parenchyma of the stem or rhizome. Even where a single fibro-vascular bundle passes outwards to the petiole it frequently subdivides into a number of others, variously distributed over the transverse section of the petiole; but even in this case we always find on the upper portion of the section two dominant ones, from which the secondary bundles going to the secondary petioles are almost exclusively given off.

The bundles, however subdivided throughout the greater part of the length of the petiole, sooner or later reunite as they approach its upper extremity, where, generally speaking, they constitute a single bundle, grooved or indented on its upper surface, and which M. Trécul has designated gutter-shaped ("gouttière vasculaire"). It is, of course, important to bear in mind the occurrence of these morphological variations within the limits of the same frond in attempting to correlate differing sections of fossil forms.

The structure and composition of the fibro-vascular bundle and the mode of orientation of the secondary fibro-vascular bundles require to be attended to. I can confirm, from personal investigation, many of M. Trécul's observations on both these points. Each bundle almost invariably consists of a combination of scalariform or reticulated, annular and spiral vessels, the two latter being located in varying special positions in the bundle in different genera and species of ferns. The vascular portion of the bundle is very distinct from the investing fibrous one, and its section describes very definite figures. The secondary petiolar bundles spring from these primary ones in various ways in the different types. In many cases (e. g. *Pteris hastata*) the upper extremity of each lateral arm of the vascular figure X becomes elongated in alternating order, , and the enlarged portion becomes detached, , to form the secondary bundle. In other cases each of these upper vascular arms terminates in a loop, the concavity of which is directed inwards, . When a secondary branch is about to be given off, this loop ("*crochet*" of Trécul) becomes elongated, (*e. g.* in *Adiantum macrophyllum*). Then a double vascular partition forms, cutting off its outer extremity, , which now separates as a distinct vascular ring, , supplying the secondary pinnule or leaflet. This ring soon opens out at its upper surface, , forming a gutter-shaped bundle; and in a few cases

it again divides inferiorly, and forms two distinct fibro-vascular bundles, but which reunite as they approach the end of the pinnule or base of the leaflet. In some species, *e. g.* in *Pteris umbrosa*, the entire loop is detached to form the secondary bundle.

In the larger number of ferns the secondary pinnules are not given off in pairs, hence the orientation of these secondary bundles alternates on opposite sides of the petiole; in other cases, where the leaflets or pinnules are opposite, we have the same process of orientation occurring simultaneously on the two sides of the primary bundle. In dealing with fossil forms, we have to guard against confounding the *two secondary bundles* thus formed *on opposite sides of the primary one* with the subdivision of a *single primary one into secondary ones* as it emerges from the parent stem or rhizome to enter the petiole. This distinction is not always easy of recognition.

There appears to be no doubt that the small outermost vessels of each arm of each bundle, whatever its form, are those first developed, and that the further growth is centripetal, the larger central vessels of the bundle being produced as the growth advances. Though these plants differ very widely in many respects from the Calamitean and Lycopodiaceous plants described in my previous memoirs, I believe that certain homologous relationships can be demonstrated to exist, enabling me to employ the same letters of reference to indicate corresponding organs and tissues, except that *a* has now been accidentally employed instead of *c* to represent the central vascular axis.

The first of these plants which I have to describe is that to which formerly I proposed to assign the name of *Edraxylon*. I have at length succeeded in connecting this petiole with its leaflets, which are either those of a *Pecopteris* or of a *Sphenopteris*. It has had the surface of its petioles covered with small warty projections, such as Brongniart has represented in several species of *Pecopteris*, especially in his *P. platyrachis**, in *Sphenopteris Hoeninghausi* †, and which appear in many recent ferns.

Plate LI. fig. 1 represents a transverse section of the matured petiole, to which I originally gave the name of *Edraxylon*‡. It consists of a mass of parenchyma containing a bundle of barred vessels, and invested by a layer of sclerenchyma, which is not continuous, but disposed in interrupted longitudinal bands. Each of these matured petioles is about a quarter of an inch in diameter.

The central vascular bundle (*a*) is somewhat of the form of the letter H, with its upper and lower vertical arms diverging outwards from their central connecting band, and its lower ones much shortened. It consists, in fact, of a horizontal bar and two vertical ones, each bar having a variable mean thickness of from ·016 to about ·12§. It consists of transverse sections of numerous vessels arranged without any definite order, and varying in size, but rarely exceeding ·0031 of an inch. I can discover no trace of a special

* Végétaux Fossiles, pl. 103. figs. 5–5 a.

† *Idem*, pl. 52. This plant is also exceedingly well figured in the 'Vorweltliche Pflanzen aus dem Steinkohlengebirge der preussischen Rheinlande und Westphalens' of Dr. Carl Justus Andrä, tab. iv. & v.

‡ Proceedings of the Royal Society, No. 136.

§ As in the previous memoirs, all these measurements are in decimal parts of an inch.

sheath, or of any fibrous elements surrounding this vascular bundle. As is so frequently the case amongst the half-dried petioles of recent ferns, the surrounding parenchymatous cells have slightly shrunk away from the bundle in several parts; but in others they extend up to it. The vertical and horizontal portions are all slightly curved; the concavities of the former being directed outwards, and that of the latter to the lower or dorsal surface of the petiole. Those who are familiar with the aspects of recent petioles of ferns will have no difficulty in identifying the superior and dorsal surfaces in this example. Much of the inner and middle portions of the cortical parenchyma (*g* and *h*) has been destroyed in the specimen; but what remains suffices to show that it was of the ordinary type, and composed of cells whose diameters ranged about ·0025 to ·003, and, so far as this individual section indicates, of nearly uniform character throughout its entire extent. Other sections will show that such was not always the case. The exterior of the petiole consists of a denser layer (*k*), having a thickness of from ·012 to ·016. This is composed of alternating wedges of sclerenchyma (*k'*) and parenchyma (*h'*), the broader parts of the former of which and the narrower ones of the latter reach the peripheral margin of the section; but the forms as well as the sizes of both are irregular and unequal. The sclerenchyma (*k'*) consists of narrow prosenchymatous cells, of from ·0012 to ·0016 in diameter. Their internal cavities are generally filled with carbonaceous matter, making them black and opaque. The intervening angles between the wedges (*k'*) are occupied by an outward extension of the middle parenchyma (*h'*), which here reaches the surface of the petiole. These cells are smaller and more uniform in size than in the interior of the section. The peripheral outline projects at numerous points (*k''*, *k''*) into little swellings. These we shall afterwards find are the little warty projections from the surface of the petiole, apparently abortive hairs, to which I have already alluded. Transverse sections, like that just described, sometimes intersect dark-coloured intercellular lacunæ; but these are much better seen in the vertical sections.

Plate LI. fig. 2 is a vertical section made a little obliquely through a section like the last. *a* represents the central vascular bundle, *g* the innermost parenchyma of the cortex, *h* its median portion, and *k* its prosenchymatous investment. The vessels composing the central bundle in this and all similar specimens are either reticulate (Plate LI. fig. 3 A), barred (fig. 3 B), or spiral (fig. 3 C). The greater portion of them belong to the first of these forms. This is especially the case with those occupying the central parts of all longitudinal sections. Towards the outer margins of the bundle in such sections we find some that are barred and a small number that are unmistakably spiral vessels. The larger ones are invariably reticulated, and in passing along the series we find connecting links between them and the smaller barred ones. On turning to transverse sections of similar bundles, we find that the central line of each bundle is occupied by vessels of larger size than its more peripheral ones. These facts accord substantially with the observations of M. Trécul upon recent Ferns, and which accord with my own. The spiral and annular vessels are both of earlier growth, and occupy a more peripheral position, as well as being of smaller size than the larger scalariform

or reticulated ones, which a centripetal mode of growth produces subsequently and in more central portions of the bundle. The special positions of these spiral vessels vary in the different species of recent ferns, but their general arrangement is that just described.

Immediately surrounding the vascular bundle we have the innermost of the cortical tissues (*g*), which varies much in thickness in different specimens. This is a delicate parenchyma (*g*), the cells of which are from ·002 to ·0025 in diameter, and which are disposed in interrupted vertical lines running parallel with the periphery of the bundle. These cells are nearly square, and must not be confounded with the prosenchymatous fibres which invest the vascular portions of each fibro-vascular bundle in living ferns. In the plant before us this fibrous sheath is wholly wanting. The line of demarcation between the innermost parenchyma and the middle one (*h*) is not very sharply defined, and yet we discern that we have passed from the one to the other from the rapid increase in the size of the cells, and in their diminished tendency to arrange themselves in vertical lines. This central parenchyma constitutes the greater portion of the bark. A large number of its component cells are fully ·005 in diameter. The striking feature of this tissue consists in its containing a large number of oblong intercellular lacunæ (*i*) of a dark colour, having their longer axes arranged radially in the horizontal direction. In Plate LI. fig. 3, two of these lacunæ (*i*) are seen enlarged and, with the cells of the surrounding parenchyma (*h*), drawn with all possible accuracy. It will be observed that whilst most of the cells of the central parenchyma assume an ordinary arrangement, such of them, in the specimen figured, as immediately surround each lacuna display a tendency to radiate from the lacuna as a centre; but this, though a frequent arrangement, is not invariably the case. These lacunæ vary in length from ·002 to ·008, the latter dimensions being the more frequent ones.

The outermost bark (*k*) is seen, in longitudinal sections, to contain a large quantity of prosenchymatous tissue, intermingled with various modifications of parenchyma. The former is arranged in longitudinal bands more or less interrupted, and which correspond with the peripheral wedges (*k*) seen in fig. 1. Plate LI. fig. 5 represents an enlarged tangential section of this part of the plant. The narrow fibres of prosenchyma (*k*) are of great and variable length, and have a diameter of about ·0028. The vertical intermediate bands of long, narrow, square-ended cells (*h*) are extreme modifications of parenchymatous forms, which gradually become shorter as we proceed towards the interior of the plant, and ultimately merge with those of the middle bark, of which they are merely prolongations separating the fibrous wedges as seen in fig. 1, *h'*. The fibrous bands are obviously modifications of the *sclerenchyma* of authors, but which, instead of forming a continuous investing layer, as is so commonly the case in recent ferns, is here arranged in a network of longitudinally disposed bands with very long and narrow meshes.

Though not appearing very prominently in the section represented by fig. 1, the outermost bark projects at innumerable points (fig. 1, *k''*) into small rounded or conical pro-

tuberances. These are generally extensions of the middle parenchyma rather than of the prosenchymatous cells, and obviously correspond with similar projections seen on the petioles of many recent ferns, as in *Alsophila excelsa* and other arborescent forms. Brongniart has also figured several fossil species of *Pecopteris* in which the rachis is covered with similar protuberances. I presume they are to be regarded as undeveloped hairs, or as the remnants of abraded ones.

Having thus ascertained the structure of the matured petiole, we may now examine the modifications which that structure undergoes as we proceed from the base towards the tip of the rachis. As we do so we find that the horizontal bar of the vascular bundle, seen in transverse sections like fig. 1, gradually diminishes in size and alters its form until we obtain the condition represented in Plate LII. fig. 6, *a*, where we have two bundles. These are widely divergent superiorly, but almost meet dorsally. In Plate LII. fig. 7, these two bundles have reunited to form the gutter-shaped bundle of Trécul. In Plate LII. fig. 8, this bundle is but slightly cordate, whilst in figs. 9 & 10 (which latter represents the extremity of the rachis) we have only a very small cylindrical bundle remaining. In all these sections we see in the bark the same features noticed as existing in fig. 1, but in a more prominent form. In the upper part of fig. 6 the cells of the middle parenchyma (*h*) are somewhat drawn out radially, whilst the prosenchymatous fibres (*k*) are very clearly shown, their cavities being filled with coally matter. In the smaller sections 7, 8, 9 & 10, the abortive hairs (*k''*) become conspicuously large in proportion to the size of the petiole. This feature reappears in all the smaller sections*. All these sections are drawn to one scale.

Some longitudinal sections which I have made of these smaller parts of the petiole reveal some important features. Plate LII. fig. 11 represents one of these, and fig. 12 a second one still more enlarged. The vascular bundle reduced in size appears at *a*. In fig. 12, *b*, we find traces of the innermost cortical parenchyma, which here appears to consist of a few layers of narrow, vertically elongated, square-ended cells, suggestive of a rapid vertical growth. The middle parenchyma (*h*) resembles that seen in sections made near the thicker base of the petiole, but in these terminal portions a remarkable difference presents itself: we find bands of small, compactly grouped cells (*i*) extending from the inner to the outermost bark; and at *i'*, *i'*, where the section has passed somewhat tangentially between the inner and outermost bark, we see that these cells are frequently arranged in quoit-shaped disks, exactly resembling, save in their greater irregularity of position, the similar disks in the young petioles of *Heterangium Grievii*, described in my fourth memoir (Phil. Trans. 1873, Pl. xxxi. figs. 45 & 47). I think there can be no doubt whatever that these are the structures which, breaking up at a later period of growth into detached masses, and having their special cells subsequently absorbed, produce the intercellular lacunæ (*i*) seen in Plate LI. figs. 2 & 4; at least I have failed to find any other explanation of the existence of these two singular arrangements in the young and the more matured parts of the petiole.

* Brongniart applies to these organs the name of "écailles sétacées" (Vég. Fossiles, p. 200).

Even had I no further evidence to offer as to the fern-like character of these petioles, it would be impossible to overlook their very close resemblances to those of many recent ferns. The form of their vascular bundles is not widely dissimilar from that of *Asplenium nidus*, whilst their bands of subepidermal fibres represent the continuous layer of the sclerenchyma of METTENIUS and other authors, so common in many ferns. The epidermal surface of *Pteris umbrosa* is covered with small rounded protuberances like those shown in the transverse sections of our fossil. But fortunately we are provided with further testimony. Many of my sections display evidence of the association of leaves with these petioles; but the two represented in Plate LII. fig. 13, and Plate LIII. fig. 14, from sections made by Mr. BUTTERWORTH, are the finest of this class that I have hitherto met with.

Plate LII. fig. 13 is evidently part of the upper extremity of a frond giving off lateral pinnules. In the large central rachis we have at a the vascular bundle, and on each side of this we have the bark with its characteristic masses of dark-coloured cells (i) and its epidermal tubercles (k''). Two lateral pinnules are given off at x, x, and fragments of similar pinnules are scattered throughout the entire section. Each pinnule divides into groups of terminal leaflets (l), each of which leaflets is supplied with a central vascular bundle (m) prolonged from the central vascular axis (a). Numerous similar but detached leaflets (l') are scattered over the section.

Plate LIII. fig. 14 represents either one of these lateral pinnules or that constituting the terminal portion of a rachis in a still more perfect state. The central vascular bundle of the rachis (a) sends branches into each of the lateral pinnules (x), and which again subdivide to reach the several terminal leaflets (l). The general aspect of these sections suggests the idea of a young half-developed frond in a semicircinate state. It is very difficult, if not impossible, to determine with absolute certainty the exact type of fern to which this specimen belongs. The mode of subdivision of the lateral pinnules is more suggestive of a *Sphenopteris* than of a *Pecopteris* or a *Neuropteris*. The leaflets seem to have been grouped in small radiating clusters rather than on the regular bilateral plan of an ordinary *Pecopteris*; at the same time this apparent irregularity may but be the result of a section passing through a plant distorted through pressure. The rounded form of the numerous transverse sections of leaflets (fig. 13, l'') seen in the specimens suggests the idea of the latter being very succulent, with a vascular bundle penetrating the centre of the parenchymatous mass.

Another circumstance connected with these objects originally suggested the possibility that the plant might have been a bipinnate frond of the type of *Pecopteris*. At an early period of my researches my attention was arrested by the frequent occurrence, in the nodules from which these specimens were obtained, of objects like Plate LII. figs. 15 & 16, associated almost invariably with long narrow lines of beaded structures like the upper parts of Plate LIII. fig. 17 and the left-hand side of fig. 18. The former of these structures (fig. 16) consisted of a doubly curved object like a section of a young leaf having a revolute vernation. These objects were generally about ·1

measured from side to side; when unrolled they would be much more. The other beaded structures (fig. 17) were fragments of various lengths, from a quarter of an inch downwards, often arranged in pairs, or even in more or less parallel sets of four, as in the upper part of fig. 17, *l'*. By making sections of these two forms at right angles to each other, as in fig. 17, I succeeded in obtaining proofs that the revolute objects (fig. 15) were merely transverse sections of structures, of which the beaded lines (fig. 17, *l'*) were longitudinal ones. After months of research, involving labour apparently very disproportionate to the end attained, I obtained further proof that these were sections of leaflets of *Pecopteris*, a nearly perfect one of which is represented in Plate LIII. fig. 19. It appears that even in their mature form these leaves were apt to assume a remarkably revolute form. Hence the great number of sections obtained like Plate LII. fig. 15. Specimens like figures 17 & 18 of Plate LIII. showed that the white bead-like spaces represented the intervals between the nervures, and the dark dividing lines were the nervures themselves. In the double-beaded line at the lower part of fig. 17 the section has only passed through one lateral half of the revolute leaf; in the upper part of the same figure, owing to the twisted position of the leaf, it has passed similarly through both halves, hence the four beaded lines *l'*, *l'*. The mid rib of the leaf appears at the points indicated by *l''*, *l''*, *l''*. In the central part of the figure the dark lines of the nervures are seen proceeding upwards and outwards from the mid rib towards the left-hand margin of the leaf, the section being here made in the plane of that half of the leaf without passing entirely through it as at *l*. The right-hand margin of fig. 18 is in the same condition as is the centre of fig. 17.

The exceeding abundance with which fragments like those just described are associated with the specimens represented by all the figures from 1 to 14, early suggested to me the possibility that they all belonged to the same plant. On this supposition figs. 13 & 14 would belong to young and half-developed fronds, whilst figs. 15, 17, 18, & 19 were parts of matured leaflets. Plate LII. fig. 16 represents one of several specimens which seemed to sustain this idea. It looks like a section of a revolute leaflet resembling fig. 15, yet it undoubtedly belongs to the leaflet series of figs. 13 & 14; as is the case with all the leaflets of the latter specimens, its obtuse margins indicate a more succulent state than do the sharply defined incurved edges of Plate LII. fig. 15. If fig. 16 is really a *transverse* section of *one* leaflet, and not a combination of *two* leaflets intersected *longitudinally*, the identity of the two forms would be almost certain; but I have not been able to link the two forms in a perfectly satisfactory manner. There is also the further difficulty, that the objects represented in figs. 15, 17, 18, & 19 are leaflets of a *Pecopteris* of the type of *P. Serlii*, none of the pinnules of fronds of which type are constructed on the plan of figs. 13 & 14. I am therefore now inclined to conclude that *Rachiopteris aspera* (as I propose provisionally to designate the petioles just described) is a *Sphenopteris*. It is an important fact in reference to this question, that we have in the Coal-measures a *Sphenopteris* with a tuberculated rachis, and which has a wide geographical range. This is the *S. Hoeninghausi* already referred to. The

pinnules of this plant exhibit aspects which resemble those of figs. 13 & 14 in the closest manner, rendering the conclusion that our Oldham fossil is generically, if not specifically, identical with the above plant, an exceedingly probable one.

The acceptance of this conclusion demonstrates two facts connected with the Carboniferous ferns:—first, that their vascular bundles are not necessarily, like those of living ferns, surrounded by a definite investing cylinder of prosenchyma; and, secondly, that the prosenchymatous fibres of their subepidermal tissues (the sclerenchyma of authors) do not, as in recent ferns, constitute an uninterrupted investing layer, but that it is broken up into a network of fibrous bands arranged longitudinally, and through the long narrow meshes of which some of the parenchymatous cells of the middle bark reach the epidermal surface. We shall find these two facts of importance, throwing light upon some other examples awaiting examination.

One of the earliest discovered of the plants from the rich storehouse of the upper foot coal-seam near Oldham was one of which Mr. Binney exhibited a section at the meeting of the Manchester Literary and Philosophical Society held on January 9th, 1872, when he proposed to give to the plant the name of *Stauropteris Oldhamia*. As in the preceding instance, I have obtained specimens of this petiole in every stage of growth, from the largest examples down to the smallest twigs. One of the most beautiful, it is also one of the most perplexing of the plants of the Coal-measures—not as regards the details of its structure, since these are clearer than in most of these objects, but the interpretation of them presents several difficulties. The largest of the transverse sections (Plate LIII. fig. 20) are usually about ·125 in diameter. The vascular bundle (*a*) consists of four clusters of vessels arranged in a crucial manner, and is imbedded in a mass of exceedingly delicate cellular tissue (*g*), which not only surrounds, but fills in all the intervals between the bundles. In the specimen fig. 20 this tissue is destroyed, but it is preserved in Plate LIII. figs. 21, 22, 23, & 24; it is invested with a broad prosenchymatous layer (*k*). Plate LIV. fig. 25 is part of a longitudinal section of a matured stem. Each of the four vascular clusters consists of a central group of barred vessels (*a*) of large size, and a more peripheral series (*a'*) of very much smaller ones. The largest of the inner series have a diameter of ·005, and the smallest of the outer ones of ·001. Both the former and the latter are barred. I am unable to detect any true spirals amongst the smaller series. They all pass through the stem with remarkable straightness and freedom from irregularity of arrangement. The cellular tissue which immediately surrounds the vessels (fig. 25, *g*) exhibits the same features as it does in the centre of the stem, where it partially separates the four vascular clusters. It consists of very delicate cells, elongated longitudinally, and having a mean diameter of ·00083. Their ends are often square, but sometimes oblique. A casual glance at many of the specimens would almost make it appear that we have here a true representative of the fibrous sheath of living ferns; but such is not the case. As we proceed from within outwards we perceive that this tissue passes gradually, though rapidly, into a coarse prosenchyma (*h*); in Plate LIII. fig. 20 this

prosenchyma appears as if it terminated abruptly at its inner margin; but this appearance is only due to the destruction of the more delicate central tissue. In Plate LIII. figs. 21 & 22, the two tissues are seen to graduate into each other. At the extreme periphery of the sections the cells become smaller; some of the larger ones have a diameter of as much as ·0025, whilst the lesser ones are not more than ·0012. This thick prosenchymatous cortical layer has every appearance of having been hard and woody, as is the case with the corresponding tissue in many of the living Adiantums. In the transverse section of a smaller stem (Plate LIII. fig. 21), the only change observable is the consolidation of the four vascular bundles into a common bundle, and in the much smaller size of its component vessels. Its transverse section still retains its crucial form. In Plate LIII. fig. 22, representing a yet smaller stem, the transverse section of the bundle has become distinctly trifid. In some specimens this tripartite feature is yet more striking than in that figured, owing to the greater length and slenderness often attained by the three divergent arms. The individual vessels of fig. 21 are now reduced to a very small size. In fig. 23 this reduction has gone further still, and the entire bundle is but a slender, cylindrical, vascular thread; whilst in Plate LIII. fig. 24 the whole stem is reduced to a small cylindrical body, looking more like the ultimate fibre of a root than a stem, and with two or three very small vessels in its centre, scarcely discernible amidst the cellular tissues that invest them. These smallest and ultimate fibres are not more than ·006 in their entire diameter.

Unlike the greater number of the non-Lepidodendroid stems of the Coal-measures, that under consideration branches freely, and we are able to trace the process in the various sections represented by Plate LIII. figs. 25 A & B and Plate LIV. figs. 26 & 27. Fig. 25 A exhibits at a' some of the small peripheral vascular tissues already seen in fig. 20, a'; but on the upper side of the section the corresponding vessels have become detached as well as increased in numbers, forming the two triangular bundles (a'', a''), the inner bark (g) still connecting them with the two uppermost of the primary bundles (a, a). In fig. 25 B we have a similar condition of things, but in a somewhat earlier stage of progress. The diverging tissues (a'') are here less completely detached from the central bundles (a, a), and they have not yet in this instance undergone the division into two separate bundles which invariably takes place before these tissues emerge from the bark. Plate LIV. fig. 26 exhibits a smaller stem in which these trifid bundles (a'') are completely detached from the central one (a). In all the above three cases the transverse sections have been made at points intermediate between those at which the central bundle first separates into three and those at which the branches dichotomize externally. This will be made sufficiently intelligible on referring to Plate LIV. fig. 27, which is a similar branching stem intersected longitudinally. In the latter section the common vascular bundle (a, a) is but slightly divided at its lower part into two bundles by some cellular tissue (g). Superiorly these two bundles (a', a') diverge—that on the right hand proceeding into the branch of its own side, whilst that on the left, and which is a somewhat smaller bundle, proceeds to the left-hand smaller

branch. In the bark investing these bundles we can still trace the various elements seen in the larger stems; but the individual layers of cells are comparatively few in number. The transverse sections of these branching examples thus exhibit some important features. Before a stem is about to branch, there is obviously a decided increase in the number of smaller vessels at the ends and corresponding side of the two arms of the crucial bundle from which the branches are about to proceed. Then these additional vessels, along with others not additional, become detached in two distinct bundles. Sometimes transverse sections of these detached bundles exhibit the same crucial form as that seen in the parent stem; but they much more frequently assume the trifid form seen in fig. 22. Though in making a longitudinal section like fig. 27 we can only pass in the same plane through the main stem and one of its branches, it is obvious that two branches of equal size are given off at each of these ramifications. Hence it would appear as if these branches had been given off in pairs, as in many species of *Pteris*.

The question remains to be asked whether or not this is a fern. In favour of this conclusion we have its general aspect, the arrangement of its vessels in each bundle, the fern-like mode of increase in the dimensions of the vascular bundles on the side from which branches are about to be given off, and the detachment of those bundles in *two* coequal groups, indicating a different mode of branching to what is common amongst the Lycopods. I have seen no Lycopod, transverse sections of which exhibit this mode of orientation of the secondary bundles from the primary ones. The single bundle given off in the Lycopods separates into two parts in the young shoot; but these two bundles always go into the same branch, continue closely parallel to each other, and are but the common points of departure for new vessels, which continue to develop centripetally until the two temporarily separated vascular foci are again united into one elliptical bundle. This is very different from what occurs in the plant just described. But in the latter case, on the other hand, we have the remarkable succession of divisions and subdivisions continued until we obtain clusters of minute twigs like fig. 24, and which look much more like cylindrical rootlets than twigs or ultimate branches of a rachis. It seems strange, supposing them to be the ultimate subdivisions of the petioles of ferns, that we should reach these ultimate divisions without discovering any trace of a leaf or of a leaf-attachment. If they bore leaves they must have been very small terminal ones, resembling in their positions those of some of the Adiantums. But notwithstanding the difficulty just stated, the arguments in favour of the affinity of this plant with ferns appear to me to preponderate over the opposite ones. I would therefore retain it in my genus *Rachiopteris*, associating with it Mr. Binney's specific name of *Oldhamia*.

Amongst the specimens sent to me from Burntisland by Mr. Grieve, I found one or two full of branching fragments of a plant apparently undistinguishable from this Oldham one. Both in size and organization the two appear to be identical.

The next plant to be described is also one which I discovered amongst the Burntisland specimens, but which I have not yet found at any other locality. It is beyond

all question one of the most beautiful structures that I have hitherto obtained from the Carboniferous beds. Plate LV. fig. 28 is a transverse section of this petiole, and Plate LIV. fig. 29 a vertical one taken through its centre and along its longer axis. Its form, as seen in transverse sections, has been oval, with a major diameter of rather more than ·5, and a minor one of about ·37. It possesses, in its matured form, a double vascular bundle, a larger one (a) filling one focal portion of the ellipse, and a smaller one (a') occupying the other. Transverse sections of the former exhibit an hourglass-shape, and similar sections of the other display a crescentic one. The vessels composing the larger bundle (a) are remarkable for the regularity of their arrangement, comparative uniformity of size, and exquisite definiteness of outline. They are generally about ·005 in diameter. At the peripheral extremity of this bundle (a'') there is a deep longitudinal groove running down the side of the bundle, so deep that it is as nearly as possible enclosed by the vessels and converted into a canal*. The vessels in its immediate neighbourhood, and especially those forming the two crescentic horns embracing the groove, are much smaller than the more central ones of the bundle, being about ·0016, and a few of them are even smaller still. These conditions are better shown in Plate LIV. fig. 30, which is a more enlarged representation of a vascular axis of this plant, the general form of which has been disturbed by pressure, but without any corresponding disturbance taking place in its individual vessels. The vessels composing the two horns enclosing the canal (a'') are again seen to be of smaller size than those constituting the rest of the bundle, and a few similar ones exist on the opposite or central side of the groove.

These distinctions are important, since on turning to longitudinal sections of these parts (Plate LIV. fig. 29) we find that these very small vessels are barred ones, whilst all the larger ones are reticulated. At the same time, as is shown by Plate LIV. figs. 31–34, there is a gradual transition from the one modification to the other. Figs. 31 & 32 represent two reticulated forms, the latter being the more common one; fig. 33 is semireticulated, and fig. 34 simply barred. All the four figures are drawn to the same scale. At the opposite or more central end of the hour-glass bundle there is a second longitudinal groove (Plate LV. fig. 28, a'''); but this is less deep, barely amounting to half a circle. A very small number of barred vessels occur in its vicinity, even the more minute ones being generally more or less reticulated. On turning to the crescentic bundle (a'), we find these conditions reversed. Generally speaking its vessels are smaller than those of the hour-glass bundle. A few of the largest are seen to be reticulated, especially as in fig. 33; but most of them are merely barred. I have been unable to detect any true spirals amongst them. In the longitudinal section (Plate LIV. fig. 29), a represents the central part of the hour-glass bundle; a' the crescentic one; a'' is the canal-like groove at the cortical end of the former bundle, with a few barred vessels on the right of it at the lower half of the figure; whilst at the upper half, where the section has just missed the groove, we find

* I have obtained one specimen (Plate LV. fig. 35) in which the incurved crescentic margins absolutely meet, virtually converting this groove into the canal a''.

its place occupied by a number of the small barred vessels which so nearly surround it. In Plate LIV. fig. 30, a', the crescentic vessel is rather different in form from what it generally appears to be. It is brought into close contact with the very slightly convex adjoining face of the hour-glass bundle, its own inner surface for nearly the central half of its length being also flattened, whilst its two extremities are incurved. These arrangements appear to me to be too symmetrical to be merely the accidental results of pressure. In Plate LV. fig. 35 this crescentic bundle is broken up into two (a', a'); and that this subdivision also is not the result of violence, but is a normal organic state, is shown by the circumstance that each half has its ends curled inwards in the same crescentic manner as characterized the two ends of the bundle a' of fig. 30—a fact which, as we shall see immediately, has a physiological significance.

The cortical investment of these bundles closely resembles that of the *Rachiopteris Oldhamia*. We have in both a delicate inner parenchyma (fig. 28, g), usually more or less destroyed, a thick middle portion consisting of a coarse parenchyma (h), and an outer dense prosenchymatous layer (k). In the plant under consideration very little of the innermost tissue (g) remains. In one example it is preserved in the canal-like groove (a''), and in several I find small portions of it in contact with the bundle. All these indications tell the same tale, viz. that it exhibits the common aspect of parenchyma, but with smaller and somewhat more delicate cells than those which constitute the middle portion of the cortex. This latter structure (h) is exquisitely preserved. It consists of a vast multitude of cells arranged, as is so commonly the case amongst recent ferns, in interrupted columns running parallel with the vascular axis. These cells, which in the perpendicular sections appear to be nearly cubical, are about ·0025 in diameter. Plate LIV. fig. 36 is an enlarged representation of three columns of them as they appear in vertical sections. Plate LIV. fig. 29, h, shows how large a portion of the bark is occupied by this tissue; but as we approach the periphery we find that its cells gradually become more elongated vertically, as shown in the enlarged figure 37, and gradually pass into the outer prosenchymatous layer (k), a few of the cells of which are seen in the longitudinal section fig. 38, which is drawn to the same scale as figs. 36 & 37. At the outer surface the prosenchymatous cells have only a diameter of from ·00083 to ·0012. Closely associated with the stems just described are considerable numbers of smaller ones of various dimensions, including mere twigs, like that represented in Plate LV. fig. 40. The structure in all these smaller stems is so exactly identical with the large ones in every respect, save in the more cylindrical form and in the shape of the vascular bundle, that, even had we no direct proof of the fact, there could be no doubt as to their being portions of the same plant. Plate LVI. fig. 39 represents one of the larger examples of this form.

The central vascular bundle (a) exhibits a crescentic section with incurved angles and with three slight tooth-like projections in the interior of the concave margin. When I first called attention to these stems, I had not ascertained the relation of the form just described to that with the hour-glass bundle (Plate LV. fig. 28). Hence I proposed

for it the separate specific name of *Arpexylon simplex*. I have now, however, indisputable evidence that the two are parts of the same plant, and that fig. 39 is but a secondary petiolar branch of fig. 28. I have traced these branches downwards, exhibiting every degree of size from fig. 39 to fig. 40, and even further, since I find abundance of yet smaller twigs in some of my slides, which I have no doubt are the terminal portions of the same structures. The latter have but a very few vessels forming a small central bundle, which loses its special crescentic shape. These conditions remind us of those already represented in figs. 22, 23, & 24, belonging to *Rachiopteris Oldhamia*.

The last described sections make it quite clear that the two halves of the subdivided crescentic bundle seen in Plate LV. fig. 35, a', a', have been so subdivided preparatory to passing outwards through the bark to supply secondary branches like fig. 39. A very slight modification of each of these two clusters of vessels in the former figure would give to it the characteristic form of fig. 39, and the entire process would be in close accordance with what takes place amongst recent ferns.

The difficulties attending the interpretation of this petiole are very great, and would have been much more so, but for a series of sections for the loan of which I am indebted to my friend Mr. CARRUTHERS. The obvious question suggested by the specimens which I have already described is this—have the secondary rachides been invariably given off from one side of the central *hour-glass* bundle? or has the primary crescentic bundle (a') been formed and disappeared in the shape of pairs of secondary bundles, alternately given off from its opposite sides? The specimens now to be described demonstrate the truth of the last of these suggestions.

Plate LV. fig. 35 A is part of an imperfect transverse section of a matured petiole. At a we have the two extremities of the hour-glass bundle, whilst at aa, aa we have, passing through the cortical layer, two crescentic secondary bundles, each of which have been exactly like that of fig. 39; on the concave side of the perfect crescentic bundle (aa), on the right of the figure, we have a yet smaller one (x), which has evidently been detached from that with which it is so closely associated. In fig. 35 B, which represents a second section of the same petiole, taken a little higher up, we obtain a clue to the relations of these several parts. At a we have the hour-glass bundle accidentally ruptured across its central portion; at a' we find the large primary crescentic bundle in its normal position as a single unbroken band. Longitudinal sections in my cabinet clearly demonstrate that the vessels of this primary crescentic bundle leave those of the hour-glass one very gradually, since the two sets run for considerable distances in perfect parallelism with each other. At aa, aa we find the two secondary crescentic bundles going off to some secondary lateral appendages, clearly demonstrating the fact that those appendages, whatever they have been, were alternately given off from the opposite convexities of the hour-glass bundle. The groove at a''' is nearly obliterated, in consequence of the recent detachment of the primary crescentic bundle a'; but the opposite side has so far recovered from the effects of the detachment of the secondary bundles aa, aa, that, instead of being truncated, it has regained its

convex form, and its canal (a'') has, in like manner, ceased to be a mere superficial groove. But we further observe that the small bundle (x) also seen in fig. 35 A has now moved away from the larger secondary bundle (aa) with which it was so closely associated in the last figure. I think I discover a similar bundle at x' in corresponding association with the twin secondary bundle (aa), to the left of the figure. The several vacant spaces (g) in this figure indicate areas from which extensions of the innermost cortical parenchyma investing the vascular bundles of the larger petioles have disappeared. The masses of the parenchyma of the middle bark (h) are continuous with the common investing cortex of the petiole.

In fig. 35 C, which is a third section of the same specimen, but made at a point yet a little higher up in the petiole, we find all the cellular tissues of the bark much more homogeneous than in fig. 35 B. The central bundle (a) and its two branches (aa, aa) retain nearly the same relative positions as before. The small bundle (x') is not very clearly seen, but its fellow on the opposite side (x'') is now very conspicuous. In a section made a little above the last, but which I have not deemed it necessary to figure, the bundle x' is seen moved still nearer to the periphery.

Plate LVI. fig. 35 D represents the next section in the ascending order, and is important to us. The various structures of this section correspond closely with those of fig. 35 C; but it will be observed that at k, k, k, k we have a broad trifid line of condensed prosenchyma enclosing the two areas of the middle cortex, within which are the two secondary bundles (aa, aa), and the left-hand one of these three divergent lines again subdivides in like manner to enclose the small ternary bundle (x''). It is sufficiently obvious that these dark bands consist of similar tissues to those constituting the periphery of the sections fig. 35 A, B, and C, and indicate that along these lines we are approaching the peripheral surface, or that, in other words, the rachis is here about to subdivide into two secondary rachides (aa, aa) and one ternary one (x''). Still further light is thrown upon this matter by another section made still higher up in the petiole, and represented in Plate LVI. fig. 35 E. We now find the two secondary bundles (aa, aa) not only moved much further away from the central hour-glass, from which they are separated by an enlarged mass of the middle cortical parenchyma, but they appear as if they were entering two secondary petioles which project from the primary one like two horns (y, y). But since these horns are chiefly composed of the prosenchyma of the outermost bark, we have no absolute proof whether they actually represent *two* separate secondary petioles, or mere longitudinal ridges of *one* secondary one. But that the former is the case is almost certain, because we see at k' the triangular line of small prosenchymatous cells, such as we find near the surface of the outermost bark, and which, I think, clearly indicates the approaching division of the stem into three distinct branches—a primary one and two secondary ones, as intimated in the description of fig. 35 D.

Unfortunately we do not possess any further sections of this most interesting specimen, hence we cannot trace the ultimate condition of these secondary branches. The

ternary branch x'' of fig. 35 D has also evidently escaped from the periphery of the petiole at the interval between the points at which the sections fig. 35 D & E were made.

On tracing the further development of one of the secondary petioles in the same series of sections, we obtain yet further information respecting their modes of branching. Plate LV. fig. 35 F is the lowermost of several sections of one petiole, which, though virtually transverse, are nevertheless slightly oblique ones. In it we have the secondary crescentic bundle (*aa*), corresponding to the bundles *aa* in figures 35 A, B, C, D, & E. At x we have a ternary bundle passing out of the petiole, the bark of which latter again is much enlarged at this point. At x' we see a second ternary bundle lying immediately outside the uncurved horn of the crescent (*aa*), and evidently preparing to pass off towards the periphery as the bundle x had done, but from the opposite horn of the crescent *aa*. At x'' we find a quaternary bundle evidently going off to supply either a lateral pinnule or a leaflet. In Plate LVI. fig. 35 G we see the ternary bundle x' of Plate LV. fig. 35 F moving outwards, but in the opposite direction to that taken by x in that figure. The latter bundle, with its enlarged mass of cortical cells, has now become altogether detached, and is seen to be a separate rachis, having the relative size and form, compared with that from which it springs, of Plate LV. fig. 40 and of Plate LVI. fig. 35 H, both of which represent sections of similar twigs. Its central bundle has acquired the gutter-shaped section seen in fig. 40*. Plate LVI. fig. 35 I is the next section in this ascending series. The central bundle retains its place at *aa*, whilst its ternary branch (x') with its investing cortex has not only become of considerable size, but is giving off at x''' a quaternary twig, which may either have been the petiole of a leaflet or, more probably, the rachis of an ultimate pinnule. The vascular bundle supplying this quaternary subdivision of the rachis has obviously corresponded to x'' in Plate LV. fig. 35 F, only opposite sides of the rachis are thus occupied in the two examples. But this section (fig. 35 I) shows that whilst this ascending branch of the petiole has thus enlarged, the central vascular bundle (*aa*) is now throwing off at x'' a corresponding bundle to x', only destined for the opposite direction, being now detached from the lower horn of the crescentic bundle as x' was from the upper one. In Plate LVI. fig. 35 K, which is the fourth of this ascending series of transverse sections, we again find the rachis much reduced in size, owing to the entire separation of the large branch x' of fig. 35 I. In the original specimen this latter is now seen to be a small, detached cylindrical rachis corresponding in size and form with fig. 35 H. In fig. 35 K the small cluster of vessels x'', seen in fig. 35 I, x'', is now larger, and more nearly detached than before from the crescent *aa*. In all these specimens, as in all similar recent ferns, the two extreme horns of the crescentic bundle (*aa*), which alternately become detached to form the ternary bundles, are entirely composed of barred vessels, whilst those forming the central part of the crescent are of the reticulated type.

* It is possible that fig. 40 may be the upper portion of a secondary rachis, since at x we find it detaching a bundle like that shown in fig. 35 K.

These latter sections (figs. 35 F, G, H, I, & K) do much to clear up the obscurities of this interesting stem. We now learn that there were certainly three sets of branches given off from the primary petiole, viz. the secondary ones with the large crescentic section (*aa*) of the above figures, the ternary ones (x') of the same series, and the quaternary ones (x'''). Those represented by the letter x' were obviously given off from the barred portions of the crescentic bundles (*aa*) in the manner usually seen amongst such recent ferns as possess a similar bundle, viz. alternately from each of the two horns of the crescent. These may have been the ultimate rachis sustaining the leaflets, one of which (fig. 35 I, x''') may have been merely a small petiole of a leaflet; but even if the latter represents a yet more peripheral or quaternary series of subdivisions, I think I have now proved the existence of that simple alternate arrangement in the orientation of the ultimate bundles, the want of which was so perplexing in the larger petioles. Since I have in no solitary instance met with a petiole containing *two* crescentic bundles, it is clear that those (*aa*, *aa*) of fig. 35 E went to two distinct subdivisions of the primary petiole. Hence it seems to become an indisputable conclusion that, whatever this arrangement may signify, these secondary rachides were given off in pairs, springing alternately from opposite sides of the primary petiole, but that they in their turn bore other subdivisions, which were arranged in the usual alternating and distichous order—these distichous branches themselves either having directly borne the leaflets, or if not having given rise to other and yet smaller distichously arranged subdivisions, upon which the leaflets were planted. It is scarcely necessary to say that this arrangement is an anomalous one. I know of no recent fern in which the secondary branches of the petiole are thus given off in pairs, which pairs are distichously arranged on the primary axis, and each of which secondary petioles sustain ternary ones arranged distichously. Not only will a somewhat similar case come before us on a later page of this Memoir in Corda's genus *Zygopteris*, but the structure of the *Rachiopteris Oldhamia* just described suggests the possibility that a somewhat similar arrangement may have existed in its case. The full meaning of these peculiar relations of the secondary branches to the primary petioles can only be cleared up by further research.

The plant represented in fig. 28 has not yet been discovered in the Lancashire district; but Plate LV. fig. 41 and Plate LVI. fig. 41 A represent two transverse sections of petioles, also with reticulated vessels, and which obviously bear some considerable resemblance to Plate LVI. fig. 39. Mr. Carruthers has favoured me with the sight of a similar section to fig. 41, from the Newcastle coal-field, and for which he was indebted to Professor King, of Galway.

The next plant to be described belongs to a type that has already been figured and described by previous observers. Cotta figured a stem from Chemnitz (Dendrolithen, tab. i. fig. 1) which he designated *Tubicaulis primarius*, and which he regarded as identical with the *Endogenites solenites* of Sprengel. The petioles attached to this stem were numerous, and each one had in its centre a vascular bundle, the transverse section of which somewhat resembled the letter H. Corda subsequently made this

plant the type of his genus *Zygopteris* under the name of *Zygopteris primæva*. Specimens of the same type were discovered by Professor Renault at Autun, and made the subject of an admirable paper which that careful observer published in the 'Annales des Sciences Naturelles,' 5ième série, Bot., tome xii. M. Renault obtained what appear to be four species of this genus, a specimen of one of which fortunately had preserved not only the petioles but the central rhizome from which the petioles sprang, leaving no room for disputing the petiolar character of these structures. Both Mr. Butterworth and Mr. Whittaker, of Oldham, placed in my hands specimens of this genus from the two-foot coal at a very early stage of my inquiries, and I subsequently obtained others from the same coal-mine. One of these closely resembles the species described by M. Renault under the name of *Zygopteris Lacattii*, and the other appears to be identical with his *Z. bibractiensis*. I have not found any of these attached to their primary stem. Mr. Carruthers has referred to this genus *Zygopteris* as being very closely allied to a group of fern-stems which he has united to form his genus *Chelepteris**. I presume he merely means by this that, judging from the two specimens figured by Cotta and Renault, these plants belong to a group in which "the persistent bases of the stipes permanently clothe the small vascular cylinder" (*loc. cit.*), since I am not aware that specimens of any of our English tree ferns have yet been found retaining their internal structure, or that our British examples of *Zygopteris* have as yet been absolutely identified with the stem of a tree fern.

Plate LVI. fig. 42 represents a transverse section of a petiole which, in its general features, closely resembles the *Zygopteris Lacattii* of Renault. These petioles always exhibit an oval section, with a major axis of about ·37 in diameter. The central vascular bundle (*a*) is usually detached from its normal position, through the disappearance of the inner cortical cellular tissue more immediately surrounding it. It consists of a complex group of vessels of various sizes, a transverse section of which group resembles the letter H, but with the extremities of the vertical bars bent inwards. The vessels of the central or horizontal bar (*a'*) are always the largest, most of them having a mean diameter of ·085. Those of each of the vertical arms (*a* & *a''*) are distinctly separable into two sets—an inner one (*a*) consisting of vessels resembling those of the central horizontal bar (*a'*), but not quite so large as the latter ones are, and an outer series (*a''*), in which the vessels are very small, rarely exceeding ·005, and many of them not having half that diameter. Those of the inner series (*a*) do not seem to undergo any material change of position, either in sections of the same stem, made at different points, or in those of different stems; but it is otherwise with the smaller ones (*a''*). In no one instance have I seen those of each of the two vertical bars occupying precisely similar positions. Thus in fig. 42 those on the right hand (fig. 42, *a''*) of the vascular group are but little removed from the vertical bar, whereas on the left hand (*a'*) they are entirely detached, and appear as two broken clusters close to the inner margin of

* "On the Tree Ferns of the Coal-measures, and their Affinities with existing Forms," Report of the British Association for the Advancement of Science for 1872, Trans. Sect. p. 98.

the outer bark. I shall be able to demonstrate that this arrangement is not the *accidental* result of some disturbing force, but is consequent upon the method in which the vascular bundles are given off to the secondary rachis or petioles. As in *Rachiopteris duplex*, these bundles going off to the secondary rachis are given off alternately from the outermost vessels of the opposite sides of the central bundle*. A specimen from Halifax, in the cabinet of Mr. Carruthers, clearly shows that the new vessels which replace those that have now become separated have the closest organic connexion with the central bundle, primarily forming a part of it. M. Renault noticed a somewhat similar condition in his *Zygopteris Lacattii*, and rightly suspected that the arrangement was connected with the formation of the secondary bundles; but owing to the small size of the only fragment which he possessed of this species he was unable to demonstrate this connexion. I happen to have been more fortunate in this respect.

M. Renault pointed out that *all* the vessels of the horizontal bar, and most of those of the two vertical arcs of *Z. Lacattii*, were of the reticulated type ("vaisseaux ponctués" or "vaisseaux aréolés"). A few only, in each of the lateral arcs, exhibited the scalariform type of structure. This result accords with my own observations, only I find in my examples rather more barred or quasi-scalariform vessels in the lateral arcs than M. Renault appears to have met with. The thin areolæ in the walls of the vessels are remarkable for their small size and definitely round or oval outlines (Plate LVII. fig. 44). The greatest diameter of the vessels is not more than ·0004, and ranges between that and ·00029. Fragments of this tissue approach nearer to those constituting the mineral charcoal so common in the Coal-measures, than any other which I have hitherto seen. The innermost cortical layer is very imperfectly represented in my examples, this tissue being generally more or less destroyed. Portions of it preserved in various specimens, as at fig. 42, *g*, exhibit a rather delicate form of parenchyma, which obviously correspond with that described and figured by M. Renault. That observer found the tissue in question penetrated longitudinally by large elongated cellules with proper walls, which he believed to have been gum-canals. The middle bark (*h*) and the outer one (*k*) are well preserved in my specimens. In the transverse sections the former appears as composed of rather thick-walled cells, which diminish in size as we proceed from within outwards. Plate LVII. fig. 43, *h*, exhibits these cells as seen in the vertical sections, where they are arranged in somewhat regular vertical lines (orthenchyma). The cells have square ends; but as we approach the periphery they gradually become elongated vertically, and their extremities become pointed and overlapping, thus passing into the very narrow elongated prosenchymatous cells (*k*) which constitute the outermost portion of the bark.

* I have recently found a specimen in which these bundles of smaller vessels have been given off simultaneously from opposite sides of the central H-shaped bundle, and are now located in similar positions immediately within the bark, but in the opposite foci of the elliptical section. In living ferns in which the distichous arrangement prevails these secondary rachides are sometimes given off in pairs. The petiole may possibly belong to a distinct species with opposite secondary pinnules.—Nov. 9, 1874.

The vertical section represented in Plate LVII. fig. 43 exhibits a vascular bundle (m) going off to some lateral appendage; and it will be observed that it has carried along with it a narrow investing cylinder of the innermost bark (g) enclosed in a broader one of the middle parenchyma (h). I have already referred to the arrangements of the outermost vessels (a'', a'') of the two lateral arcs of Plate LVI. fig. 42, as well as to the supposition of M. Renault that these tissues might possibly become detached from the main bundle to supply lateral ramifications of the frond. My specimens demonstrate the correctness of this inference. Fig. 43, m, shows that such bundles are given off, and that after leaving the central vascular axis they diverge rapidly outwards. On the right hand of fig. 42, as already observed, we see these vessels (a'') but little removed from their corresponding lateral arc, whilst to the left the similar vessels (a') are wholly detached from their arc (a), and almost reach the inner surface of the cylinder of middle bark (h). In Plate LVII. fig. 45 we have a transverse section of another of these vascular axes. At the one extremity we have a large continuous band of small vessels (a'') occupying nearly their normal position, but at the opposite end we have only two small detached clusters of such vessels (a', a'), their central connecting group having disappeared: Plate LVII. figs. 46 & 47 and Plate LVIII. fig. 48 explain these conditions. These figures represent portions of three transverse sections of the same petiole made at points about the eighth of an inch apart. Fig. 46 is a segment of the bark of the lowest of the series, and in it we see that the two vascular masses (m'', m'') have just entered the middle bark (h). At this point they are located in one common oblong areola (g), which was formerly occupied by an extension of the innermost cortical cells. Each of these vascular bundles still exhibits the irregular outline and somewhat square form that we see at a' on the left-hand side of Plate LVI. fig. 42. The middle and outer cortical layers are here unaltered. Plate LVII. fig. 47 exhibits one extremity of the central vascular bundle (a), which presents the appearances already referred to in the instance of fig. 45, a'; viz. there are two detached clusters of small vessels left (a' & a'); but those central vessels which ought to link these two detached clusters into one linear group (as at fig. 45, a'') have gone to form the two bundles already seen at fig. 46, m'', m''. In fig. 47 we find that these bundles (m'', m'') have not only penetrated further into the bark, but the section of each of them has now assumed a cylindrical form, whilst the single areola within which they were enclosed in fig. 46 is now divided into two, separated by a band of the parenchyma of the middle bark (h). The bark has also become enlarged at this point. In Plate LVIII. fig. 48 this enlargement of the petiole has increased yet more. We now find the ordinary parenchyma of the middle bark at h. At k' we have a layer of small cells like those seen near the periphery of the petiole; at h' we have a second mass of middle-bark parenchyma, and at k we have the ordinary prosenchymatous cells forming the outermost layer of the stem. The layer of small cells at k' seems to indicate the approach of the entire separation of the lateral branch or branches from the primary petiole, and the reappearance in the latter of the two layers of middle and outer barks, h & k', in their normal positions and dimen-

sions as seen in fig. 46. One of the lateral bundles (m'') continues its outward course in the more horizontal direction which the upper portion of the similar bundle is seen to follow in fig. 43, producing the more elliptical form seen both in the section of the bundle itself and of the areole representing the missing cellular tissue by which the bundle was immediately invested. The second bundle (m') is now much smaller in size, and has not advanced so far as its companion from the point at which it entered the middle bark. I presume it is either going to some secondary rachis of smaller dimensions than that to which the larger bundle m'' is proceeding, to become a secondary bundle in the same rachis as the larger one, or to become wholly abortive. It is not easy to determine which of these suggestions is the correct one. We are here brought face to face with the same difficulty as that encountered in *Rachiopteris duplex*. For double bundles to enter a common rachis is an ordinary occurrence amongst recent ferns; but unfortunately I fail to discover amongst these fossil secondary petioles any so furnished. Their bundles appear to be single ones in all the plants of the *Zygopteris* type.

Amongst many other specimens from the neighbourhood of Oldham, for which I have been indebted to Mr. J. Butterworth *, were the two represented in figures 49 & 50, and which are obviously the same plant as that described by M. Renault under the name of *Zygopteris bibractiensis*. The central vascular bundle has been somewhat disturbed, probably by the shrinking of the inner cortical layer (g), rather than by any external pressure; but it is easy to see that at a' we have the usual horizontal vascular bar, at the extremities of which we have the two vertical arcs (a, a), one arm of one of the latter being accidentally detached from its normal position at aa. Two features distinguish this vascular axis from that of the *Z. Lacattii* of Renault:—(1) all the vessels composing it are barred, none of them being reticulated; (2) each of the two vertical arcs (a) consists of a double layer of vessels, very differently arranged from those of *Z. Lacattii*. As in the latter case this double layer consists of an inner series (a), the vessels of which are large, and an outer one (a'') composed of much smaller vessels; but in *Z. bibractiensis* these do not merely constitute a double series of larger and smaller vessels in such close apposition as to constitute but two portions of a common cluster, but the two sets of vessels are so arranged as to form one unbroken linear series, constituting a loop or circle, the two sides of which have been brought into close approximation. The very small area which this loop encloses has been shown by M. Renault to be occupied by a delicate parenchyma. The following is the terse description of this arrangement given by the French botanist:—"Les bandes parallèles situées aux extrémités de la ligne

* Mr. Butterworth has recently furnished me with a second and very beautiful example of this petiole. It agrees in every respect with the above description, except that in it the innermost cells of the inner bark, *i. e.* those in immediate contact with the large vascular axis, are very much smaller even than those forming the rest of that inner tissue. They almost form a special sheath to the vascular axis. This new specimen agrees with that described above in showing that the lacunæ in the inner bark of the latter are the accidental results of desiccation. It differs, however, in not exhibiting any traces of the peripheral hairs (fig. 49, h'). The band of small vessels (fig. 49, a'') is continuous and unbroken, as well as in close contact with each vertical bar (fig. 49, a) of the transverse section of the vascular axis. I still find no gum-canals in its bark.—Nov. 9th, 1874.

horizontale du faisceau vasculaire, au lieu d'être simples et formées d'une seule ligne de faisceaux, paraissent divisées en deux lames, formées de vaisseaux de grosseurs très-différentes et séparées par du tissu cellulaire assez mal conservé; ces deux bandes se rejoignent par leurs extrémités" (*loc. cit.* p. 171). It is the loop-like continuity of the parallel bands of large and small vessels at each of their extremities that constitutes the characteristic feature of this plant. In the specimen which I have figured this arrangement is well shown at the lower end of each of the vertical arcs seen in the transverse section of the bundle. At the upper extremity of the arc to the right this arrangement has been disturbed by the accidental detachment of the fragment (*aa*).

The innermost cortical layer (*g*) is much better preserved in my specimens of this species than in those of *Z. Lacattii*, though partial desiccation or some similar disturbing agency has caused much of it to disappear and thus interrupted its continuity. In the upper part of the section about one fourth of it has so disappeared, and even in the remaining portion a ring of large lacunæ exists in it, which obviously did not characterize the plant in its normal state. It consists of a delicate parenchyma. In the vertical section (Plate LVIII. fig. 50) its cells (*g*) are seen to be arranged in the irregular vertical columns so common amongst Cryptogamic plants. This tissue evidently corresponds closely with that observed by M. Renault occupying a similar position in *Zygopteris Lacattii*, only in the latter plant numerous lines of vertically elongated cells exist—supposed by M. Renault to be reservoirs of gum, but of which I find no trace in the specimen under consideration. The middle and outer barks correspond closely with those of *Z. Lacattii*. At *h* we have the more strongly marked middle parenchyma, consisting of cells with much thicker walls than occur in the inner bark; but, as in the latter, they are arranged in irregular vertical lines. On approaching the surface of the petiole this parenchyma rapidly passes into the prosenchyma (*k*) which forms the thick outermost layer of the bark.

The latter tissue exhibits one feature not described by M. Renault, and which I have not observed in any of my specimens of *Z. Lacattii*. Both the longitudinal and transverse sections are furnished with a number of conical protuberances (figs. 49 & 50, *k'*), which are obviously abortive hairs. Transverse sections of these hairs have a perfectly circular outline, and show them to consist of some of the thick-walled prosenchymatous cells of the outer bark deflected outwards. They are evidently of the same nature as the appendages already described in connexion with Plate LII. fig. 13, and similar though smaller examples of which abound on the petioles of the living *Pteris umbrosa*, only from their size and strength in the plant under consideration they must have resembled blunted spines.

None of the writers who have described examples of Corda's genus *Zygopteris* have entertained the least doubt that the plants included in it were petioles of ferns; and my investigations have led me to the same conclusion. The discovery, both by Cotta and M. Renault, of stems to which, in some instances, these petioles were attached, led me to search diligently amongst the nodules of the Oldham two-foot coal, in hopes of

finding some traces of similar stems. The only thing of the kind which I have discovered is the central vascular axis of a stem, a transverse section of which is represented in Plate LVIII. fig. 51. This axis is surrounded by a quantity of disorganized cellular structure, which doubtless represents the cortical tissues, but their disorganized condition obscures their true nature. The axis thus transversely divided consists of five crescentic masses (*a*) of irregularly arranged vessels, enclosing a pentagonal area (*b*), the angles of which are prolonged into five long radiating arms. This area is occupied by parenchymatous cellular tissue. The maximum diameter of the entire vascular axis is about ·125. In many respects this axis approaches very closely to that of a plant described by M. Renault (*loc. cit.* p. 172, pl. 10), and which that observer has referred to Corda's genus *Anachoropteris**. This example is too incomplete to enable me to determine its exact relationships; but it is interesting to find specimens of this type at Oldham associated with *Zygopteris*, because it is an additional example illustrating the remarkable resemblances that exist between types which we have discovered associated in the Lancashire deposits, and those found by M. Renault similarly associated at Autun. That the type just described is very distinct from any of those described in my previous Memoirs is unquestionable. This plant appears to approach so near to the *Anachoropteris Decaisnii* of Renault that it will not be necessary to give it a separate name.

Since there are other fern-petioles from the Oldham beds yet to be considered, I will not make any general observations upon those now described, since that can be better done when the entire series of objects which appear to have belonged to that group of plants have been investigated. These remarks are especially applicable to a fine series of examples of the *Palmacites carbonigerus* of Corda. The plant has recently been described from specimens found at Autun by Professor Renault (Comptes Rendus), who has arrived at exactly the same conclusions respecting it as those which I had previously announced at the Bradford Meeting of the British Association for the advancement of Science in September last. I there expressed my conviction that the plant was not, as had been supposed, an *endogenous* stem, but a fern allied to the Marattiaceæ. Without being aware of this determination M. Renault has arrived at a precisely similar conclusion.

I have again to acknowledge my indebtedness to the cabinets of my friends Mr. Butterworth of Shaw, Mr. Nield and Mr. J. Whittaker of Oldham. Mr. Grieve's specimens from Burntisland have again proved of the utmost value to me, whilst Professor Renault has laid me under great obligations by kindly sending me specimens of several of his Autun fossils for the purpose of comparison with my British ones. Last, but not least, I have to thank my old friend and fellow worker in our common field, Mr. Carruthers. He has not only thrown open his cabinets for my inspection in the freest manner, but as Director of the Botanical Department of the British Museum his official aid has always been afforded to me in whatever way I sought his assistance.

* Mr. Binney has recorded the discovery of similar specimens in the lower Coal-measures of Lancashire. See 'Proceedings of the Literary and Philosophical Society of Manchester,' Jan. 21, 1873.

Description of the Plates.

Rachiopteris aspera.

Plate LI. fig. 1. Transverse section of a matured petiole, magnified 16 diameters.
" fig. 2. Longitudinal section of fig. 1, magnified 20 diameters.
" fig. 3. Vessels of vascular bundle: A, reticulate; B, barred; C, spiral.
" fig. 4. Two lacunæ of the bark, from a vertical section, magnified 65 diameters.
" fig. 5. Tangential section of the outermost layer of the bark, magnified 80 diameters.
Plate LII. fig. 6. Transverse section of the middle portion of a petiole, magnified 16 diameters.
" fig. 7. Transverse section of upper part of a petiole, with the gutter-shaped vascular bundle, magnified 16 diameters.
" fig. 8. Transverse section of the petiole yet nearer to the tip, the vascular bundle becoming circular, and the abortive hairs conspicuous, magnified 16 diameters.
" fig. 9. Transverse section of the petiole near its extremity, with small cylindrical bundle, magnified 16 diameters.
" fig. 10. Transverse section of the extreme termination of a petiole, magnified 16 diameters.
" fig. 11. Longitudinal section of a young petiole, with transverse layers of dense cells, enlarged 12 diameters.
" fig. 12. Portion of a section like fig. 11, magnified 30 diameters.
" fig. 13. Longitudinal section of the upper part of a petiole, with secondary pinnules and leaflets, magnified 16 diameters.
Plate LIII. fig. 14. Longitudinal section of a secondary pinnule, magnified 16 diameters.

Pecopteris.

Plate LII. fig. 15. Transverse section of a leaflet of a *Pecopteris*, magnified 20 diameters.

Rachiopteris aspera.

" fig. 16. Section of a leaflet or leaflets of *Rachiopteris aspera* (direction doubtful), magnified 20 diameters.

Pecopteris.

Plate LIII. fig. 17. Longitudinal and transverse section of part of a leaflet of a *Pecopteris*, magnified 20 diameters.
" fig. 18. Obliquely longitudinal section of a leaflet of a *Pecopteris*, magnified 16 diameters.
" fig. 19. Leaflet of a *Pecopteris*, enlarged 20 diameters.

Rachiopteris Oldhamia.

Plate LIII. fig. 20. Transverse section of a matured petiole, magnified 32 diameters.
" fig. 21. Transverse section of the upper part of a petiole, magnified 32 diameters.
" fig. 22. Transverse section of upper part of a petiole, with the gutter-shaped vascular bundle, magnified 32 diameters.
" fig. 23. Transverse section of a petiole near its extremity, with the vascular bundle cylindrical, magnified 32 diameters.
" fig. 24. Transverse section of the extremity of a petiole, magnified 32 diameters.
Plate LIV. fig. 25. Longitudinal section of a petiole like fig. 20, magnified 65 diameters.
Plate LIII. fig. 25 A. Transverse section of a petiole, with a cluster of small vessels at one side, preparing to become detached as two secondary bundles, magnified 32 diameters.
" fig. 25 B. Transverse section of a petiole like fig. 20, giving off trifid vascular bundles to secondary rachis, magnified 32 diameters.
Plate LIV. fig. 26. Transverse section of a smaller petiole giving off trifid secondary vascular bundles, magnified 32 diameters.
" fig. 27. Longitudinal section of a petiole giving off a large secondary rachis, magnified 32 diameters.

Rachiopteris duplex.

Plate LV. fig. 28. Transverse section of a petiole, magnified 8 diameters.
Plate LIV. fig. 29. Longitudinal section made through the longer axis of fig. 28, magnified 8 diameters.
" fig. 30. Vascular bundles of a matured petiole disturbed by pressure, but with the crescentic bundle (a') separating from the extremity of the "hour-glass" bundle, magnified 24 diameters.
" fig. 31. Reticulated vessel of the "hour-glass" bundle, enlarged 280 diameters.
" fig. 32. Another modified form of fig. 31, magnified 280 diameters.
" fig. 33. Small semireticulated vessel from the surface (a'', fig. 30) of the "hour-glass" bundle, magnified 280 diameters.
" fig. 34. Barred vessel from the cortical side of the canal (a'', fig. 30), magnified 280 diameters.
Plate LV. fig. 35. Vascular bundle of a matured petiole, the crescentic one subdividing into two secondary ones (a', a'), magnified 20 diameters.
" fig. 35 A. Part of a transverse section of a primary petiole, giving off two bundles (aa, aa) to two secondary petioles, one of which in turn is giving off the ternary bundle (x), enlarged 5 diameters.

Plate LV. fig. 35 B. A second transverse section made higher up in the petiole, displaying the same arrangements as the last, but with two ternary bundles (x, x) removed further away from the secondary ones (aa, aa), enlarged 5 diameters.

„ fig. 35 C. A third transverse section of the same made still higher up, and with the left-hand ternary bundle (x'') clearly defined, enlarged 5 diameters.

Plate LVI. fig. 35 D. The fourth section, with lines of prosenchymatous cells (k) arranged in three divergent lines, and indicating the approaching separation of the petiole into four parts, viz. a primary, two secondary, and one ternary petiole, enlarged 5 diameters.

„ fig. 35 E. The fifth section, in which the ternary branch (35 D, x'') has disappeared, and the two secondary ones (y, y) are further enlarged preparatory to becoming similarly detached.

Plate LV. fig. 35 F. Lowermost of a series of transverse sections of a secondary petiole, with its central crescentic bundle at aa; a ternary bundle (x) is proceeding outwards from one side of the petiole, and a second one is being given off at x', in the opposite direction from the opposite horn of the crescent; enlarged 8 diameters.

Plate LVI. fig. 35 G. A second section made a little above the last, with the upper ternary bundle (x') moving outwards towards the periphery, enlarged 8 diameters.

„ fig. 35 H. The ternary bundle (x of figure 35 F), from a section intermediate between F and G, and now become an independent ternary rachis, enlarged 8 diameters.

„ fig. 35 I. A yet higher section than fig. 35 G, with the ternary bundle (x') displaying a quaternary branch (x'''), and a new bundle being separated at x'' from the central crescent for liberation in the upward direction, enlarged 8 diameters.

„ fig. 35 K. A still higher transverse section, in which the ternary bundle (x' of fig. 35 I) no longer appears, it having become detached as the centre of an independent rachis, similarly, *though in the opposite direction*, to that in which the bundle x became detached from fig. 35 F. A new bundle is now preparing to be given off from 35 K, x'' in the same direction as 35 F, x. Enlarged 8 diameters.

Plate LIV. fig. 36. Portion of a vertical section of the middle bark (fig. 29, h), magnified 80 diameters.

Plate LV. fig. 37. Oblong cells of a vertical section of the outermost portion of the middle bark (h), passing into the prosenchymatous form of the outermost layer (k), magnified 80 diameters.

„ fig. 38. Prosenchymatous cells of the outermost bark, from a vertical section, magnified 80 diameters.

Plate LVI. fig. 39. Transverse section of a large secondary petiole, magnified 8 diameters.

Plate LV. fig. 40. Transverse section of a yet smaller secondary petiole, magnified 8 diameters.

„ fig. 41. Transverse section of a petiole, resembling fig. 39, from Oldham, enlarged 16 diameters.

Plate LVI. fig. 41 A. Another petiole, from Oldham, similar to fig 41, enlarged 16 diameters.

Rachiopteris Lacattii.

Plate LVI. fig. 42. Transverse section of a petiole, magnified 16 diameters.

Plate LVII. fig. 43. Longitudinal section, giving off a vascular bundle to a secondary petiole, magnified 20 diameters.

„ fig. 44. Reticulated vessel of fig. 43, magnified 280 diameters.

„ fig. 45. Transverse section of a detached vascular axis, magnified 20 diameters.

„ fig. 46. Part of the bark of a transverse section into which the two irregularly formed vascular bundles (m'', m') have entered on their way outwards towards one or two secondary petioles, magnified 20 diameters.

„ fig. 47. Portion of a transverse section of the same specimen as fig. 46, but made a little higher up in the petiole, and with the vascular bundles (m'', m'') assuming a cylindrical form, magnified 20 diameters.

Plate LVIII. fig. 48. Part of the bark of a section of the same specimen, but made yet higher up, and with one of the vascular bundles apparently becoming abortive, magnified 20 diameters.

Rachiopteris bibractiensis.

„ fig. 49. Transverse section of a petiole, magnified 20 diameters.

„ fig. 50. Longitudinal section of fig. 49, magnified 20 diameters.

Anachoropteris Decaisnii.

„ fig. 51. Transverse section of the vascular axis of a stem apparently identical with the *Anachoropteris Decaisnii* of RENAULT.

The letters of reference employed in the Plates correspond with those employed in the previous memoirs, except in the case of one or two.

a. Central vascular axis.
g. Inner cortical layer.
h. Middle cortical layer.
k. Outer cortical layer and its appendages.
x. Ultimate subdivisions of the vascular bundles supplying branches of the rachis.

XX. *On Mr.* Spottiswoode's *Contact Problems.* *By* W. K. Clifford, *M.A., Professor of Applied Mathematics and Mechanics in University College, London. Communicated by* W. Spottiswoode, *M.A., Treas. & V.P.R.S.*

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The present communication consists of two parts.

The first part treats of the contact of conics with a given surface at a given point; this class of questions was first treated by Mr. Spottiswoode in his paper "On the Contact of Conics with Surfaces," and general formulæ applicable to all such questions were given.

The results of that paper are here reproduced with some additions; with the exception of a few collateral theorems, these are all contained in the following Table:—

*Number of five-point conics through fixed point	= 6
*Order of surface formed by five-point conics through fixed axis .	= 8
Number of six-point conics through fixed axis	= 9†
*Number of seven-point conics	=70

The second part treats of the contact of a quadric surface with a surface of the order n; and in particular it determines the number of points at which a quadric (other than the tangent plane reckoned twice) can have four-branch contact with the surface. In his paper "On the Contact of Surfaces," Mr. Spottiswoode proves that at an arbitrary point on a surface there is no other solution than the doubled tangent plane, and gives the conditions that must be satisfied by those points at which another solution is possible.

The method here adopted is an extension of that applied by Joachimstal to the contact of lines with curves and surfaces. The coordinates of a point on a conic are expressed in terms of a single parameter, those of a point on a quadric by two parameters. To determine the intersection with a given surface we have an equation in the parameter or parameters, and the conditions of contact are expressed in terms of the coefficients of that equation. The special case of the intersection of a quadric with a cubic surface is treated by the method of representation on a plane.

* These results constitute the additions.

† [Note by W. Spottiswoode.—In the Memoir quoted by Professor Clifford, it was stated that the number of conics passing through a given axis and having six-pointic contact with a surface at a given point is ten. In making this statement I overlooked the fact that, in order to put in evidence that a certain quantity was a factor of the equation which determines the positions of the planes of the conics, the equation was multiplied by a quantity D which is a linear function of the position. In reckoning the degree of the equation this factor must of course be discarded. The degree is consequently less by unity than that stated in the Memoir; viz. it is 9, as proved by Professor Clifford.—July 3, 1873.]

PART I.—THE CONTACT OF CONICS WITH SURFACES OF ORDER n.

I.

The current plane-coordinates being denoted by

$$X_1, X_2, X_3, X_4,$$

let the equations of the three points A, B, C be respectively

$$0 = a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4 = \Sigma aX,$$
$$0 = \Sigma bX,$$
$$0 = \Sigma cX.$$

The quantities a_i, b_i, c_i $(i=1, 2, 3, 4)$ are the coordinates of the points A, B, C. The symbol A itself I shall use indifferently, as denoting either the form ΣaX or the differential operator

$$a_1\partial_{x_1} + a_2\partial_{x_2} + a_3\partial_{x_3} + a_4\partial_{x_4} = \Sigma a\partial_x,$$

where x_1, x_2, x_3, x_4 are the current point-coordinates. It will be seen in the sequel that this double meaning is useful, while it does not introduce any confusion. Similar interpretations are of course to be given to the symbols B, C, and the like.

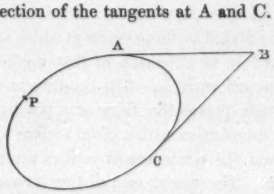
Consider now the point

$$P = A + \theta B + \theta^2 C,$$

whose coordinates are $a_i + \theta b_i + \theta^2 c_i (i=1, 2, 3, 4)$. If we suppose θ to take all possible values, the point P will describe a conic section whose tangential equation is

$$0 = 4AC - B^2 = K.$$

To the value $\theta=0$ corresponds the point A, to $\theta=\infty$ the point C; while the equation shows that B is the intersection of the tangents at A and C.



To find the point at which this conic intersects a given surface u_n of the order n, we must substitute the coordinates of P in the equation of the surface; in this way we shall form an equation in θ of the order $2n$, the solution of which will give the values of θ belonging to the $2n$ points of intersection.

If in this equation the term independent of θ vanishes, then $\theta=0$ is a root of the equation; consequently the point A is one of the intersections, or the surface u_n passes through the point A. If also the coefficient of θ vanishes, another root of the equation coincides with zero, and *two* points of intersection are at A. And generally if the

coefficient of θ^m is the first that does not vanish, m roots of the equation coincide with zero, and m points of intersection are united at A.

The result of substituting the coordinates of any point P in u_n may be conveniently represented by means of the differential operator P. It is known, in fact, that

$$(p_1\partial_{x_1}+p_2\partial_{x_2}+p_3\partial_{x_3}+p_4\partial_{x_4})^n\,.\,(x_1, x_2, x_3, x_4)^n = \underline{|n}\,.\,(p_1, p_2, p_3, p_4)^n;$$

or, which is the same thing, $P^n u_n$ is $\underline{|n}$ times the result of substituting the coordinates of P in u_n. Our result may therefore be stated in the following form:—

The necessary and sufficient conditions that the conic $K_2 \equiv 4AC - B^2$ *may have* m*-point contact with the surface* u_n *at the point* A, *are that in the expansion of* $(A+\theta B+\theta^2 C)^n\,.\,u_n$ *in powers of* θ, *the* m*th power of* θ *is the lowest whose coefficient does not vanish.*

II.

Equating to zero the coefficients of 1, θ, θ^2 in this expansion, we obtain

$$0 = A^n\,.\,u_n,$$
$$0 = nA^{n-1}B\,.\,u_n,$$
$$0 = \tfrac{1}{2}n(n-1)A^{n-2}B^2\,.\,u_n + nA^{n-1}C\,.\,u_n.$$

Before proceeding further with these equations, it is convenient to make the following remarks upon their nature, which will serve to simplify the expression of them.

In the first place, then, we have here a series of relations among the coordinates of the points A, B, C and the coefficients of the surface u_n; and the determination of the coordinates and coefficients so as to satisfy a certain number of the relations presents us with the solution of various geometrical problems. These problems fall naturally into three classes.

1. The surface u_n and the point A are given. In this case the unknowns are the ratios of the eight quantities b, c, a singly infinite number of solutions corresponding to each conic; and we are accordingly able to satisfy seven of the equations*. The problem here is to find the number of conics which have seven-point contact at a given point of a given surface.

We may, however, impose beforehand certain restrictions upon the values of the unknowns, and so consider problems which involve a less number of the equations. While the number of the septactic conics is definite, the sextactic conics form a singly infinite series; and we may ask what is the number of them:

(a) whose planes pass through a given point,
(b) which meet a given line, or
(c) which touch a given plane.

The quintactic conics, again, form a doubly infinite system, and we may inquire about the number of them which satisfy two conditions; *e. g.* which pass through a given point.

* Viz. six besides the first, which is satisfied identically.

2. The surface u_n is given, but not the point A. In this case, as the point A is only restricted to be a point on the surface, we have two more unknowns, making nine in all. The problems here are, to find the order of the curve on the surface at every point of which there is a conic having eight-point contact, and to find the number of points at which there is a conic having nine-point contact.

As before, however, there are certain derived problems coming under this head which involve a less number of equations. We may seek the order of the curve traced out by points of contact of septactic conics satisfying one condition, sextactic satisfying two, &c.; or we may seek the number of septactic conics satisfying two conditions, sextactic satisfying three, &c.

3. The surface is not wholly given. We may here assign a number of relations sufficient to eliminate the quantities a, b, c, leaving one or more relations among the coefficients of u_n. The problems here are such as:—to find the number of surfaces in a pencil $u_n+\lambda v_n$ which admit of ten-point contact with some conic, or one of whose nine-point conics meets a given line.

In the present communication only problems of class 1 will be considered; the formulæ in this case may be very considerably simplified. The quantities a and the coefficients of u_n, then, are data of the problem; so that the first of our equations, $A^n u_n=0$, is satisfied by hypothesis. The next equation, $A^{n-1}Bu_n=0$, signifies that the point B lies in the tangent plane at A, as it obviously must if the conic touch the surface at A. We shall suppose this also to be satisfied from the commencement; that is, we shall regard B as a point moving in the tangent plane, and to be determined by construction in that plane. This may be effected analytically if we substitute for B, $\lambda A+\mu B+\nu B'$, where now B, B′ are regarded as fixed points in the tangent plane, and the three unknowns λ, μ, ν take the place of the four quantities b. There is, however, as will be seen, no occasion to make the substitution explicitly.

This being so, any relation involving B only beside the data must be regarded as the equation of a curve in the tangent plane. For example, $A^{n-2}B^2u_n=0$, expressing that B lies on the quadric polar of A, is the equation of the two chief tangents at that point. Generally, $B^n u_n=0$ is the equation of the intersection with u_n of the tangent plane; and the curves $AB^{n-1}u_n=0$, $A^2B^{n-2}u_n=0$, &c. are the successive polars of A in regard to that intersection.

The terms entering into our equations are of the general form

$$\phi^{p+2q}\cdot\frac{\lfloor n}{\lfloor n-p-q\,\lfloor p\,\lfloor q}A^{n-p-q}B^pC^q\cdot u_n.$$

Any term, therefore, is completely determined by the two numbers p and q, and might, for any thing that has yet appeared, be denoted by (p, q). In view, however, of subsequent substitutions, we shall keep in evidence the manner in which B and C are involved, and denote the term in question by the symbol $\phi^{p+2q}(B^pC^q)$.

As we do not consider any higher than seven-point contact, we have only the five equations:—

$$0=(B^3)+(C). \quad (3)$$
$$0=(B^2)+(BC). \quad (4)$$
$$0=(B^4)+(B^3C)+(C^2) \quad (5)$$
$$0=(B^4)+(B^3C)+(BC^2). \quad (6)$$
$$0=(B^5)+(B^4C)+(B^3C^2)+(C^3). \quad (7)$$

III. *Conics through a fixed point.*

Combining equation (3) successively with (4) and (5), so as to obtain results homogeneous in B, we find

$$0=(B^3)(C)-(BC)(B^3),$$
$$0=(B^4)(C)^2-(B^3C)(B^3)(C)+(C^2)(B^3)^2.$$

If we regard C as a fixed point, these are equations of a cubic and a quartic curve in the tangent plane, on each of which B must lie if the conic K_2 has five-point contact. But these curves have a common node at A, and common tangents at it; for every term in each has at least one factor of the form (B^m), which we know to represent a polar of A in regard to the intersection of u_n by the tangent plane—that is, a curve touched by the chief tangents at A. Of their 12 intersections, then, 6 coincide with the point A; and there remain *six conics having five-point contact at* A *which pass through an arbitrary point* C.

IV. *Conics meeting a fixed axis through* A.

If the point C, instead of being altogether given, is movable on a fixed straight line through A, we may represent it by $A+\lambda C$; where now C is really a fixed point, and λ a quantity to be determined. When we make this substitution in our equations, they become*

$$0=(B^3)+\lambda(C),$$
$$0=(B^2)+\lambda(BC),$$
$$0=(B^4)+(B^3)+\lambda(B^3C)+2\lambda(C)+\lambda^2(C^2),$$
$$0=(B^4)+(B^3)+\lambda(B^3C)+2\lambda(BC)+\lambda^2(BC^2),$$
$$0=(B^6)+(B^4)+\lambda(B^4C)+(B^3)+2\lambda(B^3C)+\lambda^2(B^3C^2)+3\lambda(C)+3\lambda^2(C^2)+\lambda^3(C^3).$$

The last three admit of obvious simplifications by aid of the previous ones; and the system may finally be written

$$0=(B^3)+\lambda(C). \quad (3')$$
$$0=(B^2)+\lambda(BC). \quad (4')$$
$$(B^3)=(B^4)+\lambda(B^3C)+\lambda^2(C^2). \quad (5')$$
$$(B^3)=(B^4)+\lambda(B^3C)+\lambda^2(BC^2). \quad (6')$$
$$(B^4)-\lambda^2(C^2)=(B^5)+\lambda(B^4C)+\lambda^2(B^3C^2)+\lambda^3(C^3). \quad (7')$$

* It must be remembered that (B) and terms involving A only vanish by hypothesis. The formulæ have also been simplified by the omission of certain coefficients depending on n which do not affect the final results.

Locus of poles of axis in regard to four-point conics.

If we select any plane through the line AC, there will be a singly infinite number of conics in the plane having four-point contact with the surface at A. The line AC will, as is well known, have the same pole in regard to all these conics—that is to say, the point B will be the same for the whole system. If we now allow the plane to turn round the axis, the point B will trace out a curve in the tangent plane. The equation to this curve is got by eliminating λ between equations (3′) and (4′); namely, it is

$$0=(B^1)(BC)-(B^2)(C). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4'')$$

We see, therefore, that *the locus of the poles of an axis in regard to all the four-point conics whose planes pass through it is a cubic curve in the tangent plane touching the chief tangents at* A, *which point is therefore a node on the curve.*

We might have inferred this from the fact that on any line through A there is only one point B, while this point coincides with A in the case of the two chief tangents; since at a point of inflexion all the four-point conics contain the inflexional tangent.

Number of six-point conics through an axis.

We have now to determine B so that the equations (3′), (4′), (5′), (6′) may be simultaneously satisfied. We know already that B must lie on the cubic (4″); it is necessary therefore to find some other locus on which it has to lie. First of all, then, we must eliminate λ between (3′) and (5′) and between (3′) and (6′); the results are,

$$(B^1)(C)^3=(B^1)(C)^3-(B^2C)(B^1)(C)+(B^1)^2(C^2)=p_4, \text{ say};$$
$$(B^1)(C)^3=(B^1)(C)^3-(B^2C)(B^1)(C)+(B^1)^2(BC^2)=q_5, \text{ say}.$$

Here $p_4=0$ and $q_5=0$ are curves touching the chief tangents at A, and of the degrees four and five respectively. But the equations are not homogeneous; in fact only the ratios and not the absolute values of the quantities a were determined by the fixing of the point A, and they may be regarded as involving an arbitrary factor whose square affects the left-hand side of the equations. It is, however, at once eliminated, and we obtain the homogeneous result,

$$(B^1).q_5=(B^1).p_4.$$

This is a curve of order 7 having two branches in each of the chief directions at A. Of its 21 intersections with the cubic (4″), then, 12 coincide with A, and there remain *nine positions of* B *which give sextactic conics through the fixed axis*; or we may say, *of the sextactic conics at the point* A, *there are nine whose planes pass through an arbitrary point* C.

System of five-point conics through an axis.

Since there is one five-point conic in every plane, if we consider all the planes through a fixed axis we shall obtain a singly infinite number of five-point conics. Of this

system there is only one conic whose plane passes through an arbitrary point, viz. the conic determined by the plane through that point and the axis.

There are eight conics of the system which meet an arbitrary line.

The number of conics which meet an arbitrary line is clearly the same as the order of the surface which they trace out. Now, since through every point on the axis can be drawn six conics of the system (as proved in the last section), the axis is a six-fold line on the surface. The section of the surface, then, by a plane through the axis is made up of the axis taken six times over and the conic in that plane; or it is of the order *eight*. Q.E.D.

V. *Conics not subject to any condition.*

In order to get rid of the restriction of meeting a fixed axis, we must again modify our fundamental equations. We have to put them into a form in which they will represent *any* conic touching the surface u_n at the point A. For this purpose it is necessary and sufficient that C should be movable over a plane passing through A; since every conic through A must cut the plane in one other point, but this may be any point of the plane. We attain this analytically by substituting for λC in the second set of equations, $\lambda C+\mu D$, where C and D are now fixed points not in the tangent plane and not in any straight line through A. This is equivalent to still considering the conics which meet a given axis, but allowing that axis to move over a fixed plane.

The transformed equations are:—

$$0=(B^2)+\lambda(C)+\mu(D). \qquad (3''')$$

$$0=(B^3)+\lambda(BC)+\mu(BD). \qquad (4''')$$

$$(B^3)=(B^4)+\lambda(B^3C)+\mu(B^3D)+\lambda^2(C^2)+2\lambda\mu(CD)+\mu^2(D^2). \qquad (5''')$$

$$(B^3)=(B^4)+\lambda(B^3C)+\mu(B^3D)+\lambda^2(BC^2)+2\lambda\mu(BCD)+\mu^2(BD^2). \qquad (6''')$$

$$\left.\begin{aligned}(B^4)-\lambda^2(C^2)-2\lambda\mu(CD)-\mu^2(D^2)=(B^5)+\lambda(B^4C)+\mu(B^4D)+\lambda^2(B^3C^2)+2\lambda\mu(B^3CD)\\ +\mu^2(B^3D^2)+\lambda^3(C^3)+3\lambda^2\mu(C^2D)+3\lambda\mu^2(CD^2)+\mu^3(D^3).\end{aligned}\right\} \qquad (7''')$$

Locus of poles of axis in given plane in regard to sextactics.

From the first two of these equations we obtain

$$1:\lambda:\mu=(BD)(C)-(BC)(D):(B^2)(D)-(B^2)(BD):(B^2)(C)-(B^2)(BC)$$
$$=n_1:l_2:m_2, \text{ say};$$

here $n_1=0$ is the equation to a straight line passing through A, while $l_2=0$, $m_2=0$ are cubics touching the chief tangents at A.

Substituting these values in (5‴) and (6‴), we obtain

$$n_1^3(B^3)=n_1^3(B^4)+n_1l_3(B^3C)+n_1m_3(B^3D)+l_3^2(C^3)+2l_3m_3(CD)+m_3^2(D^3)$$
$$=u_6, \text{ say};$$
$$n_1^3(B^3)=n_1^3(B^3)+n_1l_3(B^3C)+n_1m_3(B^3D)+l_3^2(BC^3)+2l_3m_3(BCD)+m_3^2(BD^3)$$
$$=v_7, \text{ say}.$$

Here the curves $u_6=0$, $v_7=0$ are of the sixth and seventh orders respectively, each of them having *one* branch at A in each of the chief directions, and one other branch different for the two curves.

The equations

$$n_1^3(B^3)=u_6,$$
$$n_1^3(B^3)=v_7$$

must hold for six-point contact, but they are not homogeneous. Eliminating n_1^3, however, there results

$$(B^3).v_7=(B^3).u_6,$$

a curve of the ninth order, locus of the poles in regard to the sextactic conics of an axis moving in a fixed plane. This curve has *two* branches at A in each of the chief directions, and *one* other branch; and consequently is met by the plane ACD in *five* points coinciding with A, and in *four* other points. Now the plane ACD does not in general contain a sextactic conic; the pole of the axis can therefore only be in this plane when the axis itself is in the tangent plane. In this case there is a certain number of proper sextactic conics in planes through the axis, and it is clear that the pole of the axis in regard to any such conic is the point A. These conics, therefore, correspond to the *five* intersections of the plane ACD with the locus of poles which coincide with A; or *through an axis in the tangent plane can be drawn five proper sextactic conics.* In the tangent plane itself there are four improper conics having six-point contact; viz. the pair of chief tangents, which (as a sharp conic or line-pair reckoned among conics given tangentially) counts for two, and each chief tangent doubled.

Number of septactic conics.

There is a finite number of septactic conics at the point A; each of these meets the plane ACD in a determinate point $A+\lambda C+\mu D$, and fixes thereby a position of the point B. These positions of the point B must necessarily lie in the 9thic locus just investigated; it remains only to find a homogeneous relation which shall determine another locus for B, and to count the number of their intersections.

To this end we must first substitute for $1:\lambda:\mu$ their values $n_1:l_3:m_3$ in equation (7″). The result is

$$n_1^3(B^4)-n_1l_3^2(C^3)-2n_1l_3m_3(CD)-n_1m_3^2(D^3)=n_1^3(B^6)+n_1^2l_3(B^4C)+n_1m_3(B^4D)$$
$$+n_1l_3^2(B^3C^3)+2n_1l_3m_3(B^3CD)+n_1m_3^2(B^3D)$$
$$+l_3^2(C^3)+3l_3^2m_3(C^2D)+3l_3m_3^2(CD^2)+m_3^2(D^3),$$

or

$$n_1l_3=w_9, \text{ say}.$$

The curve w_9 has *one* branch at A in each of the chief directions and one other branch. The curve t_6 has one branch in each of the chief directions and *two* other branches.

The equations (5), (6), (7) have now become

$$n_1^2(\mathrm{B}^2)=u_6,$$
$$n_1^2(\mathrm{B}^2)=v_7,$$
$$n_1 t_6=w_9.$$

The first two of these give us the curve already considered,

$$(\mathrm{B}^2)\,.\,v_7=(\mathrm{B}^2)\,.\,u_7,$$

which has at A two branches in the chief directions and one other. The first and third give

$$n_1\,.\,(\mathrm{B}^2)\,.\,w_9\,t_6 u_6,$$

a curve of order 12, having two branches in each of the chief directions and two other branches. Of the 108 intersections of these curves, then, $24+8+4+2=38$ coincide with A, leaving 70 for the number of septactic conics.

Part II.—The Contact of Quadric Surfaces with Surfaces of Order n.

I. *Conditions of contact.*

Let A, B, C, D be four points forming a tetrahedron, whose tangential equations are

$$0=\Sigma aX,\ \Sigma bX,\ \Sigma cX,\ \Sigma dX$$

respectively, their coordinates being a_i, b_i, c_i, d_i $(i=1, 2, 3, 4)$. Then the point

$$\mathrm{P}\equiv\mathrm{A}+\theta\mathrm{B}+\varphi\mathrm{C}+\theta\varphi\mathrm{D},$$

whose coordinates are

$$p_i=a_i+\theta b_i+\varphi c_i+\theta\varphi d_i\ (i=1, 2, 3, 4),$$

will, if we suppose θ and φ to take all possible values, trace out a quadric surface whose tangential equation is

$$0=\mathrm{AD}-\mathrm{BC}\equiv\mathrm{Q}_2.$$

To the pair of values

$\theta=0$, $\varphi=0$, corresponds the point A,
$\theta=\infty$, $\varphi=0$, „ „ B,
$\theta=0$, $\varphi=\infty$, „ „ C,
$\theta=\infty$, $\varphi=\infty$, „ „ D.

The equation shows that AB, AC, BD, CD are generating lines.

If, now, we wish to find the nature of the curve in which this quadric intersects a given surface u_n of the order n, we must substitute the coordinates of P in the equation of the surface; in this way we shall form an equation which is of the order n in θ and in φ separately. If we regard θ and φ as coordinates of a point on the quadric surface, the equation just found is that of the curve of intersection.

Suppose that in this equation the term independent of θ and φ vanishes, then the equation is satisfied by the pair of values $\theta=0$, $\varphi=0$, or the curve of intersection passes through the point A. Now the various directions in which we may start from the point A are determined by the initial value of the ratio $\theta:\varphi$ when we move along them. The direction in which the intersection-curve starts from A is therefore that obtained by neglecting in the equation terms of higher order than the first; and we see that there is only one such direction.

If, however, not only the constant term but the coefficients of θ and φ in the equation vanish, the initial directions are obtained by equating to zero the terms of the second order, *i. e.* by neglecting in the equation all terms of higher order than the second. In this case, then, there are two such directions, the intersection-curve has a double point at A, and the quadric has with the surface u_n an ordinary or *two-branch* contact.

Again, if the coefficients of the terms of the third order are the first that do not vanish, the initial directions are obtained by neglecting all the terms of higher order, and there are consequently three of them. Thus the intersection-curve has a *triple* point at A, and the two surfaces have a *three-branch* contact.

And so generally, if the coefficients of the terms of the mth order are the first that do not vanish, the intersection-curve has at A a multiple point of order m, and the two surfaces have at that point an m-branch contact.

The result of these considerations may be stated as follows:—

The necessary and sufficient conditions that the quadric $Q_2 \equiv AD-BC$ *may have* m*-branch contact with the surface* u_n *at the point* A, *are that in the expansion of* $(A+\theta B+\varphi C+\theta\varphi D)^n . u_n$ *in powers and products of* θ *and* φ, *the terms of order* m *in* θ *and* φ *are the lowest whose coefficients do not vanish.*

II. *Quadrics of four-branch contact.*

The equations we shall have to employ are so simple in form that it is unnecessary to employ the abridged notation of the former Part. We shall merely omit the operand u_n, and any common factor of the binomial coefficients.

The conditions for ordinary contact are then

$$0=A^n, \qquad 0=A^{n-1}B, \qquad 0=A^{n-1}C.$$

The first of these expresses that A is a point on the surface u_n, the second and third that B and C are on the tangent plane at A.

The further conditions for three-branch contact are

$$0=A^{n-2}B^2, \qquad 0=A^{n-2}C^2, \qquad 0=A^{n-1}D+(n-1)A^{n-2}BC.$$

The first two of these show that B and C are points on the chief tangents at A. If we regard the absolute values of the coordinates of A as given, then it appears from the third equation that B and C may be chosen arbitrarily on the chief tangents and D anywhere in space, the equation giving the relation between the absolute values of their coordinates which determines the particular surface $AD-BC=0$.

For four-branch contact we have the additional equations,

$$0=A^{n-3}B^3, \qquad 0=A^{n-3}C^3,$$
$$0=2A^{n-2}BD+(n-2)A^{n-3}B^2C,$$
$$0=2A^{n-2}CD+(n-2)A^{n-3}BC^2.$$

The first two of these indicate that B and C lie on the polar cubic of A in regard to the section of u_n by the tangent plane. Now this polar cubic has a node at A, whose tangents are the chief tangents. Each of these lines therefore meets the cubic in three points at A, and cannot have any other point on the curve unless it be itself a part of the cubic. But the points B and C have to lie one on each of the chief tangents. In order, therefore, that all the equations may be satisfied, the polar cubic in question must break up into the two chief tangents and some other line.

This condition may be put into another form. For if we seek the points in which the line AB meets the surface, by substituting the coordinates of $A+\lambda B$ in $u_n=0$, the conditions $A^nu_n=0$, $A^{n-1}Bu_n=0$, $A^{n-2}B^2u_n=0$, $A^{n-3}B^3u_n=0$ indicate that *four* roots of the equation are equal to zero, or that the line meets the surface in four consecutive points at A. We find, therefore, that

Those points of a surface at which a quadric may have four-branched contact are the points at which each chief tangent meets the surface in four consecutive points, or, which is the same thing, *the points whose polar cubic contains the chief tangent.*

The number of these points has been counted by CLEBSCH, Crelle, lxiii. 14 *, and turns out to be

$$n(41n^2-162n+162).$$

A point of this nature being given, one quadric surface having four-branch contact at it may be drawn through another arbitrary point.

The coordinates of the point A being given as to their absolute values, let us substitute for B and C, $A+\lambda B$, $A+\mu C$; where now B and C are fixed points on the chief tangents, whose coordinates are given absolutely. This being so, the following equations are satisfied *ex hypothesi*:—

$$A^nu_n=0, \qquad A^{n-1}Bu_n=0, \quad A^{n-1}Cu_n=0,$$
$$A^{n-2}B^2u_n=0, \quad A^{n-2}C^2u_n=0, \quad A^{n-2}B^2u_n=0, \quad A^{n-3}C^3u_n=0;$$

from which it follows at once, for example, that

$$A^{n-3}(A+\lambda B)^3u_n=0.$$

* The investigation is given by SALMON, Geom. Three Dim. p. 444.

For D also let us substitute νD, where the coordinates of D are now given absolutely. Our three remaining equations are (omitting for shortness the mention of A)

$$0 = \nu D + (n-1)\lambda\mu \,.\, BC, \quad \ldots \quad (1)$$

$$0 = \nu D + (n-2)\lambda\mu \,.\, BC + \nu\lambda \,.\, BD + \tfrac{1}{2}(n-2)\lambda^2\mu \,.\, B^2C, \quad \ldots \quad (2)$$

$$0 = \nu D + (n-2)\lambda\mu \,.\, BC + \nu\mu \,.\, CD + \tfrac{1}{2}(n-2)\lambda\mu^2 \,.\, BC^2; \quad \ldots \quad (3)$$

and it remains only to show that these equations determine uniquely λ, μ, ν.

If we subtract (1) from (2) and (3) successively, we obtain

$$0 = -\mu \,.\, BC + \nu \,.\, BD + \tfrac{1}{2}(n-2)\lambda\mu \,.\, B^2C, \quad \ldots \quad (4)$$

$$0 = -\lambda \,.\, BC + \nu \,.\, CD + \tfrac{1}{2}(n-2)\lambda\mu \,.\, BC^2, \quad \ldots \quad (5)$$

the values $\lambda=0$, $\mu=0$ not being admissible. But if we substitute in (4) and (5) the value of $\lambda\mu$ derived from (1), we obtain two simple equations which determine the ratios $\lambda : \mu : \nu$; after which the absolute values are uniquely determined from (1).

It is otherwise obvious that if $0=q_n$ be the point-equation to a four-branch quadric, and $0=t$, to the tangent plane, $0=q_n+\rho t^n$ will be the equation to a pencil of quadrics having four-branch contact, one of which may be made to pass through an arbitrary point.

Special investigation for n=3.

In the case in which u_n is a cubic surface, the points in question are clearly the 135 points of contact of the 45 triple tangent planes, namely, the intersections of the 27 lines. This case may be conveniently studied by means of the representation of that surface on a plane. The plane sections of the surface here correspond to a system of cubics having six common points; the quadric sections therefore to sextics having nodes at these points. The problem is then to draw a sextic curve having six given nodes and a quadruple point elsewhere.

The twenty-seven lines of the cubic are represented by

the 6 fixed points,
the 15 lines joining them, and
the 6 conics each through five of them.

It shall now be proved that the sextic must include two of these; and that for each pair that meet there is a singly infinite number of sextics.

The sextic cannot be a proper curve; for six nodes and a quadruple point are equivalent to twelve nodes, and a proper sextic can have only ten. Nevertheless we may apply a quadric transformation to it, whose principal points are the quadruple point and two of the nodes. The sextic is thus reduced to a quartic, passing 2, 0, 0 times through the principal points respectively and having four other nodes. But a quartic having five nodes must be made up of a conic and two straight lines. Now, if the node at a principal point is the intersection of the two lines, the original sextic was

made up of two lines, passing each through two of the six points, and a quartic having nodes at their intersection and the remaining two points and passing through those four. Let a, b, c, d, e, f be the six points, p the intersection of ab, cd; then the sextic is in this case made up of

lines ab, cd,

quartic $p^2e^2f^2abcd$.

If, however, in the transformed figure the node at a principal point was the intersection of a line with the conic, the original sextic was made up of a line, a conic, and a nodal cubic, viz. if p be the intersection of the line ab and the conic $bcdef$, the nodal cubic is p^3acdef.

Now we know that we can draw a singly infinite number of quartics with 3 fixed nodes and 4 fixed points, or of cubics with a fixed node and 5 fixed points. Hence in both these cases the sextic includes two representatives of straight lines in the cubic, together with another curve which may be arbitrarily chosen from a pencil.

XXI. *On the Echinoidea of the 'Porcupine' Deep-sea Dredging-Expeditions.*

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I.

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The deep-sea dredging-cruises of H.M. Ships 'Lightning' and 'Porcupine' during the summers of 1868, 1869, and 1870, in the North Atlantic, were comprehended within a belt 1500 miles in length by from 100 to 150 miles in width, extending from the Færöe Islands along the northern and western coasts of Scotland and Ireland and the coasts of Portugal and Spain to the Strait of Gibraltar. In this area fifty-seven successful hauls of the dredge were made during the three summers in water exceeding 500 fathoms in depth, sixteen beyond 1000 fathoms, and two beyond 2000 fathoms.

Even at the latter extreme depths Echinodermata appeared to be abundant. At 2435 and at 2090 fathoms all the Echinoderm orders were represented—the Echinoidea by a small variety of *Echinus norvegicus*, v. Düben and Koren, and a young example of *Brissopsis lyrifera*, Forbes; the Asteridea by an undescribed species of the genus *Archaster*; the Ophiuridea by *Ophiocten sericeum*, Forbes, and *Ophiacantha spinulosa*, Müller and Troschel; the Holothuridea by *Echinocucumis typica*, Sars; and the Crinoidea by a very remarkable new form of the Apiocrinidæ, which has been noticed under the name of *Bathycrinus gracilis*, Wyville Thomson. From 2000 fathoms upwards the number of Echinoderms seems to increase rapidly; but this apparent increase may possibly be due to our greater knowledge of the fauna of shallower water; at from 300 to 800 fathoms along the coast of Britain many species of all the orders are enormously abundant, so much so as to give a very marked character to the fauna of that special zone. Several of these species (such as *Cidaris papillata*, Leske, *Toxopneustes dröbachiensis*, Müller, *Echinus norvegicus*, D. & K., *Astropecten tenuispinus*, D. & K., *Archaster Parellii*, D. & K., *A. Andromeda*, M. & T., *Euryale Linkii*, M. & T., and *Antedon celticus*, Barrett) have long been known to inhabit the deep water of the British area, and form part of a fauna which will be probably found to have a very wide lateral extension at temperatures whose minimum ranges from 0° C. to +2° C.—a fauna which crops up, as it were, within the ordinary limits of observation in the seas of Scandinavia, and which has consequently been carefully studied by the Scandinavian naturalists.

Another group of species (including *Tripylus fragilis*, D. & K., *Ctenodiscus crispatus*, Retzius, *Pteraster militaris*, M. & T., *Amphiura abyssicola*, Sars, *Antedon Eschrichtii*,

O. F. Müller, and several others) are members of the same fauna described from the seas of Scandinavia and Greenland, but not hitherto known as British. A third section, consisting of a number of undescribed Echinoideans, Asterideans, and Ophiurideans, may probably also belong to this fauna; while a fourth group, likewise undescribed, and including such forms as *Porocidaris*, *Phormosoma*, *Calveria*, *Pourtalesia*, *Neolampas*, *Zoroaster*, *Ophiomusium*, *Pentacrinus*, *Rhizocrinus*, and *Bathycrinus*, would rather appear to be referable to a special deep-sea fauna of which we as yet know only a few examples, and with whose conditions and extension we are unacquainted. This abyssal fauna is of great interest, inasmuch as nearly all the hitherto discovered forms referred to it show close relations to family types of Cretaceous or Tertiary age, and hitherto supposed to have become extinct.

Twenty-seven species of Echinoidea were procured during the cruises of 1868, 1869, and 1870, off the coasts of Britain and Portugal, at depths varying from 100 to 2435 fathoms.

Class ECHINOIDEA.

Order I. DESMOSTICHA (E. Häckel).

Family 1. CIDARIDÆ.

Test globular. The plates of the test are thick and abut against one another in vertical surfaces, showing no tendency to overlap. The ambulacral areas are very narrow, usually slightly undulating; they are composed of numerous plates which bear miliary granules only, and these usually in not more than six rows. The poriferous zones are narrow: the pores of a pair are usually contiguous; sometimes they are a little remote from one another, and are united by a shallow groove. The interambulacral areas are very wide, from three to five times the width of the ambulacral; they consist of two rows of large plates, each plate bearing one large scrobicular areola more or less depressed and encircled by a raised rim: in the centre of the areola a prominent boss, sometimes crenulate and sometimes smooth, bears a large smooth perforated mamelon.

The oral and apical openings are very large. The peristome is round or slightly pentagonal without notches. The peristomial membrane is closely mailed with twenty rows of thick calcareous scales, five double rows ambulacral and five interambulacral, imbricating towards the mouth. The ambulacral scales are perforated for the passage of tube feet, which are continued from the pore-areas of the corona in double series up to the edge of the mouth. The free edges of the scales bear granules, to which are attached small flattened spines and pedicellariæ.

The apical disk is large and rounded, and consists of five large genital plates (one of them modified to include the madreporic tubercle) and five ocular plates. The anus is central, directly opposite the mouth; and the membrane of the periproct is clothed with a varying number of irregularly formed thick plates and calcareous granules, which form a continuous pavement round the anal aperture.

The dental pyramid is strong and well developed. The outer angles of the two divi-

sions of the tooth-sockets bear short epiphyses; but these are not united into an arch, as in the *Echinidæ*. The teeth are powerful, but they are simply grooved without a median longitudinal ridge. The primary spines are remarkably large, sometimes three times the diameter of the test in length: they are cylindrical, or prismatic, or tapering, or flattened, or club-shaped; smooth or finely striated, or ornamented with longitudinal spiny ridges, or regularly or irregularly tuberculated. The secondary and smaller spines are flattened and striated. The pedicellariæ differ in form in different genera.

Thus defined, the Cidaridæ form a very natural group.

They are at once distinguished from all the other families of the "regular" Urchins, with the exception of the Saleniadæ, by the extreme narrowness of the ambulacra. They resemble the Saleniadæ in general appearance, but they differ from them in the structure of the peristome and in that of the apical disk, which, in the Saleniadæ, includes a large plate in addition to the normal double range of five "ocular" and five ovarial plates.

They approach the Echinothuridæ in the important character of the continuity of the rows of ambulacral tube feet through imbricated scales over the buccal membrane and up to the edge of the mouth, in the condition of the epiphyses of the tooth-sockets, and in the form and structure of the teeth; but they differ from them widely in the homogeneity of the tube feet, in the remarkably solid inflexible nature of the test, and in the form of the spines and pedicellariæ.

They resemble the Diadematidæ in the structure of the tooth-sockets and teeth, but they differ from them in the continuation of the ambulacral over the buccal membrane, in the absence of notches in the peristomial rim, which in the Diadematidæ are particularly deep, and in the structure of the spines and pedicellariæ.

From the Echinidæ they are distinguished by the absence of the calcareous arches uniting the halves of the tooth-sockets, by the absence of the median ridge in the concavity of the teeth, and by the absence of notches in the peristome and of branchiæ.

Genus *Cidaris*, Klein.

Test globular, slightly and equally flattened at the oral and apical poles. Ambulacral areæ very narrow, bearing miliary tubercles only, in from four to six rows. Pore-areæ simple: the pores of each pair are contiguous, or, when slightly separated, they are united by a groove. The interambulacral plates are seven to ten in a row; the areolæ are large, circular, or transversely elliptical; the bosses supporting the primary tubercles are crenulated or smooth. The ovarial plates are large, more or less rectangular or shield-shaped, with a large defined perforation for the duct of the ovary passing through the plate about one third of its width from the outer margin. The primary spines are large and variously formed in different species. The pedicellariæ are variously formed, but they are all 3-valved. The ovaries are compact and lobed, and their outer wall is supported by irregularly shaped fenestrated calcareous plates. Fenestrated plates likewise occur in the walls of the intestine.

Hitherto the attempts to subdivide the family of the Cidaridæ into genera has not

been very satisfactory. So very large a proportion of the species are extinct that the group comes rather within the domain of palæontology than that of recent zoology; and, as the two branches of the subject have not always gone hand in hand, some confusion has occurred. Among the fossil Cidaridæ, palæontologists have never regarded the smoothness or crenulation of the bosses of the primary tubercles as a character of generic value; and I believe they are entirely justified in disregarding it, for crenulation occurs in every possible degree, and we frequently find in a single specimen that some of the bosses are crenulated and others smooth. The contiguous or remote arrangement of the pores is also a very critical character.

In the fossil genus *Rhabdocidaris*, Desor, in which the remote arrangement with the connecting-groove is most marked, it is always associated with very strongly crenulated bosses, and usually with elliptical areolæ; still, although the group thus distinguished has a certain characteristic *facies* and a definite distribution in geological time, I do not think it can be regarded as of more than subgeneric value. The genus or subgenus *Phyllacanthus*, Brandt, founded upon the form of the radioles chiefly, has no sufficient basis. The genus *Leiocidaris*, established by M. Desor for the reception of certain living species with remote pores and smooth bosses, seems to me to be valueless. In these the character derived from the arrangement of the pores is so obscure that several species (as, for example, the common *Cidaris tribuloides*) occupy an uncertain intermediate place, while certain subordinate characters (such as the comparatively large size of the areolæ and the great length and the cylindrical form of the spines) are not constantly associated with it, and are shared by many fossils which still retain a place in the type genus. In his valuable 'Revision of the *Echini*,' now progressing towards completion, Mr. Alexander Agassiz sets aside the genus *Leiocidaris*; but he proposes, apparently for *Cidaris papillata* alone, the subgenus *Dorocidaris*, founded upon a number of somewhat vague characters, all of them to be found singly and in different combinations in various fossil species of the genus *Cidaris*.

This multiplication of names seems unnecessary; I would therefore propose, at all events provisionally until it is in our power to revise the whole of the fossil series from a zoological point of view (an issue which, through the excellent work of Dr. Wright, M. Cotteau, and others, may not be far distant), to restore *Leiocidaris* and *Phyllacanthus* to the genus *Cidaris*, and likewise to relegate to the type genus the species now constituting the genus *Rhabdocidaris*, retaining *Rhabdocidaris*, however, as a subgenus under *Cidaris*. *Diplocidaris* seems to have some claim to generic rank on account of its tendency to a bigeminal arrangement of the pores; and possibly the same may be accorded to *Goinocidaris* from the singular sculpture of the test. *Porocidaris* is distinguished by a remarkable character, which is evidently of some physiological import, and also differs from *Cidaris* in other less important particulars.

1. *Cidaris papillata*, Leske. (Plate LIX. figs. 1–13.)

Principal synonyms:—*Cidaris papillata*, Leske *apud* Klein, 1778; *Cidarites hystrix*,

Lamarck, An. s. Vert. 1816; *Cidaris papillata*, Fleming, Brit. Animals, 1828; *Cidaris hystrix*, Blainville, Actinologie, 1834; *Cidaris papillata*, Forbes, Brit. Starfishes, 1841; *Cidaris hystrix* and *Cidaris papillata*, Agassiz, Catalogue raisonnée, 1846; *Phyllacanthus hystrix*, Brandt, Prod. 1835; *Leiocidaris hystrix*, Dujardin et Hupé, Echinoderms, 1862; *Dorocidaris abyssicola*, A. Agassiz, Bull. M. C. Z. 1869.

The test in a mature full-sized example, from a depth of 200 fathoms off the south coast of Ireland (Plate LIX. fig. 1), is of a spheroidal form, gently depressed at the oral and apical poles. It measures 65 millims. in diameter from the centre of an ambulacral area to the centre of the opposite interambulacral area measured at the ambitus, and 50 millims. in height from the edge of the mouth to the centre of the apical disk. The ambulacra are very narrow, 7 millims. in width between the outer edges of the pore-areæ, and slightly sinuous; and they present the same character and arrangement and are nearly of uniform width from the peristome to the ocular plates. The ambulacral plates form a narrow raised sinuous band along the centre of each ambulacral area, with a row of small tubercles along either side towards the pore-areæ, and two rows of principal granules towards the centre of the band, with two or more imperfect rows of minute granules somewhat irregularly interspersed among them. The pore-areæ are depressed, forming decided grooves. The pairs of pores are ranged in single series: there are from twelve to fourteen pairs opposite each of the large interambulacral plates of the ambitus. The pores of a pair are close together but not contiguous. It can scarcely be said that they are connected together by a groove, but rather that a slight ridge runs out between the pairs of pores from the marginal tubercles of the ambulacral plates, and abuts against the edge of the adjoining interambulacral plate.

The interambulacral spaces are very wide, about 35 millims. in their widest diameter. They consist of two rows of large plates, in full-sized examples seven in each row, diminishing to either end; and the smallest plate at either end of the area belongs to the right or left row alternately. On each plate there is a large, smooth areola, greatly depressed, and occupying nearly the whole area of the plate. The areolæ round the ambitus are 12 millims. in diameter, round or slightly elliptical; they are bounded by an abruptly elevated border, and show no tendency to coalesce, except occasionally in the case of some of the smaller plates round the peristome. A very definite row of from twenty to thirty small tubercles with distinct smooth articulating surfaces border each areola; and the spaces between the areolæ are closely crowded with miliary granules, which decrease in size towards the line of junction between the two rows of plates. A boss with a smooth uncrenulated edge rises in the centre of the areola, and is surmounted by a smooth tubercle of moderate size with a large deep central perforation. The apical opening in the corona is large and nearly round. The ovarial plates are large, irregularly rectangular or shield-shaped, somewhat narrower towards the edge of the disk, where they abut against the inner halves of two terminal interambulacral plates of an area, and expanding inwards where they come in contact with the small plates of the periproct. One ovarial plate is much larger than the others, and shows the usual

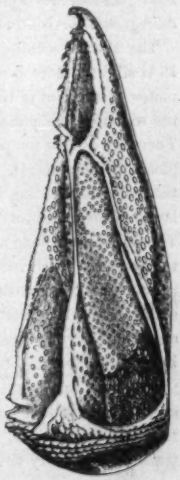
fenestrated structure, indicating its modification into the madreporic tubercle. A large definite round aperture, about one third of the width of the plate from its outer edge, is in connexion with the duct of the ovary. The "ocular plates" are heart-shaped, with a well-marked opening for the lodgment of the sense-organ. The plates of the periproct are irregular in form, diminishing in size towards the central anal opening; five of the outer row, however, are somewhat spear- or lancet-shaped, and pass outwards between the inner angles of the ovarial plates, giving the periproct a stellate form. The central portions of the ovarial, the ocular, and the larger anal plates bear close-set granules, but round the edge the plates are smooth and bare. This smooth border round the individual plates of the apical disk is a style of ornament very characteristic of this species, although not special to it.

The oral opening of the corona is 25 millims. in diameter. The peristome is markedly pentagonal. The buccal membrane is closely covered with thick and strong imbricated scales, their free edges beset with granules for the articulation of spines and pedicellariæ. The pairs of pores which traverse the ambulacral scales are smaller, and have the pores more closely set than those of the pore-areæ of the corona. The jaw-pyramid is of the ordinary form. The epiphyses of the jaw-sockets are rather longer and more developed than those of the more typical species, such as *C. tribuloides*, and the rotulæ are longer and rather more slender. The teeth are strong and sharp, formed of a simple deeply grooved band of semitransparent, very dense calcified areolar tissue. The auriculæ are large and spade-shaped; they arise entirely from the interambulacral plates; they lean slightly over the ambulacra, but they form no approach to an arch. The plates supporting the walls of the ovaries are large and closely set (Plate LIX. figs. 12, 13); they appear to originate from calcareous spicules with three rays spreading in one plane, the open angles between the branches becoming gradually filled up with thin cribriform calcareous expansions. The plates imbedding the wall of the intestine (fig. 11) and those in the suspending folds of the mesentery (fig. 10) are small and irregular in shape. The external tube feet, which are similar in character throughout the entire length of the ambulacra, have their walls supported by long curved spicules, rough with small conical projections; they terminate in sucking-disks, whose skeleton consists of a very irregular rosette of four, five, or six wedge-shaped pieces (Plate LIX. fig. 8).

The primary spines or radioles articulated to the primary tubercles on the interambulacral plates are large; those round the ambitus of large spines are sometimes 150 millims. in length by 6 or 7 millims. in greatest diameter. They are slightly enlarged just beyond the neck, and taper very gradually, rarely in full-grown spines coming to a point, but usually ending abruptly with a kind of cicatrix, which sometimes takes the form of an irregular cup. The surface of the spine is finely granular, and usually from twelve to fourteen rows of small pointed tubercles rather than spines, pointing slightly outwards, traverse its entire length at equal distances; sometimes these tubercles become regularly spine-like, and project from a continuous raised crest; and sometimes, as in a very marked variety dredged by Count Pourtales in the Gulf-stream

region, they become entirely obsolete and the spine is perfectly smooth. The radioles of the second size are about 8 millims. in length and 2 millims. in width, very much compressed and flattened, rounded at the end and finely striated longitudinally. They are articulated in a single row to the small tubercles round the edge of the areola, and in their natural attitude they lean over the naked part of the areola and cover the muscles and the head of the large radiole like a frill. A series of very much smaller spines are attached to the rows of larger granules which border the raised ambulacral band, and these are laid over to either side, covering and protecting the pore-areæ. These small pointed spines have a deep depression running down the centre, almost giving the idea of a tendency to divide into two. The miliary granules over all parts of the shell bear small flat-pointed spines. Only very minute granules bear pedicellariæ, which are of three forms. The most conspicuous of these are remarkably large, about 1·5 millim. in length and nearly a millimetre in width. The valves are greatly inflated, the central chamber large, and the ridges prominent (woodcut, fig. 1). These pedicellariæ are abundant and developed to a large size on the apical disk, particularly in northern examples (Plate LIX. fig. 5). Pedicellariæ of another form, with the valves long and slender, are ranged among the secondary spines along the borders of the ambulacral areæ and round the bases of the primary radioles (Plate LIX. fig. 6); and smaller tridactyle pedicellariæ, closely resembling a form very common in the Echinidæ, are scattered apparently irregularly all over the test.

Fig. 1.



Valve of a pedicellaria from the apical disk of *Cidaris papillata*. × 80.

Cidaris papillata has a very wide distribution. We dredged it in from 100 to 400 fathoms wherever there was a gravelly or sandy or in any way a hard bottom, in one continuous belt from the Færöe Islands to Gibraltar. Though not so abundant, it was frequent to 600 and 800 fathoms; and on one or two occasions small specimens were brought up from upwards of 1000 fathoms. In some localities the number of individuals was quite surprising. In the Shetland sea and at some stations off the south and west of Ireland, the dredge-bag was almost choked with them. *Cidaris papillata* is common off the coast of Norway. Specimens from Norway and the north of Scotland have the test usually thick and strong, the spines stout and comparatively smooth, and the large inflated pedicellariæ very numerous on the apical disk. Specimens from the Mediterranean, where the species is abundant, passing in some places into much more shallow water, are named in our Museums *Cidaris hystrix*, and are usually regarded as having the spines longer, more slender, and more markedly echinated, and the pedicellariæ less numerous than in the northern form. After examining a very large number of specimens from all

localities, I am quite satisfied that no characters of specific value distinguish the two forms. The one passes through every intermediate gradation into the other, and frequently a single locality yields a complete series, including characteristic examples of both extremes.

2. *Cidaris affinis*, Philippi. (Plate LX.)

Principal synonyms:—*Cidaris affinis*, Philippi, Wieg. Archiv, vol. i. 1845; *Cidaris Stokesii*, Agassiz, Cat. raisonnée, 1846; *Cidaris affinis*, Sars, Middelhavets Litt.-Fauna, 1857; *Leiocidaris affinis*, Dujardin and Hupé, 1862; *Cidaris* (*Dorocidaris*) *papillata*, A. Agassiz, Revision of the *Echini*, 1872.

The test of a full-grown specimen of this species is about 25 millims. in diameter. It is slightly more depressed than in *C. papillata*. The general arrangement of the plates of the test is the same, but the ambulacra are somewhat wider in proportion to the width of the ambulacral areæ. The pore-areæ are also relatively somewhat wider. There are nine or ten pairs of pores opposite the edge of one of the large interambulacral plates of the ambitus. The interambulacral plates have usually two rows of large-sized tubercles surrounding the areola, so that, instead of there being only a single row of the larger flattened spines converging over the areola, there is a double series of such spines. The radioles are long and rather delicate, ornamented as in *C. papillata*, only the pointed tubercles are much more pronounced. The spines from the larger areolæ taper to a fine point: those round the mouth are shorter, stouter, and more cylindrical; and immediately round the mouth some of them are slightly flattened. The general colour of the animal is a very brilliant cinnabar shaded with dark brown. The large spines are usually banded cinnabar and brown.

This is a very beautiful and well-marked little urchin, but I admit that its claim to specific rank is doubtful. Some varieties approach very closely to stunted, shallow-water forms of *C. papillata*, and particularly that special to the Mediterranean; and many (some of them very vivid in colour) were dredged in Tangier Bay.

Genus *Porocidaris*, Desor.

The ambulacra are narrow and strap-shaped, and slightly sinuous. The ambulacral areæ are narrow and bear miliary granules only. The pore-areæ are simple, the pores of each pair closely approximated. The interambulacral plates are large, eight to ten in a row. The areolæ are large and elliptical, and show a tendency to become confluent. The bosses of the primary tubercles are smooth or crenulated. The tubercles are of moderate size, smooth and perforated. The ovarial plates are large, halbert-shaped. A deep arch-like cleft passes inwards through about one third of the width of the plate, and a corresponding cleft separates through part of their length the two last interambulacral plates of each double series, so that the two clefts together form a large diamond-shaped opening through the test. This opening is filled up by a strong chitinous membrane, and a large pore penetrates the centre of the membrane, affording an exit to the duct of the ovary. The primary radioles are large and long, cylindrical or pointed round

the ambitus, but in the neighbourhood of the peristome becoming flattened, curved, longitudinally striated, and strongly toothed round the edge; in fact assuming a very peculiar and characteristic form. The pedicellariæ are numerous and two-valved, a structure as yet unique, so far as I am aware, among Echinoideans. The auricles are fused together through the greater part of their height in the middle of the interambulacral areæ, sending off short wing-like processes which arch over towards the ambulacra, but scarcely extend beyond their edges. The peristomial opening, the buccal membrane, and the dental pyramid and teeth have much the same character as in *Cidaris*.

I feel very little hesitation in referring the following very remarkable form to the genus *Porocidaris* of Desor, even although it is wanting in the very character upon which that genus was founded, and although the characters upon which its claims to generic distinctness are based are not shown in any of the fragments which alone have been met with in a fossil state. M. Desor gives the following characters of the genus. The areolæ, which have the form and general character of those of *Cidaris*, are pierced round the edge by a series of apertures; these pores are placed in the course of little grooves which radiate from the boss. The alveoli have a tendency to become confluent. The tubercles are perforated and crenulated. The radioles are compressed into the form of plates, and are curved, deeply grooved longitudinally, and strongly toothed along the edges. From their very remarkable form several species have been founded upon these radioles alone.

Some separate plates have been found associated with the radioles of one species only, *Porocidaris verronensis*, Schmidel, from the Nummulitic Tertiaries of Verona and of the neighbourhood of Nice; the other species (*P. serrata*, D'Archiac, from beds of the same age near Biarritz; *P. serraria*, Broun, from the Miocenes of Castel Arguato; and *P. Schmidelii*, Münster, from the Lower Oolite of the valley of the Frick) rest upon the evidence of the spines only. The last species M. Desor quotes with some hesitation, as it would be somewhat remarkable to meet with the same type at such distant periods, while they do not seem to occur in intervening beds. The recent species dredged off the north of Scotland has the spines, the perforated and crenulated bosses, and the coalescing areolæ of *Porocidaris*, which genus is known to come down as far as the Miocene Tertiaries; and it must be remarked that many animal forms hitherto known only as Miocene fossils are met with living at great depths in the present sea.

M. Desor attached much significance to the ring of perforations in the scrobicular area. In this I cannot concur. Several species of *Cidaris*, and even more markedly the species of *Porocidaris* under discussion, have radiating muscular impressions for the insertion of the muscles which move the large spines. It is perfectly conceivable that these depressions may sometimes actually go through the test, or at all events leave it so thin that the action of water and attrition during the process of fossilization may be well supposed to have completed the perforation.

Porocidaris purpurata, Wy. T. (Plate LXI. & Plate LIX. figs. 14 & 15.)

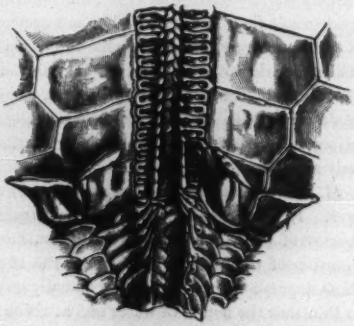
Synonyms:—*Porocidaris purpurata*, Wyville Thomson, Proc. R. S. 1869, 1872; The Depths of the Sea, 1873; *Porocidaris purpurata*, Alexander Agassiz, Revision of the *Echini*, 1872.

The test (Plate LXI. figs. 2, 3), in what appears to be a full-sized example of this species, from a depth of 542 fathoms about 100 miles to the north of the Butt of the Lews, is generally globular, but decidedly pentagonal in outline, owing to the central portions of the interambulacral areæ projecting considerably beyond the lateral portions and the ambulacra. It is 50 millims. in diameter at the ambitus, and greatly depressed, its polar axis not being more than 35 millims. in length. The ambulacra are narrow and band-like, 4 millims. in width between the outer edges of the pore-areæ, which occupy the greater part of the space, seeing that the central miliary strap of the ambulacra bears only two marginal rows of larger miliary tubercles with a few minute granules irregularly scattered between them.

The interambulacral areæ are wide, about 22 millims. in width at the ambitus. The plates are large, eight or nine in longitudinal series. The areolæ, which are transversely elliptical, slightly depressed, and surrounded by a raised border, occupy the greater part of the surface of the plates, the interspaces being crowded with miliary granules. A single row of large granules occupies the raised border of the areola. The bosses for the primary tubercles are rather prominent, those on the oral aspect of the test smooth, those on the apical aspect strongly crenulated, the change taking place quite suddenly at the ambitus. The tubercles are of moderate size, smooth, and perforated.

The peristomial opening in the test is proportionally rather small, in the specimen described 7 millims. in diameter. The peristome is subpentagonal, not notched. The buccal membrane is paved with imbricated scales, ten double rows ambulacral, and perforated up to the edge of the mouth with double pores in continuation of the pore-areas of the corona. The interambulacral spaces are filled up by a single row, or an imperfect double row, of imperforate scales, which alternate in imbrication with the neighbouring ambulacral scales. The free edges of all the scales are crowded with granules for the articulation of small spines of a special form. On the inner surface the ambulacral scales of the buccal membrane rise up into a series of curved spines on each side of the ambulacral vessel, so as to form a kind of channel for the passage of the

Fig. 2.



Auricles and buccal membrane of *Porocidaris purpurata*. ×5.

vessel continuous with the channel formed by processes of the ambulacral plates towards the oral end of the ambulacral areæ of the corona. The apical disk is very wide, 23 millims. in diameter. The ovarial plates are halbert-shaped; the cleft in their outer border is 3 millims. in depth and 3 millims. in width at the base, where it meets the corresponding cleft between the interambulacral plates. One of the ovarial plates is modified, as in *Cidaris*, to include the madreporic tubercle. The ocular plates are very regularly heart-shaped. The pore for the sense-organ is very distinct, about one fourth of the width of the plate from its outer edge. The peri-proct is pentagonal, the outer row of plates large and somewhat irregular in form, the inner plates becoming smaller towards the central anal orifice, round which the final two or three rows converge almost in the form of spines.

The primary radioles are longest round the ambitus, from 100 to 120 millims. in length (Plate LXI. figs. 4, 5), cylindrical or pointed, finely striated longitudinally, with some of the ridges prominent and rising into irregular lines of strong spines. The radioles on the apical aspect of the test become gradually somewhat shorter and are more pointed (Plate LXI. figs. 6, 7); on the oral surface they diminish in size towards the mouth, and assume the flattened, curved, grooved, toothed, paddle-like shape characteristic of the genus (Plate LXI. figs. 8, 9, 10). The spines of the second series form single rows converging towards the base of the principal spines over the alveolæ; they are flattened and striated, and about 10 millims. in length. The spines of the third series are narrower and more pointed, about 8 millims. in length, and diverge from the outer edges of the central band of the ambulacra over the pore-areæ. Smaller pointed spines are articulated to the smaller miliary granules over the surface of the test. The pedicellariæ are very remarkable. They consist of two long pointed valves, which, when closed, resemble very closely the smaller forms of the flattened spines (Plate LIX. fig. 14). Their structure, however, is in every way the same as that of the ordinary three-valved pedicellariæ, except in the number of the valves. All the usual chambers and ridges are developed, and the different muscles are very evident through the transparent walls; they are congregated chiefly on the apical disk and along the edges of the pore-areæ.

The basal portions of most of the large spines for about one third of the length of the spines are of a rich deep purple, and the remainder of the spines pale pink. Some of the large spines are of a uniform purplish brown, and all the smaller spines and the pedicellariæ are of a rich purple-brown.

The dental pyramid is lower and a little wider in proportion than in *Cidaris*; but it is constructed on the same plan, the epiphyses of the tooth-sockets forming ear-like appendages, but not uniting into an arch. The teeth are simply channelled, as in *Cidaris*. The auricles start from the interambulacral areæ, in the centre of which the two adjacent buttresses of two auricles are soldered together through nearly their whole height (woodcut, fig. 2), and only send out a small curved expansion which projects slightly towards the ambulacral groove. The tube feet are provided with suckers, which are supported by small and irregularly formed calcareous rosettes, and very small calca-

reous styles are scattered in the walls of the vessel. The ovaries are botryoidal, loosely lobed, and of rather small size. The ovarial ducts are very wide, and the ova are remarkably large. No calcareous framework could be detected in the investing membrane of the ovaries, but small irregular fenestrated plates are imbedded in the wall of the intestine.

With the general facies of *Cidaris*, *Porocidaris purpurata* differs in many remarkable details. Probably the character of the highest physiological import is the structure and great size of the ovarial openings, particularly when taken in connexion with the unusual size of the ova. The form of the pedicellariæ and the form and arrangement of the scales of the buccal membrane are likewise highly characteristic.

Four specimens were taken with the dredge at one station about 100 miles to the north of the Hebrides. One or two spines of apparently the same species were dredged by Mr. Gwyn Jeffreys off Cape Espichel. As I have already said, I have no doubt that this species must be associated, at all events generically, with the tertiary species of *Porocidaris*, and that it is one of the many examples, which are now forcing themselves upon our notice daily, of the persistence of tertiary, and even of older, types in deep water to the present day.

No representatives of the Saleniadæ or or the Diadematidæ were procured in these dredgings.

II.

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Family 2. Echinothuridæ, fam. nov.

Regular Echinoideans with the plates of the test imbricated and the test flexible. The plates of the interambulacral areas overlap from the mouth towards the apex, and the ambulacral plates in the opposite direction. The plates of the ambulacral areas are within those of the interambulacral areas, and their ends are overlapped by the ends of the adjacent interambulacral plates. The peristome is covered by ten double rows of imbricated calcareous scales, and five alternate double rows of these are perforated for the passage of tube feet in continuous series with those of the ambulacral areas of the corona, as in the Cidaridæ. The edge of the peristome is entire, as in the Cidaridæ. The spines are hollow, as in the Diadematidæ. The ambulacral tubes on the oral surface of the test are provided with a sucking-disk supported by a rosette of fenestrated calcareous plates, while those on the apical surface are long, conical, and pointed, without either sucking-disks or terminal pits. On both aspects of the body the walls of the tubes contain fenestrated calcareous spicules.

The dental pyramid is large and strong. Epiphyses are developed on the upper and outer angles of the pairs of tooth-sockets, but there is no arch formed. The teeth are deeply and symmetrically channelled within. The auriculæ are large and strong, forming closed arches, and their bases are united across the interambulacral spaces by strong

calcareous ridges. The auriculæ have the effect of springing from the edges of the ambulacral plates; but this character seems to have but little value, as the ring of calcified elements forming the auriculæ and their uniting ridges appears to be entirely distinct, merely forming adhesions with the ambulacral and interambulacral elements of the perisom in its course.

The peristome and the periproct are unusually large; the peristome entire in outline, and the peristomial membrane mailed, as in the Cidaridæ; and the periproct somewhat irregular in form, owing to the large size of the ovarial plates. The pores of the ambulacral areas are trigeminal; but the two pairs of each arc nearest the central line of the area are approximate to one another, and each pair penetrates an accessory plate, the two accessory pore-plates being intercalated between two of the ambulacral plates, while the third pair of pores of the arc is remote from the others, and penetrates the substance of the ambulacral plate near its outer edge. The ambulacral areas are rather wide, usually more than one half the width of the interambulacral areas.

The Echinothuridæ occupy an intermediate position between the Cidaridæ and the Diadematidæ, certain very important anatomical characters associating them with the former family, while in general facies and habit they more nearly resemble the latter. As in *Cidaris* the edge of the peristomial opening is entire, and the oral branchiæ are absent; the peristomial membrane is covered with imbricated calcareous scales, with their free edges studded with tubercles for spines and pedicellariæ, and perforated for lines of tube-feet which carry the ambulacra up to the edge of the mouth. As in *Cidaris*, *Diadema*, and *Echinocidaris*, the tube feet on the apical surface of the test are conical, with neither sucking-disks nor terminal pits; and as in *Cidaris*, *Diadema*, and *Echinocidaris*, the epiphyses of the tooth-sockets do not unite into an arch over the roots of the teeth. As in *Cidaris* and *Diadema*, there is no central ridge dividing the inner tooth-groove, and, as in many species of the Diadematidæ, the spines are hollow.

From the Cidaridæ, the Echinothuridæ differ in the arrangement and in the delicacy of the calcareous structures of the perisom, in the relations of the auriculæ (which start from the edges of the ambulacral areas, and not from the interambulacral plates), and in the structure of the spines. From the Diadematidæ, they differ in the absence of external oral branchiæ and branchial notches, and in the continuity of the ranges of pores up to the edge of the mouth; and they differ from the Echinidæ in the absence of longitudinal ridges dividing the concavity of the teeth.

The flexibility of the shell, the continuity of the ambulacra over the membrane of the peristome, the hollow spines, the peculiar arrangement of the pairs of pores, the singularly large ovarial openings, the absence of calcified arches uniting the elements of the tooth-sockets, the simply grooved teeth, the duck-billed pedicellariæ, and the singular arrangement of the walls of fenestrated fascia bounding the ambulacral spaces, form together an assemblage of characters which altogether preclude the fusion of the group with any family hitherto defined. It is very possible that *Asthenosoma varium*, Grube (45ter Jahres-Bericht d. schles. Gesell. f. vat. Cult. Breslau, 1868), may belong to this

family; but, if so, all the important anatomical characters, the overlapping of the plates of the perisom, the absence of branchial notches, the heterogeneous tube feet and their occurrence on the peristomial membrane up to the edge of the mouth, are omitted in the description.

The genus *Astropyga*, rightly placed among the Diadematidæ, certainly approaches the Echinothuridæ in many respects, and resembles them closely in general appearance. The plates of the perisom in *Astropyga* overlap slightly, but they all overlap in one direction, towards the apical pole; so that the arrangement in the two forms, though presenting a certain resemblance, is essentially different. The peristomial membrane in *Astropyga* is studded with irregular calcareous granules, with ten perforated bosses for the five pairs of large mouth-tentacles; and the edge of the peristome is deeply notched for the external branchiæ. The groove on the inner surface of the teeth becomes gradually filled up towards the tip. The spines are solid. The pairs of pores are trigeminate, arranged in symmetrical arcs as in the typical Diadematidæ. There is a marked difference in character between the apical and the oral surfaces of the test, which recalls this peculiarity in *Phormosoma*; and the structure of the small pedicellariæ is somewhat similar to that of the pedicellariæ in the Echinothuridæ. It appears, therefore, that while some characters would seem to indicate a tendency to a passage from the Diadematidæ to the Echinothuridæ through such forms as *Astropyga*, the resemblances are for the most part superficial, and very important anatomical characters maintain, according to our present knowledge, a broad line of distinction between the families.

Phormosoma, gen. nov.

Plates of the corona only slightly overlapping, and forming a continuous shell without membranous interspaces. Ambulacral and interambulacral areas of the apical surface of the test with irregular rows of primary tubercles with small areolæ. Oral surface of the test different in character from the apical, with the areolæ of the primary spines large and deep, occupying a large portion of the surface both of the interambulacral and of the ambulacral plates.

Phormosoma placenta, sp. nov. (Plate LXII. and Plate LXIII. figs. 1–8.)

The test is extremely thin, round, and much depressed. The only example which was procured in a condition sufficiently perfect for measurement is 80 millims. in diameter by about 18 millims. in height. The interambulacral areas are 32 millims. in width at the edge of the disk-like body, and the ambulacral areas 16 millims., exactly half. The peristome is 30 millims. in diameter, nearly circular, without notches, and the periproct 30 millims. from the outer edge of an ovarial group of granules to the outer edge of the opposite ocular plate.

The apical surface of the corona (Plate LXII. fig. 2) is slightly arched, not much more so apparently than is necessary for the accommodation of the masticatory pyramid. At the circumference the test is angular, almost carinated, and the ventral surface of the

body is nearly flat. The interambulacral plates of the apical surface are transversely oblong, the alternating double row meeting in the central line of the interambulacral area in a serrated suture; thus the plates of the opposite sides of the area slightly overlap, and all the plates of the interambulacral area slightly overlap one another from the circumference of the test towards the apex. The outer edges of the interambulacral plates are truncated, and form together nearly a straight line, the whole series overlapping the outer edges of the plates of the adjacent ambulacral areas (Plate LXIII. figs. 6 & 7). The plates of the ambulacral areas are thus essentially *within* those of the interambulacral areas. The first four or five interambulacral plates of each area towards the apex are separated—in four of the areas to accommodate the outer angle of the groups of tubercles which represent the genital plates in this genus, and in the fifth to admit the madreporic tubercle (Plate LXIII. fig. 4).

The surface of the interambulacral plates is smooth, with sparsely scattered primary tubercles, a few secondary tubercles, and a few minute miliary grains (Plate LXIII. fig. 5). The primary tubercles are one or two on each plate towards the apical end of the area; but they become more numerous, two at least, and often three, on each plate towards the circumference. The primary tubercles are perforated. The mamelon is surmounted by a distinct porcellanous ring, but is not crenulated. The areola is well marked, slightly excavated, and about 1·5 millim. in diameter. The number of interambulacral plates in series, from the circumference of the test to the edge of the periproct, is about ten.

The ambulacral plates of the apical surface are likewise oblong, in double series; meeting in the centre of the ambulacral area in a serrated suture. The outer edges of the plates are slightly rounded. The plates overlap from the apex towards the circumference. The outer edges of the ambulacral plates are entirely covered by the outer ends of the plates of the adjacent interambulacral series. Nearly midway between the central line of the ambulacral area and the outer edge, two small accessory plates, each perforated with a double pore, are intercalated between each contiguous pair of ambulacral plates; and another pair of pores, forming the lowest pair of an arc, penetrates the substance of the ambulacral plate a little nearer the outer edge. The pores are therefore trigeminal (Plate LXIII. figs. 5 & 6). Each ambulacral plate usually bears towards the centre a single primary tubercle or two smaller-sized tubercles, or, in some cases, only miliary granules. The number of ambulacral plates in series from the edge of the periproct to the circumference is about fifteen.

At the circumference there is an abrupt change of character. Just above the edge there is a nearly continuous band, running through all the plates, of large miliary tubercles bearing five spines, which form a kind of marginal fringe; and beneath that, towards the lower surface, an interrupted smooth band with neither tubercles nor granules; and the ambulacral and interambulacral areas then proceed towards the edge of the peristome. Each interambulacral plate is deeply excavated or impressed with two or three disproportionately large and deep areolæ, which appear on the outside like deep cups

upwards of 4 millims. in diameter, surrounded by a distinct areolar ridge with miliary tubercles, the areolæ sometimes tending to become confluent. On the inside of the test, owing to the thinness of the shell, the areolæ have the appearance of raised bullæ (Plate LXII. fig. 1, Plate LXIII. figs. 1, 2, 3). The mamelon is large and hollow, with an entire porcellanous ring; the tubercle is rather small and perforated. The general arrangement of the plates is the same on the lower as on the upper surface, but it is difficult to make out the sutures from the extraordinary size of the scrobiculæ. The pairs of pores are jostled into single file, and find their way to the edge of the peristome as best they can.

The apical opening (Plate LXIII. fig. 5) is very large and star-shaped, owing to the passing down into the interambulacral areas of a triangular prolongation of the membrane of the periproct, in the centre of which the ovarial duct opens. Over each ovarial aperture there is a small crescentic calcareous plate; and this, with a set of small calcareous granules imbedded in the membrane round the opening, represents the large ovarial plate, which is usually developed in this position. The madreporic tubercle, which is reniform and of large size, is in the position of the crescentic plate above one of the ovarial openings. The ocular plates are quadrate, with the angles rounded. The anal aperture is central, and the membrane of the periproct is thickly studded with small round plates, most of them bearing a miliary granule and a spine, the interstices between them filled up with calcareous granules thickly set. The plates decrease in size towards the anus, and immediately round the opening they become lengthened, and have almost the character of spicules converging towards it.

The peristome is mailed with ten double rows of thick calcareous imbricated scales, five double rows ambulacral and five interambulacral, all overlapping towards the mouth. The ambulacral scales are perforated for the passage of processes from the ambulacral canals which pass right up to the edge of the mouth-opening (Plate LXII. fig. 1).

There are two kinds of spines, one articulated to the primary tubercles, and the other to the minute miliary tubercles. The larger spines (Plate LXII. fig. 3) are from 10 to 15 millims. in length, with the shaft 5 millims. in diameter; they are hollow, consisting of a fenestrated calcareous tube with eight to ten projecting ridges, which are here and there raised into obliquely ascending spines. Sometimes a spiral arrangement may be detected in their projections, as we see in the very small spines of *Diadema*. The proximal end of the spine is enlarged, with a marked ring, and a space for the insertion of the large muscular mass which fills the deep alveola. The acetabulum is shallow, with a smooth margin and a central pit for the "round ligament." The distal end of the spine is drawn to a transparent point, like the drawn-out and sealed end of a glass tube (Plate LXII. fig. 3). The smaller spines are excessively fine hair-like fenestrated calcareous tubes. The spines are most numerous towards the edge and on the oral surface of the body, the small spines, mixed with pedicellariæ, clothing the scales of the peristomial membrane close up to the mouth. It is difficult to imagine the object of the peculiar arrangement on the oral portion of the corona; the size of the areolæ and of the masses

of muscle which fill them seem disproportionate to the rather small and very delicate and fragile spines.

There are two forms of pedicellariæ: one (Plate LXII. fig. 5), which occurs in small number in rows obscurely parallel with those of the larger spines, approaches in form the ordinary tridactyle pedicellariæ of the Echinidæ, though presenting some special characters; the second is a peculiar form of the so-called "ophiocephalous" pedicellariæ, peculiar, so far as I am aware, to the Echinothuridæ and the Diadematidæ (Plate LXII. fig. 6). The head of this pedicellaria is three-valved and very minute—in the present species not more than ·5 millim. in length. The distal end of the valve is broad and thin, and the three valves when closed meet only at the edges, and leave a small triangular space in the centre. The proximal end of each valve is truncated, ending in a triangular fluted plate. These three plates, meeting in the centre, form the base of the head of the pedicellaria, and are attached beneath to a gelatinous column about equal in length to the valve of the pedicellaria, which intervenes between the head and the end of the delicate stem. About the middle of each valve the fenestrated calcareous plate of which it is formed appears to curl inwards on each side, and the valve becomes double, forming a small conical chamber. A triangular keel runs from the inner angle of the basal plate up the face of this chamber in the middle line nearly to the point where the chamber ceases and the plate becomes single. The calcareous rod supporting the pedicellaria is of considerable length, from 3 to 5 millims. Pedicellariæ of this description are excessively numerous, scattered apparently irregularly all over the test; they are particularly abundant on the lower surface towards the edge of the peristome.

The tube feet on the lower surface of the test have their walls supported by large, broad, fenestrated plates (Plate LXIII. fig. 8), and are provided with a sucker with a well-developed calcareous rosette of four or five pieces. The tube feet on the apical surface of the test are much longer, conical, and come to a point: these are probably subservient to the function of respiration only.

The auriculæ are well developed and strong, though not very heavy. The two sides meet in a complete arch. Each pair of adjacent auriculæ are connected together by a raised calcareous ridge, which passes across, soldered to the interambulacral plates (Plate LXIII. fig. 1). The jaws are short and very wide, forming together a wide depressed pyramid. The two sides of each jaw-pair are broad and wing-like, and their outer angles present scarcely a trace even of epiphyses. The teeth are simply channelled, and come to a fine sharp point, as in some species of *Diadema*.

The specimen from which the above description was taken, the only one procured with the exception of some fragments, was torn open and the soft parts were greatly injured. The ovaries were long and narrow, running, closely attached to the test, from the ovarial opening to the periphery, along the middle line of the interambulacral space. The colour of the test is a pale grey flecked with purple. The tube feet are purple, and the spines are nearly colourless.

Locality. One imperfect specimen from a depth of about 500 fathoms, lat. 59° 43′ N.,

long. 7° 40′ W., about 100 miles to the north of the Butt of the Lews. Numerous fragments and spines from depths from 500 to 800 fathoms off the west coasts of Scotland and Ireland.

This form derives a special interest from its evident relation to the fossil *Echinothuria floris* from the White Chalk, described by the late Dr. S. P. Woodward.

Two imperfect specimens of this singular fossil had been obtained—the first, many years ago, by Mr. Wickham Flower, of Park Hill, Croydon, from the Upper Chalk of Higham, near Rochester, and the second by the Rev. Norman Glass, from Charlton, in Kent.

Mr. Flower's specimen consisted of several series of imbricated plates converging towards a point, and some of them showing the characteristic double pores and spine-tubercles of the Echinoderms, which at once set aside the first idea that it might be part of a large cirriped allied to *Loricula*. Edward Forbes examined the Higham specimen, but was unable to come to a decision. Afterwards Dr. Woodward examined it, and felt in a like dilemma. The general impression was that it might be a group of the scales of the peristome or the periproct of some large Echinoid allied to *Diadema* or *Cyphosoma*.

While Dr. Woodward was hesitating about publishing Mr. Flower's specimen, the second example was procured by the Rev. Mr. Glass, and it seemed to solve the problem; for it consisted of a well-developed dental apparatus with portions of several series of imbricating plates radiating from it, thus apparently supplying the peristome and lantern of the same great *Diadema* or *Cyphosoma* of which Mr. Flower's specimen might be the apex or periproct.

Still there were difficulties in the way of accepting this explanation. Dr. Woodward writes:—"In Mr. Flower's specimen the imperforate plates imbricate towards the centre, where the smaller ends of the several series converge. In Mr. Glass's specimen they slope away from the centre or mouth, that is also towards the apex. The perforated or ambulacral plates which overlap one another outwardly in fig. 1 are seen in fig. 2 sloping towards the dental cone and reclining upon it. The portion of an ambulacral area (*a*, fig. 2) consists of seven plates, diminishing in size in a line not accurately directed towards the centre. This portion exhibits the interior surface of the plates, known by their curved surfaces destitute of ornamental granules; it is not, however, the oral end of one of the segments turned over (a thing scarcely possible to happen), for in that case the dip of the plates would be reversed; but it must be the opposite (or apical) extremity of a series folded back upon its origin and exposed to view by the damage which the surface of the specimen has sustained. From this circumstance it seems probable that the whole fossil when complete was not elongated nor even spherical, but somewhat depressed in a vertical direction, though doubtless admitting of a moderate degree of flexure. At the last hour, after making this drawing, I ventured to clear away the chalk from the side of Mr. Glass's fossil, where the ambulacral segment is seen to curve as if it might be continued round to the other surface. The attempt was successful; for the ambulacrum and also the adjacent interambulacral segment were found conti-

nuous, though crowded and displaced at the turns, falling again into regular order and diminishing in size, though not nearly so complete as in Mr. FLOWER's specimen.

"After this apparently conclusive demonstration it appears desirable to give a name to this fossil, and to attempt a short description, although its rank and affinities are to us still a matter of conjecture. At present it is one of those anomalous organizations which MILNE-EDWARDS compares to solitary stars belonging to no constellation in particular. The disciples of VON BAER may regard it as 'a generalized form' of Echinoderm, coming, however, rather late in the geological day. The publication of it should be acceptable to those who base their hopes on the 'imperfection of the geological record,' as it seems to indicate the former existence of a family or tribe whose full history must ever remain unknown.

"Order ECHINIDÆ.

"Genus *Echinothuria*.

"*Echinothuria floris*, n. sp. Test globular ?, diameter of compressed specimen 4 inches, thickness ½ an inch, lantern projecting ½ an inch; composed of ten segments or double series of imbricating plates, ornamented with obscure miliary granules and small spine-bearing tubercles, a few larger than the rest; *interambulacral* plates narrow, slightly curved, with the convex edge upwards and overlapping; the alternate plates bearing one large extero-lateral tubercle, perforated and surrounded by a raised ring and smooth areola; largest plates measuring 6 lines in length, the smallest 3 lines or less (the longest in second specimen equalling 7 lines); *ambulacral* plates 7 lines long, equalling the breadth of the exposed portions of eight plates, similar to the former, but curving and imbricating downwards towards the dental orifice, and having two small plates, each perforated by a pair of pores, intercalated in a notch of the middle of the lower margin; a third pair of pores perforating the plate itself a little external to the centre; primary tubercles few, irregularly distributed.

"*Spines* of three kinds, those adhering to the plates minute and striated; fragments of larger spines (not certainly belonging to the species) striated, annulated, and furnished with a prominent collar to the articular end (fig. C); the third kind minute, clavate, and truncate, articulated (?) to a slender stalk (fig. E *d*)"*.

Calveria, gen. nov.

Plates of the corona greatly expanded towards the middle line of the interambulacral and ambulacral areas, the wide expanded portions overlapping. The plates curve abruptly about one third of their length from the middle line in the interambulacral spaces towards the mouth, and in the ambulacral areas towards the apex (the direction opposite to that in which they overlap). The outer portions of the plate in each area are so narrow as to leave spaces between the plates covered by membrane only. There is no special difference in character between the apical and the oral surfaces of the test.

* "On *Echinothuria floris*, a new and anomalous Echinoderm from the Chalk of Kent," by S. P. WOODWARD, Geologist, vol. vi. (1863) p. 330.

1. *Calveria hystrix*, sp. nov. (Plates LXIV. & LXV.)

The test is circular and depressed. The calcareous plates are very thin, but they are imbedded in a firm leathery perisom, so that the body-wall is tough and resistant. The only perfect specimen of the species procured is 130 millims. in diameter and about 25 millims. in height. The interambulacral areas are about one third wider than the ambulacral. The peristome is 35 millims. in diameter, and the periproct 20 millims. from the outer edge of an ocular plate to the outer edge of the ovarial plate opposite. Both the apical and the oral surfaces are nearly flat. The edge is round and full, with no tendency to become angular; and the plates, both ambulacral and interambulacral, retain, in passing round the edge, the same character, both as to form and position and as to the distribution of primary and secondary tubercles, which they have on other parts of the test. The number of interambulacral plates in a row extending from the edge of the peristome to that of the periproct is about forty-four, and that of the ambulacral plates considerably greater, as very frequently two ambulacral plates terminate under one interambulacral plate. Near the circumference, on the oral surface, the interambulacral plates are 25 millims. in length, and the narrowest part of the plate is 2·5 millims. in width, expanding at the middle line to 4 millims. The narrow strap-shaped portion of the plate occupying the outer part of the interambulacral area is 20 millims. in length, and the wide expansion which over- or underlaps the like expansion of the corresponding plate on the other side of the area in the middle line is about 5 millims.; so that although the total length of each interambulacral plate is 25 millims., the total width of the area is not much more than 40 millims. Each plate is rounded and somewhat expanded towards the outer edge, where it covers the end of one or of two ambulacral plates; and at this point it bears a primary tubercle with a large smooth scrobicular area. It then continues for about two thirds of its length narrow and strap-shaped, bearing a few secondary tubercles and miliary grains. It then expands again and bears another primary tubercle not quite so large as the former, and then abruptly curves towards the oral pole, passing in that direction under the expanded end of its fellow on the opposite side. The outer surface of the expanded portions of the plate, in the centre of the area, is also studded with miliary granules (Plate LXV. figs. 1 & 2).

The strap-shaped portions of the plate between the two primary tubercles is so much narrowed that the plates of a series are not in contact at this point, a narrow linear space being left between them, closed only by the soft perisom and the membrane in which the calcareous plates are formed.

The ambulacral plates are much narrower than the interambulacral. They likewise have narrow fenestræ between them towards their outer ends, and they overlap, though not to so great an extent, in the centre of the ambulacral area. Each ambulacral plate is studded with secondary tubercles and miliary granules, and at about two thirds of its length from the outer edge it bears a small primary tubercle. As in *Phormosoma*, the double pores are arranged in arcs of three, and, as in that genus, the two inner pairs of pores pass through special pore-plates intercalated between the ambulacral plates, while

the third pair, remote from the others, penetrates the ambulacral plate near its outer end.

About two thirds of their length from the outer edge, the ambulacral plates bend strongly towards the apical pole, as in the case of the interambulacral plates, in the direction contrary to that in which they overlap. The outer edges of the series of ambulacral plates are covered by the ends of the plates of the interambulacral series. Frequently only one ambulacral plate slips under the end of one interambulacral plate; but perhaps once out of thrice the outer ends of two ambulacral plates approach one another, and together are covered by the end of one interambulacral plate. The ambulacral plates are thus considerably more numerous than the interambulacral. As already stated, the interambulacral and ambulacral plates do not differ materially either in their individual form or in their arrangement in combination on the opposing surfaces of the flattened corona.

The primary tubercles are perforated. The mamelon is surrounded by a slightly marked ring, but is not crenulated. The areola is large and is not depressed, or very slightly so. The tubercle, mamelon, and scrobicule seem to be formed of a separate calcareous piece, cemented, as it were, to the surface of the plate which passes behind it, leaving the mamelon hollow, and a small conical space between the tubercle and the inner calcified layer of the plate.

The larger spines (those which are articulated to the primary tubercles) are from 15 to 20 millims. in length and ·8 millim. in diameter. They are hollow as in *Phormosoma*, the fenestrated calcareous tube rising into from sixteen to twenty ridges, between which are uniform rows of fenestræ. As in the former genus, the ridges rise here and there into processes which have a distinctly spiral arrangement in the larger spines. Near the proximal end the spine is markedly enlarged, running out a kind of collar; and from this point it diminishes, forming a regular cone down to the acetabulum, which is shallow, with a puncture in the centre for the insertion of the ligament of the spine (Plate LXIV. fig. 2). The fine spines articulated to the miliary tubercles are extremely delicate fenestrated calcareous tubes running to a fine point.

The pedicellariæ are very numerous and of three forms. The largest are the least numerous; they occur in irregular rows parallel with and near the rows of large spines, and are also sparsely scattered, apparently irregularly, over the surface of the test. The head (Plate LXIV. fig. 4) is not less than 1·2 millim. in length; and as these pedicellariæ are attached to stalks as long as medium-sized spines, they can be seen quite distinctly with the naked eye (Plate LXIV. fig. 1). They have the general character of the tridactyle pedicellariæ of the Echinoideans, only the basal triangular portion of each valve is unusually broad; the middle part is arched outwards and narrow, and the terminal expanded part is shorter and more regularly spoon-shaped than in most species. The second form (Plate LXIV. fig. 5), which is a modification of the first more or less reduced in size and lengthened in its proportions, is like some of the common varieties in the Cidaridæ. The third (Plate LXIV. fig. 6) resembles in all essential respects the

small pedicellariæ in *Phormosoma*. The valves are somewhat thicker and stronger and wider at the distal end in proportion to the width of the base, therefore widening out more. The head of the pedicellaria has thus more the form of an inverted cone.

The tube feet on the oral surface of the test are cylindrical, expanding distally into a sucker, which is supported by a fenestrated calcareous disk in four, five, or six pieces, with the edge produced into an irregular spiny border. Irregular cribriform plates (Plate LXIV. fig. 3) are imbedded in the walls of the tubes on both surfaces of the body. The tube feet on the apical surface are conical and pointed, with neither sucking-disk nor terminal pit.

The ovarial plates are complete and triangular in form; the aperture is very large, and filled in with a membrane of a dark purplish colour. As has been already said, the interambulacral plates, while overlapping towards the apical pole, bend strongly near the middle line of the area towards the mouth. The triangular space thus left between the first pair of plates of the series is occupied in four of the areas by the large ovarial plates, and in the fifth by the still larger ovarial plate, which is partly modified into the madreporiform tubercle. The membrane of the periproct is thickly set with discoid plates, becoming gradually smaller towards the circumference of the anus, and studded with small spines and small pedicellariæ. The ocular plates are club-shaped, and touch the outer angles of the ovarial plates on either side. A strong calcareous ridge runs across each ambulacral area at the border of the peristomial opening of the test, soldering its plates together, and rising up at the edge of the ambulacral area into a rod which forms one half of the auricle, which arches over the ambulacral space by uniting by a small anchylosed suture with the corresponding rod on the other side.

The peristomial membrane is covered by twenty rows of densely imbricated scales in ten double rows, corresponding with the ambulacral and interambulacral areas of the test. The plates of the peristome are all imbricated towards the mouth. The scales of the five double rows, corresponding with the ambulacra, are perforated towards their outer angles with double pores, which carry the tube feet up to the edge of the oral opening. The peristomial scales are studded with small tubercles bearing small spines and many pedicellariæ. The edge of the peristomial aperture is entire, as in *Cidaris*, without branchial notches.

The dentary pyramid is broad and low. The pairs of jaws are broadly triangular; the epiphyses are small, and show no tendency to form an arch. The rotulæ are wide and strong. The teeth have much the character of those of *Diadema*, being narrow and coming to a long point, and being simply grooved without the characteristic inner ridge of the Echinidæ.

The colour of the perisom is a brilliant deep rose inclining to claret-colour; twenty bands of deeper shade run in pairs, alternately closer and more remote from one another, along the ambulacral and interambulacral spaces. The ends of the spines are pale pink, and the tube feet are nearly white.

The specimen of *Calveria hystrix* from which the above description was taken, the

only one of its species which we met with, was dredged about 100 miles off the Butt of the Lews in 445 fathoms water on the edge of the warm area, with a temperature of 7°·5 C. It came up perfect and living. The spines were moving freely; and after the animal had rolled out of the dredge upon the deck, and assumed what appeared to be its normal form and attitude, curious vermicular movements passed through its test. When handled the test moved and shrank from the touch, and had much the feel of the disk of a *Polaster* or other large starfish.

As this specimen was exceptionally good it was reserved nearly intact to show the general characters of the group, a small segment of the ventral surface only having been removed to allow the description of the structure of the test and pyramid. The disposition of the plates and the structure and arrangement of the soft parts will be more fully detailed in the description of the next species, for which there were fuller materials. Fragments of plates and spines apparently belonging to *Calveria hystrix* occurred in several of the dredgings on the west coasts of Scotland and Ireland.

2. *Calveria fenestrata*, sp. nov. (Plate LXIII. figs. 9 & 10 and Plates LXVI. & LXVII.)

The test is rounded, slightly pentagonal in outline; in the largest specimen procured 110 millims. in diameter. It is greatly depressed, being little more than 20 millims. in extreme height in the centre, from which the upper surface droops uniformly to the edge, which is sharper than in *C. hystrix*, although it does not form any thing like the keel of *Phormosoma*. The form and general arrangement of the plates is the same on the apical and oral surfaces of the test. The structure of the ambulacral and interambulacral areas is the same as in *C. hystrix*; the overlapping portion of the plates is, however, much larger and wider, and the strap-shaped portion of the plate is much narrower, narrowed in some cases almost to a rod, and thus leaving wide membranous fenestræ between the plates. The ambulacral areas are nearly equal to the interambulacral in width, while in *C. hystrix* they are at least one third narrower, and the plates of the perisom generally are lighter and more delicate (Plate LXVII. figs. 1, 2, 3).

As in *C. hystrix* the outer surface of each plate bears tubercles of three kinds:—primary tubercles with a large smooth areola, a smooth conical mammillary eminence, and a perforated mamelon; smaller tubercles of a similar character; and miliary granules. The arrangement of the primary and larger tubercles is nearly the same as in *C. hystrix*, but they are fewer and more remote.

The spines are, as in the species previously described, of two kinds. The large spines articulated to the primary tubercle are like those of *C. hystrix*, only they are somewhat more delicate; the ridges on the shaft of the spine are more numerous, and the projecting processes are irregular and less strongly marked (Plate LXVII. fig. 4); and the collar at the tip of the conical portion of the spine to which the muscle is attached is more irregular in outline. The smaller spines are extremely delicate transparent fenestrated tubes.

The pedicellariæ are of three kinds. The largest of these are arranged, both on the

interambulacral and the ambulacral plates, in irregular rows parallel to the rows of the larger spines. A long style (Plate LXVII. figs. 5 & 6) is articulated to one of the miliary granules. This style enlarges somewhat at the distal end; and from the enlargement there spring four delicate rods which gradually widen and become tubular, their walls being at length perforated by wide fenestræ (Plate LXVII. fig. 6). Each rod is surmounted by a hollow disk very gracefully twisted, reminding one somewhat of a frustule of *Campylodiscus*. The walls of the disk are fenestrated, and from the margin a delicate calcareous border is thrown backwards and downwards, edged with a fringe of singularly elegant design. A large mass of red muscle, covering the end of the main stem and the bases of the four branches, no doubt for the purpose of imparting motion to the latter, renders these pedicellariæ very conspicuous. They are strikingly beautiful objects, and differ entirely from any pedicellariæ which have hitherto been described.

The pedicellariæ next in size are of the ordinary tridactyle type, with a special modification in form, the bases of the valves being very broad, and the spoon-shaped extremities unusually long and close-fitting (Plate LXVII. fig. 7). The third set resemble closely the minute pedicellariæ of *C. hystrix*; but they are more delicate in their proportions, corresponding in this with the test and its other appendages.

The apical disk (Plate LXVI. figs. 1, 2) is 18 millims. in diameter from the outer angle of an ovarial plate to the outer edge of the ocular plate opposite. The ovarial plates are large, and have nearly the form of an equilateral triangle; the outer angle, which is produced somewhat more than the others, is wedged between the first pair of interambulacral plates in the space formed by their common inclination towards the oral pole. The ovarial openings are remarkably large, 2·5 millims. in diameter, so large as to occupy the whole of the central part of the ovarial plate, leaving a narrow rim only. Even this rim is sometimes encroached upon and the side of the plate separated into two parts. The ocular plates are large and mallet-shaped. A transversely oval portion continues the line of the upper edges of the ovarial plates, coming into contact with one of these on either side, while a narrower process, perforated in the centre by the minute ocular pore, meets the two upper plates of the ambulacral series. On the inside of the test a raised calcareous ridge runs continuously along the free edge of the ovarial plates through the ocular plates, with a suture at the points of junction, forming a kind of ring supporting the edge of the apical space. The madreporic tubercle is large, formed of the modified upper portion of one of the ovarial plates. The membrane of the periproct is supported by irregularly shaped scattered plates bearing small tubercles and miliary grains and small spines and pedicellariæ. These plates are of considerable size, and are lenticular in form towards the circumference, but they become smaller and finally almost linear as they approach the anus.

As in *C. hystrix*, a calcareous keel runs across the interambulacral spaces at the edge of the oral opening, and complete auriculæ span the ambulacra; but these auriculæ, like the other part of the shell, are very light. The oral aperture is 25 millims. in diameter with an entire margin. As in the other members of the family, the peristomial

membrane is mailed with densely imbricated plates, those which continue the ambulacral series of the test being perforated by double pores up to the edge of the mouth; and the whole are studded, on their outer exposed edges, with tubercles for short spines and for pedicellariæ of the two smaller forms.

The dental pyramid is rather higher in proportion to its width than in *C. hystrix*, and much higher than in *Phormosoma placenta*. The epiphyses of the jaws are much larger, and form curved ascending processes almost as in *Diadema*, but they do not unite into an arch. The teeth are broad and simply and uniformly grooved; they come somewhat rapidly to a point at the extremity.

The arrangement of the internal organs is remarkable. It is probably the same, or nearly the same, in all the members of the family which have been described; but the largest specimen of the present species which we procured was torn open and a good deal injured, which gave an opportunity of working out the general arrangement of its parts without sacrificing a unique specimen. A strong fibrous fascia (Plate LXVI. fig. 3) rises from the inner surface of the ambulacral plates near their outer ends, and forms a kind of diaphragm nearly dividing the cavity of the test perpendicularly on each side of the ambulacral area, and thus inclosing the ambulacral organs (nerves, vessel, and vesicles) and cutting them off from the rest of the body-cavity. This diaphragm does not run up as a complete wall to the edge of the peristome and periproct, but a kind of notch is left towards the centre of the body-cavity to allow the passage of the intestine. At this point the fascia rises up from each side of the ambulacral area as elsewhere; but instead of joining the fascia from the opposite wall of the test, its fibres unite with those on the opposite side of the same ambulacral area, and take the form of a cribriform membrane, which thus closely arches over the ambulacral organs. The intestine, in passing round the body-cavity, is thus restricted, as it passes the ambulacral areas, to these notches, while in the interambulacral spaces it forms wide loops suspended from the interior of the test by delicate mesenteric membranes and filaments (Plate LXVI. fig. 4). This arrangement of fibrous fasciæ reminds one strongly of the arrangement of the calcareous plates and trabeculæ in the Clypeasters. The fascia is continued at the apical pole into a broad band which passes round the margin of the periproct, including in a common sheath the circular ambulacral vessel and the genital ducts (Plate LXVI. fig. 3). The ovaries are very long and narrow (Plate LXVI. fig. 3), and occupy the centre of the interambulacral areas to the circumference.

The ambulacral vessels send off a straight simple branch to each pair of pores, the branch dilating as it approaches the pores into a pyriform vesicle (Plate LXVI. fig. 5). The tube feet on the ventral surface are cylindrical; their walls are supported by irregularly shaped cribriform plates (Plate LXVII. fig. 9), and are provided with a terminal sucker with a calcareous rosette. The tube feet on the apical surface are long and conical, and taper to a point, without a terminal sucker.

The colour of the test generally is greyish, but ten wide bands of purplish brown radiate from the apical pole, shading off into the grey of the test, and giving a rich effect

of colour to the upper surface. The spines are whitish, and the ambulacral part grey tinged with purple. A large mutilated specimen of this species, and a smaller one nearly perfect, were dredged by Mr. Gwyn Jeffreys off the coast of Portugal. There was a fragment of the test taken by the dredge off Rockall, and some spines which must be referred to it were sifted out of the mud brought up by the dredge in the Bay of Biscay.

III.

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Family 3. Echinidæ.

Genus *Echinus*, Link.

1. *Echinus melo*, Lamarck.

Two specimens were dredged off the coast of Portugal in shallow water.

2. *Echinus Flemingii*, Ball. (Plate LXVIII. fig. 14.)

The large typical form of this species was met with in deep water off the Shetlands, but not in large numbers.

3. *Echinus rarispina*, G. O. Sars.

4. *Echinus elegans*, Von Düben and Koren. (Plate LXVIII. figs. 11–13.)

5. *Echinus norvegicus*, Von Düben and Koren.

The last four are critical species; and although the extreme forms are very dissimilar, in a large series there are so many intermediate links that it is difficult to tell where the one begins and the other ends. It is possible that when our information is more complete all these may come to be regarded as varieties, and may be grouped together under Lamarck's name, *Echinus acutus*.

6. *Echinus microstoma*, nov. sp. (Plate LXVIII. figs. 1–10.)

Although I have great hesitation in proposing an addition to the genus *Echinus*, I feel compelled to separate, in the meantime, this very distinct form with a thin depressed test, a remarkably large apical system and periproct, a small peristome with the margin curved inwards and a uniform salmon-colour. *Echinus microstoma* is very abundant at depths between 150 and 400 fathoms off the western coasts of Scotland and Ireland.

7. *Echinus esculentus*, L.

A marked variety with a tall narrow test occurred in deep water off the Færöes.

Genus *Toxopneustes*, Agassiz.

1. *Toxopneustes dröbachiensis*, Müller.

Of this species *T. pictus*, Norman, and *T. pallidus*, G. O. Sars, can only be regarded as varieties. It is generally distributed at depths beyond 100 fathoms in the northern seas.

2. *Toxopneustes brevispinosus*, Reiss, sp.

In shallow water off the coast of Spain.

Genus *Psammechinus*, Agassiz.

1. *Psammechinus miliaris*, Lamarck, sp.

Abundant in the British area at depths less than 50 fathoms.

2. *Psammechinus microtuberculatus*, Agassiz.

A few examples were dredged in shallow water off the coast of Portugal.

Order II. PETALOSTICHA (E. Häckel).

Family 1. CASSIDULIDÆ.

Genus *Neolampas*, A. Agassiz.

Outline of the test pyriform from above, prolonged posteriorly into a short blunt rostrum and truncated. The test arches nearly uniformly from the anterior margin to the posterior rostrum. The oral surface is slightly concave. The ambulacra are flush with the surface of the test; they are reduced to the greatest simplicity, showing no rosette or other indication of a petaloid arrangement, but running from the apex to the peristome as a uniform double row of simple pores passing between the ambulacral plates and giving exit to minute conical tube feet. The mouth is pentagonal, nearly in the centre of the oral surface of the test, a little nearer the anterior border. It is surrounded by a well-marked floscelle and distinct bourrelets. The apical disk is on the upper surface of the test, nearly directly vertically over the mouth; it is compact, with three large ovarial openings, five very distinct pores for the sense-organs, and a rather small madreporic tubercle in the middle. The anal opening in the test is large, occupying a deep depression in the truncated posterior extremity of the shell.

Neolampas rostellata, A. Agassiz. (Plate LXIX.)

Synonym:—*Neolampas rostellata*, A. Agassiz, Bulletin of the Museum of Comparative Zoology, 1869.

The single specimen procured in the 'Porcupine' Expedition, which was dredged living in 690 fathoms at the mouth of the English Channel, is 20 millims. in length, 16 millims. in extreme width across the ambitus, and 7 millims. in height. The outline from above is not quite symmetrically oval or pyriform, the shell bulging on either side somewhat irregularly towards the posterior extremity. In profile, the anterior end of

the test is the thinnest (Plate LXIX. fig. 3); the outline rises to the apex and then sinks gradually to the truncated posterior rostrum, along the top of which it coincides with a slight longitudinal ridge. The oral surface of the test also rises slightly from the anterior border to produce the depression in which the mouth is placed, and from the mouth it sinks towards the truncated extremity forming the floor of the rostrum. This truncated end is occupied by a deep inversion (Plate LXIX. fig. 4), deepest above, at the bottom of which the anal orifice opens. The periproct is oval, large, and plated with small scales. In one specimen there is no trace of the exserted anal tube described by Prof. A. AGASSIZ as occurring in the specimens dredged by Count POURTALES in the Strait of Florida.

The ambulacra have all precisely the same character. Those of the bivium are apparently in slight depressions. This is, however, only an effect produced by the slight projection of the sides of the posterior rostrum (Plate LXIX. fig. 4). The ambulacra are not very easily seen, the pores are so minute; by holding the shell up to the light, however, they become sufficiently apparent as rows of simple pores passing between irregularly hexagonal plates, in double series, from the apex to the mouth. The ambulacral areæ widen somewhat from the apex to the ambitus, and become slightly narrower from the ambitus towards the mouth; they are about 4 millims. in width at the widest point, the lateral interambulacral spaces being 12 millims. At the mouth the ambulacra expand into a very distinct floscelle (Plate LXIX. fig. 6), and the interambulacral areæ end in bourrelets crowded with tubercles and bearing combs of long spatulate spines. The plates of the apical disk are so compacted and fused together that it is difficult to trace their outline. Eight holes, nearly of equal size, surround a central madreporic tubercle (Plate LXIX. fig. 5); of these, five terminate the ambulacra and are the pits for the sense-organs, the other three are ovarial. The posterior and the right anterior ovaries and ducts are undeveloped.

The surface of the test is crowded with minute tubercles for the articulation of the larger spines and many small granules (Plate LXIX. fig. 5). The tubercles are imperforate, with a smooth mammillary boss; they are placed in circular scrobicular depressions, but they project somewhat above these and above the surface of the test. The larger spines, articulated to the tubercles, are cylindrical, fenestrated, with slight asperities on the longitudinal calcareous shafts (Plate LXIX. figs. 7, 8, 10).

The small spines, which, attached to the minute granules, form a close underfelting all over the test, are fenestrated and slightly roughened, and expand at the end into a rosette of pointed tubercles (Plate LXIX. fig. 9). The pedicellariæ, articulated to some of the granules, are three-valved; they are very small and of a somewhat peculiar form, though resembling generally the smallest pedicellariæ in *Echinus*. The bases of the valves are wider, the valves themselves are rounder and more arched and toothed round the edge (Plate LXIX. fig. 11). Round the mouth there are groups of three and four very small pedicellariæ, differing in form somewhat from the others, and with the bases of the valves apparently fused together (Plate LXIX. fig. 12).

Family 2. CLYPEASTRIDÆ, Agassiz.

Genus *Echinocyamus*, Van Phelsum.

Echinocyamus angulatus, Leske.

Generally distributed, but not found living beyond 150 fathoms.

Family 3. SPATANGIDÆ, Agassiz.

Genus *Pourtalesia*, A. Agassiz.

The body is flask-shaped, the anterior extremity abruptly truncated; the test is then almost cylindrical for half its length, contracting gradually from the middle to the posterior extremity, where it is produced into a long rostrum. A narrow flattened plastron occupies the ventral surface, while on the dorsal surface a well-marked ridge runs along the middle line from the apex to a point a little way in front of the posterior rostrum, where it forms an arched projection protecting a deep pit, at the bottom of which the anal opening lies.

The apical system is produced or disjunct. The three ambulacra of the trivium, after passing up from the oral ring, the odd one along the centre and the two others along the outer sides of the truncated anterior face, meet directly above the mouth at a point where are also four large ovarial openings and a central madreporic tubercle. The ambulacra of the bivium, starting from beneath the mouth, diverge and pass one along either side of the ventral (interambulacral) plastron until they come opposite the anal opening, where they form loops, and return along either side of the dorsal ridge, and unite at a point about one fourth of the length of the test behind the point of junction of the ambulacra of the trivium.

The mouth is at the bottom of a deep inversion of the truncated anterior end of the test; it is elliptical, the peristome simple, not bilabiate. Teeth are entirely wanting, and there is no trace of either phyllodes or bourrelets. The anal opening is quadrate, the membrane of the periproct plated with irregular calcareous scales.

1. *Pourtalesia Jeffreysi*, n. sp. (Plate LXX. figs. 1–10 and Plate LXXI.)

One apparently mature example of this very remarkable urchin was dredged in 640 fathoms, halfway between Færöe and Shetland, at a bottom temperature of —1°·1 C. The total length of the test is 45 millims., and its extreme width 18 millims.; the height of the test from the ventral plastron to the dorsal ridge at its highest point is 20 millims.

The groove at the bottom of which the mouth lies is very deep—a tunnel-like inversion of the test in the centre of the lower part of the truncated anterior end. The mouth-opening is elliptical, the peristome simple. The buccal membrane is bare towards the centre, and covered towards the outer edge with scales, the larger of which are granulated and give attachment to small spines and pedicellariæ. The odd ambulacrum of the trivium starts from the upper end of the mouth, forms, with a double row of somewhat irregularly shaped plates, the roof of the mouth-inversion (Plate LXXI. figs. 4, 5),

then passes up the centre of the anterior end of the test to the apex (Plate LXXI. fig. 3). The two lateral ambulacra of the trivium pass from the sides of the peristome upwards over the sides of the shell, just behind the lateral borders of the anterior end, to meet at the same point (Plate LXXI. fig. 3). The ambulacra of the bivium start very close together from this lower end of the mouth (Plate LXXI. fig. 4); they pass along on either side of the ventral plastron to a little beyond the anus, nearly to the end of the shell; then they turn upwards and, still separated by the interambulacrum of the bivium, run forwards on the dorsal aspect of the test to the secondary or posterior pole of the apical system (Plate LXXI. figs. 6, 8). The tube feet in connexion with the odd ambulacral vessel of the trivium are conical and of considerable size; those springing from the other four ambulacral vessels are uniform throughout, very small and pointed.

The interambulacral areas of the trivium are comparatively narrow, and form the lateral portions of the anterior truncated extremity running up to the principal apical pole. The lateral interambulacra are wide, and correspond with the space between the primary and secondary apical poles. The odd interambulacrum of the bivium is most remarkably developed, and chiefly contributes to the peculiar form of the shell. Starting from below the mouth, it forms a narrow plastron along the middle line of the ventral surface of the test; it then widens out to form the posterior rostrum, arching upwards to the edge of the periproct; it forms the projecting arch over the anal opening, whence it continues along the median ridge of the back to end with the bivial ambulacra in the secondary apical pole (Plate LXXI. figs. 1, 2, & 3).

The apical system is disjunct, as in *Dysaster*. The anterior portion consists of a series of coalesced ovarial plates, with four large and distinct apertures, the two anterior closer to one another than the two posterior, all of them inclining markedly forwards in passing through the test (Plate LXX. fig. 4). The ocular plates are undistinguishable, and there are no evident ocular pores. The posterior pole of the apical system consists of two odd plates terminating the ambulacra of the bivium, and therefore representing ocular plates. These plates show no pores, and they are so small and indistinct that this "secondary pole" is chiefly indicated by the junction of the bivial ambulacra.

The anal opening is small; the periproct is oblong, transverse, and plated with irregular unequal calcareous scales (Plate LXXI. fig. 7).

The surface of the test is somewhat sparsely and irregularly sprinkled with primary tubercles; the scrobicular area is rounded or slightly angular, roughened and slightly depressed; the mammillary boss is low, and peculiarly crenulated by a chain of minute rounded elevations. The tubercle is perforated, and likewise slightly crenulated (Plate LXX. figs. 6, 7). Between the primary tubercles there are numerous minute tubercles for the articulation of the small spines and the pedicellariæ. The larger tubercles are specially crowded near the apex and round the border of the anterior flattened surface (Plate LXX. figs. 1, 2).

The primary spines are cylindrical, fenestrated, with about twelve smooth ribs; the milled ring is very prominent, and gives attachment to a circle of very strong and

elastic muscular fibres (Plate LXX. fig. 7); some smaller spines have small pointed elevations on the ridges (Plate LXXI. fig. 9); the secondary spines are very minute, paddle-shaped, and ornamented with crenated ribs (Plate LXX. fig. 8). The pedicellariæ are of two kinds; the larger (Plate LXX. figs. 9, 9*a*) are very peculiar in form. Three thick stems are articulated to a short stalk, and from each of these curves inwards a calcareous disk, bordered by an extremely elegant fringe of spines. The stems and the disks which represent the valves of pedicellariæ of the ordinary form contain cavities which occupy the usual positions. The smaller pedicellariæ resemble generally the smaller forms in the Echinidæ.

The general colour of *Pourtalesia Jeffreysi* is purple, rather light over the greater part of the test, but dark and rich from the accumulation of large pigment-spots round the raised border of the anterior surface, and especially about the posterior rostrum. The test of *Pourtalesia* is so remarkably thin that it will scarcely bear its own weight.

Pourtalesia Jeffreysi differs from *P. miranda*, the species described by Prof. Alexander Agassiz, in having the test more uniformly cylindrical, and particularly in the form of the rostrum, which is much smaller than in *P. miranda*, with the usual depression much less marked. Another character which, in these aberrant forms having a tendency to the extreme modification of various parts, may have less importance than we should at first sight be inclined to assign to it, might be supposed to separate the two species widely: Prof. Agassiz states that in *P. miranda* the apical system is elongate, but not disconnected. In *P. Jeffreysi* the apical system is certainly disjunct, several extra plates which cannot be referred to the apical system being intercalated between its parts (Plate LXXI. fig. 3).

A single specimen only of *Pourtalesia Jeffreysi* was dredged during the 'Porcupine' cruises. On one or two occasions, however, fragments of a thin purple test and fragments of spines, which we referred to this species, were sifted out of the mud. The shell is so excessively fragile that it may be regarded as a fortunate incident to have recovered any thing like a complete example.

2. *Pourtalesia phyale*, n. sp. (Plate LXX. fig. 11.)

Test very much prolonged, almost tubular; posterior rostrum more produced than in *P. Jeffreysi*; anterior surface oblique to the axis of the test; mouth-depression nearly central in the anterior end of the shell, surrounded by an oval raised rim or border.

The specimen of *P. miranda* described by Prof. A. Agassiz is very small, but it already approaches *P. Jeffreysi* closely in form, and gives no indication of having undergone any very great change during the later stages of the process of growth. I therefore cannot imagine that the singular little urchin to which I have given provisionally the name *Pourtalesia phyale* is the young of the species previously described. Both of the two specimens procured by Mr. Gwyn Jeffreys at a depth of 1215 fathoms in the Rockall Channel are immature, and their characters are too undefined for satisfactory description.

Genus *Brissopsis*, Agassiz.

Brissopsis lyrifera, Forbes.

Large specimens of this species are abundant at from 50 to 250 fathoms. Beyond this latter depth the specimens decrease in size, and at extreme depths only examples which have all the appearance of being very young are met with. These small delicate specimens are found at all depths, even down to 2090 fathoms.

Genus *Tripylus*, Philippi.

Tripylus fragilis, Von Düben and Koren.

At from 400 to 500 fathoms, between Scotland and Færöe. Hitherto known from the Scandinavian seas.

Genus *Schizaster*, Agassiz.

Schizaster canaliferus, Lamarck.

A single small specimen from the coast of Spain.

Genus *Amphidetus*, Agassiz.

Amphidetus ovatus, Leske, sp.

Abundant at moderate depths.

Genus *Spatangus*, Klein.

1. *Spatangus purpureus*, O. F. Müller.
2. *Spatangus Raschi*, Lovén.

This species is apparently gregarious, and is enormously abundant, in patches here and there, from the Færöes to the Strait of Gibraltar, at depths of from 100 to 300 fathoms.

Of the twenty-six species observed, six (namely, *Echinus Flemingii*, *E. esculentus*, *Psammechinus miliaris*, *Echinocyamus angulatus*, *Amphidetus ovatus*, and *Spatangus purpureus*) may be regarded as denizens of moderate depths in the "Celtic province," recent observations having merely shown that they have a somewhat greater range in depth than was previously supposed. *Spatangus Raschi* may probably be an essentially deeper-water form having its headquarters in the same region. Eight species (*Cidaris papillata*, *Echinus elegans*, *E. norvegicus*, *E. rarispina*, *E. microstoma*, *Toxopneustes dröbachiensis*, *Brissopsis lyrifera*, and *Tripylus fragilis*) are members of a fauna of intermediate depth; and all, with the doubtful exception of *Echinus microstoma*, have been observed in comparatively shallow water off the coasts of Scandinavia. Five species (*Cidaris affinis*, *Echinus melo*, *Toxopneustes brevispinosus*, *Psammechinus microtuberculatus*, and *Schizaster canaliferus*) are recognized members of the Lusitanian and Mediterranean faunæ, and seven (*Porocidaris purpurata*, *Phormosoma placenta*, *Calveria hystrix*,

C. fenestrata, *Neolampas rostellata*, *Pourtalesia Jeffreysi*, and *P. phyale*) are forms which have been for the first time brought to light during the late deep-sea dredging-operations, whether on this or on the other side of the Atlantic: there seems little doubt that these must be referred to the abyssal fauna, upon whose confines we are only now beginning to encroach. Three of the most remarkable generic forms, *Calveria*, *Neolampas*, and *Pourtalesia*, have been described by Prof. ALEXANDER AGASSIZ among the results of the deep-dredging operations of Count POURTALES in the Strait of Florida, showing a wide lateral distribution; and even a deeper interest attaches to the fact that while one family type, the ECHINOTHURIDÆ, has been hitherto known only in a fossil state, the entire group find nearer allies in the extinct faunæ of the Chalk, or of the earlier Tertiaries, than in that of the present day.

DESCRIPTION OF THE PLATES.

PLATE LIX.

Cidaris papillata.

Fig. 1. Test of a full-sized specimen, showing the arrangement of the apical disk. Natural size.

Fig. 2. Portion of the test, showing the arrangement of the scales of the buccal membrane. × 2.

Fig. 3. Radials from the region of the ambitus. Natural size, and fig. 3 *a* × 2.

Fig. 4. Small spine from the ambulacral area. × 20.

Fig. 5. One of the inflated pedicellariæ from the apical disk. × 25.

Fig. 6. Tridactyle pedicellaria from the edge of the pore-area. × 25.

Fig. 7. Tridactyle pedicellariæ from the miliary area of the test.

Fig. 8. One of the ambulacral tube feet, showing the calcareous spicules of the wall and the rosette of the sucker. × 50.

Fig. 9. Spicules from the ambulacral tube feet. × 100.

Fig. 10. Portion of the mesenteric membrane, showing the fenestrated supporting plates. × 15.

Fig. 11. The wall of the intestine, showing the calcareous plates.

Fig. 12. One of the ultimate branches of the ovary, showing the fenestrated plates. × 25.

Fig. 13. Plates from the wall of the ovary. × 50.

Porocidaris purpurata.

Fig. 14. Didactyle pedicellaria from the edge of the pore-area. × 50.

Fig. 15. Fenestrated plates in the wall of the intestine.

PLATE LX.

Cidaris affinis.

Fig. 1. Full-sized specimen from off Cape Spartel. Natural size.
Fig. 2. Apical aspect of test. × 2.
Fig. 3. Portion of the test, showing the pore-areæ. × 4.
Fig. 4. One of the radials from the region of the ambitus. × 3.
Fig. 4*a*. A portion of a primary radiole, showing the sculpture. × 5.
Fig. 5. Smaller spine from the neighbourhood of the mouth. × 2.
Fig. 6. Spine from the border of an areola. × 6.
Fig. 7. Dental pyramid. × 3.

PLATE LXI.

Porocidaris purpurata.

Fig. 1. A mature specimen. Natural size.
Fig. 2. The apical aspect of the denuded test. Natural size.
Fig. 3. A portion of the oral aspect of the test. × 2.
Fig. 4. Radiole from the region of the ambitus. Natural size.
Figs. 5–7. Smaller radioles from different parts of the corona.
Figs. 8–10. Paddle-shaped radioles from the neighbourhood of the mouth. Natural size.
Fig. 11. Spine from the margin of an areola. × 3.

PLATE LXII.

Phormosoma placenta, Wy. T.

Fig. 1. Oral surface of the test. Natural size.
Fig. 2. Apical surface of the test. Natural size.
Fig. 3. One of the smaller spines. × 50.
Fig. 4. Portion of one of the larger spines. × 25.
Fig. 5. Tridactyle pedicellaria. × 50.
Fig. 6. Ophiocephalous pedicellaria. × 150.

PLATE LXIII.

Phormosoma placenta.

Fig. 1. Interior of the oral portion of the test, showing the peculiar character of that part of the test, the auriculæ, the arrangement of the scales of the peristome, and the dental pyramid. × 2.
Fig. 2. A portion of the ventral surface of the test, seen from without. × 2.

Fig. 3. A portion of the ventral part of the test, seen from within. × 2.
Fig. 4. The apical plates and the periproct, seen from within. × 2.
Fig. 5. The outer aspect of a portion of the apical surface of the test. × 2.
Fig. 6. Inner aspect of the same. × 2.
Fig. 7. A cribriform plate from the wall of one of the tube feet. × 60.
Fig. 8. Appearance of a thin slice of one of the plates of the perisom under a low power.

Calveria fenestrata, Wy. T.

Fig. 9. A pair of tooth-sockets, showing the form of the epiphyses.
Fig. 10. Lateral view of the same.

PLATE LXIV.

Calveria hystrix, Wy. T.

Fig. 1. Natural size.
Fig. 2. Portion of one of the large spines. × 30.
Fig. 3. One of the tube feet from the ventral surface of the test. × 30.
Fig. 4. A large tridactyle pedicellaria. × 30.
Fig. 5. One of the smaller tridactyle pedicellariæ. × 50.
Fig. 6. An ophiocephalous pedicellaria. × 50.

PLATE LXV.

Calveria hystrix.

Fig. 1. Outer aspect of a portion of the test of the ventral surface. × 2.
Fig. 2. The same, seen from within. × 2.

PLATE LXVI.

Calveria fenestrata, Wy. T.

Fig. 1. Plates of the apical disk, from within. × 2.
Fig. 2. The external surface of the same. × 2.
Fig. 3. The test opened to show the arrangement of the fasciæ which enclose the ambulacral organs and the position of the ovaries. × 2.
Fig. 4. The test thrown back to show the mesenteric membranes and fibres which suspend and retain in position the coils of the intestine. × 2.
Fig. 5. A portion of a series of ambulacral plates, showing the arrangement of the ampullæ and the distribution of the branches of the ambulacral vessel to the pores.

PLATE LXVII.

Calveria fenestrata.

Fig. 1. A portion of an ambulacral area from the oral surface of the test. × 2.
Fig. 2. A portion of an interambulacral series from the same part of the test. × 2.
Fig. 3. A part of an interambulacral series from the apical surface of the test near the edge of the periproct. × 2.
Fig. 4. A portion of the base of one of the larger spines. × 20.
Fig. 5. One of the tetradactyle pedicellariæ. × 35.
Fig. 6. A like pedicellaria. × 60.
Fig. 7. A tridactyle pedicellaria. × 40.
Figs. 8, 8 *a*. An ophiocephalous pedicellaria. × 60.
Fig. 9. One of the tube feet from the oral surface of the test.

PLATE LXVIII.

Echinus microstoma.

Fig. 1. General view of the test from above.
Fig. 2. A portion of the test removed to show the form of the auricles and the position of the dental pyramid.
Fig. 3. Dental pyramid. × 2.
Fig. 4. One of the small flattened spines from the neighbourhood of the mouth. × 6.
Fig. 5. A pedicellaria. × 25.
Fig. 6. A pedicellaria, the valves provided with large external fleshy lobes containing curved calcareous spicules. × 40.
Fig. 7. The separated valves of a pedicellaria of this form. × 50.
Fig. 8. Stems of pedicellariæ with curved calcareous spicules imbedded in their investing membrane. × 80; the separate spicule × 100.
Fig. 9. A portion of one of the tube feet, showing the curved spicules imbedded in the wall. × 60.
Fig. 10. A portion of the wall of the intestine. × 100.

Echinus elegans.

Fig. 11. View of the test from above. × 2.
Fig. 12. One of the primary spines. × 5.
Fig. 13. The dental pyramid. × 4.
Fig. 14. The dental pyramid of *Echinus Flemingii*, introduced for comparison. × 2.

PLATE LXIX.

Neolampas rostellata.

Fig. 1. Oral aspect of the test. × 4.
Fig. 2. Apical aspect of the test. × 4.
Fig. 3. The test seen in profile. × 2.
Fig. 4. The posterior aspect of the test, showing the deep anal inversion. × 2.
Fig. 5. A portion of the test, showing the terminations of the ambulacra and the apical system. × 8.
Fig. 6. The mouth, the floscelle, and the bourrelets. × 8.
Figs. 7, 8. The larger spines. × 50.
Fig. 9. One of the smaller spines, showing the terminal rosette of pointed tubercles. × 90.
Figs. 10, 11, 12. Pedicellariæ with their stalks. × 90.

PLATE LXX.

Pourtalesia Jeffreysi.

Fig. 1. Lateral view of the test. × 2.
Fig. 2. View of the truncated anterior extremity of the test, showing the position of the oral inversion. × 2.
Fig. 3. Dorsal aspect of the posterior extremity of the test, showing the anal depression and the rostrum. × 2.
Fig. 4. Portion of the test, showing the arrangement of the apical system. × 8.
Fig. 5. An interambulacral plate, showing the distribution of the primary tubercles and of the granules.
Fig. 6. One of the primary tubercles. × 20.
Fig. 7. The base of one of the primary spines. × 40.
Fig. 8. One of the small spatulate spines. × 150.
Figs. 9, 9 *a*. One of the larger pedicellariæ. × 90.
Fig. 10. A small pedicellaria. × 100.

Pourtalesia phyale.

Fig. 11. Lateral view of the test.

PLATE LXXI.

Pourtalesia Jeffreysi.

Fig. 1. Test denuded to show the arrangement of the plates. × 2.
Fig. 2. Oblique view of the test. Natural size.

Fig. 3. Inner aspect of the anterior portion of the test, showing the mouth, the oral ring of the ambulacral vascular system, the ambulacral vessels of the trivium, and the disjunct apical system. × 2.

Fig. 4. The mouth from within, showing the ring of the ambulacral vascular system and the mode of origin of the five ambulacral vessels. × 4.

Fig. 5. Lateral view of the oral inversion of the test, seen from within. × 4.

Fig. 6. Inner aspect of the posterior portion of the test, seen from below. × 2.

Fig. 7. Anal opening, and the arrangement of the plates and scales of the periproct, from within. × 4.

Fig. 8. The looping of the vessels of the bivium opposite the anal opening.

Fig. 9. A portion of a spine.

XXII. *On the Structure and Development of* Peripatus capensis. *By* H. N. Moseley, *M.A., Naturalist to the 'Challenger' Expedition. Communicated by* Professor Wyville Thomson, *M.A., F.R.S., &c., Director of the Scientific Civilian Staff of the Expedition.*

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Introduction.

At the outset I wish to express my obligations to Prof. Wyville Thomson, by whom this paper is communicated to the Royal Society. Prof. Thomson kindly examined a series of preparations of various structures in *Peripatus*, and gave me the benefit of his long experience in such matters, and especially confirmed my views as to the identity of the tracheæ, which, of course, I had some hesitation at first in admitting as such, since they occurred in an animal in which, from what was at that time known concerning it, such structures were so little to be expected. Prof. Thomson further gave every encouragement to the prosecution of my further investigations on the subject.

Peripatus has always been regarded as of such especial zoological interest that no apology is necessary for the present paper. *Peripatus* was naturally the first animal sought after by the naturalists of the 'Challenger' expedition on their arrival at the Cape of Good Hope, and I was lucky enough to find a considerable number of specimens on the very first occasion of searching for them. My intention had been only to try to keep the animals alive so as to obtain their eggs and watch their development, but on opening one large specimen I immediately recognized the presence of tracheæ, and found the animal to be viviparous and full of far-advanced embryos. I therefore commenced as careful an examination of the structure and development of *Peripatus* as my available time during our stay at the Cape allowed; and although the investigation is far from complete, the results embody so much that is novel and important that I consider it better to publish them now, leaving the gaps to be filled in by other observers, or by myself by further work at the subject during the Antarctic cruise of the 'Challenger,' should such work be then found practicable.

Literature.

No original papers on *Peripatus* are available to me here at the Cape of Good Hope, but from such text-books as Claus's 'Grundzüge der Zoologie,' Schmarda's 'Zoologie,' Gegenbaur's 'Grundzüge der vergleichenden Anatomie,' tolerably complete information

about what is known or believed concerning the structure and affinities of the animal can be gathered. The following are the papers quoted on the subject:—

L. Guilding, "An Account of a new Genus of Mollusca," Zool. Journ. ii. 1826.

M. Gervais, "Études pour servir à l'histoire des *Myriapodes*," Ann. des Sc. Nat. 1837.

E. Blanchard, "Sur l'organisation des Vers," Ann. des Sc. Nat. 3 sér. tom. viii. 1847.

De Blainville, Suppl. au Dict. des Sc. Nat. t. i.

M. de Quatrefages, "Anat. des Hermelles," Ann. des Sc. Nat. 3 sér. tom. x.

E. Grube, "Ueber den Bau des *Peripatus Edwardsii*," Müller's Archiv, 1853.

Grube, whose paper on the subject is the best known and most searching, came to the conclusion that *Peripatus* was hermaphrodite, probably from want of sufficient material, or because his specimens were not collected at a period when the reproductive functions were in activity. He further failed to see the tracheal system, as could hardly be avoided, since it becomes almost invisible when the air has been removed from the tracheal tubes by long soaking of the animal in spirit. Grube thus placed *Peripatus* (amongst the Bristle-worms) in a separate order, Onycophora; and he is followed by Claus and Schmarda and most zoologists, although the position of *Peripatus* has always been considered doubtful. Gegenbaur (Grundzüge der vergleichenden Anatomie, p. 199) speaks of the placing of *Peripatus* amongst the worms as by no means certain, but adds that *Peripatus* certainly connects the Ringed worms and Arthropods with Flat worms. The older writers on the subject come nearer the mark. Newport* says, "De Blainville first connected Myriopoda with Annelida by means of the bristly genera, the Annelida Errantia; but subsequently remarked a closer connexion between the two classes in the singular *iuliform* genus *Peripatus*."

De Quatrefages† does not place *Peripatus* with the worms, but in his 'Histoire Nat. des Annelés' gives a short account of these animals in the Appendix. He dwells on the fact that *Peripatus* has feet armed with nails, to which muscles are attached, and not bristles lodged in a follicle. He considers that De Blainville‡ was near the mark when he formed a special class "*Malacopodes*" for *Peripatus*, but that M. Gervais was most in the right when he placed the group in affinity with the Myriopods.

M. de Quatrefages was evidently quite right in following M. Gervais in this matter. The affinities of *Peripatus* will be discussed shortly at the close of this paper.

De Quatrefages† gives a list of four species of *Peripatus*, viz. *P. iuliformis* of Cuba with 33, *P. d'Edwards* of Cayenne with 30, *P. Blainville* of Chili with 19, and *P. brevis* of the Cape with 14 pairs of feet.

The specimens obtained and examined by me here cannot be of the species "*brevis*," since, instead of 14 pairs of feet, they invariably, whatever their size, have 17 pairs of ambulatory members besides the labial and anal papillæ, and the young embryos very

* "Monograph of the Class Myriopoda, Order Chilopoda, with Observations on the General Arrangement of the Articulata, by George Newport," Linn. Trans. vol. xix. 1843, p. 268.

† Hist. Nat. des Annelés, Paris Libraire Encyclopédique de Roret, 1865, t. xi. Appendice, pp. 675–6.

‡ Suppl. au Dict. des Sc. Nat. t. i.

early obtain the full number of members possessed by the adults. The largest specimen found was about 7 centims. long when in a state of rest, the smallest 1·6 centim. in length. In no specimen, however, were the reproductive organs immature. Most of the smallest specimens were found to be males, but one of only 2 centims. in length contained embryos in a far advanced condition. A species of *Peripatus* from the Cape is, I believe, described, in the zoological publications of the 'Novara' expedition, by GRUBE as *Peripatus capensis*. Probably there is only one species at the Cape, *P. capensis* and *P. brevis* being the same badly described. Specimens of the one here treated of are forwarded with this paper for determination of the species.

Locality.—The specimens made use of were all found at Wynberg, a village situate on the road between Simon's Town and Cape Town, and not far from Constantia, and all in one spot in that place. This was a plantation surrounding the house of Mr. MAYNARD, and situated just behind Cogill's hotel. These particulars are given because *Peripatus* seems to be remarkably local, and is not by any means easy to find. A single specimen was found in Dr. BLEEK's garden at Mowbray, and a further locality is known to Mr. TRIMEN, the Curator of the Cape Museum.

Habits of Peripatus.

Peripatus is to be sought for in places where the soil is rich in vegetable matter and kept constantly moist and shady by a thick growth of trees and the proximity of some stream or source of water. The first and largest specimen found was resting under a piece of half-rotten board; others were met with under fallen logs; but the greatest number were found by tearing to pieces stumps and fallen logs of willow-wood, which were very damp, half buried in the ground, and in the condition known as touchwood. As many as twelve specimens of various sizes were thus found in one log. The willow-wood seemed to be preferred to the poplar-wood, of which there was a large quantity in the locality, but which was always unproductive. The animals were always met with in a quiescent condition, with the tentacles retracted and feet drawn together, the body being thus arched above. The animals are roused into activity with some little difficulty at first. They avoid the light, and crawl away to the first dark corner. They are easily kept alive in confinement amongst damp earth and pieces of dead wood; but if the supply of moisture be not kept up, they shrivel up and die at once. They are nocturnal in their habits, remaining concealed amongst the earth and dead wood, with which they are kept during the day, but leaving this and crawling about their box at night. When suddenly handled or irritated, they shoot out fine threads of a remarkably viscid and tenacious milky fluid. The threads of fluid are emitted with such remarkable suddenness that it is almost impossible to observe their passage from the animal's head; but on close observation with a lens, especially in the case of large specimens, they may be seen to be projected from the tips of the oral papillæ. The threads cross one another in various directions, and form a sort of mesh-work, often of considerable complexity, which suddenly appears, as if by magic,

suspended from objects lying in front of the animal, and having the appearance presented by a bit of spider's web with the dew upon it. When examined under the microscope the threads are seen to be fine and hyaline, with variously sized highly refractile spindle-shaped globules situate at intervals upon them (Plate LXXIII. fig. 4). They are thus very like the viscid threads of the spider's web. The fluid of the globules is seen under a high power to contain a few fine granules. As it dries under the microscope, it forms into a tenacious mass, showing extremely fine lines pervading the threads in the direction of their length, and giving them a fibrillated appearance. The fluid is not perceptibly irritant when applied to the tongue, but has a slightly bitter and at the same time somewhat astringent taste. Small specimens of *Peripatus* soon exhaust their immediate supply of the fluid, and cannot be induced to make more than two or three discharges at one time even when squeezed hard; but large specimens can make at least a dozen discharges one after another. The animals evidently use the fluid as a means of defence; for when they are pricked with a needle or forceps about the side or middle of the body, they turn their head round and aim their discharge at the place at which the injury is being received. The tenacity of the threads formed by the fluid is so great and their viscidity so remarkable, that the meshwork of them thrown out over an insect or other such enemy would entangle it, and render it powerless for some time, even if it were of considerable size. The fluid adheres most tenaciously to the fingers, just like bird-lime; and when a large *Peripatus*, when first found and handled, shoots out its fluid over its own back and the fingers of the finder, it requires a very hard shake to free it from the hand. Whilst I am writing several flies have walked into some of the fluid which I caused a large *Peripatus* to discharge upon a glass slide in order that I might test the action of the fluid on my tongue. The flies are helplessly stuck fast; and I believe that the fluid is quite sticky enough to hold small birds, though it dries too rapidly to be used for that purpose. The glands containing the fluid give some little difficulty in dissection of the animal, when this is conducted under water or in the dry condition, since they protrude out of the first incision, adhere tightly to forceps, scissors, or scalpel, and can hardly be disengaged without being torn, or very often without carrying away, together with themselves, the surrounding viscera, intestine, generative ducts, &c.

Peripatus coils itself up, often, like *Iulus*, in a spiral with the head in the centre of the coil; and small specimens always assume this posture when they are severely pricked or maltreated. When in motion the animal extends its tentacles and body to their full length, thus becoming of nearly twice the length which it measures when at rest and proportionately narrower. The tentacles are kept in motion in search of obstacles, and the feet are moved rapidly in pairs, the alternate pairs moving together with considerable regularity, but at the same time with the peculiar progressive movement of a caterpillar. The gait is, in fact, extremely like that of a caterpillar. There is never any wriggling or sinuous movement of the body like that in worms. The ventral surface of the body is entirely supported free of the ground by the feet. The animals

are readily killed by immersion for four or five hours in water; and when thus treated they die in an extended condition, and are obtained in the most favourable condition for dissection or preservation in alcohol as museum specimens. They are apparently paralyzed after immersion for only a quarter of an hour in water, and remain extended and motionless, but they soon recover when placed again in the air. They can also be killed in an extended condition by being placed in a weak solution of chromic acid; and this is the best method of commencing to harden them for the preparation of sections.

Anatomy.

Only those points in the anatomy of *Peripatus* will here be dwelt upon in which correction of former accounts appears to be necessary, or those which have been hitherto entirely unnoticed. No attempt will be made at a description of the whole structure of the animal.

Digestive System.—The intestinal tract of *Peripatus* is described by GRUBE as straight and enlarged in each segment of the body. This is far from being the case, at least in the Cape species. The digestive tract is here longer than the body, and in all the specimens which I have examined has been considerably distended, so that it protruded itself in a loop, together with the attached slime-glands, from the first incision. The digestive tract is displayed in Plate LXXII. fig. 1*. It commences as an ovoid muscular pharynx, which is separated by a short contracted œsophagus from the elongate and capacious stomach, from which a very short piece of intestine or rectum leads to the anus.

From the posterior part of the lateral surfaces of the pharynx arise a pair of small muscles, which probably are protractors of the pharynx, and serve to push forwards the jaws. The under surface of the pharynx, and also of the rectum, receive a large and conspicuous supply of tracheæ: these are shown in the figure. The division between the stomach and intestine is much more marked in some specimens than others, and is best seen in quite fresh ones, where the stomach is distinguished by its pale pink colour. In some specimens there is a distinct shoulder at the point of junction. The wall of the stomach is plicated, and the organ has a convoluted appearance. In many specimens the stomach was folded in a single short dorsal loop, at about the junction of its first with its second third. The loop lies folded flat over the upper surface of the succeeding part of the stomach. It lies in the middle line, and is not at all inclined to either side. It was only about half an inch long, and was not observed in all specimens, but is probably always present when the body is in the contracted condition, since the excess in length of the intestinal tract over that of the body necessitates a folding of some kind. As Mr. TRIMEN, Curator of the Cape Museum, who long ago watched the habits of *Peripatus*, has pointed out to me, the great power of extension of

* The beautiful drawings in Plate LXXII. were made for me from my dissections by Mr. J. J. WILD, Artist to the 'Challenger' Expedition.

its body is very remarkable in *Peripatus*. When the animal is crawling, the body is extended to about twice the length which it has when the animal is at rest. When the body is thus fully extended, the digestive tract is probably quite straight, becoming looped again when the body is drawn together. The convolutions and folds of the stomach do not in any way correspond to the segments of the body. The outer surface of the stomach has upon it a pattern composed of small hexagons, which is shown by Mr. WILD in the figure. This appearance is caused by the glandular follicles of the mucous lining of the organ, which are seen through its outer wall. The glandular lining of the stomach has always a light pinkish colour * (in the embryo the digestive tract is red). It is very thick, and composed mainly of large cells (Plate LXXIII. fig. 3, *a*) with a polygonal outline, usually pentagonal, showing a granular structure with a distinct nucleus and nucleolus. These cells probably have an hepatic function. Besides them there are slime-cells and masses made up of aggregations of small fat-particles (Plate LXXIII. figs. 3, *b* and *c*) in abundance in the stomach. Vegetable cells and portions of woody fibre in small quantity were found in the stomachs of two specimens; and there can thus be little doubt that their food, like that of *Iulus*, is the decayed wood amongst which they are so constantly found. The stomachs of several specimens were quite empty. It is very possible that the animals feed very little or not at all during the breeding-season, but rest, as does *Iulus* according to NEWPORT, at the time of the production of the eggs.

Some very small encysted Gregarinæ were found in the stomachs of all the specimens examined.

The separation between the stomach and rectum is marked by the difference of colour in the fresh state, the rectum being pink. The alteration in diameter of the tube is more sudden in some specimens than in others.

Slime-glands.—The glands, which are those described by GRUBE as the testes, are present in both sexes; they lie on each side of the digestive tract, and stretch down nearly its whole length. The glands are shown in Plate LXXII. fig. 1, *s.g*, *s.g*. They consist of a series of elongate ramified tubes, which twist round the stomach in a sort of meshwork, and entangle themselves around the male or female generative organs. The ramified tubes terminate in ducts, which open one on each side of the body in the tips of the oral papillæ. Along the greater part of their course these ducts are enlarged into wide sacs, which serve to store up the secretion of the gland, and to eject it in the form of the viscid thread as already described. These reservoirs or ejaculatory sacs (Plate LXXII. fig. 1, *e.s*) are represented in the figure in a collapsed condition, the animal dissected in this instance having been killed by immersion in alcohol vapour, and having discharged its store of viscid fluid. When distended the sacs appear as cylindrical bodies, with a tense transparent wall, very like the sacs on the ducts of the salivary glands in *Blatta* and several other Orthoptera. The sacs are provided with spirally arranged muscular fibres, disposed in two layers and twisted in opposite directions.

* Just the colour of the excreta constantly voided by moths immediately after leaving the pupa-case.

The interior of the sac is lined with large flat endothelial cells, with a polygonal outline, and resembling the well-known endothelial cells of the frog (Plate LXXIII. fig. 6). The secreting tubes of the gland itself (Plate LXXIII. fig. 7) are covered externally with a layer of hexagonal plaster-cells (not shown in the figure), and internally are lined with a thick layer of slime-cells, which are elongate, often pear-shaped, disposed transversely in the gland-tubes, with their narrower bases resting on its wall and their pear-shaped ends projecting into its cavity. The cells have the usual granular structure and a very distinct nucleus and nucleolus. The riper cells become transparent and almost free from granules. An irregular space leading down the axes of the tubes is filled with fluid containing fine granules. The slime-cells probably swell gradually to the transparent condition, and then bursting, discharge their contents into the cavity of the gland. The glands exhibit all the viscous properties of the fluid ejected by the animal, and are so sticky as to be of considerable hindrance in the dissection of the animal in the fresh condition or under water. The oral papillæ in which the glands open, as will be seen in the sequel, are modifications of the second pair of body-members of the embryo. The glands are probably homologous with the silk-glands of caterpillars and the poison-glands of *Scolopendra*. There are no excretory glands representing the Malpighian tubes of insects and Myriopods in *Peripatus*.

Fat-bodies.—The lateral bodies (*x*, Plate LXXII. fig. 1) were described as two canals imbedded in the muscular wall of the body, which might perhaps belong to the vascular system (Claus, Zoologie, p. 387). These bodies vary very much in different specimens. In some they are narrow and terminate about the middle of the body; in others they are much longer and broader, and stretch to the very end of the body. They are composed of small round oily particles, and apparently are homologous with the fat-bodies of other Tracheata.

Circulatory System.—The dorsal vessel, when examined in the fresh state or in glycerine, appears to be of very simple structure. As in insects, it is imbedded in fatty tissue. I have been unable as yet to see any valvular arrangements in it or system of transverse fibres. I intend to examine its structure more carefully by means of sections &c. When the vessel is dissected away from the body-wall, a series of openings are seen in the middle line of the body-wall, directly above it, exactly like those which exist on either side of the ventral wall of the body along the line of origin of the feet. The ventral openings were described by Grube. They are figured in Plate LXXII. fig. 2, where they are seen situate on either side, immediately externally to the ventral nerve-cords (*v.c*, *v.c*).

Respiratory System.—The tracheal system in *Peripatus* renders itself conspicuous at once when the animal is opened in the fresh condition. The tracheæ have the characteristic pearly lustre given by all fine transparent tubes when filled with air and seen by reflected light. In specimens which have become saturated with spirit, the air becomes entirely removed from the vessels, and they are consequently hardly to be seen

at all, and, in consequence of the great imperfection of their spiral fibre, are with great difficulty to be made out, even with a high power, under the microscope. It is thus easy to understand how they escaped the observation of GRUBE. The tracheæ arise from the skin all over the surface of the body, but are specially developed in certain situations. When the animal is opened under water in the fresh state, the groups of them appear to the naked eye as pearly white dots scattered over the inner surface of the body-wall, and, when viewed with a lens, show themselves as small tree-like ramifications coming up through the tissue and ramifying on the surface. These are shown in Plate LXXII. fig. 2, where they are represented as dark on a light ground for convenience of drawing: in reality they appear glistening and pearly.

Especially large and conspicuous groups of tracheæ are supplied to the rectum and pharynx (Plate LXXII. fig. 1, *tr*, *tr*). These tracheæ arise from the middle line of the ventral surface.

The structure of the tracheal system resembles very much that of *Iulus*. Plate LXXIII. fig. 5 shows the mode of origin of the tracheæ from the skin-surface. A short wide tube leads from a simple opening between the cells of the epidermis directly inwards through the skin. The tube is contracted at its opening, and swells out somewhat at its inward termination. From the inner end of this tube arise all together a vast number of fine tracheal vessels in a sort of brush. The vessels run for the first part of their course in a densely packed cylindrical mass, apparently united together side by side, running parallel, but never communicating. The cylindrical mass of vessels soon divides into several trunks, which again subdivide, and so give off the bundles of vessels which are distributed to the viscera, &c. The tubes never anastomose in any part of their course. In the first part of their course, where they are closely packed together, they run in almost straight lines; but as soon as they pass into smaller bundles, or are distributed to their destinations, they take a tortuous or zigzag course (Plate LXXIII. fig. 2). The closely packed masses of tracheæ at their origin, when filled with air and viewed by transmitted light under the microscope, are very conspicuous objects, and appear quite black, transmitting no light. In order to see the mode of origin of the tracheæ well, a vertical section of the body-wall of a *Peripatus* must be made in the fresh condition, and placed in glycerine for examination. It should be rather thick, or otherwise most of the air-vessels are cut across, the air escapes, and the vessels become almost invisible. When a thick section is thus prepared, and pressure is brought to bear on the covering-glass, several small bubbles of air may be caused to pass out from the spiracular opening of one of the tracheal bundles. Sections prepared from specimens hardened in spirit show almost nothing of the tracheal system, and it is hence difficult to get transverse sections to show the structure of the primary short air-tube. It must, however, terminate internally in a finely perforated plate. Some few tracheal vessels are given off at once from the large primary mass to the immediately surrounding tissue; but the larger part run a long course in parallel bundles before they separate for their final disposition. All the

iscera are supplied richly with tracheæ. Their manner of distribution is shown in Plate LXXIII. fig. 2, *a*, which shows the arrangement on one of the secreting-tubes of the slime-gland.

The vessels run in bundles of two, three, or sometimes as many as six together in an irregular zigzag course, crossing one another in all directions, so as to form a tangled irregular meshwork. They usually run separately for the last part of their course, and they terminate simply without enlargement, apparently with closed ends. The vessels are very fine, being only about ·003 millim. in diameter, but are very conspicuous when filled with air and viewed by transmitted light. When examined with very high powers they show transverse markings, which appear to indicate the existence of an imperfectly developed spiral fibre within them (Plate LXXIII. fig. 2, *b*). The fibre, or rather band, is broad in proportion to the diameter of the vessel; it is apparently very imperfectly differentiated, and not tenacious enough to be partially uncoiled when a vessel is torn across, as is the case in the tracheæ of insects.

The distribution of the tracheæ to the uterus and oviducts is seen in Plate LXXIV. fig. 1. The tracheæ very rarely branch; but branching does occur. Plate LXXIII. fig. 2 represents an instance of branching in the tracheæ of the uterus.

The spiracles, or openings of the tubes from which the tracheæ take origin, lie in the depressions between the conical warty excrescences with which the skin is covered. They are small and difficult to see when the epidermis is spread out and examined with the microscope. They seem to be mere cracks between the superficial epidermic cells; and no definite arrangement of these cells around them was observed, although most carefully sought for. The skin contracts when removed from the body and placed on a slide, and the spiracles close up and are most difficult to find. If a large specimen of *Peripatus* be killed by drowning, the animal dies in a state of extension, and the spiracles remain patent. A row of minute oval openings may then be seen with a lens extending along the median line of the ventral surface of the body. The openings are situate with tolerable regularity in the centres of the interspaces between the pairs of members, but additional ones occur at irregular intervals. Other similar openings occur in depressions on the inner sides of the conical foot protuberances. If the skin in which these openings occur be dissected off and spread out under the microscope no definite arrangement of cells around them is to be observed, and they collapse at once, having no chitinous ring or such structure.

Generative Organs.—Peripatus was described as hermaphrodite by Grube, that observer having mistaken the slime-glands of the animal for testes; in reality the sexes are distinct. There appear to be no outward characteristics by which a male can be distinguished from a female. Out of about thirty specimens dissected two thirds were females. All the males but one were very small, not more than 2 centims. in length; and it was at first concluded that the males were always smaller than the females; but one female was met with of only 2 centims. in length; and the last specimen dissected, which was 5 centims. in length, turned out to be a male. The

animals vary in colour, some being of a dusky brick-red, with a narrow black dorsal stripe, whilst the majority are entirely black; but both males and females were found of the light-coloured variety. Of the twenty female specimens dissected only one was found which did not contain embryos in some stage of development. In this one the ova were immature, and there were no spermatozoa in the ovary. The breeding-period of *Peripatus capensis* is thus probably the months of November, December, and January, the three Cape summer months. Observations are required on the mode of congress of the sexes, and on the time and manner of the birth of the young.

Female Organs.—The female organs were described by GRUBE as consisting of a pair of tubular ovaries running on the ventral surface of the intestine, and opening by a common aperture in the last segment but one. The organs are much more complicated than this in *Peripatus capensis*. They are figured in Plate LXXII. fig. 1. They consist of an ovary and pair of oviducts, which in the lower part of their course are dilated and perform a uterine function.

The ovary is a small elongate body, divided by a median septum into two lateral halves, which lies underneath the lower part of the stomach, and between that viscus and the ventral wall of the body. The position of the ovary varies somewhat. It is sometimes higher up the body than at others; but it is usually situate distant from the hinder extremity of the body about one sixth of the animal's length. The ovary is usually bound to the under surface of the intestine by tracheæ and the meshes of the slime-gland. From the ovary lead a pair of oviducts, connected with it in some cases by an elongate pedicle, as in Plate LXXII. fig. 1, in others (as in that figured in Plate LXXIV. fig. 1) arising directly from the base of the organ. The oviducts are slender tubes which pass upwards in the body-cavity, and then turn downwards again to run to the vulva, forming thus a loop.

At a short distance, but one very variable in length from their commencement, the oviducts become enlarged and perform the function of a uterus, and when filled with embryos have the appearance of a string of sausages, each dilatation containing one embryo. The ducts, finally passing inwards towards the middle line under the nerve-cords, unite under the rectum into a very short common tube, which terminates in the vulva. The embryos are contained in the duct up to its very termination. The number of embryos present varies greatly. In the specimen figured (Plate LXXII. fig. 1) there were fifteen embryos on one side, and seventeen on the other. One small specimen had only ten embryos altogether, and another only eight.

The oviducts and uterine tubes are not attached to the body-walls, but lie loosely in its cavity; hence the loops become twisted about into various postures, and a most extraordinary condition exists with such frequency and regularity that it must almost be considered normal. This is one in which the oviduct, or sometimes the uterine tube itself, is tied in a knot round its own loop. The knot, which is shown in Plate LXXII. fig. 1, *k*, is what is called by sailors an "overhand knot on a bight," and is, of course, such a one as can be tied on a loop of string which has both ends made fast. The

formation of the knot is evidently connected with the great power of extension of the body already referred to as possessed by *Peripatus*. The body being constantly protracted and then again shortened, the loops of the oviduct must be constantly pushed up and down inside the body-cavity, and thus get knotted. In order that the knot should be formed, the primary loop requires to receive a single twist, and then through the loop thus closed by the twist a second loop of the part of the duct beneath the twist must be pushed up. The movements of the body soon draw the knot tight. In the first specimen of *Peripatus* dissected, the knot was found on both sides of the body, in each case drawn so tight that it had to be very carefully loosened with forceps before its exact nature could be made out. In the one drawn for me by Mr. WILD there was a knot only on one side of the body as figured; but it is represented much looser than it was in reality, in order to show the way in which it runs. The knot was found in about ten other specimens, in some cases on one side, in others on both. It is evident at once to what pathological complications such a knotting of the duct must give rise. As the embryos swell, the uterus will become constricted at the knot, and the embryos will be unable to be extruded. In one of the largest specimens dissected this very thing had occurred, and in consequence the loops of the uterus on both sides had become dilated into wide sacs, which had lost all the normal successive constrictions originally separating the embryos from one another, and were filled with the decomposed remains of embryos and masses of fatty matter. The two sacs had lost all connexion with the lower parts of the uterine canals, these having apparently mortified off at the knots, and were attached only to the ovary. The remnants of the lower parts of the uterine canals were short and empty, having probably discharged their ripe contents before their upper portion perished.

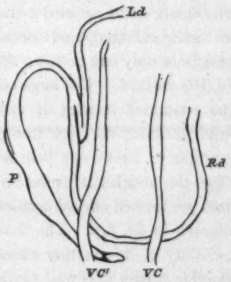
The structure of the ovary is shown in Plate LXXIV. fig. 1, *a*. It is composed of a stroma, fine fibres with numerous small rounded nucleated cells containing granules, and other clear round cells, with neither nucleus nor granular contents, and other elongate nucleated cells like those seen in the figure in great abundance in the oviduct. The ovary has in it two elongate sacs separated from one another by a median septum; and in nearly all the pregnant specimens examined, the sacs were found crammed with spermatozoa felted together into masses, as seen in the figure. Pear-shaped sacs hang from the margins of the ovary in which the ova are developed.

The ripe ovarian ova are about ·17 millim. in diameter. They have a thin transparent vitelline membrane, which encloses them within the sacs. They have an abundant finely granular yelk, a large, very transparent clear spherical germinal vesicle, and a finely granular germinal spot, which has a somewhat irregular outline. Some of the full-sized ovisacs are to be seen containing irregularly shaped masses of yelk, without any germinal vesicle. The ovary contains between thirty and forty ova. The ova appear to ripen all at about the same time. Stages in the development of the ova were observed in one female specimen which contained no spermatozoa, in which none of the ova had reached maturity. These are represented in Plate LXXIV. fig. 1, *b*, *c*, *d*.

The young ovisac (*b*) is filled with the peculiar round clear cells described as existing in the stroma of the ovary, and with a small quantity of coarsely granular yelk-matter, derived probably from the granulated cells of the ovary, which are to be seen occasionally in the necks of the ovisacs in a semidisintegrated condition, appearing as mere aggregations of granules around a clear spot representing the nucleus. In the next stage (*c*) the vitelline membrane is drawn across the cavity of the ovisac, separating off a space containing the relatively large germinal vesicle and the small quantity of coarsely granular yelk which float in the clear fluid which fills the greater part of the cavity. The germinal spot is composed of small rounded particles, and is irregular in form. In the next stage (*d*) the fine granular yelk entirely fills the cavity within the vitelline membrane. In all the ovaries examined, spermatozoa were found attached in tangled groups and masses amongst the ovisacs on the exterior of the ovary, and apparently in some cases the long filaments of the spermatozoa penetrated the ovisacs with one of their ends, whilst the other was in active motion. The filaments are, however, so fine and hyaline, that they are difficult to follow when lying amongst other tissue; and the supposed cases of penetration observed may have been merely cases of superposition. Free spermatozoa were also met with amongst the tissue of the lateral bodies termed here fat-bodies; indeed they were found first of all in this situation: only two were seen; they probably commonly escape amongst the viscera. Spermatozoa were found in abundance in the ovaries, even in specimens containing far advanced embryos. The uterine portion of the oviduct is very richly supplied with tracheæ, which appear to be more abundant in a uterus with advanced embryos than in one which is empty or has very early embryos. Perhaps the tracheal supply increases with the growth of the embryo and consequent increased development of the uterine wall.

Male organs.—The male generative organs consist of the prostates, the testes, with their vesiculæ seminales and vasa deferentia, and a pair of bodies, apparently accessory glands. The male organs are displayed in fig. 3, Plate LXXII., which is a drawing made by Mr. Wild from a dissection. The testes (*t*, *t*) are ovoid bodies of large size, which, as well as their ducts, prostates, and vesiculæ seminales, are of a glistening white colour and very conspicuous. A short but wide convoluted tube, the prostate (*pr*), surmounts each testis, and communicates with it by means of a narrowed neck, which springs from a spot on the somewhat flattened surface of the testis, and not from its apex, but is situate somewhat below that point. The vas deferens arises as a fine thread in the same manner from the same surface as the vesiculæ seminales lower down, but above the inferior margin of the organ. The thread-like duct enlarges directly after quitting the testis, and forms a spiral coil, which probably serves as a reservoir for the semen, and may be called vesicula seminalis. From this coil the long continuation of the duct, somewhat reduced again in diameter, twisting about in all sorts of irregular loops and turns, passes down to the hinder end of the body. Here the duct of one testis passes transversely across the body underneath the ventral nerve-cords,

passes back on the other side, up the body again a short distance, and then turns down again, forming a loop to reach the male generative opening, which lies in the same position as that of the female. The duct thus passes under one of the ventral cords again. The lower part of the duct, from the loop downwards, is enlarged, muscular, and supplied much more richly than the remainder with tracheæ. It is therefore an ejactulatory duct, and perhaps is somewhat exerted in copulation as a penis. In only two specimens of males were the terminal ducts dissected out with care. In one case the terminal duct was that from the right testis, in the other that from the left. The vas deferens of the other testis of the pair, after leaving its vesicula seminalis, becomes twisted up with the spiral coil of the testis of the other side, and after coiling about in the lower part of the body-cavity like its fellow, passes to that side of the terminal loop of its fellow duct which is opposite to that which is enlarged into the penis-like organ, and is there bent itself into a sharp loop, which is closely applied against the side of the large terminal loop and fused to it, the two ducts here anastomosing. This arrangement was observed to be exactly similar in both the dissections made, though the sides were reversed. The enlarged part of the duct or penis (*p*) appears, as described, to be a continuation of one of the ducts only, the other duct being cut short and entering from the side. But from the way in which one duct passes under the nerve-cord and not the other, and from the curious sharply turned loop formed by this latter duct on entering its fellow, it would appear that the original condition had been almost exactly similar to that existing in the female organs. A common duct (now the lateral penis) lay in the middle line on the inner surface of the ventral wall of the body, the tube being much longer in the male than in the female. The two vasa deferentia passed inwards, one on each side under the nerve-cords, to enter the common median tube together at its upper extremity, as do the oviducts in the female. The common duct, however, in the male has become longer and longer, and thus formed an upward loop in the body, which looping has necessarily brought about the sharp turn in the one vas deferens; the large loop at the same time slipped out sideways under the nerve-cord, and lay in future always on the side of the body; and the other vas deferens following it was dragged under the second nerve-cord, and thus now passes in the present singular manner under both nerve-cords. The diagrammatic woodcut may serve to make this plain:—*VC*, *VC* are the ventral nerve-cords, *P* the penis or enlarged common terminal duct. The right vas deferens (*Rd*) passes right across the body at its very end under the ventral cords, and, turning up, is apparently continued in a sweeping curve into the penis. The left vas deferens (*Ld*) enters the loop of the penis with its peculiar sharp



bend. If the right duct (*Rd*) were gradually pulled upon and every thing supposed free, the original condition of affairs would be restored, the penis would lie in the middle line, and the sharp turn be taken out of the left duct; both ducts would pass under a single nerve-cord, one on each side. The present arrangement appears to be a commencement of the unilaterality found in Scolopendridæ. In two of the males opened, merely to observe the sex, one testis with its accessory gland and vesicula seminalis was much larger than the other, showing a further step towards unilaterality. The testes are packed in the body one a little way above the other beside and behind the intestine.

The convoluted tube above the testis contains no spermatozoa or vesicles of evolution, but only large fine granulated or transparent gland-cells, being, in fact, very like in structure to the slime-glands of the animal. These tubes are probably diluent accessory glands of the testis, and might be called "prostates."

The testes examined contained abundance of vesicles of evolution, with spermatozoa in various stages of development. The structures observed are shown in Plate LXXIV. figs. 2, 3, & 4. The earliest forms of the cells which produce the spermatozoa appear to be those seen in fig. 2, connected together in a row and filled simply with fine granules. The cells are flattened by mutual appressure, and probably multiply by division. The cells enlarge and acquire a transparent nucleus. In the next stage observed the cells are again enlarged, and contain from three to as many as six vesicles of evolution, sometimes only one or two; these are shown in fig. 3, *a*, *b*, *c*, as seen with a HARTNACK'S No. 10, ocular 4. The large cells have perfectly transparent contents without granules. The contained vesicles of evolution are ovoid in form, and have a fine granulated ovoid nucleus: they are transparent over the greater part of their area, but arranged irregularly; about one pole is a single layer of fine distinct granules. In the next stages the vesicles of evolution are full of granules, and the filaments of the spermatozoa are formed curved around the ovoid nuclei. This stage is represented very much enlarged in fig. 3, *d*. The short tails of the spermatozoa project from the parent cell. *a*, *b*, *a*, fig. 4, show further stages, these being the most abundant forms in the testis. The granular matter and wall of the vesicles of evolution have almost disappeared. The tails of the spermatozoa are longer. The number of spermatozoa developed in each cell is very variable; cells with only one, as in Plate LXXIV. fig. 4, *a*, are very common. In the next (fig. 4, *c*, *d*, and *e*) stage the spermatozoon is free from the cell; the nucleus of the vesicle merely remains attached to it laterally within its loop. Some fine granules are often to be seen in the space between the nucleus and the loop of the spermatozoon, the remnants of the granulation of the vesicle of evolution. None of the spermatozoa are found freed from their nucleus within the testis, and most of them retain the nucleus in the vesiculæ seminales. In the mass of spermatozoa contained within the penis or terminal ejaculatory duct, about half of the spermatozoa had thrown off the nucleus, and the other half retained it still. Free nuclei abounded in the mass. The ripe spermatozoa met with in the ovary of the female are always simply filamentary, like

those of insects and Chilopods, and never retain a nucleus; they are about ·5 millim. in length; but their length is very difficult to estimate, since the excessively fine undulating terminations of their filaments are only to be followed with high powers and by constant alteration of focus.

The spermatozoa met with in the ovaries were, wherever space allowed it, in constant motion. The long tails showed an undulating movement, accompanied by a spiral corkscrew-twisting motion. As the filaments turned round partially or wholly on their axis, it could be seen that they are, in their broader median part, not cylindrical and circular in section, but flat bands, much broader along one transverse axis than the other. The spermatozoa are found always twisted up into loops, which show all kinds of varieties of form. The commonest form is a simple loop near one end of the ribbon-like part of the long filament, the spermatozoon thus having one long and one much shorter tail (Plate LXXIV. fig. 5, 1). In other cases a double loop is formed at one end of the ribbon (2). Sometimes the ribbon forms a loop at one end, and the two ends twist round one another spirally in opposite directions (3). In other cases, not so common, the loop is near the centre of the ribbon (4).

Besides the glands already described there are in the male a pair of simple rod-like bodies lying at the hinder end of the body outside the nerve-cords, and running down towards the generative aperture. These bodies (Plate LXXII. fig. 3, *a. g*) are in the fresh state conspicuous from their pearly whiteness. When examined under the microscope they appeared to consist of simple tubes lined with a single layer of gland-cells. They are probably accessory generative glands; but their mode of termination was not made out, nor was much attention paid to their structure.

Development.—When the uterus filled with embryos was found in the first specimen of *Peripatus* dissected, it was hoped that all the stages in development, or many of them, might be observed amongst the many embryos in the one specimen. It was, however, found that the whole of the thirty or more embryos were in exactly the same stage, which is that shown in Plate LXXV. fig. 3.

In only two cases were embryos found in the same mother, some of which were more advanced than others. In these two the period of pregnancy appeared to be just the same, and the embryos were just in that condition in which the first pair of members were turning in to become the jaws (figs. 6, 7, 8). A regular gradation showing this change was found in each case. No serial arrangement of the embryos of various stages in the uterine tubes was observed. The youngest stages in development were not met with in the twenty specimens dissected, though, as before mentioned, one still virgin female was found, and therefore the earliest stages should still have been to be met with. The stage apparently youngest, and in which the ovum was smallest, is that shown in Plate LXXIV. fig. 1, *a*; but the embryos in this case, or ova, appeared to have perished or become injured. There were very few in the uterine tubes, and they all appeared somewhat collapsed. In several cases embryos which had perished and become formed into opaque masses of fatty and fibrous tissue were met with situate in their somewhat shrunken uterine enlargements between other perfectly healthy embryos.

The embryos lie within the ovoid cavities of the uterine tube, quite free and unattached. They are enveloped in a single very thin pellucid envelope, which encloses an ovoid cavity within which the embryo lies coiled up.

The earliest stage observed is that shown in Plate LXXV. fig. 1. The embryo is worm-like; the hinder part of the body is coiled up spirally with the ventral surface inwards, the anterior extremity being free and nearly straight. The body shows a distinct segmentation about its middle, but there are no transverse lines of segmentation between the large rounded cephalic lobes and the next segment, nor in the hinder part of the body. The three layers of the embryo are already differentiated, and the body-cavities closed in. A line following the contours of the body shows the line of separation of the cutaneous layer. The future intestinal tract is represented by a dark elongated mass of pigmented matter stretching from a pointed anterior extremity, which protrudes within the cephalic lobes, all along the body. The intestine shows bulgings corresponding to the segmental elevations of the cutaneous layer. Three rounded elevations of the cutaneous layer, marking the commencement of the three first pairs of members, are distinctly to be seen projecting from the under surface of the body on its ventral aspect. There is as yet no trace of antennæ, and no trace of the mouth was to be found. The as yet rudimentary members marked 1, 2 probably correspond to those marked 1, 2 in fig. 2.

The next stage is to be seen in fig. 2. The antennæ have budded out and show themselves as broad, blunt-ended, unsegmented processes arising from the upper surfaces of the cephalic lobes. In rising up from the lobes they carry processes of the cephalic cavity with them. In the roof and sides of the cephalic cavity a fold rises up and indicates a separation of the cavity into two. The mouth appears as a simple opening situate between the cephalic lobes inferiorly, and bounded by a line of thickened tissue. The lateral members become more and more developed serially from before backwards; they do not bud off from the end of the body, but transverse lines of segmentation first appear between the terminal portion of the body and the already formed members. A lateral rounded swelling gradually forms above these lines, becomes more and more prominent, and thus forms at last a projecting fold of tissue, the basis of the future member. The transverse lines disappear as the members become formed. The second pair of members is larger and longer than the rest. The first pair is larger than the remainder, and becomes a conspicuous mark by which to determine the identity of those on each side of them in further stages. The intestinal tract shows an enlargement at its anterior extremity; it sends out processes of its lateral wall to meet the tissue at the interspaces of the feet. Between the first pairs of members it shows a slight bulging. There is as yet no anus formed. The lateral undulations or swellings of the body-wall, which are the first stages of the limbs, are hollow, a somewhat oval cavity being formed between them and a corresponding but opposite inward curve of the digestive tract (Plate LXXIII. fig. 9). The undulation of the limb itself is built up of two layers of cells, separated by a very distinct line. The inner layer is composed of small rounded

or spindle-shaped cells. The lateral projections of the digestive tract adhere to the inner layer at the interspaces between the undulations. The wall of the digestive tract is covered with a layer of cells continuous with and similar to those of the inner layer of the members, and probably a reflection of the same layer. The members as they project further and further outwards become hollow loops, which later become solid by proliferation of the cells of the inner layer. The large cavity in the cephalic lobes corresponds exactly to the cavities formed in the members. The digestive tract is in this stage filled with darkly pigmented rounded particles and fine granules. The tract assumes gradually a brick-red colour, which increases in intensity, and eventually in later stages renders the embryo very conspicuous.

In the next stage observed (Plate LXXV. fig. 3) the embryos have reached a length of from 5 to 5·5 millims. Since the most fully developed specimen was no longer than this, it is probable that this is about the length of the young when born. All the pairs of members are formed, but the anterior are more perfectly shaped than the posterior. The antennæ show a commencement of jointing; the cephalic lobes show a line of depression, indicating more plainly a division into two segments. The second pair of members is still the largest, but the first is somewhat smaller than the third. A bifid saddle of yelk is seen on the dorsal surface of the embryo opposite the seventh, eighth, and ninth pairs of members. More investigation is required about this yelk-mass; it was not observed in the early stage (fig. 1) at all. It was observed in one specimen of the very late stage (fig. 10), but often not in intermediate stages. It may have been overlooked, but possibly is not constant. The embryo in the stage shown in fig. 3 is coiled up within its envelope in the manner shown in fig. 4, and retains this posture till the latest stage observed. The body is bent round in an oval form, and the posterior extremity bent up and applied against the front of the cephalic lobes between the pair of tentacles. The full number of body-members having been attained at this early age, the further advance of the front segments only will be followed, viz. those which form the parts about the mouth. Buds (fig. 5, *m*) grow out backwards from the hinder part of the cephalic lobes. In one case, shown in fig. 5, the buds appeared double; at all events, only the larger one develops. The first pair of limbs have shrunk a little in proportion to the rest, though the second still remains very large, as is seen in fig 6, which represents a lateral view of nearly the same stage as fig. 5. When the members are said to shrink, it is intended of course that the other members outgrow them. The first pair of members gradually turn inwards (fig. 7, ı), and are overlapped by the elongations of the processes of the cephalic lobes (*m*). A rounded process grows downwards over the mouth from the front wall of the head in the median line; a transverse elevation arises beneath the mouth opposite to this and between the upper edges of the second pair of members. The eyes appear (*e*) at this stage as oval aggregations of cells derived from the internal masses of cells of the head, and covered externally by transparent rounded elevations of the epidermic layer, the future corneæ.

The first pair of members pass further inwards (fig. 8), and their claws appear. The

head-processes (*m*) grow right down over their bases; the superior and inferior median projections (labrum and labium) become more prominent, the labrum covering the mouth gradually over. At the bases of the second pair of members (oral papillæ) are seen through the skin the openings of the ducts of the slime-glands, which will eventually terminate at their tips.

A defined secondary cavity or mouth is already shut in around the true mouth by the rising up of surrounding structures. The head-processes become lengthened or raised into tumid folds on their inner margin, the future tumid lips of the adult; they grow on and fuse with the structures about the bases of the oral papillæ (Plate LXXV. figs. 9 & 10), and the first pair with their developing claws become shut entirely within the secondary mouth. The head-processes extend their inner margin up on to the front of the head, where there rise up tumid processes which swell up above the labrum, and eventually shut it out from view, join the swellings on the inner margin of the head-processes, and thus complete the characteristic tumid lip-ring of *Peripatus*. The latest stage observed is seen in fig. 10. A sulcus persists in the lips of the adult *Peripatus*, which probably represents the line of separation between the lips of the mouth-processes and the structures with which they fuse. The antennæ become gradually more slender in proportion to their length, and acquire a greater and greater number of joints. The number of joints in the antennæ is a sure indicator of the stage of development of the parts of the mouth. The most developed embryo observed possessed seventeen joints in the antennæ; in the adult certainly as many as thirty are present. It would be of interest to observe whether the embryos at the time of birth possess the full number of joints in their antennæ.

The members which become ambulacral in function and the oral papillæ, become in development at first two- or three-jointed and eventually five-jointed. They thus have the number of joints so remarkably constant in insects. In the adult five joints can also be made out in the ambulacral members, the first joint or coxa being very long proportionately to the rest, and marked with closely approximated rings. The other joints are short, except the last claw-bearing joint or tarsus, which is fine and neat like that of many insects. In *Scolopendra* the first pair of feet, modified into prehensile jaws, has five joints, including the claw, and perhaps the more numerous joints of the other feet are derived by secondary jointing from a like number.

The claws are developed at the bottom of depressions or invaginations formed at ends of members, and are derived from the epidermic layer (Plate LXXV. fig. 12). They appear late, having been first observed in the latest stage which was found. The mouth-claws, or jaws, are represented in fig. 11, from an embryo in the stage shown in fig. 10. I have no papers at hand in which the development of the parts of the mouth in Myriopods or Insects are described at length, but am only able to refer to Newport's conclusions on the subject in his well-known papers already quoted*.

* Newport, Linn. Trans. vol. xix. 1843, p. 265.

Newport came to the conclusion that the head in *Scolopendra* was built up of eight segments, with appendages and outgrowths as follows:—

First segment.		Antennæ.
Second	„	Eyes and labrum.
Third	„	Dental plates="mandibles" of modern writers.
Fourth	„	First pair of maxillæ.
Fifth	„	Second pair of maxillæ.
Sixth	„	Mandibles="foot-jaws" of modern writers.
Seventh	„	0
Eighth	„	0

The actual mode of development of the parts is not described. The eye-segment may be disregarded, the eyes of Arthropods being now for so many reasons not regarded as homologous with appendages. The cephalic lobes, as already described, show early traces of a division into two segments, anterior and posterior. The downward growths of the hinder parts of the head would appear, then, to be homologous with the mandibles of *Scolopendra*, and the process between them with the labrum. The oral papillæ or shrunken second pair of members must most probably represent the foot-jaws of *Scolopendra*, their glands representing their poison-glands, and probably also the silk-glands of caterpillars. If this be the case, then the jaws developed from the first pair of members must be maxillæ; but we have two pairs of jaws developed from one pair of appendages, the jaws being evidently merely the modified claws of the ambulacral members. In *Scolopendra*, according to Newport, the biting-parts of the maxillæ are sternal and episternal in origin, the claws terminating the palpi. Are these two pairs of jaws homologous with the two pairs of maxillæ of *Scolopendra*, or do they only represent one pair? If only one pair, then do the oral papillæ represent the second? On these questions appears to depend the homology of the mesially indented line showing a ridge stretching between the bases of the oral papillæ. This, if the oral papillæ are homologous with the foot-jaws of *Scolopendra*, represents the second under lip of that animal, which is formed of the fused sternal and episternal plates of the segment to which the foot-jaws belong, and is slightly notched anteriorly as here.

Conclusion.

In the present state of our knowledge concerning the structure of *Peripatus*, the most remarkable fact is the wide divarication of the ventral nerve-cords. The fact was considered remarkable and dwelt upon in all mention of *Peripatus* before the existence of tracheæ in the animal was known, and when it was thought to be hermaphrodite, but it is doubly remarkable now. The fact shuts off at once all idea of *Peripatus* being a degenerate Myriopod, the evidence against which possibility is overwhelming. The bilateral symmetry and duplicity of the organs of the body, the absence of striation in the muscles, the absence of periodical moults of the larval skin

in development, and the absence of any trace of primitive three-legged condition, taken in conjunction with the divarication of the nerve-cords, are conclusive. The parts of the mouth are not to be regarded as degraded to any great degree; and homologies for some of them, at least, may perhaps be found amongst the higher Annelids. The structure of the skin is not at all unlike that in some worms, especially in its chitinous epidermic layer, which strips off in large pieces occasionally as a thin transparent pellicle. The many points of resemblance of *Peripatus* to Annelids need not be dwelt upon; they led to its former placing in classification; but it is difficult to understand how the very unannelid-like structure of the foot-claws did not lead others, like De Quatrefages, to draw a line between the two. In being unisexual, *Peripatus* is like the higher Annelids, as well as the whole of the higher Tracheata. To Insects *Peripatus* shows affinities in the form of the spermatozoa, and the elaboration, structure, and bilateral symmetry of the generative organs, though there is a very slight tendency towards the unilaterality of Myriopods in the male organs.

To Insects, again, it is allied by the five-jointing of the feet and oral papillæ and the form and number of its claws. It should be remembered that spiders' feet are two-clawed, and those of some Tardigrades, and that some of these latter forms have two-clawed feet in the early condition, even when they possess more claws in the adult state. In Newport's well-known figure of the young *Iulus* with three pairs of limbs, the tips of these latter are drawn with *two* hair-like claws; these are not mentioned in the text.

To the ordinary lepidopterous larva the resemblances of *Peripatus* are striking—the gait, the glands (so like in their function and position to silk-glands), the form of the intestine, the less perfect concentration of the nervous organs in larval insects. To Myriopods *Peripatus* is allied by the great variety in number of segments in the various species and in its habits, and especially in these to *Iulus*. The parts of the mouth perhaps show a form out of which were derived by modification those of *Scolopendra*; but the resemblance may be superficial. Our knowledge is not yet sufficient to determine such points. The usual difficulties occur in the matter. Segments may have dropped out or fused, and their original condition may not be represented at all in the process of development. In structure *Peripatus* is more like *Scolopendra* than *Iulus*, viz. in the many joints to the antennæ (Chilognaths having never more than fourteen), in the form of the spermatozoa, and in being viviparous, as are some Scolopendras; in the position of the orifices of the generative glands, and in the less perfect concentration mesially in *Scolopendra* of the nerve-cords. *Peripatus* thus shows affinities in some points to all the main branches of the family tree of Tracheata; but a gulf is fixed between it and them by the divarication of the nerve-cords, and borne out somewhat by such facts as the non-striation of the muscles, great power of extension of the body, arrangement of the digestive tract in the early stage, persistence of metamorphosis, and in the parts of their mouth, the full history of the manner of origin of these being reserved.

Speculations as to the mode of origin of the tracheæ themselves in the Tracheata are many. Prof. HAECKEL* follows GEGENBAUR, whose opinion is expressed in his 'Grundzüge der vergleichenden Anatomie,' 2nd edit. p. 441. GEGENBAUR concludes that tracheæ were developed from originally closed tracheal systems through the intervention of the tracheal gills of primæval aquatic insects now represented as larvæ. If *Peripatus* be as ancient in origin as is here supposed, the condition of the tracheal system in it throws a very different light on the matter. *Peripatus* is the only Tracheate with tracheal stems opening diffusely all over the body. The Protracheata probably had their tracheæ thus diffused, and the separate small systems afterwards became concentrated along especial lines and formed into wide main branching trunks. In some forms the spiracular openings concentrated towards a more ventral line (*Iulus*); in others they took a more lateral position (Lepidopterous larvæ, &c.). A concentration along two lines of the body, ventral and lateral, has already commenced in *Peripatus*. The original Protracheate being supposed to have had numerous small tracheæ diffused all over its body, the question as to their mode of origin again presents itself. The peculiar form of the tracheal bundles in *Peripatus*, a number of fine tubes opening into the extremity of a single short common duct leading to the exterior of the body, seems to give a clue. The tracheæ are, very probably, modified cutaneous glands, the homologues of those so abundant all over the body in such forms as *Bipalium* or *Hirudo*. The pumping extension and contraction of the body may well have drawn a very little air, to begin with, into the mouths of the ducts; and this having been found beneficial by the ancestor of the Protracheate, further development is easy to imagine. The exact mode of development of the tracheæ in present forms must be carefully studied; there was no trace of them in the most perfect stage of developing *Peripatus* which I obtained.

Prof. GEGENBAUR'S† opinion on the position of *Peripatus* is, that the placing of it amongst the worms is not certain, but that, at any rate, it connects ringed worms with Arthropods and flat worms. The general result of the present inquiry is to bear out Prof. GEGENBAUR'S opinion, but points to the connexion of the ringed and flat worms, by means of this intermediate step, with three classes only of the Arthropods—the Myriopods, Spiders, and Insects, *i. e.* the Tracheata. From the primitive condition of the tracheæ in *Iulus*, and the many relations between *Peripatus* and *Scolopendra*, it would seem that the Myriopods may be most nearly allied to *Peripatus*, and form a distinct branch arising from it, and not passing through Insects. The early three-legged stage may turn out as of not so much significance as supposed. If these speculations be correct, the Crustacea have a different origin from the Tracheata. *Peripatus* itself may well be placed amongst Prof. HAECKEL'S Protracheata; GRUBE'S term Onychophora becomes no more significant than DE BLAINVILLE'S Malacopoda. Some notions of the actual history of the origin of *Peripatus* itself may be gathered from its development.

In conclusion I would beg indulgence for the many defects in this paper, due to the hurry with which it is written (all available time, almost up to the last moment of our

* Biologische Studien, p. 491.

† Grundzüge der vergleichenden Anatomie, p. 199.

sailing for the Antarctic regions, having been consumed in actual examination of the structure of *Peripatus*), and due, further, to the impossibility of referring in any scientific library to original papers. At all events it is hoped that *Peripatus* has been shown to be of very great zoological interest, as lying near one of the main stems of the great zoological family tree, and that further examination of the most minute character into the structure of this animal will well be repaid.

Description of the Plates.

(The whole of the Plates represent structures occurring in *Peripatus*.)

PLATE LXXII.

Dissection of *Peripatus*, drawn by Mr. J. J. Wild.

Fig. 1. Female laid open from the back, to show the general arrangement of the viscera.

a. Antennæ; *c.g*. Cephalic ganglia; *v.c*. Ventral nerve-cords; *op*. Optic nerve; *n*. Nerve to the papilla, in which the ejaculatory duct of the slime-gland terminates.

x. Lateral canals of Grube = "fat-bodies."

s.g. Slime-glands = testes of Grube.

e.s. Muscular ejaculatory sacs of these glands.

r.m. Retractor muscles of the head.

p. Pharynx with a pair of small linear protractor muscles on its upper surface. *œ*. Œsophagus.

s. Stomach.

i. Intestine.

an. Anus.

o. Ovary.

od. Oviduct.

u. Tubular uterus. The successive constrictions separate off sacs, each of which contains an embryo in process of development.

k. Overhand knot formed by the oviduct over the bight of the uterus. The two tubes of the uterus join into a common canal behind the intestine close to the vulva.

On the inner surfaces of the reflected walls of the body are seen the numerous minute bunches of tracheæ. Especially large groups of tracheæ (*tr. tr*.) pass to the underside of the intestine and of the pharynx from the middle line of the ventral surface.

Fig. 2. Inner surface of a portion of the ventral and lateral walls of the body spread out flat and magnified four times, to show the manner in which the tracheæ form tree-like appearances.

The tracheæ here are represented, for convenience of drawing, as dark on a light ground; in reality, when observed by reflected light, they have the usual pearly lustre so characteristic of tracheæ when full of air. Exteriorly to the ventral nerve-cords (*v.c. v.c.*) are seen the rows of openings communicating with the feet, and probably vascular, described by GRUBE.

Fig. 3. Hinder part of the body of male *Peripatus*, laid open from the back. The stomach and short intestine have been turned down, in order to show the arrangement of parts behind them.

v.s. Vesiculæ seminales; *t.* Testes; *pr.* Prostate; *v.d.* Vasa deferentia; *p.* Enlargement of azygos terminal portion of male duct, ejaculatory duct, or penis? *a.g.* Accessory gland; *v.c.* Ventral nerve-cords; *s.* Stomach; *i.* Intestine.

PLATE LXXIII.

Fig. 1. Tracheal twig from the wall of the uterus, to show the manner of branching. Branching, however, is very exceptional.

Fig. 2. *a.* Arrangement of the tracheæ on wall of one of the fine tubes of the slime-gland. There is here no branching.

b. Small portion of one of these tracheæ as seen with HARTNACK'S No. 8, ocular 3. Transverse lines indicate the existence of an internal spiral band, which is broad in proportion to the diameter of the tube. Actual diameter of the tube ·003 millim.

Fig. 3. Cells from the mucous surface of the stomach.

a. Liver-cell; *b.* Slime-cell; *c.* Aggregation of oil-globules. Actual diameter of liver-cell ·04 millim.; others in proportion.

Fig. 4. Portion of one of the viscid threads which are shot out by *Peripatus* and form meshworks.

Fig. 5. Vertical and transverse section through the ventral wall of the body, to show the origin of the tracheæ.

a. Epidermis, by reason of the thickness of the section, showing to the left two of the warty protuberances with which the surface of the animal's body is covered.

b. Region occupied by oblique or decussating muscular fibres.

c. Layer of transverse muscles.

From a pit at the base of one of the cutaneous warts arises a simple short tube, from the inner end of which pass off an enormous number of fine tracheæ. These latter do not branch, but arise all directly from the end of the tube, and at their origin are so closely packed as, with their contained air, to prevent the transmission of any light. Some of the tracheæ of the bundle are distributed at once to the skin and muscles.

Fig. 6. Polygonal flat endothelium lining the ejaculatory sac of the slime-gland. Length of one of the longest cells ·06 millim.

Fig. 7. Structure of one of the ultimate ramifications of the slime-gland. Actual diameter of the tube ·2 millim.

Fig. 8. Muscular fibres.

a. Fibre in state of contraction, which has taken a somewhat spiral form. The shading gives somewhat the appearance of transverse striation.

b. Portion of a fibre as seen under HARTNACK'S No. 10, ocular 3.

Fig. 9. Portion of an embryo *Peripatus* in the condition seen in Plate LXXV. fig. 5, viewed in optical section to show the cell arrangement. The figure is taken from the hinder portion of the body, and represents one of the undulations of the lateral margin constituting the commencement of a member.

a. External or epidermic layer.

b. Middle layer.

c. Dark mass of pigmented granules and oil-globules, out of which is to be formed the intestine.

The middle layer is reflected over the intestine, which projects outwards to meet the body-wall in the interspaces between the members.

PLATE LXXIV.

Fig. 1. *a*. Ovary, oviducts, and commencement of uterus. The oviduct, as represented, is somewhat too short. Actual diameter of the ovarian ova ·175 millim. The ovary is divided by a median longitudinal partition into two sacs. The dark longitudinal masses within these sacs are composed of felted spermatozoa. Other groups of spermatozoa are seen at the margins of the ovary in relation with the ova. The oviducts at a short distance from the ovary are well supplied with tracheæ, which are, however, wanting on the ovary itself. On the left in the commencing uterus is seen a developing ovum; its peculiar shape is believed to be due to partial collapse.

b, *c*, *d*. Successive stages in the development of the ovum from a female specimen the ovary of which was not yet ripe and contained no spermatozoa, and in which there were no embryos in the uterus.

Figs. 2, 3. Cells from the testis of a male specimen, as seen in the fresh state, showing the mode of development of the spermatozoa.

Fig. 4. Further development of the spermatozoa.

a, *b*. Vesicles of evolution, with the heads of the spermatozoa coiled within them and the long tails projecting freely. Each spermatozoon has its nucleus attached.

c, *d*, *e*. Spermatozoa freed from the vesicle of evolution, but with the nucleus still attached. A slightly granular substance is visible, in most cases intervening between the nucleus and edges of the loop in which it lies.

Fig. 5. Spermatozoa without a nucleus, from the ovary of a female specimen, and as they occur in the lower part of the vas deferens of the male, mingled, however, there with others retaining the nucleus and free nuclei.

The actual length of the ripe spermatozoa appears to be about three times that of an ovarian ovum, or about ·5 millim.

PLATE LXXV.

Various stages in the development of *Peripatus*, from embryos taken from the uterus of various female specimens.

Fig. 1. Earliest stage of the embryo observed, as seen within its envelope; the cephalic lobes are formed; the anterior part of the body shows a definite segmentation. On the three segments immediately succeeding the cephalic, the outer layer is rising up to form the members.

Fig. 2. Embryo, somewhat more advanced, removed from the envelope and viewed from the ventral surface by transmitted light in optical section.

a. Antennæ. *c*. Cephalic lobes. 1. First pair of body-members, forming later the maxillæ. 2. Second pair, perforated later by the ejaculatory duct of the slime-gland. 3. Third pair, afterwards first pair of ambulacral appendages.

o. Mouth. *i*. Intestine. (The same numbers and letters in the succeeding figures have the same signification.)

The second pair of members are the longest, and continue so for a considerable period; ultimately in the adult each is represented only by a small papilla.

The cephalic lobes are hollow; a fold rising up in the roof of their cavity divides them into two. Their structure is identical with that of the posterior members (see Plate LXXIII. fig. 9). The limbs first appear as undulations of the body-wall; they form from before backwards, and anteriorly to the segments of the body. Before the appearance of the members segmentation of the body is marked by a transverse line. The segmentation disappears after the members are formed. The intestine is connected by lateral projections with the folded interspaces between the limbs (see Plate LXXIII. fig. 9).

Fig. 3. Embryo in a further state of advancement removed from the envelope, and as seen by reflected light: *a*, from behind; *b*, from in front. The number of body-members is complete, viz. nineteen, of which the first becomes the oral and the nineteenth the anal papilla. A saddle-shaped mass of yelk remains on the dorsal surface opposite the seventh, eighth, and ninth pairs of members. The mouth and anus are sketched out; the cephalic lobes (*c*) show a lateral depression dividing them into two regions, anterior and posterior. The anterior members are most perfectly formed; the antennæ (*a*) already show jointing. Actual length 5·5 millims.

Fig. 4. Embryo in the same stage as the preceding, to show the manner in which it lies coiled up in its envelope within the uterus. The tip of the posterior extremity rests in the space between the antennæ and against the front of the cephalic lobes.

Fig. 5. A very slightly more advanced stage, much magnified.

m. The buds which form the processes corresponding to the mandibles of Myriopods.

Fig. 6. Slightly further advanced stage, seen from the side. *m* as before.

Fig. 7. More advanced. The first pair of body-members (1) is turning inwards towards the mouth, and is overlapped by the down-growing processes (*m*). The labrum (*l*) begins to cover over the mouth.

Fig. 8. Further stage. The processes (*m*) have come further down over the members (1). These members have turned further inwards towards the mouth; and on each there is seen one claw of the pair of claws which they bear in common with the ambulatory members, and which are modified into the horny jaws = maxillæ.

e. Commencement of the eye; *d*. End of duct of slime-gland.

Fig. 9. Further stage. Both masticating claws are visible on each side. The process (*m*) is much folded on its inner margin, and its folds extend up on to the cephalic lobes. It has fused with the superficial structures about the base of the first and second pair of members (1 and 2), and thus the formation of the tumid plications surrounding the external oral aperture in the adult is nearly completed.

Fig. 10. The plications extending from the base of the processes (*m*) are continuous across the cephalic lobes, and the labrum is thus shut in with the maxillæ within the external oral aperture. The antennæ exhibit a very advanced state of jointing.

Fig. 11. Maxillary claws from the specimen figured in fig. 10.

a. Anterior; *b*. Posterior claw.

Fig. 12. Ambulatory member from the same, showing the way in which the claws appear in induplications of the extremities of the members.

XXIII. *On the Fossil Mammals of Australia.*—Part IX. *Family* Macropodidæ; *Genera* Macropus, Pachysiagon, Leptosiagon, Procoptodon, *and* Palorchestes. *By Professor* Owen, *F.R.S. &c.*

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§ 1. *Macropus Titan.*—In illustration of certain fossils from the freshwater beds of the Queensland province, showing the hindmost mandibular molars with characters of the general type of those of the *Macropus Titan*, but with modifications indicative of specific difference, it appears requisite to premise figures and descriptions of the corresponding molars at a similar stage of wear and age, which are plainly referable to *Macropus Titan*.

In the subjects of figs. 13 & 15, Plate xxii. of a former Part*, the last mandibular molar had but recently come into use; the edges of the two main lobes showed the backwardly oblique abrasion of the enamel, but not carried so far as to expose the dentine; the links were entire from their origin at the fore and outer angles of the two lobes; the hind surface of the second lobe showed the whole of the angular fossa on its inner half, and the small faint vertical notch external thereto.

In the subjects of Plate LXXVI. figs. 2, 4, the direction of wear against the upper molar from above downward and backward has extended the abrasion from the second lobe to its hind basal surface, where the bottom of the hind fossa (*g*) alone remains; with the edges of the wedge-shaped lobes the origins of the link (*r*) are gone, and only the lower antero-posterior ends remain, dividing the valley into a smaller outer and a larger inner depression. The prebasal ridge (*f*) is correspondingly reduced in fore-and-aft extent.

That these changes or modifications of working-surface of m_2 and m_3 are due to age and wear will be clear to any one comparing these teeth in Plate LXXVI. figs. 2 & 4, with those before quoted of large full-grown individuals of *Macropus Titan*, described and figured in Part VIII., and with m_2 of fig. 1, Plate LXXVI.

The size and form of the portions of mandible preserved closely agree with those of the corresponding parts of the more entire mandibles of *Macropus Titan*, figured in Plate xxii. figs. 13 & 15 and Plate xxvi. figs. 11 & 13 of Part VIII. And I here subjoin a view from above (Plate LXXVI. fig. 1), not before given, of the subject of the last-cited figure, to illustrate the correspondence in the breadth and direction of the "ectalveolar groove" (*u*) and in the thickness of the mandible, including the fore part of the base of the coronoid, with corresponding parts of the bone from the more aged animals that have afforded the subjects of figures 2 & 4, Plate LXXVI.

* Part VIII., Philosophical Transactions, 1874, p. 256.

The only difference of note is in the more advanced position of the last molar; and this is due to the well-known general forward movement of the serviceable teeth when those anterior to them have been worn down and shed—a movement which reaches its maximum in the Elephants.

§ 2. *Macropus Ferragus*, Ow.—A portion of a right mandibular ramus with the last two molars in place and worn (Plate LXXXI. fig. 4, and Plate LXXXII. figs. 3 & 4) indicates, both by the size of the teeth and that of the jaw itself, a Kangaroo too much exceeding in size the largest individuals of *Macropus Titan* to belong to that species. A comparison of the above-cited figures with figures 13, 15 in Plate XXII. of the previous Part* will demonstrate this fact.

The jaw-bone of *Macropus Ferragus* is relatively thicker in proportion to the teeth, and the last molar (Plate LXXXII. fig. 3, m_3) rises more in advance of the origin of the coronoid process; but this may relate to the greater age of the individual as shown by the exposure of the dentine in both lobes (ib. fig. 4) of that tooth, though they are not worn to quite the same degree as in fig. 2, Plate LXXVI. The base of the notch (Plate LXXXII. fig. 4, *g*) remains on the hind part of m_3, and internal thereto are indications of two vertical ridges which I have not found in any fossil of *Macropus Titan*.

§ 3. *Pachysiagon*† *Otuel*‡, Ow.—In the portion of right mandible with the three hindmost molars (Plate LXXVI. figs. 7–10), in which the dentine is exposed on the summits of the lobes of m_3 and in the usual proportionally greater degree in m_2 and m_1, the three molars, corresponding in fore-and-aft extent with those in *Macropus Titan* (Plate XXVI. fig. 9, Philosophical Transactions, 1874), and only exceeding by a line the same teeth in the subject of figure 11, Plate XXVI., ib., are implanted in a jaw of considerably greater thickness. This jaw likewise shows a wider ectalveolar channel (Plate LXXVI. fig. 9, *u*); the anterior border of the base of the coronoid also rises more vertically and more in advance of the last molar, a difference not to be accounted for by age, as is manifest in a comparison of the present fossil with the mandible in younger individuals of *Macropus Titan*.

With these mandibular indications of, at least, specific distinction, the molars themselves show the following differences. The prebasal ridge does not extend so far forward as in *Macropus* proper; the hind surface of the molars has two vertical narrow grooves, to which a third shorter and feebler one is added in m_3 (ib. fig. 10, *g*). These grooves do not extend below the base of the transverse lobe, and below them the common base of the crown of the tooth rises smoothly and convex for a line and a half clear of the socket; the innermost notch is very little deeper or wider than the next—contrasting strongly with the large and deep notch on the inner half of the hind surface of the molars in *Macropus Titan*, which descends nearer to the origin of the hind fang of the tooth.

Another character is shown in the mid link (Plate LXXVI. fig. 9, *r*), which sends off

* Philosophical Transactions, 1874, p. 245.

† From παχὺς, thick; σιαγὼν, jaw-bone.

‡ A giant in the 'Romance of Roland and Ferragus.'

from the inner side near its termination in the hind lobe a short and low ridge of enamel passing downward, forward, and inward to be lost in the inner division of the transverse valley. There is likewise a small vertical ridge of enamel from the fore part of the hind lobe, internal to the link and its accessory process, which ridge descends straight to the bottom of the valley. These productions of enamel cause corresponding complexities of the grinding-surface, which are correlatively associated with the more massive bone wielding the grinding-instruments.

On the inner surface of this portion of mandible (Plate LXXVI. fig. 8) the super-angular channel (ib. *b*) is shorter, deeper, defining a narrower and shorter beginning of the inflected angle than in *Macropus*, the channel commencing anteriorly at a vertical parallel with the interval between the last and penultimate molars. By the above-defined characters I infer from the present fossil a distinct subgenus of *Macropodidæ*, for which the massiveness of the mandible in proportion to the teeth it supports has suggested the name above given.

§ 4. *Leptosiagon** *gracilis*, Ow.—On similar grounds the present genus and species of extinct Kangaroo are based upon a portion of a right mandibular ramus with m_1 and m_2 in place, part of the socket of m_3, and the whole of that of d_4, from which the tooth appears to have been naturally shed or worn away (Plate LXXVI. figs. 11–15). The transverse fracture of the mandible anterior to d_4 (ib. fig. 15) shows the bone to be unusually thin in proportion to its depth; the partial thickening on the inner side is almost ridge-like; the outer thickening, beginning opposite the inner one, is continued with a convex curve to the lower border of the ramus.

The distinctive dental character of the present subgenus is in the sculpturing of the hind surface of the molars (ib. fig. 14); two slender well-defined pyramids of enamel, in high relief, rise from the base of that surface at its inner half, and terminate in points, the inner one within a line of the unworn summit of the hind lobe, the outer one within two lines of the same. A deep reversed, narrow, angular or pyramidal notch divides the inner pyramid from the inner side of the hind lobe, a deeper notch of corresponding form divides the inner from the outer pyramid, and a fainter, narrow, short, linear groove separates the outer pyramid from the rest of the outer part of the hind surface, which is continued by a bold convexity into the outer side of the molar.

The outer end of both transverse lobes incline more forward than in *Macropus Titan*; but, as in that species, the outer bases of the two lobes meet to define an acute-angled pointed lower termination of the interval or valley there (ib. fig. 11, m_1, m_2). In *Sthenurus* and *Protemnodon* the lower termination of the same interspace is rounded, not pointed, the outer bases of the lobes not being in the same degree extended antero-posteriorly.

There is no accessory ridge from the inner side of the mid link; but in the smaller outer part of the hollow between the front lobe and the prebasal ridge, defined by the fore link, there is in m_2 a vertical enamel ridge: it is not present in m_1 (Plate LXXVI. fig. 13).

* From λεπτὸς, slender; σιαγὼν, jaw-bone.

I have seen none such in the corresponding part of any molar of *Macropus Titan*; and it may be a variety in the individual specimen exemplifying or indicating the present subgenus.

But the character of the back part of the molar is decisive, so far as my present observation of existing and extinct Kangaroos has extended, of the taxonomic distinction which I have assigned to *Leptosiagon gracilis*.

§ 5. Genus *Procoptodon**, Ow.—The indications of this genus, at first fragmentary, have been raised, by evidences successively received since the year 1845, to demonstration, under, at least, three specific modifications.

The first of these is exemplified, mainly, by maxillary fossils. The most instructive are portions of the right and left maxillary bones, each with three molars (d_4, m_1, m_2) and part of a fourth, d_3 (Plate LXXVII. figs. 2–6), of the same skull.

In antero-posterior extent and in breadth the two molars (m_1, m_2) do not exceed their homologues in the upper jaw of *Sthenurus Brehus*†; but the configuration of the grinding-surface is different and much more complex, of itself indicating a distinct genus if not subfamily of *Macropodidæ*.

The prebasal ridge (*f*) is narrower than in *Sthenurus*; it descends (the tooth being viewed prone) from the fore part of the outer angle of the front lobe (*a*) and then passes inward with a slighter descent to the fore part of the base of the inner half of the front lobe, subsiding before attaining the inner side of that lobe. The fore link (*s*) is feebly represented by a vertical ridge on the fore part of the front lobe, nearer its outer than inner end, the rest of that fore surface being marked by more minute vertical ridges and grooves. The hind surface of the front lobe is bounded by sharp subvertical ridges, respectively descending, with a slight degree of convergence, from the hind part of the outer and inner angles of the front lobe. In the transverse concavity of the hind surface of that lobe, so bounded, two stronger sharp vertical ridges of enamel descend toward the intervening valley, the outer ridge being continued backward across the valley as the 'mid link' (*r*), with a slightly sinuous course, convexly outward. The hind surface of the hind lobe (ib. fig. 5, m_3) almost repeats the characters of that of the front lobe; the inner (*g*) of the two submedian vertical ridges in the concavity of such surface expands at its base into the convexity of that part of the molar; the outer submedian ridge is minute. From the outer lateral or boundary-ridge a sharp vertical plate of enamel (*h*) is directed inward, and there are some minor sculpturings of this part of the hind surface. Its basal part is somewhat bulging and smooth, as are the outer and inner swollen ends of the two main transverse ridges or lobes (ib. fig. 6, *a*, *b*).

The second molar (m_2) is rather larger than the first; both are feebly abraded along the fore part of the transverse summits of the wedge-shaped lobes, but not so far as to expose the dentine; both teeth are behind the back surface of the masseteric process (m^p). These signs of immaturity were associated with and confirmed by the presence of

* From πρὸ, before; κόπτω, to pound; ὀδούς, tooth.

† Philosophical Transactions, 1874, Plate XXVII. fig. 6, m_1, m_2.

the undeveloped and unextricated premolar (ib. figs. 2, 3, 4, p^3); it was so far advanced as to have pushed its crown to near the level of the base of d^3, at the interval between the diverging pairs of the fore and hind fangs of that deciduous tooth (ib. fig. 6, d^3). The major part of d^4 had been broken away on the right side, but the crown remained in the left maxillary. The mid link and the accessory ridge are here present, and the hind surface of the second lobe shows the complex accentuation, which is further carried out in the succeeding teeth. On the summits of both lobes of d^3 and d^4 the dentine had been exposed by masticatory abrasion.

The thin smooth partition-wall between the sockets of m^3 (the hindmost tooth in place) and of m^2 was manifest at the back part of each maxillary (fig. 5, m^3), confirming, with the more decisive evidences at the fore part, the homologies of the more or less complete teeth, and demonstrating the immaturity of the individual from which the fossils had been derived; the usual process for exposing the premolar had the usual result.

The crown of the premolar (ib. figs. 2, 3, 4, p^3, and fig. 7, p^3), in its antero-posterior extent, resembles that of *Macropus* proper, not being quite equal to that of the crown of the next tooth (d^4); but it is thicker transversely than in *Macropus* or *Sthenurus*, with a transversely ridged complexity of the broad working-surface between the outer (*a*) and inner (*b*) longitudinal ridges or walls (fig. 7), which shows it to have acted as a pounder rather than a divider of the vegetable food, a character which suggested the name proposed for the genus (Note *, p. 786).

The vertical extent of the enamelled crown of the premolar does not exceed the breadth of the hind part of the tooth. The outer surface shows three conical elevations, in low relief, of the enamel, one behind another (fig. 2, p^3). The apex of the foremost constitutes the anterior prominence of the outer longitudinal ridge; that of the second slightly projects from the middle of that ridge (*a*, p^3, fig. 7); that of the hindmost one subsides before it gains the ridge, which is continued, sharply, upon the outer and back part of the crown of the premolar. The foremost of the three conical low reliefs of the enamel forms the outer part of the fore swollen end of the crown (fig. 4, p^3), which is divided by a depression from the lower and narrower, basally swollen beginning of the inner longitudinal ridge (*b*). This slightly diverges as it recedes from the outer ridge, obeying the hinder enlargement of the crown, and it is united near that end by a transverse ridge with the outer longitudinal one. The transverse ridge is a miniature or rudiment of a hind lobe, and its hind surface is excavated and ridged in a feeble or rudimental way like that part of the normal hind lobe of the bilophodont molars. The horizontal triturating surface of the premolar between the outer and inner longitudinal ridges is sculptured by transverse sharp enamel-ridges and deep depressions.

There is a character in the upper jaw of *Procoptodon* which is not present in the large existing Kangaroos of the subgenera *Macropus* and *Osphranter*. It is present, under some modifications, in *Boriogale* and in the smaller Kangaroos or Wallabies of the subgenera *Halmaturus* and *Petrogale*. I find, indeed, in *Petrogale xanthopus*, Gd., and *Halmaturus brachyurus*, Wth. (Plate LXXVII. fig. 1, *b b*), the

nearest approach to the structure in question. It is a large unossified tract of the palate, shown in both maxillaries of the fossil (Plate LXXVII. fig. 6, *b b*) by the passage of the palatine (fig. 3, *n*) into the nasal (*n*) plate at a distance of from $2\frac{1}{2}$ lines to 3 lines from the inner wall of the alveoli of d_4, m_1, and m_2. This upward bend of the palatal plate (ib. fig. 3, *n*) is at nearly a right angle with the narrow horizontal palatal strip, and in both maxillæ the continuation of the nasal plate of the maxillary into the orbital one by the large spheno-palatine foramen (fig. 3, *s*) is shown.

Arrested ossification of the palate is, however, a marsupial rather than a macropodal character, and is exemplified, in my "Osteology of the Marsupialia"*, in the Thylacine, the Sarcophile, the Dasyure, the Bandicoots, the Peragale, and the Potoroo, as well as in a small species of Kangaroo (*Halmaturus Bennettii*). In this species the palatal vacuities are four in number, in two lateral pairs†. In *Halmaturus brachyurus* (Plate LXXVII. fig. 1, *b b*) the intervening strip of bone between the fore and hind vacuity is wanting, and each pair is blended into one large lateral unossified tract, which either falls into its fellow, or is separated from it by a mere filament of bone. Such was the structure in the great procoptodont Kangaroos, the vacuity, moreover, extending more forward than in *Halmaturus* or *Petrogale*.

I need scarcely say that the dentition of the small existing species of these genera is of the ordinary halmatural type; the premolar is trenchant, the inner ridge being low or 'basal.'

The front pier of the zygoma (Plate LXXVII. figs. 2–5, zy', zy''), extending from the fore part of the orbit obliquely downward and backward, terminates below, in the present immature specimen of *Procoptodon*, above the interval between d_4 and m_1; it might recede in relative position to the molar series when this was fully in place; but so much of the pier as is preserved is more compressed, and stands out relatively further than in any existing Kangaroo. I shall subsequently be able to show that the more advanced position of the descending masseteric process is constant at all ages in *Procoptodon*, and may be added to the palatal and dental characters of the genus.

§ 6. *Procoptodon Pusio*.—These characters, as above defined, added to size, exemplify the present species.

The antero-posterior extent of the three molars d_4, m_1, m_2, Plate LXXVII. in the subject of figures 2, 3, 6, is 1 inch 10 lines; the four teeth, the premolar being in place, would reach along 2 inches 4 lines: taking the fore-and-aft diameter of the last molar, m_2, to equal that of the penultimate one, m_1, here in place, the permanent upper molar series of *Procoptodon Pusio* (ib. fig. 7) would be 3 inches 2 lines in longitudinal extent. All things being equal this Kangaroo was one third larger than *Osphranter robustus*, Gd.‡

§ 7. *Procoptodon Rapha*, Ow.—The size of the premolar (Plate LXXVII. fig. 8, p_3) in the fragment of lower jaw of the immature *Procoptodon* next to be described indi-

* Zoological Transactions, 4to, vol. ii. part 1, 1838, p. 388, plates 70 & 71. † *Op. cit.* pl. 71. fig. 5.

‡ Compare figure 7, Plate LXXVII. with figure 3, Plate XXI., Philosophical Transactions, 1874.

cates it to have come from another and larger species than *Procoptodon Pusio*, yet of inferior size to *Procopt. Goliah* (Plate LXXX. fig. 7, p_3); the shape of the incisor indicates likewise a difference of species.

The size of the remains of the two anterior fully developed molars (Plate LXXVII. fig. 8, d_3, d_3), in proportion to the jaw supporting them, suggested immaturity, and the excavation of the bone beneath them exposed the crown and beginning of the fangs of a large and thick premolar, of the crushing type of the present genus.

The outer part of the crown (ib. fig. 8, p_3) is divided into two subequal lobes by a marginal notch, from the apex of which a linear groove is continued down the outer side of the crown. The height of this is 6 lines, the fore-and-aft extent is 7 lines; the breadth behind is 5 lines. The crown bulges out above the roots, the hinder one of which bifurcates 6 lines below the crown. The enamel is smooth, white, and the exposed parts show more or less convexity.

The base of the broken crown of the first deciduous molar (ib. fig. 11, d_3) is triangular, with the angles rounded off and the base turned backward; the length of the triangle is 4 lines, the basal breadth 3 lines. The base of the next bilophodont tooth (d_3) is subquadrate, $5\frac{1}{2}$ lines in length, 5 lines in breadth.

The diastemal tract (figs. 8, 9, & 11, *l*, *s'*) has a characteristic shortness; it is continued with, at first, a feeble concavity almost straight on to the outlet of the incisive alveolus, to which it slightly bends down. The outlet of the dental canal (fig. 8, *v*), about 2 lines below the diastemal border, would be about 7 lines from the alveolar outlet were this entire. The lower or symphysial border (figs. 8, 9, *s*, *s''*) rises to the alveolar outlet at a less open angle with the lower border of the horizontal ramus than in *Sthenurus**. The syndesmotic articular surface is flat, rough, and is continued, well defined, to the outlet; it indicates the generic firmer union of the rami than in the type Kangaroos, and a greater development of the uniting or binding structure than in *Sthenurus*†.

The thickness of the ramus at the formative cell of the premolar, in *Procoptodon Rapha*, is $10\frac{1}{2}$ lines (Plate LXXVII. fig. 10). The base of the incisor (ib. figs. 8 & 9, *i*) indicates a tooth of less relative thickness than in *Procoptodon Goliah*. The fractured end gives an ovoid section $5\frac{1}{2}$ lines in vertical, 3 lines in transverse diameter; the outer side is moderately convex, the inner side less so: both upper and lower borders are thick and obtuse; a thin coat of enamel extends from the lower upon the outer sides of the tooth. The shape of the working crown cannot be inferred from the basal part here preserved; but this shows faint longitudinal broad impressions on the outer side, and also that the incisor must have risen obliquely upward at a low angle (140°) with the horizontal line of the molar series.

Thus the present, like the following evidence of the genus *Procoptodon*, indicates, with the shortness and depth of the symphysial part of the jaw, the thickness of the crown of the lower premolar especially behind, and the thickness and apparent depth

* Philosophical Transactions, 1874, Plate XXII. fig. 6. † Ib. ib.

of the ramus, a nearer approach to the characters of *Nototherium* than is shown in any of the foregoing extinct genera of the Macropodal group of Marsupialia.

What is wanting in the above-described fragment of jaw with the small proportion of dentition, by which is indicated a second species of *Procoptodon*, has been supplied by a plaster cast and photographs of an almost entire right ramus, with the molar series. The complex characters of the hindmost and least worn teeth (Plate LXXVIII. fig. 3, m_3, m_3) unmistakably repeat the generic type of those of the upper molars of *Procoptodon Pusio* (Plate LXXVII. fig. 7); but the difference of dimension is, to my experience, specific; and since it is in degree that shown by the premolar germ in the fragment of immature jaw (Plate LXXVII. fig. 8), I regard the specimen in the Sydney Museum, from which the cast and photographs were taken, as part of a mandible of a mature individual of *Procoptodon Rapha*. The longitudinal extent of the upper molars (d–m_3) in *Proc. Pusio* is 2 inches 6 lines; that of their homotypes in the mandible of *Proc. Rapha* is 3 inches. The breadth of these lower molars similarly exceeds that of the upper ones in the subjects of Plate LXXVII. figs. 6, 7, which is fatal to the hypothesis of those of Plate LXXVIII. belonging to the same species.

The mandibular ramus of *Procoptodon Rapha* is short, straight, deep, and thick, in the latter dimension (fig. 3) increasing from the symphysis to the ascending branch more gradually but to a greater extent than in preceding genera of Kangaroos. The lower border is convex across, thickest behind the symphysis, and suddenly becomes thinner at its hinder third (ib. fig. 2, d, d'), where a longitudinal channel (ib. b') along its inner side gives it the appearance of being slightly inflected that way; but the channel (b') is interrupted by a more sudden or direct inflection of the proper "angle of the jaw" (ib. a), which here, as in the other macropodal fossils, has been broken away.

The fore border of the symphysial end of the ramus bends up at about an angle of 135° with the axis of the mandible. On the outer side (ib. fig. 1) the alveolar tract is impressed by an irregular longitudinal moderately deep channel, below which the bone swells out and the vertical convexity to the lower border augments from below the first molar (p_3) to the origin of the coronoid process (q). A little way behind the anterior border of this process it is feebly impressed, or the surface subsides; but the low rough ridge (fig. 1, h), indicative of the lower boundary of the ectocrotaphyte fossa, is at a higher level.

The ramus gains slightly in depth or vertical extent from the groove (b', fig. 2) to the symphysis (s, s'). The articular syndesmotic surface is broad, strongly roughened and sculptured, and extends from the lower border of the ramus (s) to the outlet of the incisor (i); the hind border of the symphysial surface is feebly impressed, as in *Phascolomys* (Philosophical Transactions, 1872, Plate XXI.).

The union of the two rami must have been firm, not admitting of any reciprocal movement; indeed the fore part of the joint had become obliterated by anchylosis, and part of the left ramus is here adherent in the fossil. The inner surface of the ramus is almost flat to where it bends to the under border; the narrow continuation forward of

that surface between the symphysial joint and the diastemal tract (fig. 2, *l*, *s'*) is traversed by a ridge, between which and the joint is a narrow groove. The hind part of the ectalveolar groove (fig. 3, *u*) is deep and moderately wide; the postalveolar angle (ib. *t*) is better marked than in the type Kangaroos.

The crown of the premolar has been worn or broken off in the original of Plate LXXVIII.; the length and breadth of its base are the same as in the undeveloped lower premolar (Plate LXXVII. fig. 6, p_3). The strong fore and hind roots are longitudinally striate.

The crown of d_4 is worn down to the base; each lobe shows a broad tract of dentine united by a linear tract along the remains of the mid link. The third tooth (m_1) shows a broad prebasal ridge; the fore link has been worn down to its base, and the dentine there is continuous with that exposed on the anterior lobe. The tract continued along the mid link to the hind lobe is much narrower, being finely linear. A ridge from the inner side of the link is indicated, and a second ridge from the fore part of the hind lobe internal to the confluence therewith of the link. Indications of the procoptodont sculpturing of the back of the hind lobe are unmistakable, but the pattern is better shown in the succeeding molars; the outer and inner smooth convex enamelled ends or sides of the hind lobe are continued a short way upon the back of that lobe and terminate there in well-defined borders, between which the surface sinks. This broad, shallow fossa is divided in two by a pyramidal process of enamel in low relief, the basis below being coextensive with the breadth of the fossa, the apex reaching to the working ridge, in its present degree of wear, of the hind lobe of the last molar.

In the penultimate tooth (m_2) the greater degree of wear gives a strong undulatory course to the enamel-ridge bounding behind the working-surface of the complex molar.

From what may be discerned, in the cast, of the implanted base and outlet of the socket of the incisor, that tooth was relatively small, as in the subject of Plate LXXVII. fig. 8, and was directed obliquely upward and forward at nearly the same angle as that of the posterior border of the symphysis; but the cast does not supply safe data for further particulars.

We have thus in the foregoing evidences of the singular phytophagous Marsupial genus, with the dentition, as to the formula and fundamental pattern of the molars, of the bilophodont *Macropodidæ*, the extremest deviation from the characters of the existing subgenera which the fossil remains of the family have yet exhibited.

I infer from the proportions and shape of the mandible that the rest of the skeleton may have presented a more robust character, with thicker and shorter extremities and with less inequality between the fore and hind pairs of limbs than in the living Kangaroos. Of these, as before remarked, *Procoptodon Pusio* must have exceeded in bulk the largest known. This excess is greater in *Procoptodon Rapha*. But the following fossils show that their generic modifications have been exemplified on a more gigantic scale.

§ 8. *Procoptodon Goliah*, Ow.—The present extinct species of *Macropodidæ* was

first indicated by a fragment of a maxillary bone with three molars, transmitted to me by Sir THOMAS L. MITCHELL, C.B., in 1844.

As the two smallest of these measured rather more than one inch and a half in the antero-posterior direction, and the least transverse diameter was 7½ lines, I provisionally assigned to the species the name "*Macropus Goliah*," and communicated my brief notes on the fossil to my friend and present colleague, GEORGE R. WATERHOUSE, Esq., at that time engaged in his excellent work, 'The Natural History of the Mammalia'*. The type specimen is figured in Plate LXXIX. fig. 1.

My hopes of further elucidating the singular extinct gigantic Kangaroo so indicated have been, through the kindness of friends and correspondents in Australia, abundantly fulfilled. The grounds of the formation of a distinct genus of *Macropodidæ*, of which the "Goliah" is the type, are given in the section (§ 5) descriptive of the smaller species, *Procoptodon Pusio*. As in that species, the two lobes of the molar (Plate LXXIX. fig. 1, $m_{1, 2, 3}$) have a more regular elliptic basal section than in the foregoing genera; their outer and inner ends are more convex or bulging; they are not separated by so deep a valley, and this lodges a greater proportion of hard enamel in the form of irregular subundulate ridges, affecting mostly the longitudinal direction, the chief of which may, however, be homologized with the mid link of more normal Macropods. The fore surface of the front lobe and the hind surface of the back lobe are similarly sculptured by irregular ridges, of which two are more conspicuous than the rest at the back of the molar, within a depressed area defined by the abrupt sharp margins, inflected upon that surface of the outer and inner convex ends of the hind lobe. The unworn summits of both lobes are less regularly or extensively transverse than in ordinary Kangaroos, the inner angle curved backward. There is a low and short prebasal ridge, but no such definite production from the back part of the hind lobe.

The fore pier of the zygoma, at least so much of the hind part thereof as was preserved in the present fragment, stands out opposite the antepenultimate tooth (m_1), and its hind curve subsides before reaching the last molar. The narrow plate extending from the palatal wall of the alveoli terminates in a smooth and seemingly natural border. But this and other characters of the species are more fully and plainly illustrated by the specimens next to be described, of which I commence with the portion of a left maxillary, forming the subject of figures 2, 3, 4 of Plate LXXIX.

This portion of jaw retains the five permanent molars (p_2 to m_3), but with some mutilation as well as wear of their crowns. The fore pier of the zygoma (ib. fig. 2, zy^p) springs out above the interval between the second (d_4) and third (m_1) molars; and as the last (m_3) is in place and shows wear, it may be concluded that this anterior portion of that pier is characteristic of *Procoptodon*.

On the palatal surface of the bone (ib. figs. 3 and 4) the evidence of the same large vacuity as is indicated in the smaller species (*Procoptodon Pusio*, Plate LXXVII. fig. 6) is unmistakable. The anterior ridge begins to curve to join its fellow opposite the

* Part 2, *Macropodidæ*, 8vo, 1845, "*Macropus Goliah*, Ow.," p. 59.

second molar (d_4), the junction taking place at the mid interval between the right and left premolars (p_3); the posterior ridge similarly bridges the palate between the alveoli, right and left, of m_3 & m_3, fig. 4, Plate LXXIX.

On the working-surface of the crown of p_3 the transverse ridges of enamel along the tract occupying the fore and inner part of that surface appear. This tooth is broadest behind; it is supported by three stout fangs; the foremost diverges from the hind pair as it ascends into the substance of the jaw, where two thirds of an inch of it is exposed; it is thick, and the cemental coat is longitudinally and irregularly striate.

At the back part of m_1 two of the deep vertical notches (ib. fig. 9, *h*) and the intervening strong ridge of enamel (ib. *g*) are conspicuous. The fore link (*s*) and the accessory link to its inner side, also the mid link and the accessory ridges on the surfaces of the lobes internal to it, are shown. The prebasal ridge (*f*), with the fore link (*s*) of the last molar (m_3, fig. 4), has been broken off and slightly displaced from the rest of that tooth, in which the indents and ridges on the hind surface are well displayed. At every part of the worn and fractured surface of this molar series the stalactitic-like complexity of the thick and hard enamel is exemplified. The figures being of the natural size preclude the need of giving dimensions of the several teeth.

In the smaller portion of maxilla and molar series (Plate LXXIX. figs. 5, 6, 7) the coronal modifications of m_1 and m_2 are instructively displayed. Viewing the working-surface from above, the postbasal ridge seems to be represented by a short bar of enamel (figs. 7 & 9, *g*) supporting the bases of the narrow but strongly developed pyramidal ridges (ib. *h*), sending their apices to the working-surface of the hind lobe; but the basal bar does not project like a true basal ridge from the hinder convex surface of the crown (fig. 9).

The correspondence of the complex pattern of the crown of the lower with the upper molars, combined with the usual modifications of greater length of the prebasal ridge, is patent in the subject of figs. 1 & 2, Plate LXXX. In this fossil the last molar (m_3) had but recently risen into place, and the antecedent one (m_2) showed slight traces of abrasion. In d_4 the front lobe (*a*) is narrower than the hind one (*b*); these proportions are reversed, but in a minor degree, in the last molar (m_3). The fore (*s*) and mid (*r*) links are well defined. Deep clefts on the back of the hind lobe mark out the vertical ridges of enamel, of which one is shown at *g*, fig. 2. The base of the coronoid process (fig. 1, *q*) advances forward a little beyond m_3.

The characters of the lower molars of *Procoptodon Goliah* are further exemplified in a smaller fragment of a large, apparently male, animal, with the last two molars showing a greater degree of wear. This specimen (Plate LXXX. figs. 3 & 4, Plate LXXIX. fig. 8), now in the Museum of Natural History of Sydney, New South Wales, has been made known to me by a good plaster cast. The penultimate molar (m_2) is slightly worn, the last (m_3) in a less degree; but both had been in use and indicate a full-grown though not old individual.

The prebasal ridge (ib. ib. fig. 8, *f*), commencing externally at the outer and fore

part of the base of the front lobe, rapidly rises as it curves inward, and extends as a thick ridge three lines in advance of the front lobe, rising nearly to its level, and assuming more of the character of an accessory lobe than a mere basal ridge or developed portion of the "cingulum." Its lobular appearance is strengthened by the well-developed link (*s*) which connects its outer angle with the fore part of the front lobe (*a*) near its swollen end. The accessory ridge (*n*), internal to the link, is well marked, but subsides before reaching the prebasal ridge. This sinks more gradually and with less loss of thickness to the inner and fore part of the base of the front lobe than at its outer end or beginning. The anterior surface of this lobe is broken by minor and less regular ridges and grooves of enamel. The mid link (*r*) has the same relative position to the transverse eminences it connects as has the fore link; the inner accessory ridge (*o*) subsides in the valley; smaller subsidiary ridges and grooves accentuate and multiply the hard enamel-constituent of the grinding-surface of the lower molars, as in those of the upper jaw.

The last molar (m_4), with a very slight decrease of breadth of the hind lobe, repeats the characters of the preceding tooth; in both, the hind surface (Plate LXXIX. fig. 10) is impressed with a pair of narrow conical excavations, with apices not reaching to the basal beginning of the crown; these impressions define or leave three ridges (*h*), not extending back beyond the smooth basal part of the tooth, but which, when worn down to near that part, give a strongly undulating course to the enamelled summit of the hind lobe of the molar.

The most complete example of a lower jaw of *Procoptodon* is that which was presented to the Museum of the Royal College of Surgeons of England in the year 1853, by Dr. Falder, with a note of its discovery in the freshwater or drift beds of Darling Downs, Queensland, "at a depth of 40 feet from the surface of the soil."

Each ramus of the jaw is preserved as far as the beginning of the horizontal branch; a small portion of the alveoli of the two incisors, with the corresponding part of the diastemal ridges, has been broken away; the lower border of the right ramus has likewise suffered. But the most instructive element in the jaw, viz. the molar series, is almost entire in both rami.

This series includes six teeth. The last molar is represented by a partially developed crown in its formative cell; the next in advance is not fully risen into place, and the summits of its two lobes are unworn. The antepenultimate molar, with these ridges just touched, shows on each a polished linear tract of enamel; the three anterior teeth, with progressively increasing degrees of wear and decrease of size as they precede in position, are therefore deciduous molars. Of this phase of dentition, as seen from the outer side view of the right ramus, a figure is given in Plate LXXX. fig. 6.

The foremost tooth (d_4) shows a working-surface of a triangular form, the base turned backward, the apex forward and obtuse. The outer border has a submedian notch corresponding to a groove terminating there, which divides the outer surface into a fore and hind lobe, both of which are convex transversely, and in a minor degree vertically.

The inner side of the tooth is flatter, and is impressed at its middle third by three shallow parallel grooves descending rather obliquely backward. The coronal ends of these grooves give a crenate character to that part of the enamel. A field of dentine is exposed on the broader hind lobe.

The crown of the second molar (Plate LXXX. figs. 6 & 7, d_2) has the two transverse lobes and a prebasal production; a continuous tract of dentine is exposed on these parts and along the mid link connecting the two lobes. Certain longitudinal impressions on the outer surface of the crown give a wavy character to parts of the peripheral coat of enamel on the working-surface of the tooth.

The third molar (d_4) is mutilated by fracture in both rami, but least so in the right. The prebasal ridge is narrow transversely in proportion to its length; its enamel coat is indented; the fore limb is feebly indicated. The hind surface of the hind lobe shows three vertical enamel ridges, two of which attain the grinding-surface. The mid link is strengthened by a vertical ridge on its inner side.

In the fourth molar (m_1) the prebasal ridge has a distinct link passing from its outer angle toward the middle of the fore part of the front lobe, ending nearer the outer side. Short ridges project from this surface on the inner side, marked off by the fore link. The mid link passes from near the outer side of the fore lobe to near the middle of the hind lobe. The link is complicated by a vertical ridge or fold of enamel on its inner surface. Similar ridges mark the inner half of the fore surface of the hind lobe. On the hind surface of this lobe, internal to the ridge by which the outer convex end there terminates, is a narrow pyramidal tract of enamel in low relief, the apex of which reaches the working-ridge of that lobe; internal to this pyramid is a second, less strongly defined by two sharp linear grooves. The inner side or end of each transverse lobe is narrower.

The fresh and unworn crown of the penultimate molar (m_2) shows well the accessory ridges which complicate the enamel cap of its crown. In this tooth, as in m_1, the prebasal ridge beginning near the base of the inner end of the fore lobe curves forward and outward, rising rapidly to a narrow summit, which seems to represent a third short transverse lobe; this, as soon as it has received or sent off the fore link, subsides suddenly upon the base of the fore lobe before attaining its outer end.

The position of the germ of the last molar is such that the transverse lobes turn their working-edges obliquely inward, rather forward, and, with but a very slight inclination, upward, indicating the semirotating movement by which, in the course of the growth of both tooth and jaw, the crown is brought into the latter direction, and in a working line with the antecedent grinders. The superadded complexities noted in these teeth are multiplied in the last molar; at least so much of the hind surface of the hind lobe as is formed shows, in the left ramus, not fewer than six accessory ridges in the flattened and rather depressed tract included between the hinder terminal ridges or angles of the outer and inner convex ends of the hind lobe. Of these six vertical ridges two are larger than the rest, and of the narrow pyramidal shape before mentioned (they answer to those marked *h* in fig. 10, Plate LXXIX.).

In the unworn molar of this young jaw (Plate LXXX. fig. 6, m_1) the mid link does not come to the grinding-surface as in the worn molars from the mandible of the adult *Procoptodon* (ib. fig. 4); they show a sharp concave border slightly notched midway. Notwithstanding some minor differences to be expected in the modifications of the enamel which complicate the crown of the molars in the present genus, the dimensions of d_4, m_1, m_2, and m_3 led me to view this mandible as having been derived from a young of *Procoptodon Goliah*.

The germ of the premolar was accordingly sought for, and the right ramus (Plate LXXX. fig. 7) was fortunately so far preserved as to include its formative cell*. The crown of the tooth therein lodged (ib. fig. 3) has a fore-and-aft extent one third greater than the larger of the two teeth (d_3) which it displaces; the outer side of the crown is divided, as in *Procoptodon Rapha*, by a vertical groove, but more equally; a short second vertical groove or notch indents the hinder division, near the main groove. The convexity of the exposed part of the crown testifies to the procoptodont character of this premolar.

The incisors, both broken away near their issue from the alveoli, show a full elliptic section, 6 lines in the vertical, 5 lines in the transverse direction; they come almost into contact medially; the transverse extent of the pair is 10 lines; their direction is obliquely forward and upward, as in *Nototherium*, not procumbent as in the typical *Macropodidæ*.

The symphysis is continued, broadly, to the incisive outlets; it has insured, apparently, an attachment to each other of the rami of this instructive mandible, too intimate to be disturbed by posthumous movements, although anchylosis had not been completed, if it had commenced. From the anterior molar to the incisor measures but 10 lines, though it might be 1 inch or a little more if the incisive alveoli were entire. The outlet of the dental canal (ib. fig. 6, *v*) is small; it is 3 lines below the diastemal border, and 5 lines from the fore part of the socket of d_3.

Each ramus is thick vertically, convex externally, in a degree increasing to the origin of the coronoid plate. The thick lower border, where it curves from the horizontal to the ascending parts of the ramus, is slightly inflected; a rather wide shallow groove extends above it. The inner surface of the ramus in advance of the inflection is feebly convex vertically. The base of the coronoid process describes a curve convex inward as it extends from before backward.

The lower border of the left ramus is entire from below (m_1) backward to the hind fracture; it is obtuse. The thickness or transverse diameter of the ramus, below the middle of m_3, is rather more than four fifths the vertical diameter at that part. Thus the characters of the mandible of the smaller species of *Procoptodon* are closely repeated in that of the immature specimen of the larger species.

If the present mandible of *Procoptodon Goliah* be compared with that of *Nototherium Mitchelli* in a like state of preservation, which is the subject of Plate IV. Philosophical

* Permission to remove the requisite portion of bone was accorded by the Museum Committee of the Royal College of Surgeons.

Transactions, 1872, or if a reference be made to that Plate and Plate VIII. (op. cit. *Nototherium inerme*), the interesting and instructive modification of the present extinct Marsupial, as transitional between *Macropus* and *Nototherium*, will be obvious. But the true macropodal character comes strongly out in the different result of the quest in the substance of the jaw carried out in the subject of fig. 7, Plate LXXX. of the present memoir, and in fig. 5, Plate VI. (*loc. cit.*) of a *Nototherium* at a similar stage of immaturity.

I have been favoured with a photograph of a portion of a left mandibular ramus of a young *Procoptodon Goliah* at a corresponding stage of dentition (Plate LXXX. fig. 5). This was obtained from the Breccia-cave of Wellington Valley; the original is in the Museum of Natural History of Sydney, New South Wales.

§ 9. Genus *Palorchestes**, Ow.—The finest fossil evidence which has yet reached me of an extinct Kangaroo is the portion of skull figured, of the natural size, in Plates LXXXI., LXXXII., & LXXXIII.

It was discovered in the year 1851 by Dr. LUDWIG BECKER, "in a bed of yellowish sand and clay mixed with very small shells," in the Province of Victoria, Australia. The matrix had been cleared off before the fossil reached me.

I am indebted for the opportunity of now describing and figuring this specimen to the kindness and liberality of my esteemed friend and fellow labourer in palæontology, the late estimable Dr. KAUP, of Darmstadt, to whom the fossil was in the first instance transmitted.

It is much petrified, heavy, massive, like most of the fossils from the freshwater formations of Australia; but it partakes of the colour of its matrix, which is lighter than that of the fossils from the drift-beds of Queensland.

It includes the facial or fore part of the skull with the bony palate and both right and left series of molars. The sockets of the three incisors are preserved in the right premaxillary (Plate LXXXII. fig. 1, i 1, 2, 3); the left has suffered fracture in that part.

Comparing the least-worn molar in this skull (Plate LXXXII. fig. 1, m 3, and fig. 2, restored) with the corresponding tooth in the upper jaw of *Procoptodon* (Plate LXXIX. fig. 1, m 3), the coronal enamel is disposed on a simpler pattern, more in accordance with that of the normal Kangaroos†. Moreover, as in the larger living and extinct species of *Macropus*, the bony palate is entire (Plate LXXXII. fig. 1).

The usual permanent or adult series of five molars have remained in place and use in our present fossil, whereas at the degree of wear shown by the last grinder, the first, if not also the second, would have been shed in *Macropus* proper‡.

In the maintenance of the adult series of five grinders the fossil resembles *Osphranter*, *Halmaturus*, and *Sthenurus*, but the premolar (ib. ib. p 3) differs in shape and proportion. Its antero-posterior extent is but three fourths that of the next tooth (ib. d 4); in *Osphranter* and *Petrogale* (Plate LXXVII. fig. 1) the premolar equals, in *Halmaturus*

* παλαιὸς, ancient; ὀρχηστὴς, leaper.

† Philosophical Transactions, 1874, Plate XXI. fig. 10, m 3 (*Macropus Titan*).

‡ Ib. ib. fig. 15 (*Macropus Titan*).

and *Sthenurus* it exceeds, d_4 in that dimension. But in the fossil the transverse diameter of the premolar in proportion to the length of the crown is greater than in any known existing Kangaroo, and in this respect is equalled only by the extinct Procoptodons. As in these, also, the front pier of the zygoma (ib. zy^a) has a more advanced position in relation to the molar series than in *Macropus*, *Osphranter*, or *Halmaturus*. But the chief distinctive character of the present fossil is the great proportional length of the premaxillaries (Plates LXXXI.–LXXXIII. m, m') and the concomitant backward position of the incisive or premaxillary palatine foramina (ib. *a*), which are closer together, and were separated by narrower processes of these bones, if so separated at all; for the foramina in the fossil seem to have been confluent, and were certainly short and parallel; whereas in *Macropus*, *Osphranter*, *Halmaturus*, and *Petrogale* (Plate LXXVII. fig. 1, *a a*) they are relatively longer, are wider apart, and usually with their narrow or pointed hind ends convergent.

The dental formula, $i\,\frac{3-3}{}$, $c\,\frac{0}{}$, $m\,\frac{4-4}{}$, and pattern of the grinding-surface of the molars (Plate LXXXII. fig. 2) being macropodal, the differential characters from known recent and fossil Kangaroos are of subgeneric value, and call for the usual taxonomic indication which heads the present section.

§ 10. *Palorchestes Azael*, Ow.—The fore end of the premaxillaries is not quite entire, but sufficient of the sockets of the three incisors of the right side remain to show that not more than half an inch of the bone can there be wanting; the teeth have dropped out prior to fossilization. From the fore end of the skull to the premolar (p^3) is 5 inches; the length of the molar series is 4 inches 6 lines. These two admeasurements are relatively almost the same in *Osphranter robustus*, the length of its series of five molars being 2 inches 2 lines, and that of the skull in advance of the premolar being 2 inches 3 lines; the part of the skull behind the last molar in the fossil is 2 inches 9 lines in length. On this basis the entire skull of *Palorchestes Azael* may be reckoned at about 16 inches in length.

The extent of the diastema in the upper jaw supports an inference of a like extent in the lower, and would show a nearer resemblance, in the latter respect, of *Palorchestes* to *Macropus*, *Osphranter*, and other genera of normal Kangaroos, than to the extinct *Sthenuri*, *Protemnodons*, and *Procoptodons*, which, in their shorter and deeper diastemal and symphysial part of the mandible, exemplify the transition to *Nototherium*.

The extent of the premaxillaries (m) from the alveolar outlets to the anterior bony nostril (n) is relatively greater than in any skull of a living Kangaroo I have yet seen (compare fig. 2 with fig. 3, Plate LXXXI.). The tract in question slopes backward as it rises and curves, more vertically near the nostril, where it swells into a pair of low tuberosities (ib. fig. 1, n). So far the union of the two premaxillaries is close, seemingly anchylosed, though the mid line of their symphysis is traceable. In this character *Palorchestes* offers a resemblance to the Koala and Wombat*. A little behind the tuberosities each premaxillary sends upward its plate to form the outer walls of the

* Philosophical Transactions, 1872, Plate II. figs. 3 & 4.

nostril. The fore or free edge of this plate is thick, rounded, and slopes backward as it rises in a greater degree than the subnasal coalesced plate.

Of the right side-plate but an inch and a half is preserved; the left one (ib. ib. $_{mx}$) is continued to its junction with the maxillary ($_{mx'}$); and this extends backward to near the orbit, 7½ inches from the incisive alveoli. The upper border of this plate is, at first, thinner than the anterior or ascending border, but gains in thickness as it approaches the orbit (Plate LXXXIII. fig. 1, $_{mx'}$); it may have been broken and the fractured margin worn smooth and obtuse; as it is, it shows no trace of the junction with the nasal bone. As much of the plate as remains is nearly vertical, neither swelling outward, as in *Macropus major* and *Osphranter*, nor bending inward at its upper part to join the nasal, as in all recent Kangaroos. The external nostril must have been relatively narrower, and seems to have been longer and more upward in aspect than in other known *Macropodidæ*.

The floor of the nostril is continued backward by the coalesced premaxillaries 2 inches behind the lower border of the aperture (Plate LXXXIII. fig. 1, *n*); it is here half an inch in breadth, concave and pitted along the line of the interpremaxillary suture. Then, seemingly, it has been broken off, the thinner vertical plates rising from the partition of the confluent prepalatine openings (ib. *a*), which are seen at a lower level. Behind these the bony base of the "septum narium" rises for about half an inch before its fracture; and it can be traced back three inches (as at *n*) before it suddenly sinks down to the lower level of the upper surface of the bony palate (ib. *b b*). This subsidence is more abrupt than in living Kangaroos (ib. fig. 2, *b b*), and each lateral division of the superpalatal part of the floor of the nasal chamber forms, anteriorly, in *Palorchestes* a blind fossa (ib. fig. 1, *b b*) below the level of the floor of the antorbital part (ib. *n'*) of that chamber.

Some traces are discernible of the suture between the premaxillary and maxillary upon the palate (Plate LXXXII. fig. 1, $_{mx'}$) and upon the side of the facial part of the skull. Behind the third incisive alveolus, on the right side, a narrow oblique cavity like an alveolus (Plate LXXXI. fig. 1, *c*) is exposed by fracture or attrition of the outer palate of the premaxillary, into the base of which cavity opens a small vascular or nervo-vascular canal; this may have contained the germ or aborted rudiment of a canine.

The bony palate (Plate LXXXII. fig. 1) loses a little breadth behind the incisive foramen (*a*), then expands with an outward curve to the sockets of the premolars (p^3). A mid tract of the breadth of the incisive foramen is continued backward as a very shallow channel to opposite the third grinder (m^1), where the uniform level of the bony palate is gained. A similar but rather deeper channel is continued forward from the fore part of the incisive foramen, and slightly expands to its termination at the outlets of the mid incisors (i^1). The intermolar tract of the bony palate is less concave longitudinally than in *Macropus* and *Osphranter*, and less so transversely than in *Macropus major*. *Osphranter* more resembles *Palorchestes* in this direction.

There is no trace in *Palorchestes* of the pair of small oblong holes where the palato-maxillary suture (ib. ib. $_{mx}$) begins to bend backward near the socket of the last molar.

In *Palorchestes* this suture describes a less angular, more semicircular course than in existing Kangaroos. The hind border of the bony palate appears to have been more concave than in *Macropus major* or *Osphranter robustus*, in which respect it resembles more some of the *Halmaturi*. A pair of small vascular foramina, 1 inch 3 lines apart, occur at the interval between the right and left premolar (p^2).

The anterior pier of the zygoma (Plates LXXXII., LXXXIII., zy^a) is subtrihedal, the hind surface or side being the broadest, nearly flat vertically, concave transversely. The fore part is subequally divided into an upper and lower facet by a forwardly directed rounded angle. A narrow semicircular notch (Plate LXXXI. fig. 1, *o*) at the upper part of the base of this pier indicates a part of an orbit relatively smaller than in the Kangaroos. Above and in advance of this notch is the ectorbital aperture of the lacrymal canal characteristic of the Marsupialia. There does not appear to be, in either side of the skull, a trace of an antorbital foramen; but I incline to believe in some accidental obliteration of that issue, rather than that it never existed.

Each molar series describes a slight curve convex outward. The base of the broken crown of the premolar is subelliptic (Plate LXXXII, fig. 1, p^2), $7\frac{1}{2}$ lines in long diameter, 6 lines in transverse diameter, which is at its middle; it is not broadest behind as in most other Kangaroos; the inner side is more convex than the outer one. The second molar (ib. ib. d^4) is bilophodont, with a prebasal ridge; a fold of enamel (ib. *f*) indicative of the ridge remains on the inner side; the rest of the ridge is lost in the continuous hollow tract of dentine occupying the base of the worn-down anterior lobe (*a*). Opposite folds of enamel, the inner one the longer, indicate the interlobal valley; the dentine at the base of the link (*r*) leads across from the anterior to the posterior field of attrition. The third molar (ib. ib. m^1) has been worn down to the same degree. In both these molars the inner side of the basal part of the front lobe (*a*) encroaches further than that of the hind lobe (*b*) upon the bony palate, forming the origin of a very strong antero-internal root. In the fourth molar (m^2) and in the last (m^3), the corresponding root makes a projection beyond the inner side of the hind lobe of the tooth in advance. The base of the mid link (*r*) remains in m^2; in m^3 that link shows a median notch. More of the narrow prebasal ridge (*f*) is apparent in the last two teeth. The hind surface of the penultimate and last molars seems to have been slightly depressed at *g*, fig. 2, but much less so than in *Macropus major* and *Osphranter robustus*; the hind lobe of m^3 is narrower than that of m^2. The greatest breadth of the crown of the third molar (m^1) is 10 lines. The surface of the enamel in all the grinders appears to be finely rugous.

Thus, so much of the dental characters as can be defined in the present unique fossil concurs with the cranial ones in showing *Palorchestes Azael* to have deviated less from the type of the existing bilophodont *Macropodidæ* than have the species of *Procoptodon*, some of which (*Proc. Goliah*, for example) were its rivals in bulk.

Description of the Plates.

PLATE LXXVI.

Fig. 1. Upper view of portion of right mandibular ramus, with the hindmost molar, of *Macropus Titan*.
Fig. 2. Upper view of an aged individual of *Macropus Titan*.
Fig. 3. Upper view of portion of left mandibular ramus, with the hindmost molar and débris of antecedent ones, of an aged *Macropus Titan*.
Fig. 4. Outside view of figure 2.
Fig. 5. Inner side view of figure 3.
Fig. 6. Outer side view of the teeth in ditto; 6*a*, hind surface of much-worn last molar of *Macropus Titan*.
Fig. 7. Outside view of portion of right ramus, with three hindmost molars, of *Pachysiagon Otuel*, Ow.
Fig. 8. Inside view of the same fossil.
Fig. 9. Upper view of the same fossil.
Fig. 10. Hind view of the same fossil.
Fig. 11. Outside view of part of right mandibular ramus, with the penultimate and antepenultimate molars, of *Leptosiagon gracilis*, Ow.
Fig. 12. Inner view of the same fossil.
Fig. 13. Upper view of the same fossil.
Fig. 14. Hind view of the same fossil.
Fig. 15. Section of middle of mandibular ramus of the same fossil.

PLATE LXXVII.

Fig. 1. Under view of cranium of *Halmaturus brachyurus*, Wth.
Fig. 2. Outside view of part of the upper jaw and teeth of *Procoptodon Pusio*, Ow.
Fig. 3. Inside view of the same fossil.
Fig. 4. Front view of the same fossil.
Fig. 5. Back view of the same fossil.
Fig. 6. Under view of right and left maxillæ of the same young *Procoptodon Pusio*.
Fig. 7. Left upper molar series of adult *Procoptodon Pusio*.
Fig. 8. Outer side view of fore part of left mandibular ramus, with the premolar exposed, of *Procoptodon Rapha*, Ow.
Fig. 9. Inner side view of the same fossil.
Fig. 10. Back view of the same fossil, showing relation of the premolar (p_3) to the milk-molar, d_3.
Fig. 11. Upper view of the same fossil: *a*, section of incisor.
Fig. 12. Under view of the same fossil.

PLATE LXXVIII.

Fig. 1. Outside view of right mandibular ramus of adult *Procoptodon Rapha*.
Fig. 2. Inside view of the same fossil.
Fig. 3. Upper view of the same fossil.

PLATE LXXIX.

Fig. 1. Portion of upper jaw and hinder molars of *Procoptodon Goliah*.
Fig. 2. Outer side view of left maxillary and teeth of an old *Procoptodon Goliah*.
Fig. 3. Inner side view of the same fossil.
Fig. 4. Under view of the same fossil.
Fig. 5. Outer side view of a portion of left maxillary and teeth of a less aged *Procoptodon Goliah*.
Fig. 6. Inner side view of the same fossil.
Fig. 7. Working-surface of the grinders of the same fossil.
Fig. 8. Upper view of hind part of left mandibular ramus, with two hind molars (m_3, m_4), of *Procoptodon Goliah*.
Fig. 9. Hind view of last upper molar of *Procoptodon Goliah*.
Fig. 10. Hind view of last lower molar, ditto.
Fig. 11. Outer side view of adult series of grinders, right side, lower jaw, of *Procoptodon Goliah*.

PLATE LXXX.

Fig. 1. Inside view of part of right mandibular ramus of a mature *Procoptodon Goliah*.
Fig. 2. Grinding-surface of the teeth in the same fossil.
Fig. 3. Outside view of a fragment of the left mandibular ramus, with the penultimate molar, of a mature *Procoptodon Goliah*.
Fig. 4. Inside view of the same fragment, showing the last two grinders.
Fig. 5. Inside view of part of the right mandibular ramus of an immature *Procoptodon Goliah* (from a photograph).
Fig. 6. Outer side view of right mandibular ramus of an immature *Procoptodon Goliah*.
Fig. 7. Portion of the same ramus, with the premolar (p_3) exposed in its formative cell.
Fig. 8. Grinding-surface of worn penultimate lower molar of *Procoptodon Goliah*.

PLATE LXXXI.

Fig. 1. Right side view of fore part of the skull of *Palorchestes Azael*, Ow.
Fig. 2. Fore end of the same fossil.
Fig. 3. Fore end of the skull of *Osphranter robustus*, Gd.
Fig. 4. Inner side view of a fragment of the right mandibular ramus of *Macropus Ferragus*, Ow.

PLATE LXXXII.

Fig. 1. Under view of the fore part of the cranium of *Palorchestes Azael*, Ow.
Fig. 2. Pattern of working-surface of penultimate upper molar of ditto.
Fig. 3. Outer side view of portion of right mandibular ramus and last two molars of *Macropus Ferragus*, Ow.
Fig. 4. Upper view of the same fossil.

PLATE LXXXIII.

Fig. 1. Upper view of the fore part of the cranium of *Palorchestes Azael*.
Fig. 2. Upper view of corresponding part of the skull of *Macropus ruficollis*, Gd.
Fig. 3. Back view of mandibular fragment, with last grinder, of *Macropus Ferragus*.

All the figures are of the natural size.

XXIV. *Researches in Spectrum-Analysis in connexion with the Spectrum of the Sun.*—No. IV. *By* J. Norman Lockyer, *F.R.S.*

Received May 11,—Read June 18, 1874.

I beg to lay before the Royal Society a Map of that portion of the spectra of calcium, strontium, and barium comprised between wave-length 3900 and 4500. This Map has been constructed from photographs of the spectra taken by the method described by me in a former communication (the third paper of this series) to the Society, and I am induced to send it in as a specimen of the results to be obtained by the method in the hope that other observers will co-operate; for I am of opinion that it will be necessary to construct similar maps for all the metallic elements before either our knowledge of the composition of the sun's reversing layer can be said to be in any way perfect, or we can be said to have a ready means of determining cyclical changes in its composition. The great labour attending and long time required for the construction of these maps results from the universal presence of impurities, even in the purest specimens of the metals or metallic salts prepared by the ordinary chemical methods; and although the method employed is the only one which enables us to eliminate them eventually, this elimination necessitates a photographic comparison of the spectrum mapped with those of all the substances present as impurities. Hence there are numerous records to be discussed, and the discussion requires special treatment.

Method of Mapping.

The method of treatment which I have employed in constructing the Maps is as follows:—

1. *Elimination of lines due to impurities.*—The spectrum of the element is first confronted with the spectra of the substances most likely to be present as impurities, and with those of metals which, according to Thalén's measurements, contain in their spectra coincident lines. Lines due to impurities, if any are thus traced, are marked for omission from the Map and their true sources recorded, while any line that is observed to vary in length and thickness in the various photographs is at once suspected to be an impurity line, and if traced to such is likewise marked for omission.

The retention or rejection of lines coincident in two or more spectra is determined by observing in which spectrum the line is thickest; and it is then, as indicated in my last paper, assigned to that element. Where, as in the present case, several elements are mapped at once, all their spectra are confronted on the same Plate, as by this means the presence of one of the substances as an impurity in the others can be at once detected.

Many lines due to impurities have in this manner been traced in the photographed

spectra of calcium, strontium, and barium. For instance, the two longest aluminium lines (3943 and 3961) are invariably present in all three spectra, and the long iron lines (4045, 4063, and 4071) can in most cases be likewise detected. The lines 4076·9 and 4215·4 present in all photographs of calcium, and assigned to this element both by Ångström and Thalén, are, as I showed in my last paper, really due to strontium.

Similarly the two H lines (3933 and 3968), assigned both to iron and calcium by Ångström, are proved to belong to calcium by the following observations:—

a. The lines are well represented in the spectrum of commercial wrought iron, but are absolutely coincident with two thick lines in the spectrum of calcium chloride with which the iron spectrum has been confronted.

b. The lines are represented by mere traces in the spectrum of a specimen of pure iron prepared by the late Dr. Matthiessen, and obligingly placed at my disposal by Dr. Russell. In this photograph (Plate LXXXV. Spectrum 7) both poles of the lamp were of iron, the lower pole consisting of an ingot of the metal which had been cast in a lime-mould.

c. The lines are altogether absent in a photograph of pure iron, where both poles of the lamp were of the pure metal *not* cast in lime, and they are likewise absent in a photograph of the spectrum of the Lenarto meteorite (Plate LXXXV. Spectrum 1)*.

These examples serve to illustrate the manner in which large numbers of the coincidences recorded by former observers have been disposed of in the course of mapping by the photographic method.

In determining the coincidence of very thick lines, such as the H lines just mentioned for example, the centre of the thick line is taken, except in instances where the whole of a spectrum has been slightly displaced to the right or left by the displacement of the apparatus during the act of photographing, in which case a correction has to be made for the position of every line in such displaced spectrum. It not unfrequently happens that a very thick line will reverse itself, a circumstance which greatly facilitates its comparison with confronted lines, since a thin dark line then runs down the centre of the thicker bright one (Plate LXXXV. Spectra 2 & 3)†.

2. *Determination of the positions and lengths of new lines.*—By eliminating lines due to impurities in the manner just described, a spectrum is at length obtained, of which every line is assignable to the particular element photographed.

These lines are then entered in a map, those measured by Ångström or Thalén being placed in the positions assigned by those authors, while the new lines are at first entered approximately. The wave-lengths assigned to the new lines, although probably not far from the truth, must, in the absence of actual measurement, be regarded only as approximate. They have been found in the following manner:—

The admirable photographic print of the solar spectrum, from H to F, obtained some

* In both these photographs, however, the *longest line* of calcium is to be traced.

† The absorption-line does not always occupy the *exact* centre of the bright band. This point is occupying my attention, as it raises a very interesting question connected with molecular vibrations.

time ago by Mr. RUTHERFURD (and which I have not yet been able to equal*), was first divided (from 3900 to 5000) into portions corresponding each to a difference of wave-length of $\frac{1}{100,000}$ millim. by identifying in the print the more prominent of the FRAUNHOFER lines mapped by ÅNGSTRÖM, and attaching the proper numbers to the consecutive divisions. The spaces between these larger divisions were then, by a similar use of ÅNGSTRÖM'S map, divided into tenths, each of these divisions corresponding consequently to a difference of wave-length of $\frac{1}{1,000,000}$ millim. The millimetres of ÅNGSTRÖM'S scale (equal to a difference of wave-length of $\frac{1}{10,000,000}$ millim. each) were found by dividing the $\frac{1}{1,000,000}$ millim. divisions into ten equal parts. From this standard print the position of any new line is found by carefully observing (in a photograph of the spectrum of the metal confronted with the solar spectrum) the precise position of the new line with respect to the solar spectrum, and then finding the corresponding position in the print†. Although absolute accuracy cannot be claimed for the positions of lines determined in this way, I have nevertheless expressed the wave-lengths in the Tables in the usual scale ($1–10^{10}$ metre) in order to distinguish between lines which occur close together, of which there are several instances in the present Map.

The lengths of the new lines are determined from special photographs (Plate LXXXIV.), to obtain which a horizontal arc is employed, as explained in my last paper.

REVERSAL OF THE NEW LINES IN THE SOLAR SPECTRUM.

With regard to the reversal or non-reversal in the solar spectrum of the new lines revealed by the photographs, I refrain for the present from making any positive statements, as I have not yet been able to obtain sufficiently good photographs of the metallic and solar spectra confronted. I may add, however, that lines are to be found in Mr. RUTHERFURD'S photograph of the solar spectrum in the positions, within the limits of error, of nearly all the new lines; but the close approach to coincidence between lines undoubtedly distinct, which I have observed in many cases in the course of constructing the accompanying Map, convinces me that in future much greater precision will be necessary before assigning any line in the solar spectrum to a particular metal, and I therefore reserve the question of the reversal of the new lines till our knowledge of the solar spectrum is more complete. For a similar reason it is obvious that greater attention will have to be given to the precise *character* as well as to the position of each of the FRAUNHOFER lines, in the thickness of which I have already observed several anomalies. I may refer more particularly at present to the two H lines 3933 and 3968 belonging to calcium, which are much thicker in all photographs of the solar spectrum than the longest calcium line of this region (4226·3), this latter being invariably thicker than the H lines in all photographs of the calcium spectrum, and remaining, moreover, visible in the spectrum of substances containing calcium in such small quantities as not to show any traces of the H lines. How far this and other similar variations between

* Owing, among other causes, to constant vibrations incident to observations in London.

† A photograph in which all the metals mapped are confronted on the same plate is a useful guide for checking the positions of the lines when found as above.

photographic records and the solar spectrum are due to causes incident to the photographic record itself, or to variations of the intensities of the various molecular vibrations under solar and terrestrial conditions, are questions which up to the present time I have been unable to discuss.

Observations on the Maps.

In the accompanying Map I have placed under each spectrum the spectra mapped by Ångström and Thalén for the sake of comparison. In constructing the Map only those lines distinctly referable to the metal have been inserted; minute traces of faint lines and the banded structure visible in most of the photographs, and belonging most probably to the carbon of the lamp-poles, have been omitted.

The lines described in the Index as "nebulous" are of a peculiar and distinct nature, presenting the appearance of being out of focus, and suggesting that they arise from the surface of the arc. Whether these lines are due to the metal itself or to some compound of the metal stable at the temperature of the electric arc, I am at present unable to say. Special experiments will have to be made upon this point. The calcium lines 4091·8, 4093·3, and 4097·5, and the barium lines 4081, 4084, and 4087, are of the nature indicated.

In certain cases lines have been found of equal thickness, and absolutely coincident in two confronted spectra, without being referable to any known impurity common to the two metals. In these instances the lines have been assigned to both metals, but must be considered as subject to correction. The lines 4282·5* (common to calcium and barium) and 4325 (common to barium, strontium, and iron) are instances of coincidences occurring in the present Map.

The scale of lengths given in the Index is that made use of by Thalén† for indicating the intensities of the lines, 1 being the longest and 5 the shortest line. In my former papers, where eye-observations were made use of, the scale of length ranged only from 1 to 4; but in the present case we have photographs to decide from, and these permit a more satisfactory determination.

The Map and accompanying Tables have been constructed by my assistant, Mr. R. Meldola, to whom I am much indebted for the skill and patience he has brought to bear upon the inquiry.

The photographs which accompany this paper have been taken in the room in Dr. Frankland's private laboratory, to which I made reference in my former communication. In again tendering my thanks to Dr. Frankland for having placed it at my disposal, I cannot refrain from pointing out that without such aid my present researches would have been impossible.

* It has since been found by using a camera of 6 feet focus that this line is not absolutely coincident in both spectra, the calcium line being very slightly more refrangible. The Map and Index have been altered accordingly.—J. N. L., September 26, 1874.

† "Mémoire sur la détermination des longueurs d'onde des raies métalliques," Nova Acta, Upsala, 1868.

INDEX TO MAP. (Plate LXXXVI.)

CALCIUM.

Lines marked * are those in the Maps of other observers which have been omitted from the present Map on the photographic evidence.

Wave-length.	State in Ångström's Maps.	State in Thalén's Maps.	Length in Calcium Photographs.	Remarks.
3932·8 [H_{11}]	Present. Assigned also to Fe.	Present.	1	One of the longest Ca lines. In all photographs of substances containing the smallest trace of Ca.
3956·0	Not present.	Not present.	5	Short Ca line nearly coincident with an Fe line. In all photographs.
3968·0 [H_1]	Present. Assigned also to Fe.	Present.	1	Long Ca line in the centre of a complex group of Fe lines. This line is grazed on its right by an Fe line, but is not coincident as in Ångström's Map.
3974·0	Not present.	Not present.	4	Short Ca line in all photographs.
*4077·0	Present.	Present.	3	One of the longest Sr lines present in all photographs of Sr and Ca, but invariably thicker in the Sr.
*4078·5	Not present.	,,	....	No line in this position in any of the photographs. It is probable that this is identical with the foregoing line.
4091·8	Present.	,,	5	A nebulous line in all photographs.
4093·3	Not present.	Not present.		
*4095·5	Present.	Present.	....	Not in any photograph.
4097·5	,,	,,	4	Nebulous line in all photographs.
*4098·0	Not present.	,,	....	This line is probably identical with the foregoing.
*4131·5	Present. Assigned also to Fe.	Present. Assigned also to Fe.	....	Present as mere traces in some photographs, absent altogether from others.
*4143·0	,,	,,		
*4188·5	Present.	Present.	....	Not in photographs.
*4192·2	,,	,,		
*4192·5	Not present.	,,	....	Not in photographs. ? Identical with foregoing line.
*4215·3	Present.	,,	2	One of the longest Sr lines present in all photographs of Sr and Ca, but invariably thicker in Sr.
4226·3	,,	Present. Assigned also to Sr.	1	Longest Ca line in this part of the spectrum. In all photographs of substances containing the smallest trace of Ca.
*4233·0	Present. Assigned also to Fe.	Present. Assigned also to Fe.	....	Not a Ca line.
4237·5	,,	Present.	5	Nebulous line in all photographs. Not an Fe line.
*4247·5	,,	Present. Assigned also to Fe.	....	Not in photographs.
*4249·8	,,	,,	....	
*4253·9	Present. Assigned also to Fe and Cr.	Present. Assigned also to Cr.	....	

CALCIUM (continued).

Wave-length.	State in Ångström's Maps.	State in Thalén's Maps.	Length in Calcium Photographs.	Remarks.
*4271·5	Present. Assigned also to Fe and Cr.	Present. Assigned also to Cr.	....	Not in photographs.
*4274·5	Present. Assigned also to Cr.	Present. Assigned also to Cr.		
4282·4	Present.	Present.	3	Ca line in all photographs. Very nearly coincident with a Ba line, and almost coincident with an Fe line.
*4285·5	,,	Not present.	....	Not in photographs.
4289·4	Present. Assigned also to Cr.	Present. Assigned also to Cr.	3	In all photographs. Just to left of a Ba line, and apparently coincident with a line in Al photograph.
4298·5	Present. Assigned also to Fe.	Present. Assigned also to Fe.	3	In all photographs. Almost but not quite identical with Fe line, as in Ångström's and Thalén's Maps.
*4300·2	Present.	Not present.	....	Not in photographs.
4302·0	Present. Assigned also to Fe.	,,	3	In all photographs. Not assignable to Fe, as in Ångström's Map.
*4302·3	Not present.	Present.	....	Probably identical with preceding line.
4306·5	Present.	,,	3	In all photographs. Nearly coincident with an Fe line.
4318·0	,,	,,	3	In all photographs.
4354·0	Not present.	Not present.	4	Nebulous line in all photographs.
*4379·1	Present. Assigned also to Fe.	Present.		
*4384·7	Present.	,,		
*4389·4	,,	,,		
*4393·0	,,	,,	....	Not in photographs.
*4405·7	,,	,,		
*4407·0	,,	,,		
*4407·7	Present. Assigned also to Fe.	,,		
4425·0	Present.	,,	3	In all photographs.
4434·5	,,	,,	2	
4435·3	Not present.	,,	....	Not in photographs, unless identical with preceding line. Although both lines are down in Thalén's Tables only one is shown in his Map.
4454·2	Present.	Present [4454·0]	2	In all photographs.
*4455·2	Not present.	Present.	....	Not in photographs unless identical with preceding line. Here also Thalén has the two lines in the Table, but only one in the Map.

STRONTIUM.

Wave-length.	State in Ångström's Maps.	State in Thalén's Maps.	Length in Strontium Photographs.	Remarks.
3940·0	Not present.	Not present.	5	Short line in all photographs. Nearly coincident with a faint line in Fe spectrum.
4029·4	Present. Assigned to Mn.	Present. Assigned to Mn.	2	In all photographs. Nearly coincident with a long Mn line.
4031·5	Present. Assigned to Fe.	Not present.	4	In all photographs of Sr; faint in some. Apparently coincident with lines in Mn and Fe spectra. Another line occurs in photographs of Sr about W. L. 4033, but is very faint, and is moreover common to Fe and Mn, and has not therefore been inserted in Map. The spectrum is much confused at this region.
4031·7	Present. Assigned to Mn.	Present. Assigned to Mn.		
4077·0	Present. Assigned to Ca.	Present. Assigned to Ca.	1	The longest Sr line in this portion of the spectrum; generally present also in Ca photographs.
*4078·5	Not present.	Present.	....	No line in this position in any of the photographs unless identical with preceding.
4161·0	A line at 4160·8. Not assigned to any thing.	„	3	In all photographs.
4215·3	Present. Assigned to Ca.	Present. Assigned to Ca and Sr.	1	One of the longest Sr lines in this portion of the spectrum; generally present also in Ca photographs.
*4226·3	Present. Assigned to Ca.	„	2	Longest Ca line. Present in all photographs of Ca and Sr, but invariably thicker in Ca.
4305·3	Not present.	Present.	2	In all photographs.
4235·0	Present. Assigned to Fe.	A line of W. L. 4325·2. Assigned to Fe.	3	In all photographs. Coincident with a thick Fe line and with a Ba line.
4336·0	Not present.	Not present.	3	In all photographs. Just to right of a line in Fe spectrum.
4365·0	„	„	4	In all photographs.
4437·0	„	„	5	

BARIUM.

Wave-length.	State in Ångström's Maps.	State in Thalén's Maps.	Length in Barium Photographs.	Remarks.
3934·0	Not present.	Not present.	3	In all photographs. Apparently coincident with a very faint line in Fe spectrum.
3937·0	,,	,,	4	In all photographs.
3994·5	,,	,,	2	In all photographs. A double line.
3996·2	,,	,,	3	In all photographs. Almost coincident with a short Fe line.
4081·0	,,	,,	5	Very nebulous line in all photographs. Just to right of a line in Fe spectrum.
4084·0	,,	,,	5	Broad and very nebulous line in all photographs. Coincident with a thick Fe line.
4087·0	,,	,,	5	Very nebulous line in all photographs. Just to right of a faint double line in Fe spectrum.
4130·5	,,	Present.	1	Longest Ba line in this portion of the spectrum.
4131·5	,,	Present. Assigned to Fe.	3	In all photographs. Almost coincident with a thick Fe line.
4165·5	Not present.	Present.	2	In all photographs.
4224·0	,,	Not present.	3	In all photographs. Just to left of a thin line in Fe spectrum.
4239·0	,,	,,	4	In all photographs. Almost coincident with a thin line in Fe spectrum.
4241·5	A line at 4241·6. Not assigned to any thing.	,,	4	In all photographs.
4264·0	A line at 4264·1. Assigned to Fe.	,,	3	In all photographs. Between two thick Fe lines. Slightly nebulous.
4282·5	Present. Assigned to Ca.	Present. Assigned to Ca.	1	In all photographs. Very nearly coincident with a Ca line.
4290·6	Not present.	Not present.	3	In all photographs. Just to right of a Ca line.
4323·0	,,	,,	3	In all photographs. A nebulous line.
4325·0	Present. Assigned to Fe.	A line of W. L. 4325·2. Assigned to Fe.	3	In all photographs. Coincident with a Sr and thick Fe line.
4332·0	Not present.	Not present.	4	In all photographs.
4351·0	,,	,,	2	In all photographs. To left of a Ca line, and nearly coincident with a thin line in Fe spectrum.
4401·5	,,	,,	4	In all photographs. To left of a thick Fe line.
4433·0	,,	,,	5	In all photographs. Between two Ca lines.
4488·0	,,	,,	5	In all photographs. A nebulous line.
4493·0	,,	,,	5	In all photographs. A nebulous line. Apparently coincident with a thin line in Fe spectrum.

Description of the Photographs.

PLATE LXXXIV.

Spectrum 1. Long and short lines of calcium.
Spectrum 2. Long and short lines of strontium.
Spectrum 3. Long and short lines of barium.

Note.—In these photographs the impurity lines which have been eliminated from the map by the process described in the paper are present.

The photographs illustrate the use of the horizontal arc.

PLATE LXXXV.

Spectrum 1. Comparison of the spectra of the Lenarto meteorite, calcium, and aluminium.

1, spectrum of the meteorite.
2, „ calcium.
3, „ aluminium.

Spectrum 2. Comparison of the spectra of nearly pure iron and strontium.

4, spectrum of iron.
5, „ strontium.

Spectrum 3. Comparison of the spectra of Matthiessen's iron and calcium.

6, spectrum of calcium.
7, „ iron when cast into ingot.
8, „ „ before casting.

INDEX

TO THE

PHILOSOPHICAL TRANSACTIONS

FOR THE YEAR 1874.

A.

Abel (F. A.). Contributions to the History of Explosive Agents.—Second Memoir, 337.—On the transmission of detonation, 340; interference with the transmission of detonation by tubes, 353; conclusions regarding the transmission of detonation by tubes, 357; development of detonation as distinguished from explosion, 358; influence of dilution by solids and by liquids on the susceptibility of explosive compounds to detonation, 362; employment of water as a means of transmitting detonation, and of applying the force developed by explosion, 372; on the velocity of detonation, or the rate at which detonation is transmitted, 377; on circumstances which influence the behaviour of explosive agents when exposed to high temperatures, 388.

Alloys, quantitative analysis of, by the spectroscope, 495.

Analysis, quantitative, by means of the spectroscope, 481, 495.

Asterophyllites, 41.

Attraction and repulsion resulting from radiation, 501.

Australia, fossil mammals of, 245, 783 (see Owen).

B.

Bacteria, development of (see Roberts).

Bakerian Lecture.—Researches in Spectrum-Analysis in connexion with the Spectrum of the Sun, 479 (see Lockyer).

Ball (R. S.). Researches in the Dynamics of a Rigid Body by the aid of the Theory of Screws, 15 (for contents see p. 15).

Battery, on a standard voltaic, 1.

Biogenesis, 457 (see Roberts).

Blanford (H. F.). The Winds of Northern India in relation to the Temperature and Vapour-constituent of the Atmosphere, 563 (for contents see p. 563).

Brodie (Sir B. C.). On the Action of Electricity on Gases.—II. On the Electric Decomposition of Carbonic-acid Gas, 83.

C.

Carbonic-acid gas, electric decomposition of, 83.
Cayley (A.). A Memoir on the Transformation of Elliptic Functions, 397.—The general problem, 398; the Ωk-modular equations, 400; equation-systems for the cases $n=3, 5, 7, 9, 11, 402$; the Ωk-form, order of the systems, 404; the modular equation, 409; the multiplier equation, 420; the multiplier as a rational function of u, v, 423; theorem in connexion with the multiplication of elliptic functions, 427; the transformations $n=3, 5, 7, 11, 429$; the general theory of q-transcendents, 436; the four forms of the modular equation, and the curves represented thereby, 450.
Chlorine, analogy of, to ozone, 103.
Clark (L.). On a Standard Voltaic Battery, 1.
Clifford (W. K.). On Mr. Spottiswoode's Contact Problems, 705.—The contact of conics with surfaces of the order n, 706; the contact of quadric surfaces with surfaces of order n, 713.
Coal-measures, fossil plants of the, 41, 675.
Contact of conics with surfaces of order n, 706; *of quadric surfaces with surfaces of order* n, 713.
Crookes (W.). On Attraction and Repulsion resulting from Radiation, 501.

D.

Daylight, total intensity of, 655 (see Roscoe).
Dendrocœla, 106 (see Moseley).
Detonation, 337 (see Abel).
Dynamite, detonation of (see Abel).

E.

Echinoidea, on the, of the 'Porcupine' Deep-sea Dredging-Expeditions, 719.
Electromotive force, a practical standard of, 1.
Electrotorsion, 529, 560.
Elliptic functions, 397 (see Cayley).
Explosive agents, 337 (see Abel).

F.

Flashes as fog-signals, 232.
Flower (W. H.). On a newly discovered Extinct Ungulate Mammal from Patagonia, *Homalodontotherium Cunninghami*, 173.
Fog, action of, on sound, 184, 209, 214, 216.
Fog-signals, 186 &c.
Fossil plants of the Coal-measures, 41, 675.

G.

Gore (G.). On Electrotorsion, 529.—Note by Sir W. Thomson, 560.
Gun-cotton, detonation of (see Abel).

H.

Homalodontotherium Cunninghami, 173.

I.

India, winds of Northern, 563.

L.

Leptosiagon, 785.

Lockyer (J. N.). The Bakerian Lecture.—Researches in Spectrum-Analysis in connexion with the Spectrum of the Sun.—No. III., 479.—Introduction, 479; the experiments made on a possible quantitative spectrum-analysis, 481; the method of photographing spectra, 484; on the coincidences of spectral lines, 487; preliminary inquiry into the existence in the sun of elements not previously traced, 490; explanation of the plates, 494.

———— Researches in Spectrum-Analysis in connexion with the Spectrum of the Sun.—No. IV., 805.

Lockyer (J. N.) and Roberts (W. C.). On the Quantitative Analysis of certain Alloys by means of the Spectroscope, 495.

M.

Macropodidæ, 245, 783 (see Owen).

Macropus, 245, 783.

Moseley (H. N.). On the Anatomy and Histology of the Land-Planarians of Ceylon, with some account of their Habits, and a Description of two new Species, and with Notes on the Anatomy of some European Aquatic Species, 105.—Preface, 105; Dendrocœla, 106; habits of land-planarians, 111; methods of investigation, 116; anatomy, 117; summary, 146; description of plates, 150.

———— On the Structure and Development of *Peripatus capensis*, 757.—Introduction, 757; literature, 757; habits, 759; anatomy, 761; conclusion, 775; explanation of the plates, 778.

O.

Osphranter, 261.

Owen (R.). On the Fossil Mammals of Australia.—Part VIII. Family Macropodidæ: Genera *Macropus*, *Osphranter*, *Phascolagus*, *Sthenurus*, and *Protemnodon*, 245.—Introduction, 245; Macropus, 245; Osphranter, 261; Phascolagus, 261; Sthenurus, 265; Protemnodon, 274; description of the plates, 282.

———— On the Fossil Mammals of Australia.—Part IX. Family Macropodidæ: Genera *Macropus*, *Pachysiagon*, *Leptosiagon*, *Procoptodon*, and *Palorchestes*, 783.—Macropus, 783; Pachysiagon, 784; Leptosiagon, 785; Procoptodon, 786; Palorchestes, 797; description of the plates, 801.

Ozone, formation of, from carbonic acid gas, 84; analogy of, to chlorine, 103.

P.

Pachysiagon, 784.

Palorchestes, 797.

Parker (W. K.). On the Structure and Development of the Skull in the Pig (*Sus scrofa*), 289.

Peripatus capensis, 757.

Phascolagus, 261.

Pig, development of the skull in the, 289.

Planarians, land-, of Ceylon, 105 (see Moseley).

Procoptodon, 786.

Protemnodon, 274.

R.

Radiation, attraction and repulsion arising from, 501.

Reversal of lines in solar spectrum, 807.

Rigid body, dynamics of a, 15.

ROBERTS (W.). Studies on Biogenesis, 457.—Introduction, 457; on the sterilization by heat of organic liquids and mixtures, 458; on the capacity of the juices and tissues of animals and plants to generate *Bacteria* and *Torulæ* without extraneous infection, 465; on the bearing of the facts adduced on the origin of *Bacteria* and *Torulæ*, and on the real explanation of some of the alleged cases of abiogenesis, 471; conclusions, 475.

ROBERTS (W. C.) and LOCKYER (J. N.) (see LOCKYER).

ROSCOE (H. E.). On a Self-recording Method of Measuring the Total Intensity of the Chemical Action of Total Daylight, 655.

S.

Screws, application of theory of, to the dynamics of a rigid body, 15.

Skull, development of, in the Pig, 289.

Sound, aërial reflection of, 194; aërial echoes, 197; stoppage of, by aërial reflection, 195, 202; action of hail and rain on, 205; of snow, 207; of fog, 184, 209, 214, 216; of wind, 224.

Spectra of calcium, strontium, and barium, map of, 809.

Spectral lines, application of, to quantitative analysis, 481, 495; photographs of, 484; coincidences in, discussed, 487; reversal of, 807 (see LOCKYER, and LOCKYER and ROBERTS).

SPOTTISWOODE (W.). On his Contact Problems, 705 (see CLIFFORD).

Sthenurus, 265.

Sun, spectrum of, 479, 805; new elements in, 490.

Syren, steam-, use of, as fog-signal, 186 &c.

T.

THOMSON (Sir W.). Note on Mr. GORE's paper on Electrotorsion, 560.

THOMSON (W.). On the Echinoidea of the 'Porcupine' Deep-sea Dredging-Expeditions, 719.

TYNDALL (J.). On the Atmosphere as a Vehicle of Sound, 183 (for contents see p. 244).

W.

WILLIAMSON (W. C.). On the Organization of the Fossil Plants of the Coal-measures.—Part V. *Asterophyllites*, 41.—Description of the plates, 77.

———— On the Organization of the Fossil Plants of the Coal-measures.—Part VI. Ferns, 675.—Description of the plates, 700.

Winds of Northern India, 563.

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Fig. 1.

Fig. 2.

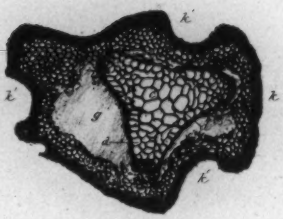
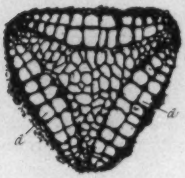


Fig. 3.



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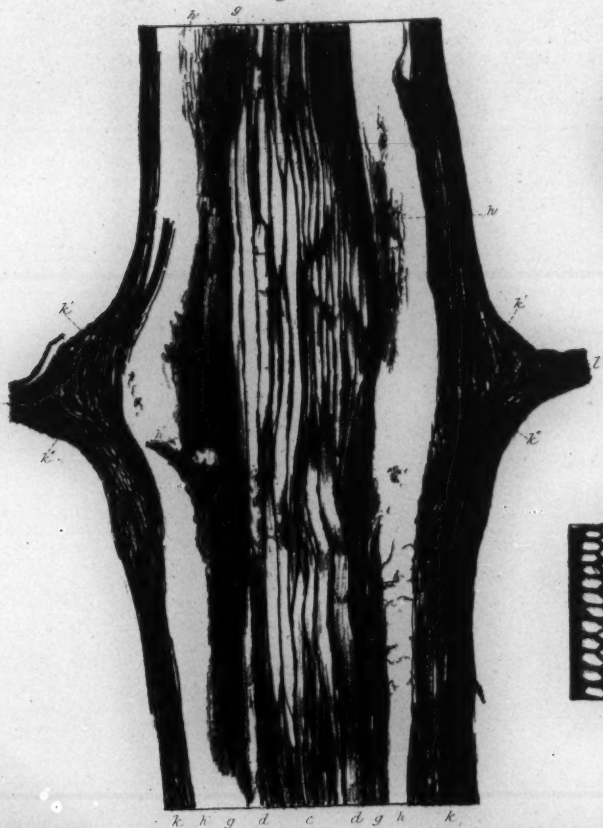


Fig. 4.

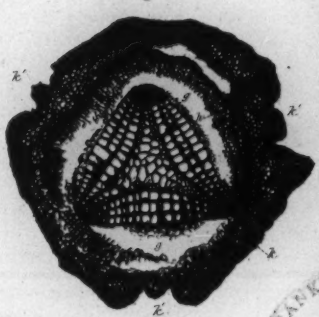
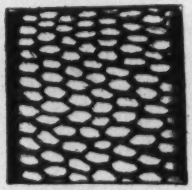


Fig. 6.



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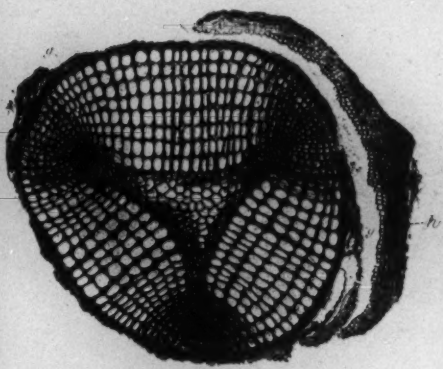


Fig. 10.

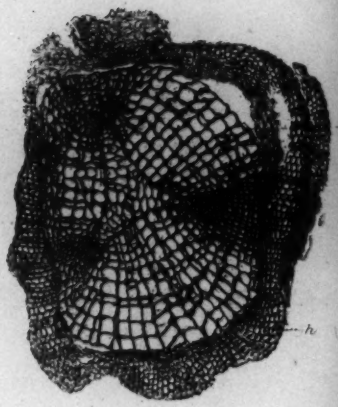


Fig. 12.

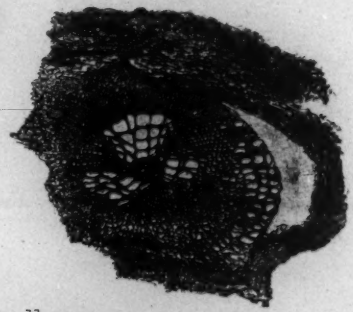
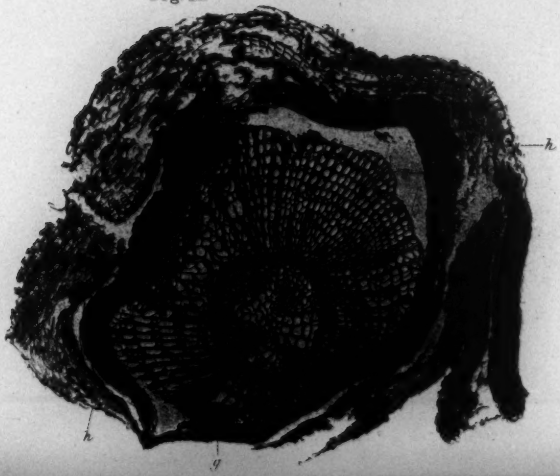


Fig. 13.

Fig. 11.



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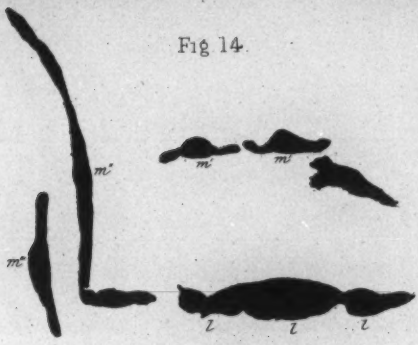

Fig. 15.

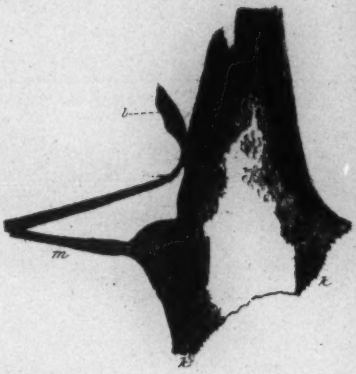

Fig. 16.

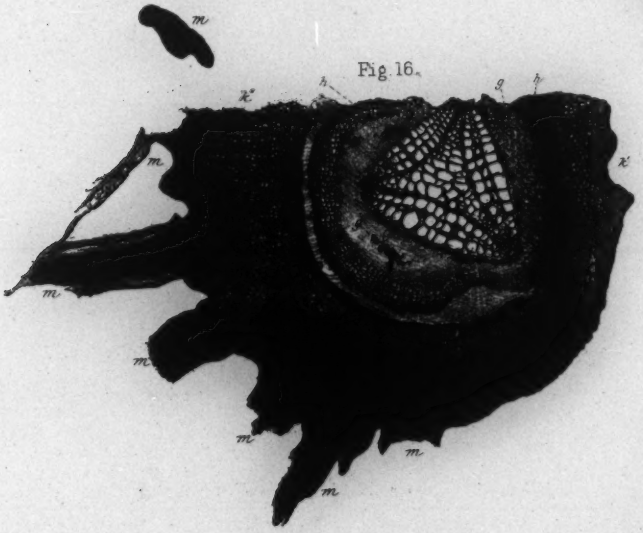

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Fig. 17.

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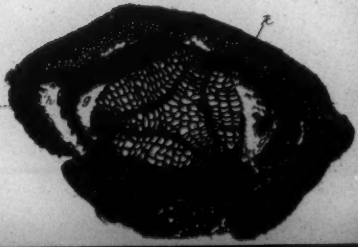

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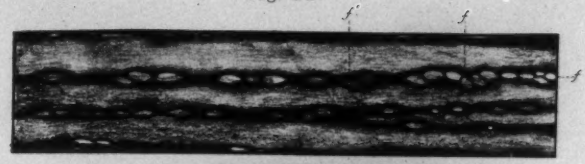

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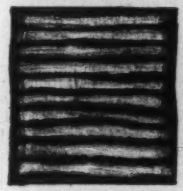

Fig. 25.

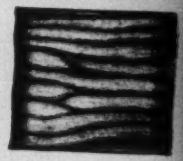

Fig. 21.

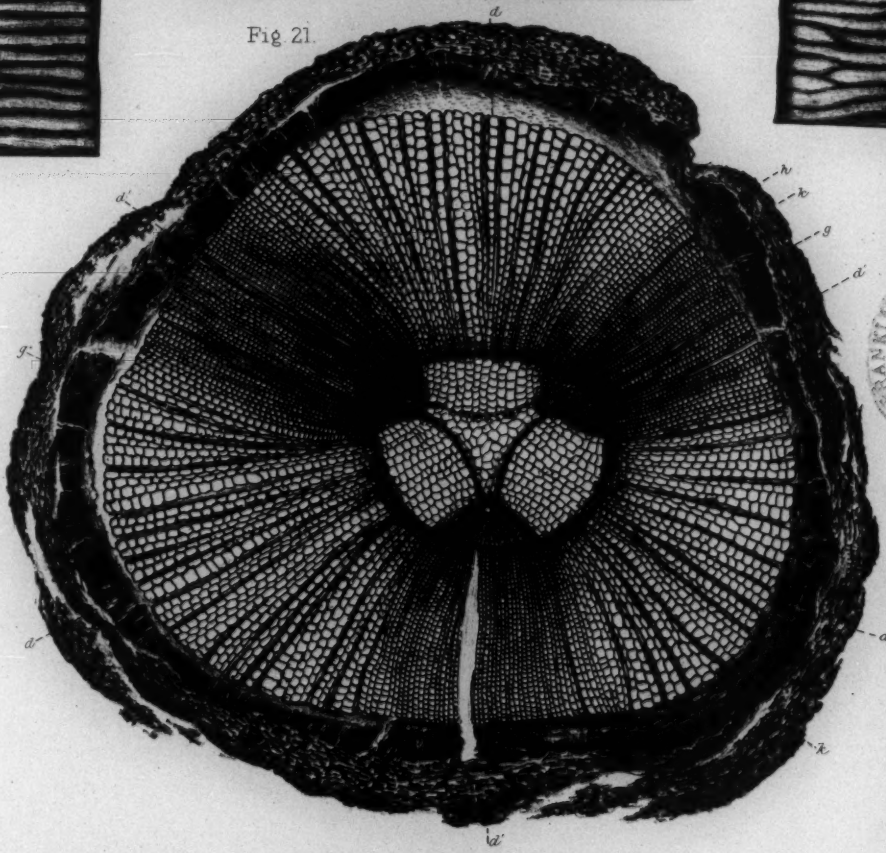

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Fig. 23.

Fig. 26.

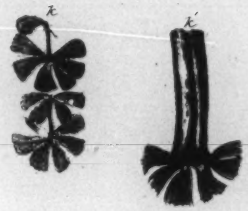


Fig. 27.

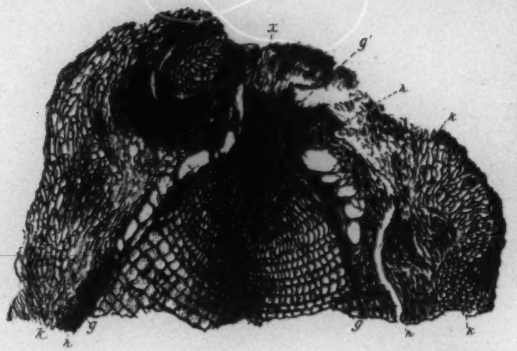


Fig. 31.

Fig. 28.

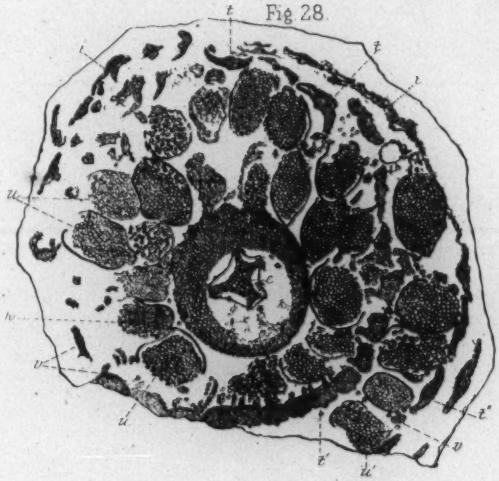
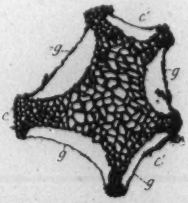


Fig. 29.



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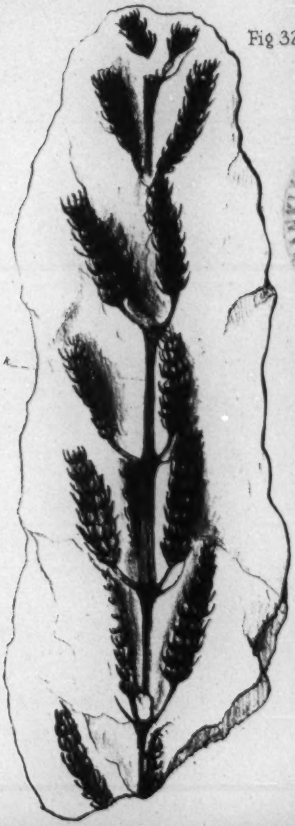

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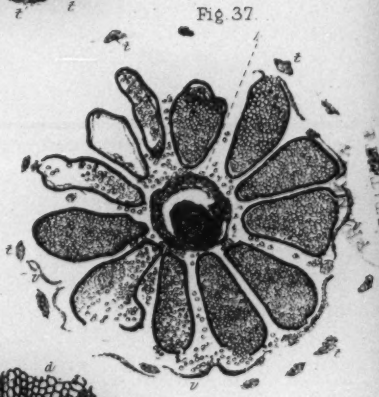
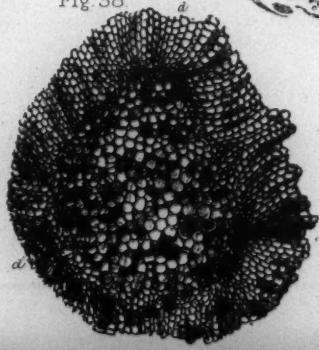


Fig. 38.



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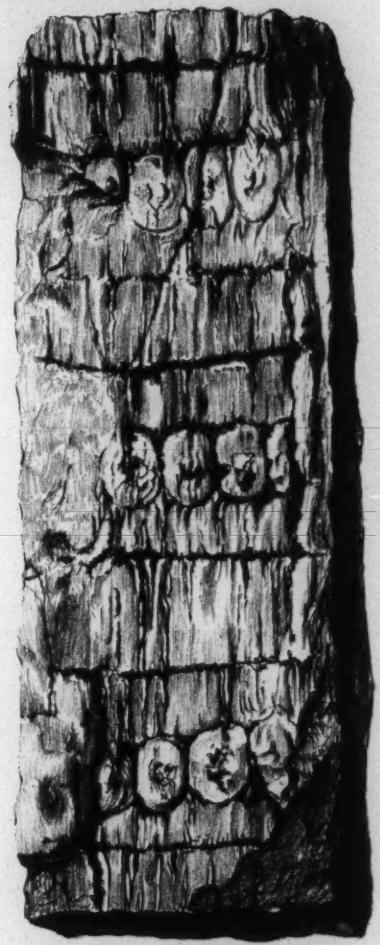

Fig. 46

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Fig 44

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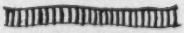

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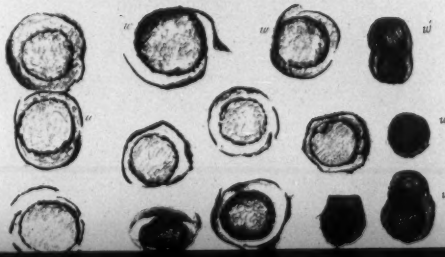
Fig. 43.



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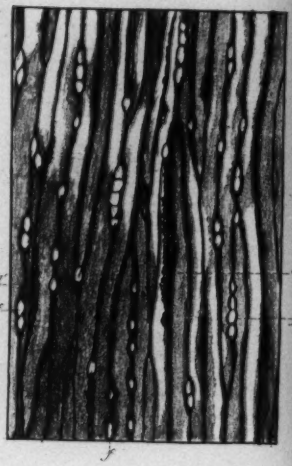

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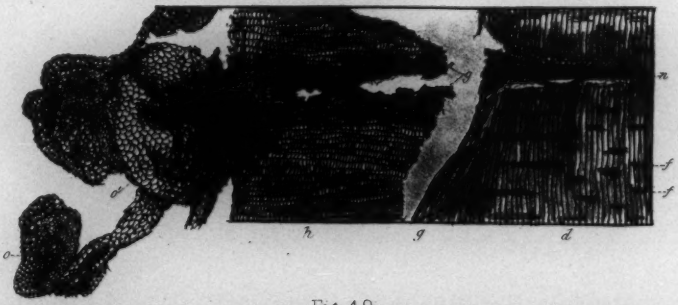

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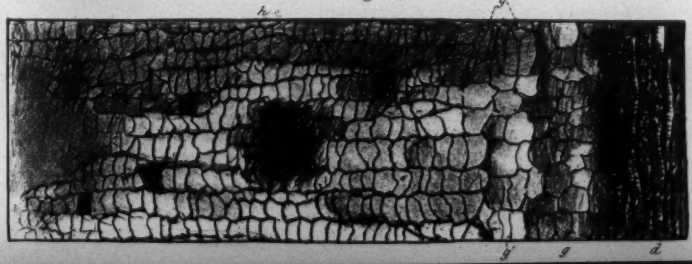
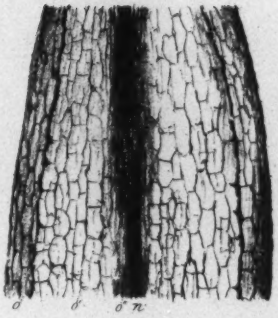


Fig. 53.



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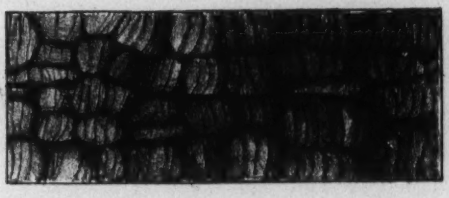


Fig. 56.

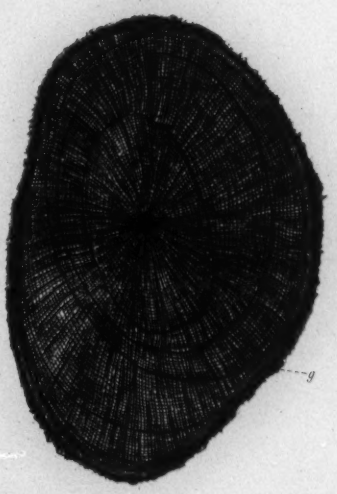


Fig. 59.

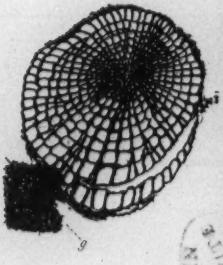


Fig. 55.

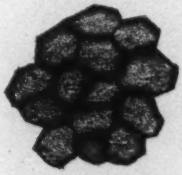


Fig. 57.

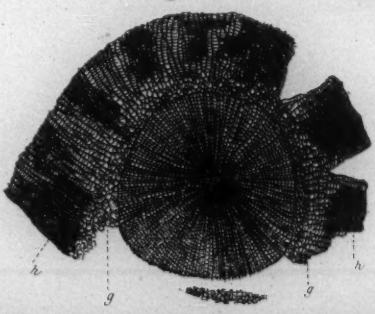
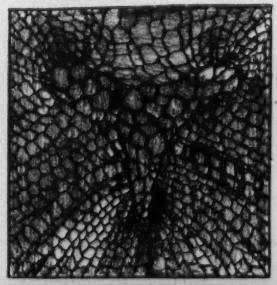
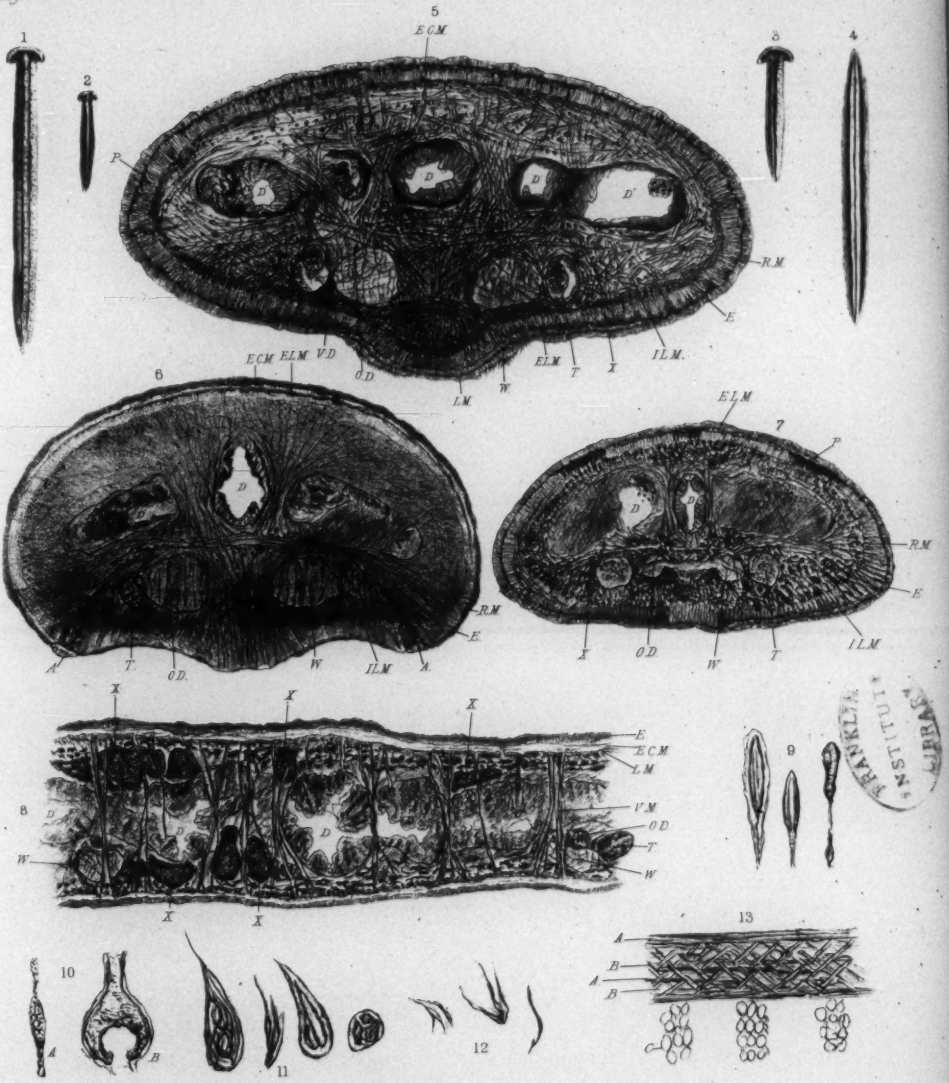


Fig. 58.





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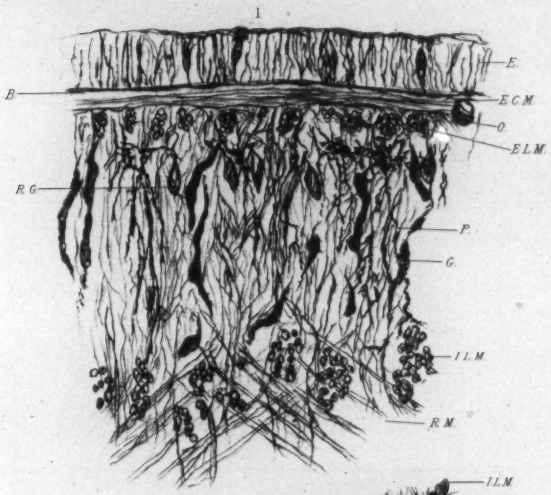

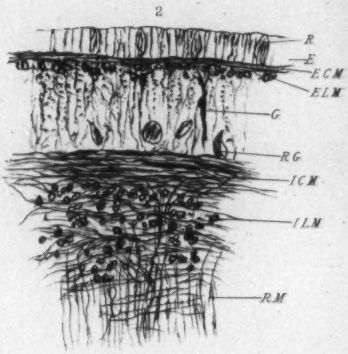

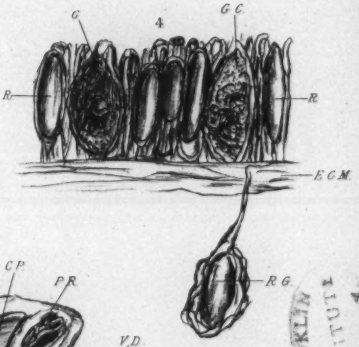

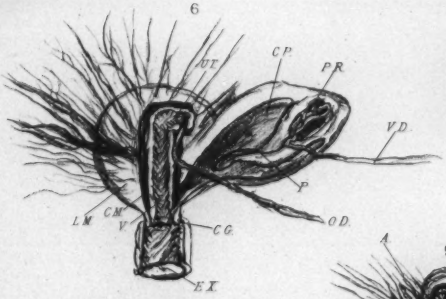

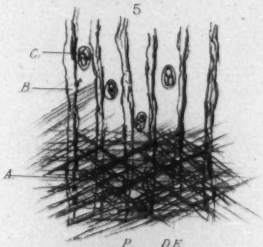

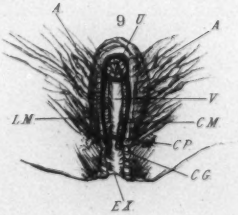

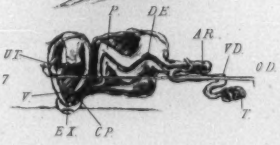

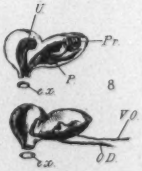

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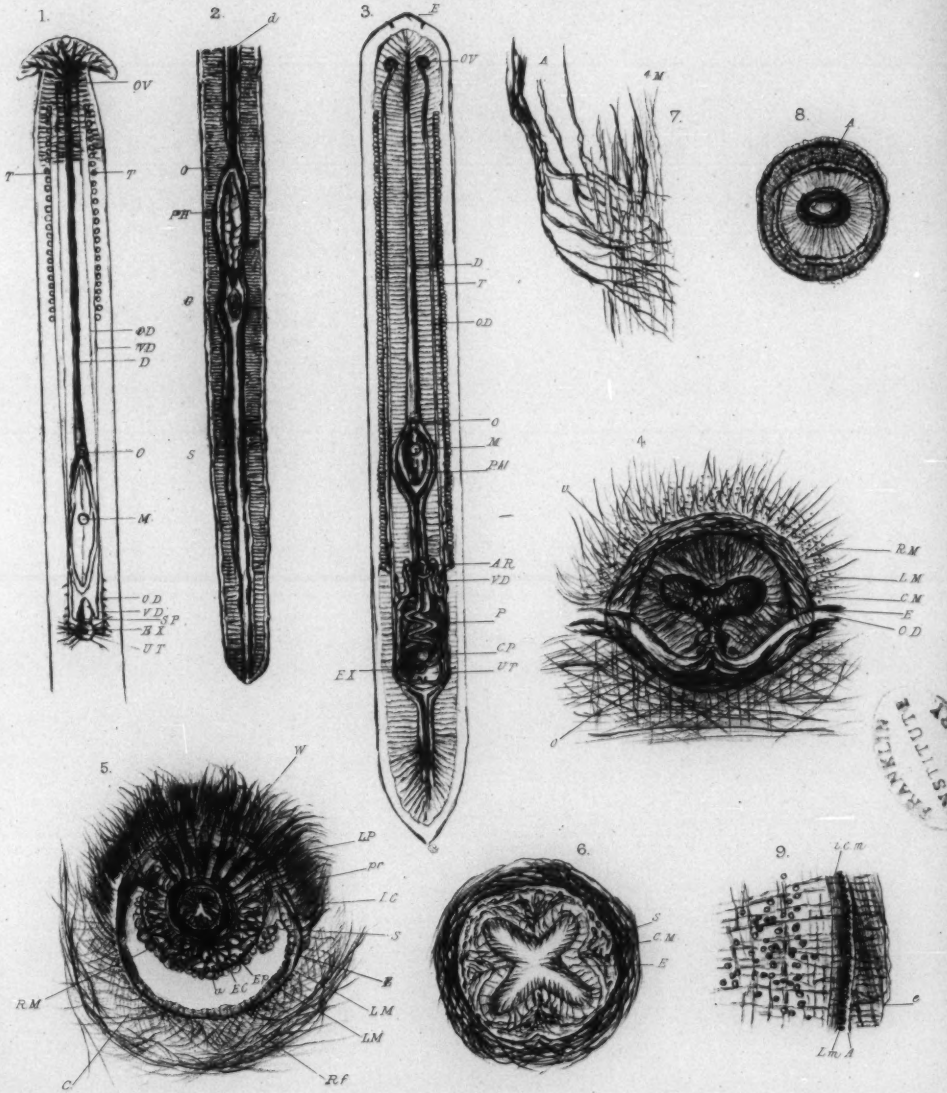

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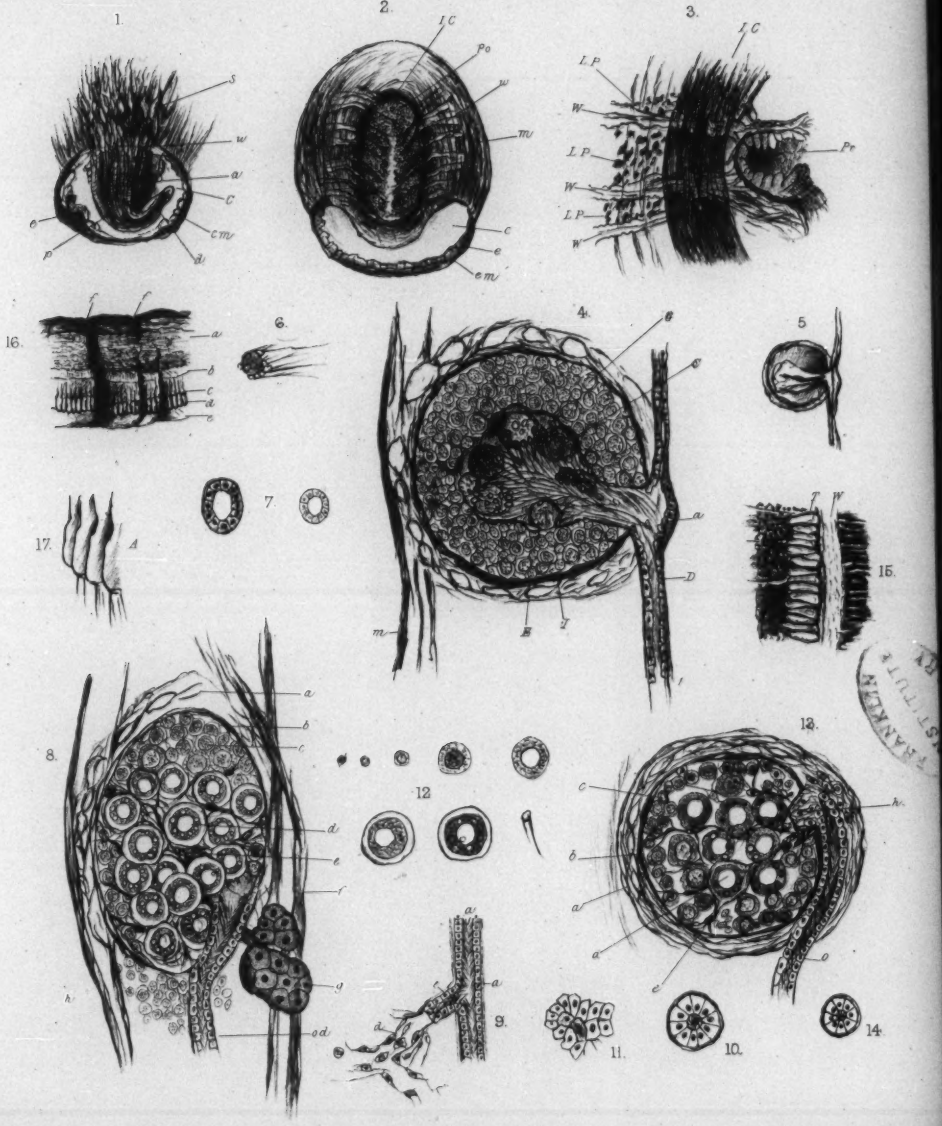
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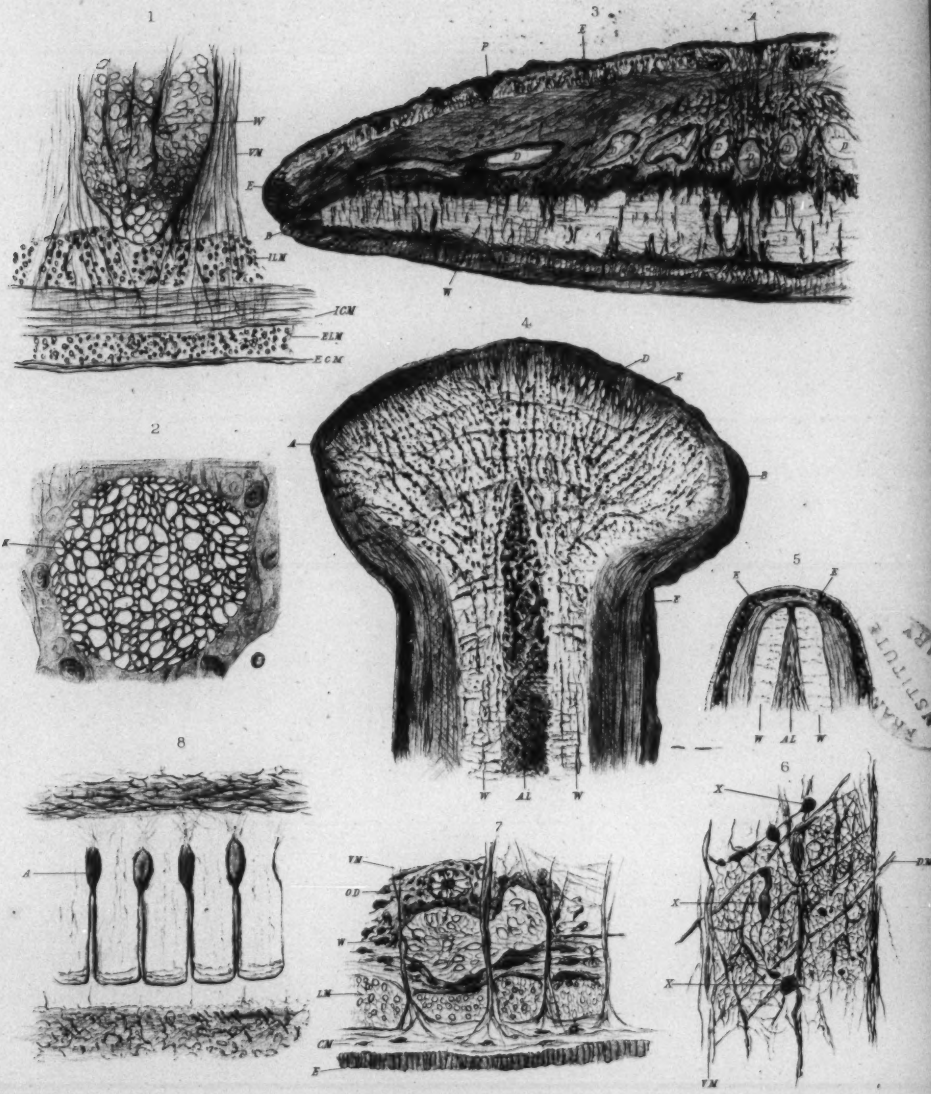
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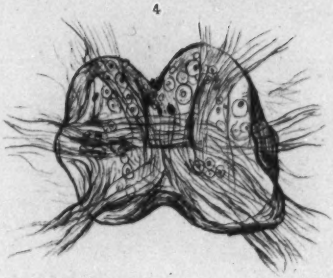
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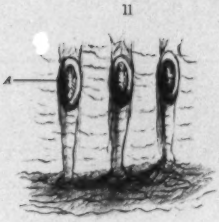
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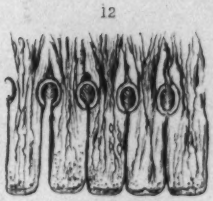
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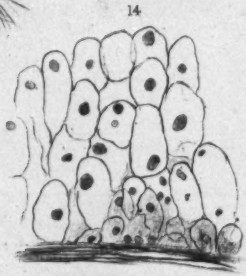
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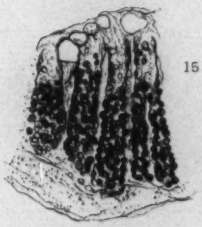
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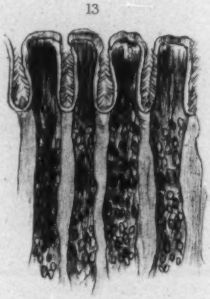
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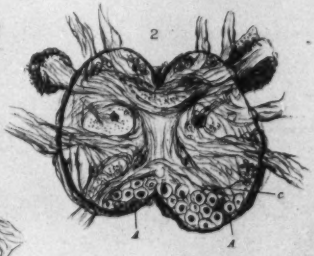
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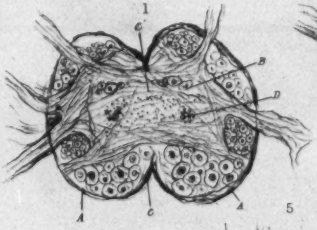
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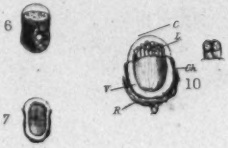
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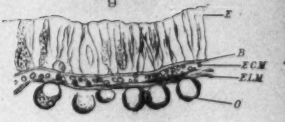
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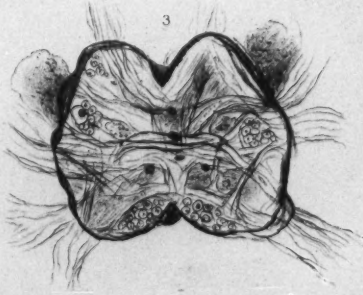
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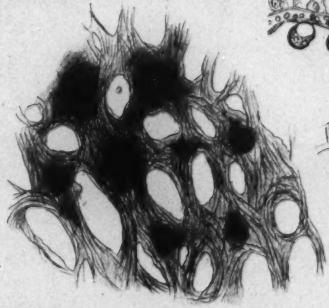

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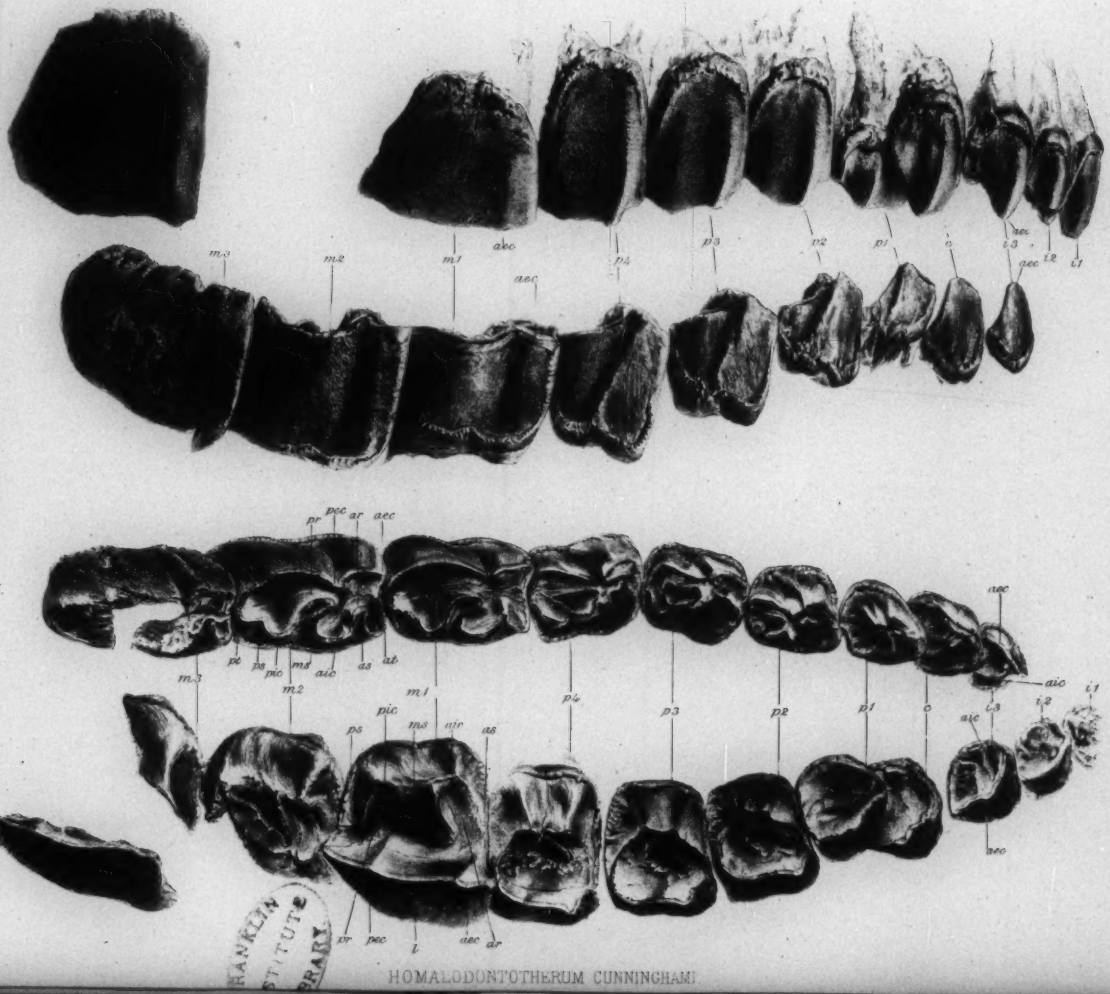

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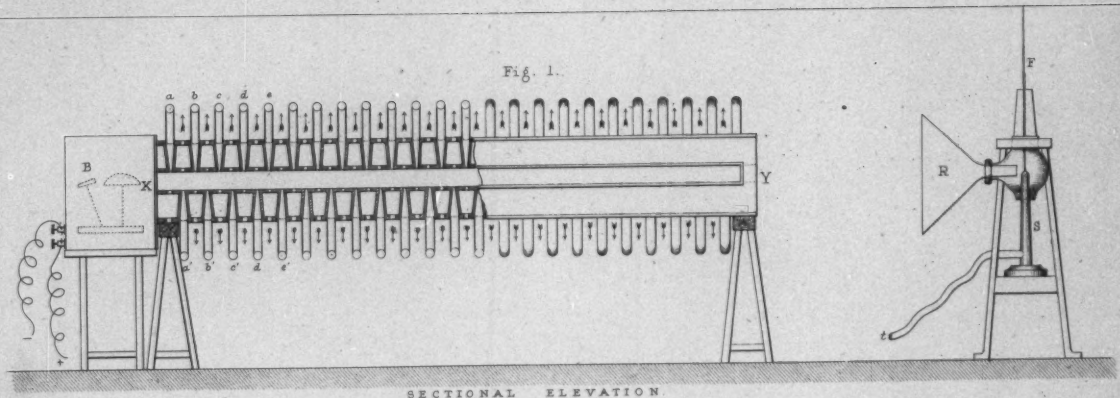
W.H. Mosley, ad nat. lith.
Mclure & Macdonald, Lith. London.
Flower.

KENT
NORTH GODWIN
SOUTH GODWIN
GOODWIN SANDS
THE VARNE
VARNE RIDGE
Telegraph Cable Dover to Calais
The distance between each Circle is one Nautical mile
Sandwich
Deal
Walmer
Kingsdown
Ramsgate
Margate
Broadstairs
Dover
Folkestone
Sandgate
Hythe
Romney
Dungeness
Boulogne
Calais
Grisnez
Gris Nez
Belle Bouy
Les Etaples
C. Blanc Nez
Dover Cliffs
Calais Cliffs
Varne Buoy
Varne Light Vessel
Ridge Buoy
Ridge Light Vessel
Goodwin Light Vessel
Gull Stream
Brake Sand
Ruckinge Church
Dungeness Point
Dungeness Beacon
Dungeness Light House
Dungeness Point
Dungeness Shoal
Dungeness Rocks
Dungeness Buoy

Tyndall.

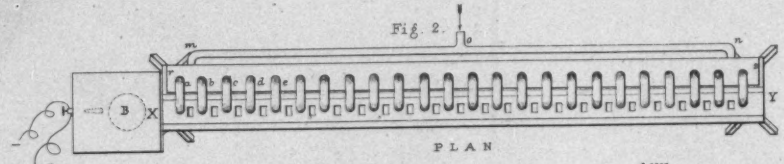
Phil. Trans. MDCCCLXXIV. Plate XVIII.

Fig. 1.



SECTIONAL ELEVATION

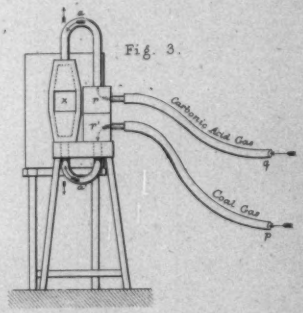
Fig. 2.



PLAN

Scale for Figs 1, 2, & 3.

Fig. 3.



END VIEW.

Scale ½ inch = 1 Foot.

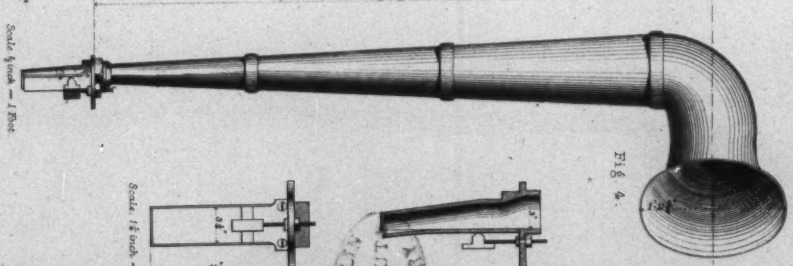
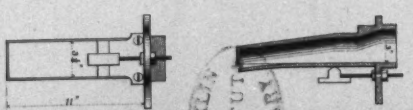


Fig. 4.

Scale ½ inch = 1 Foot.



W. West & Co. lith.

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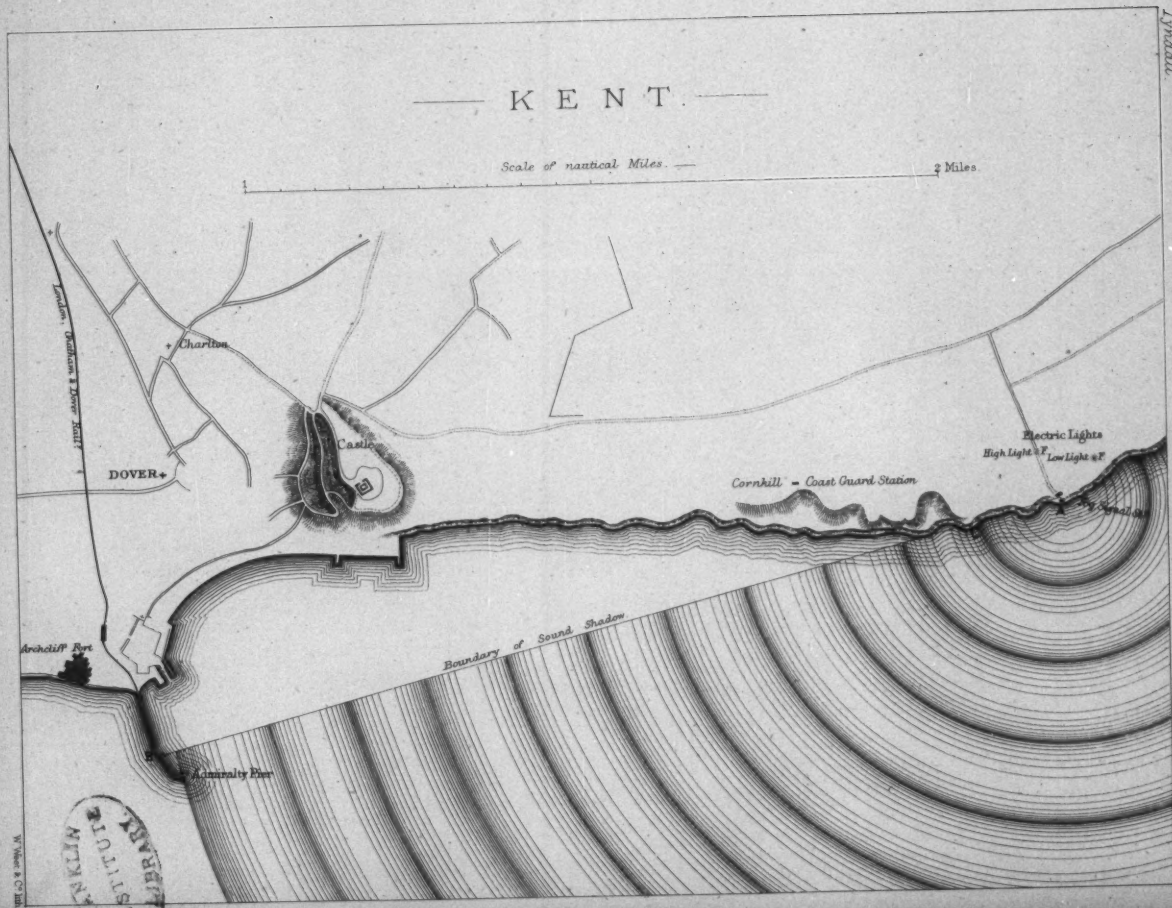
Scale of nautical Miles.
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2 Miles.
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Charlton
DOVER
Castle
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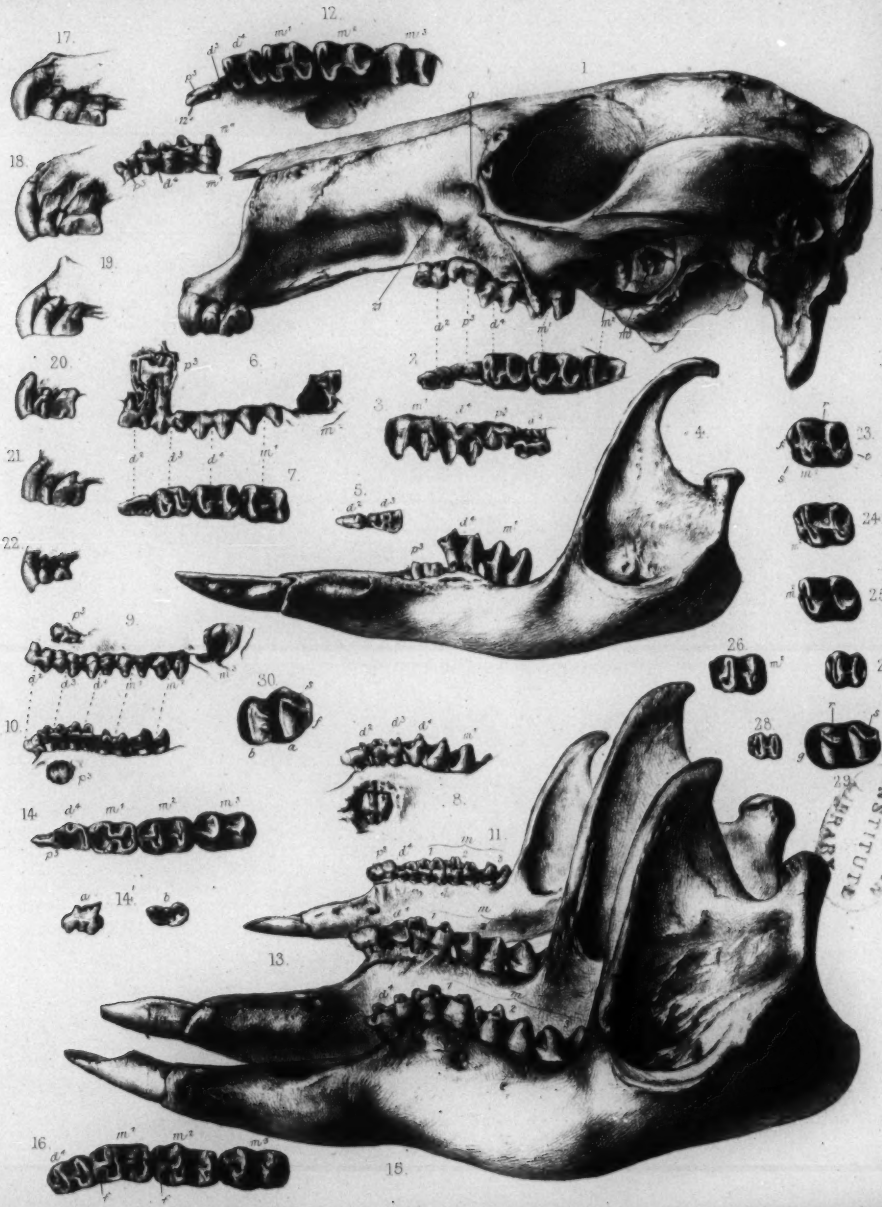
High Light F.
Low Light F.
Cornhill = Coast Guard Station
Boundary of Sound Shadow
Archcliff Fort
Admiralty Pier



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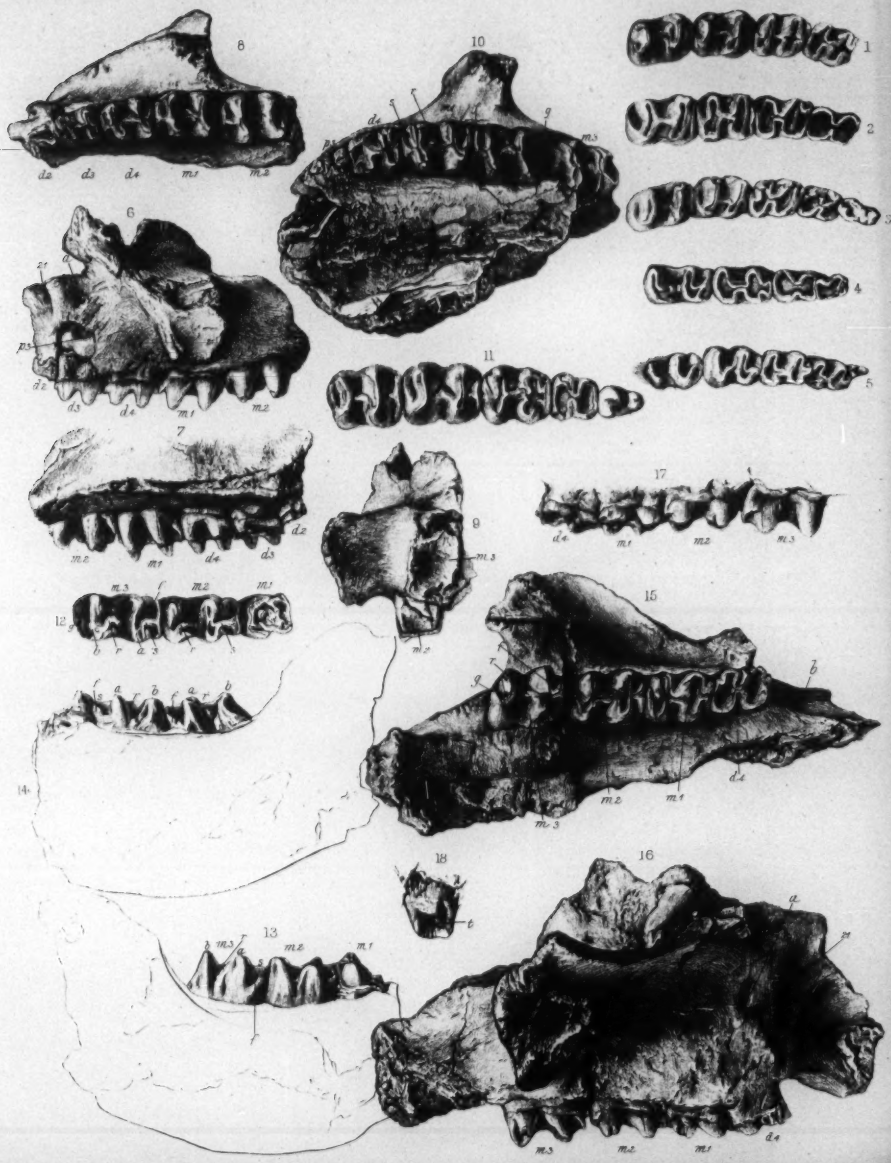
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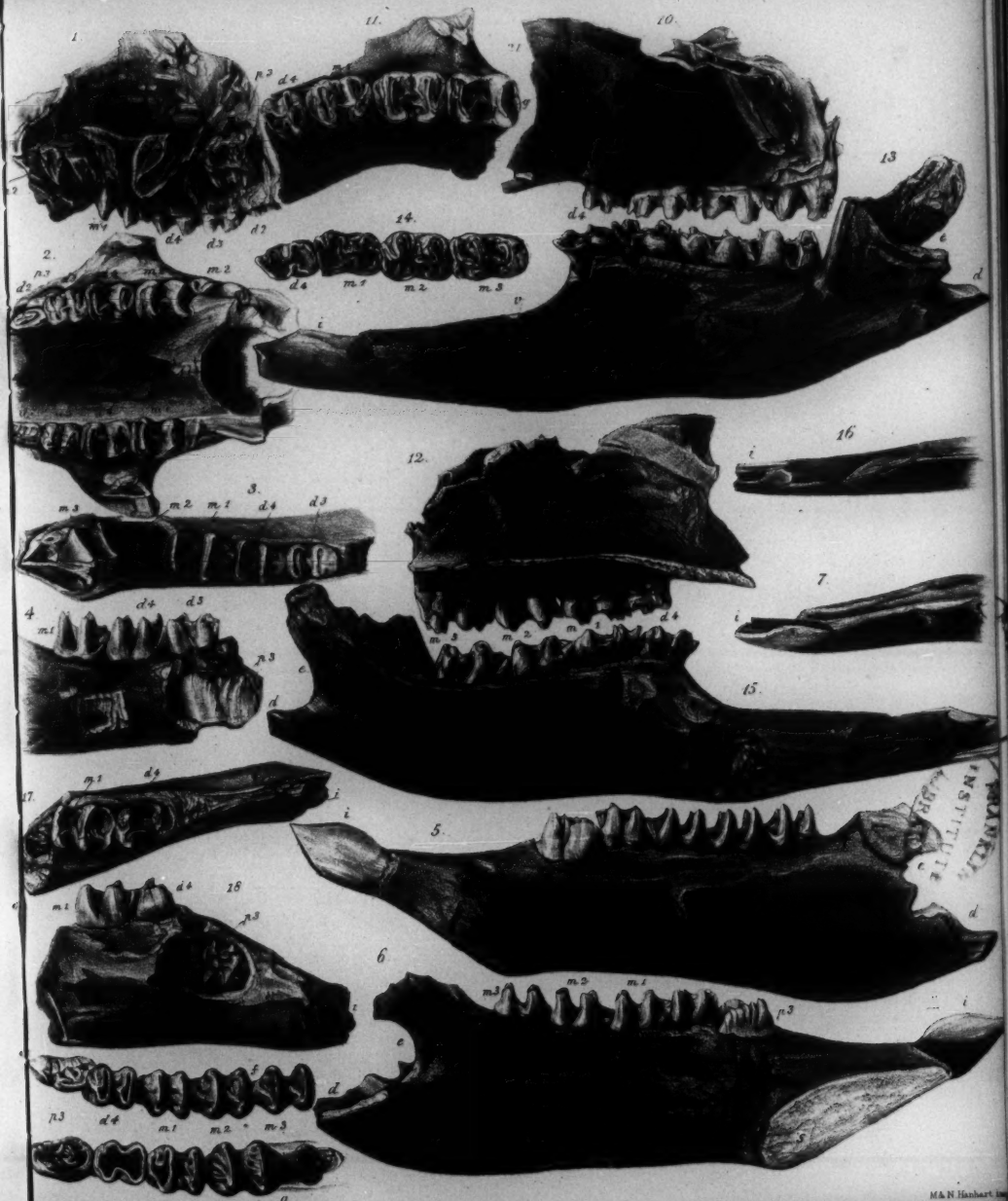

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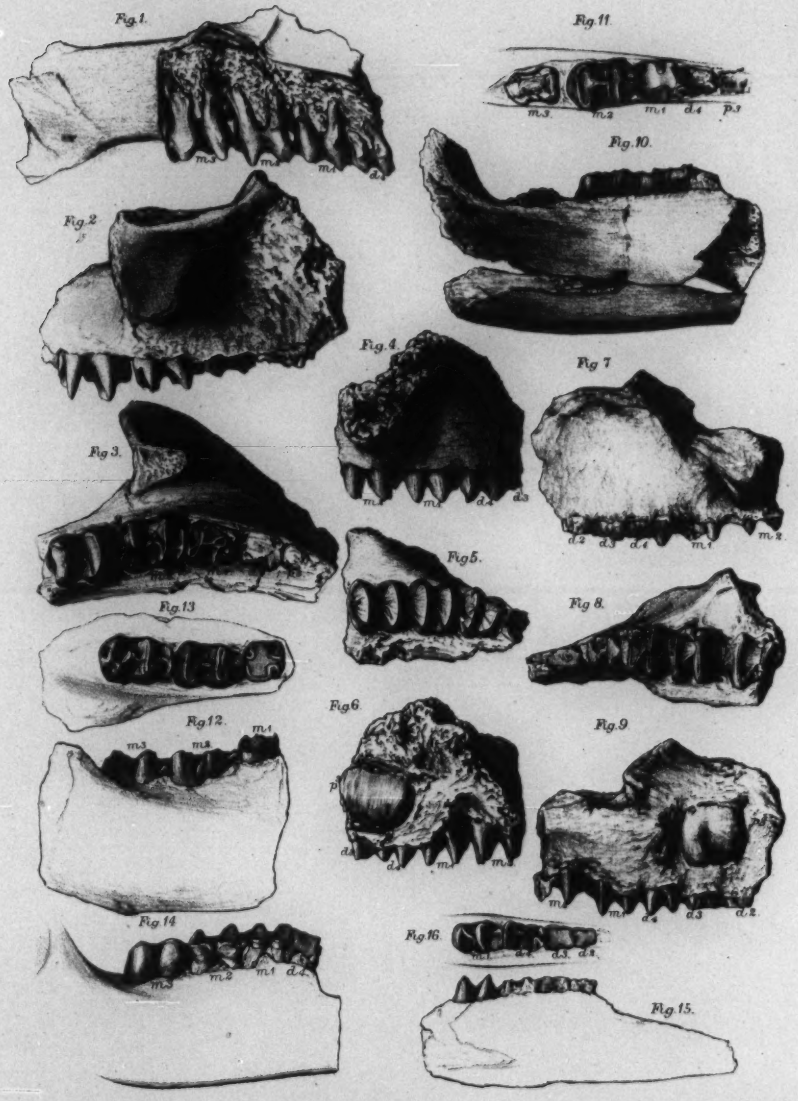
Drawn and on Stone by J. Erxleben.



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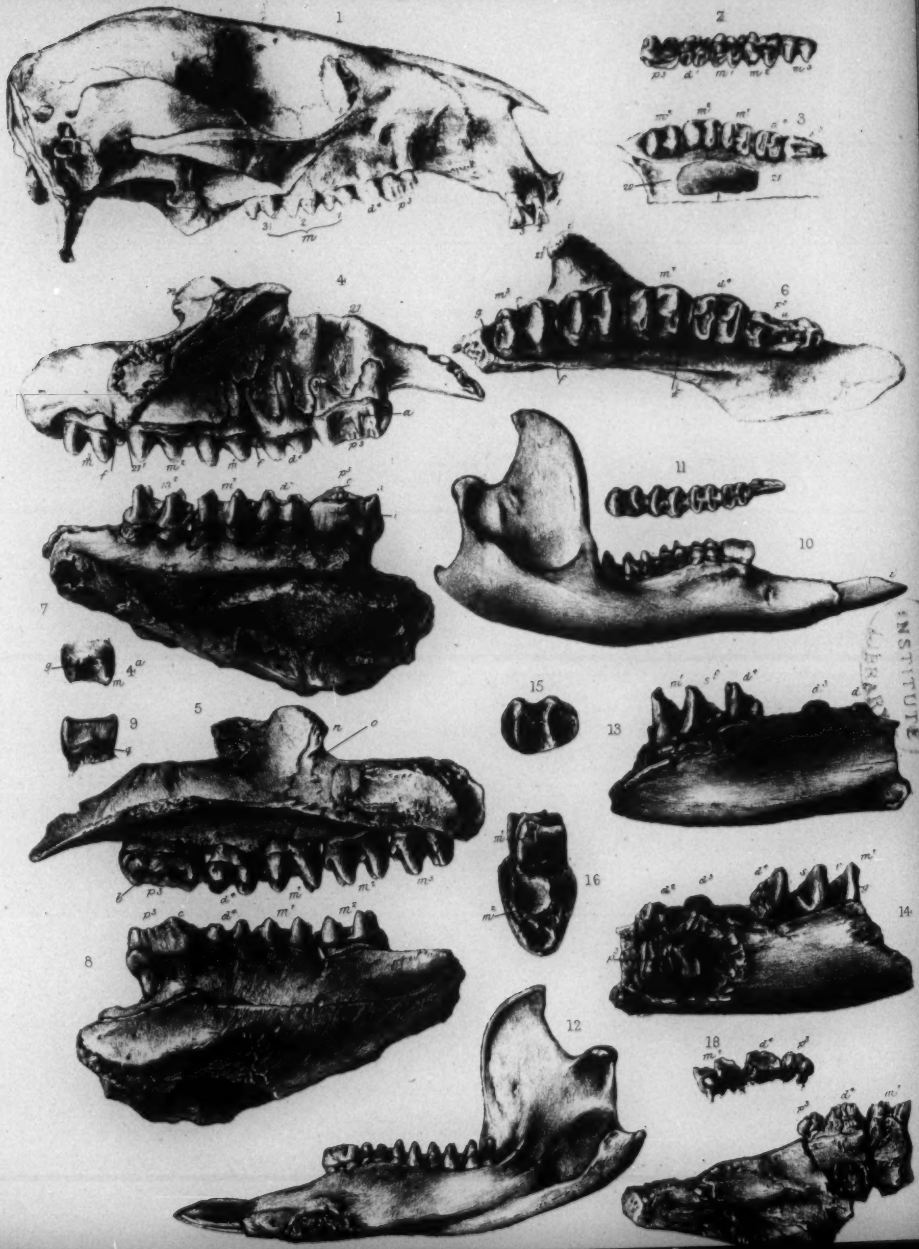
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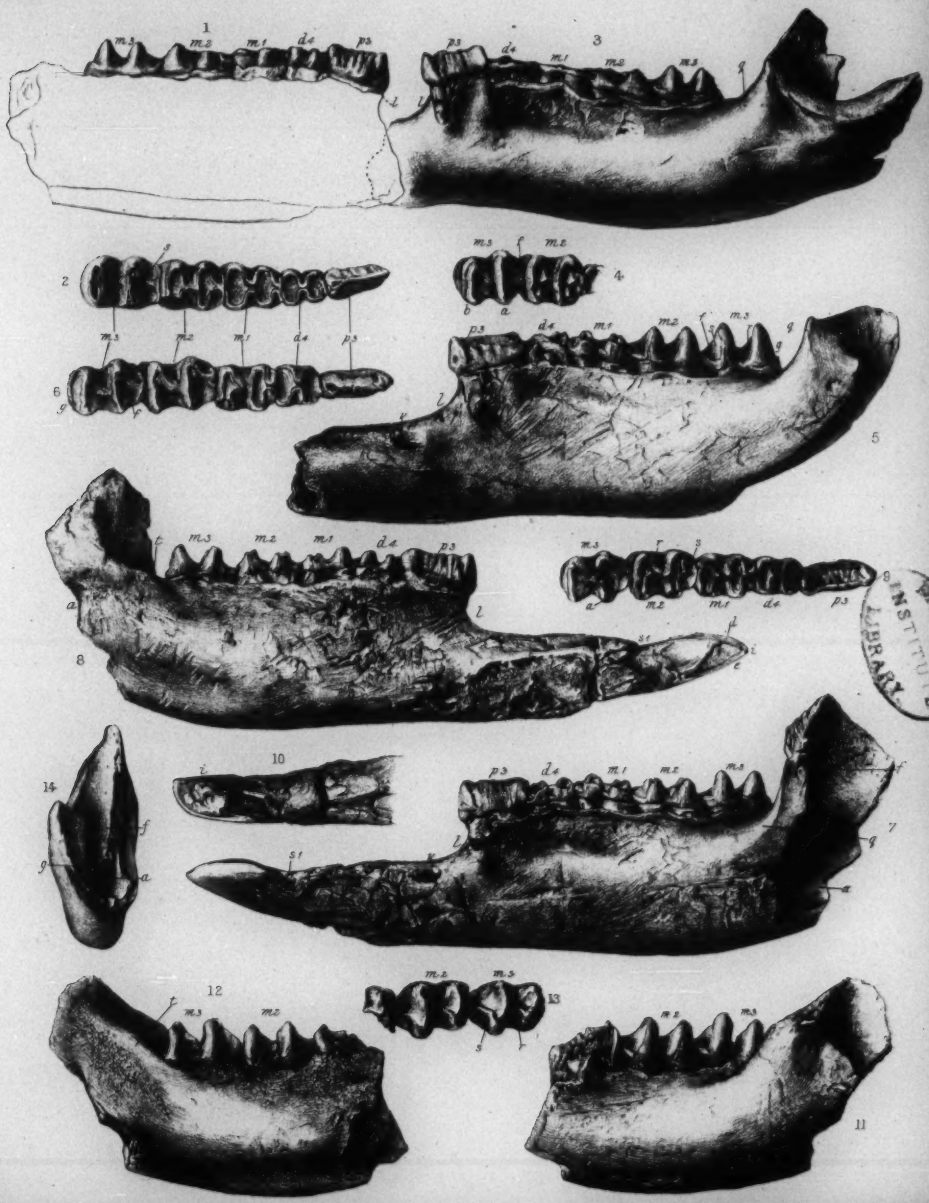

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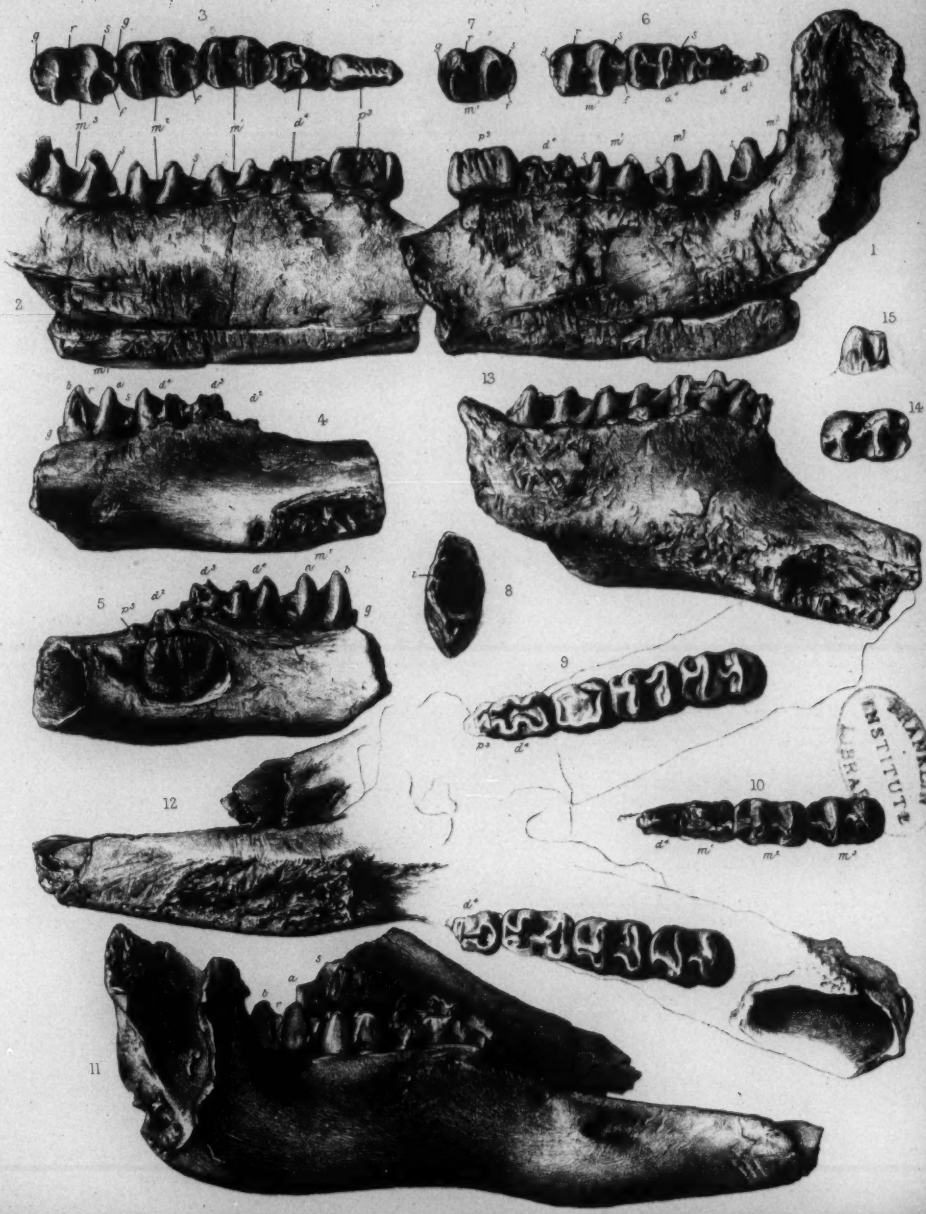
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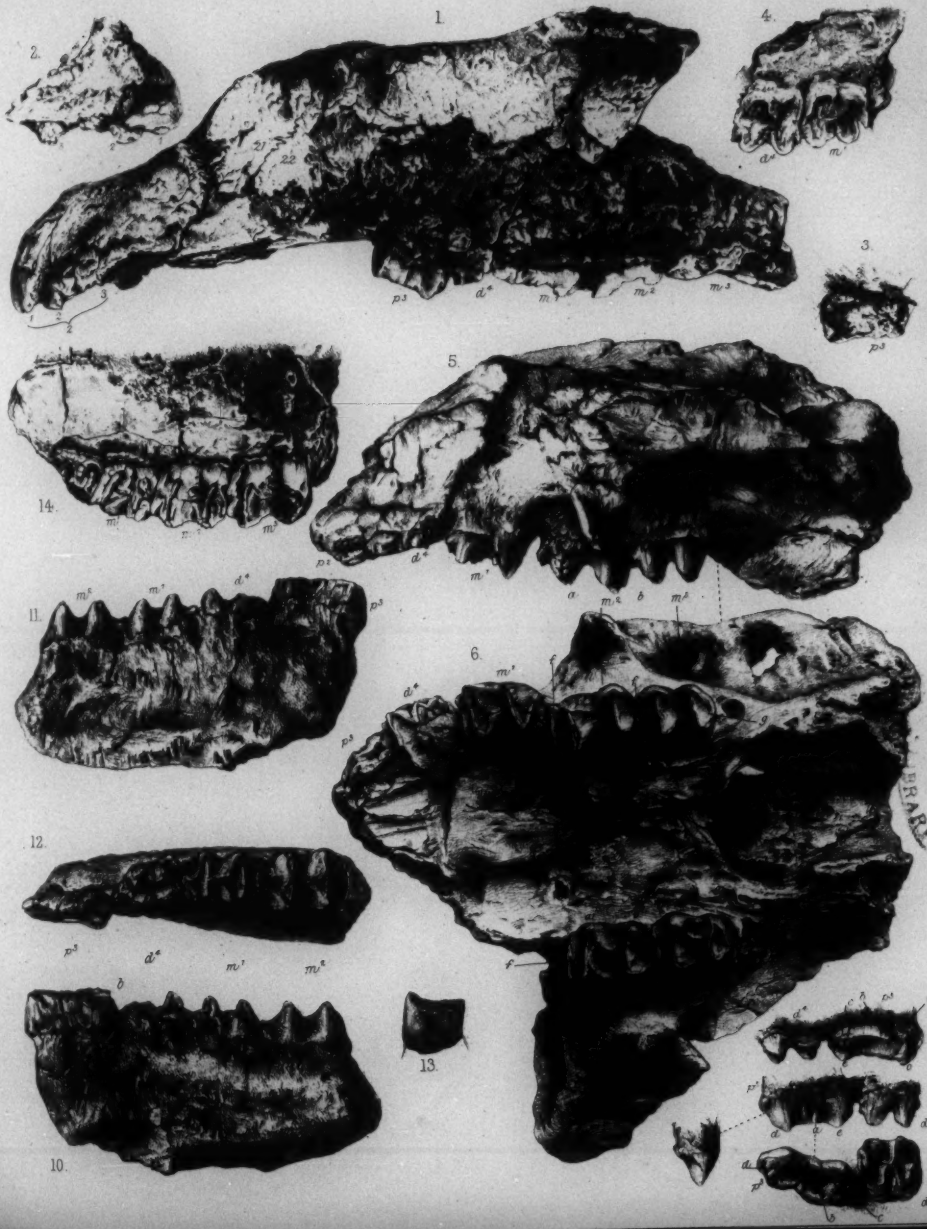
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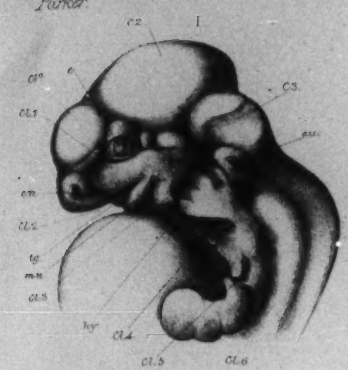

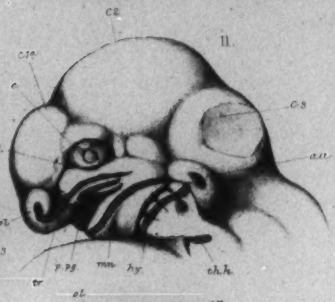

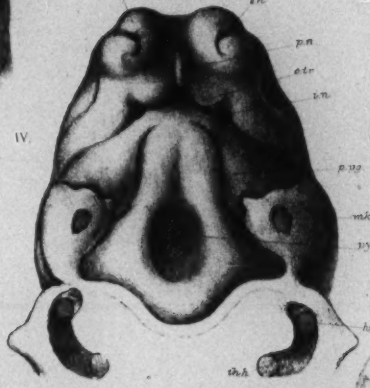

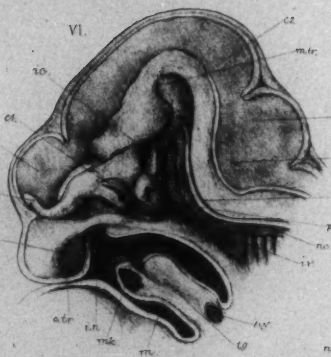

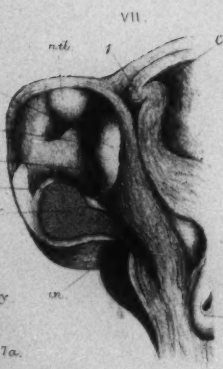

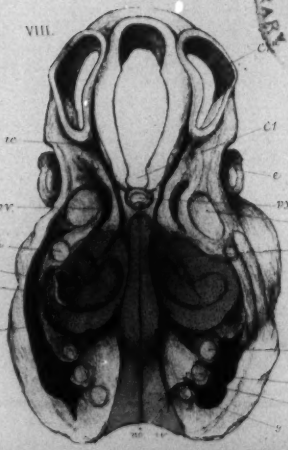

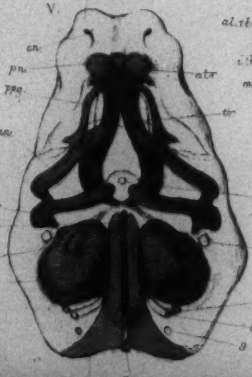

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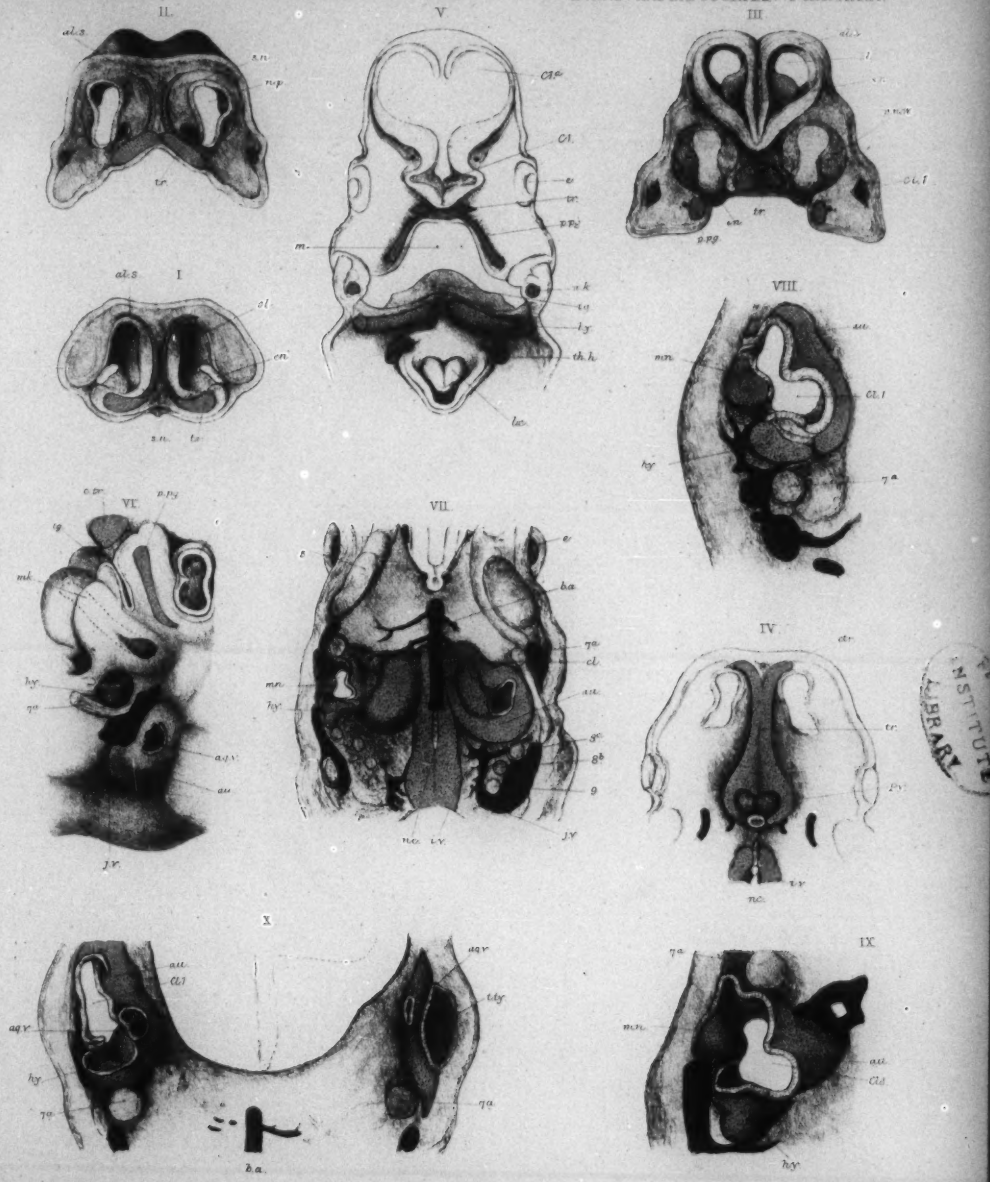

W.K.P. del. ad. nat. & West. lith.

W. West & Co. imp.

Pig's Skull 1st Stage.

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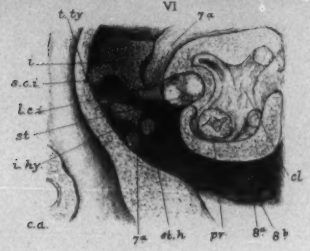

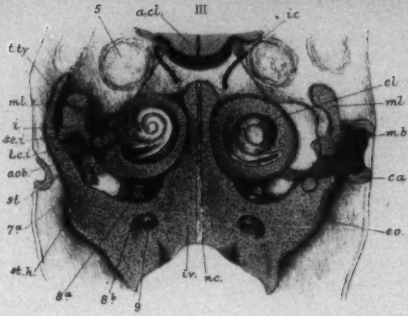

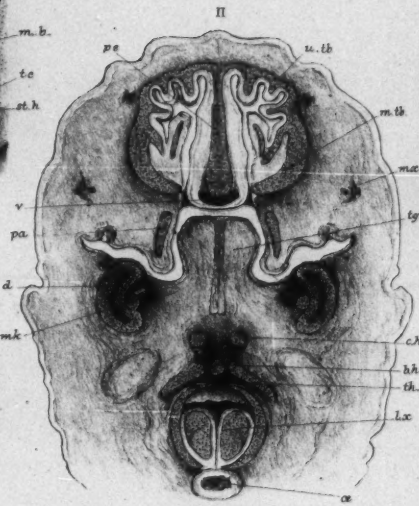

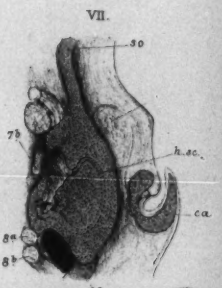

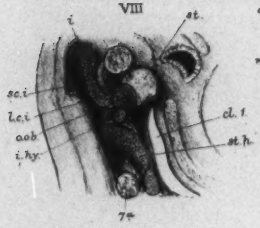

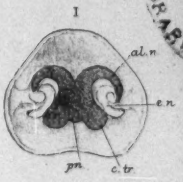

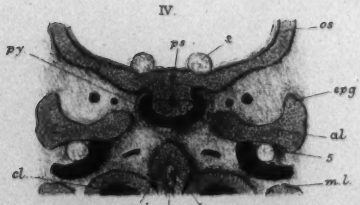

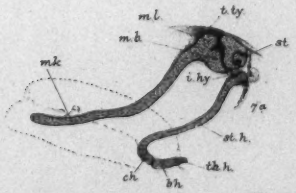

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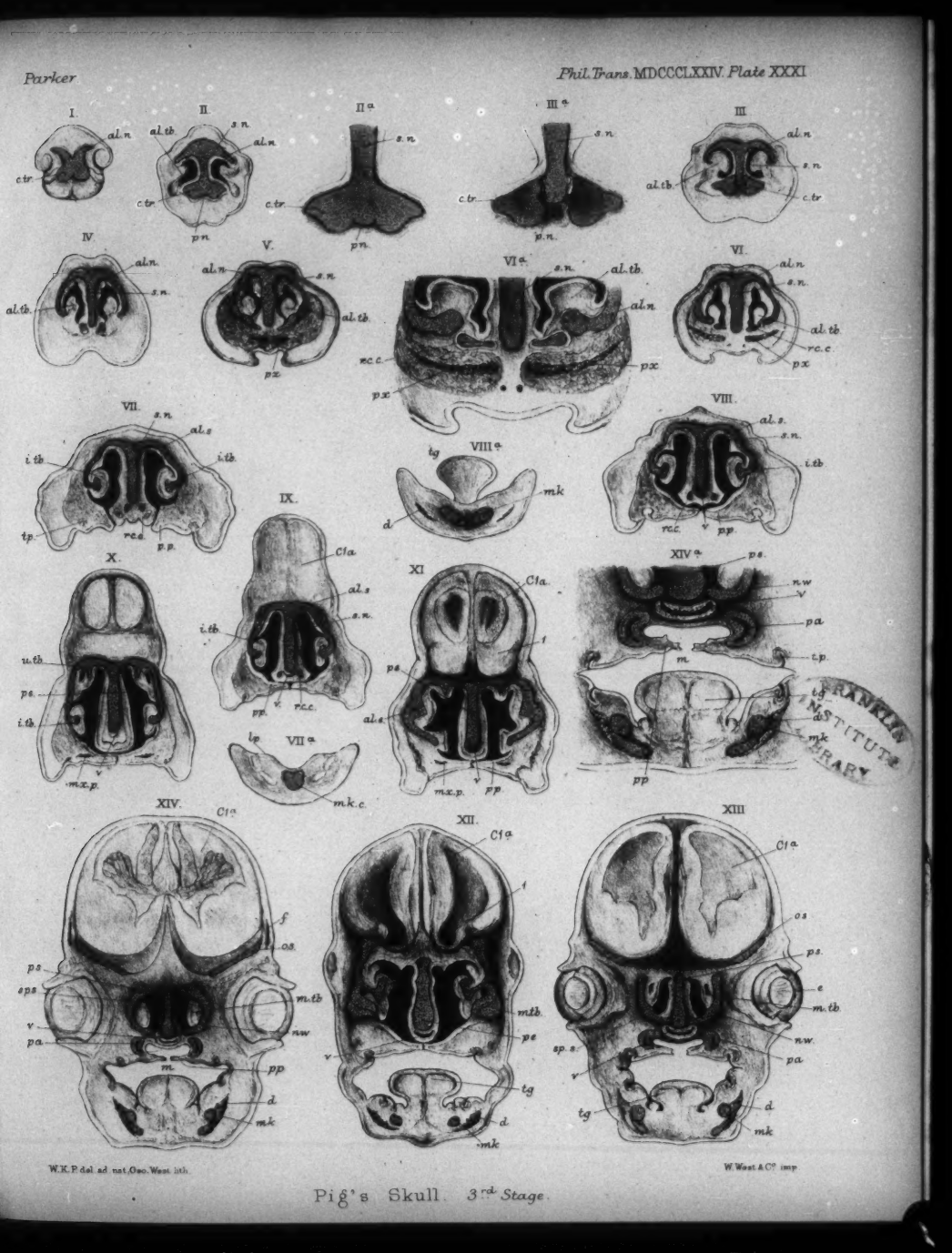
W. West & Co. imp.

Pig's Skull.

Figs. 1—7. 2nd Stage. Figs. 8, 9. 3rd Stage.



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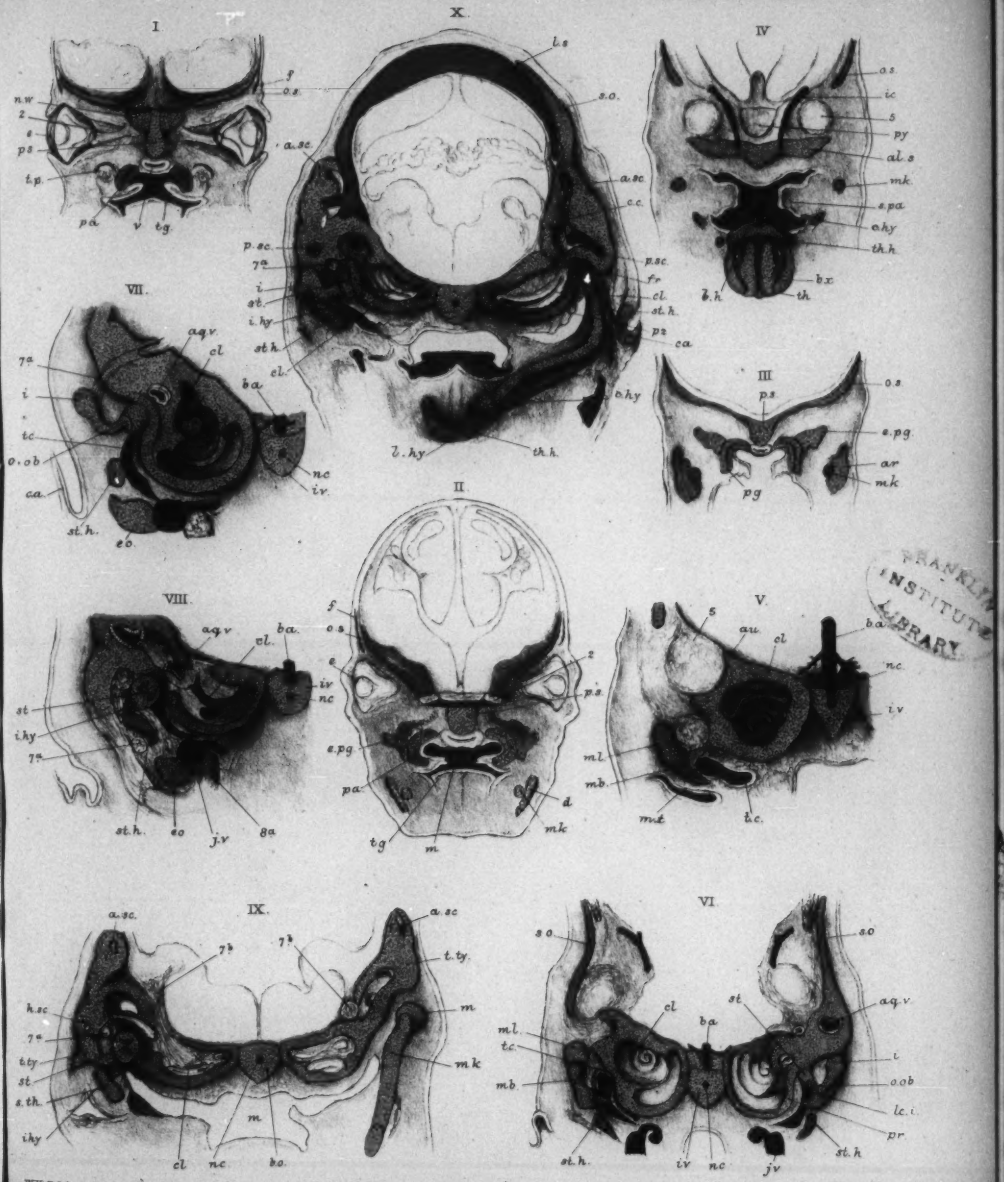
W.K.P. del ad nat. Oec. Wast. lith.

W. West & Co. imp

Pig's Skull. 3rd Stage.



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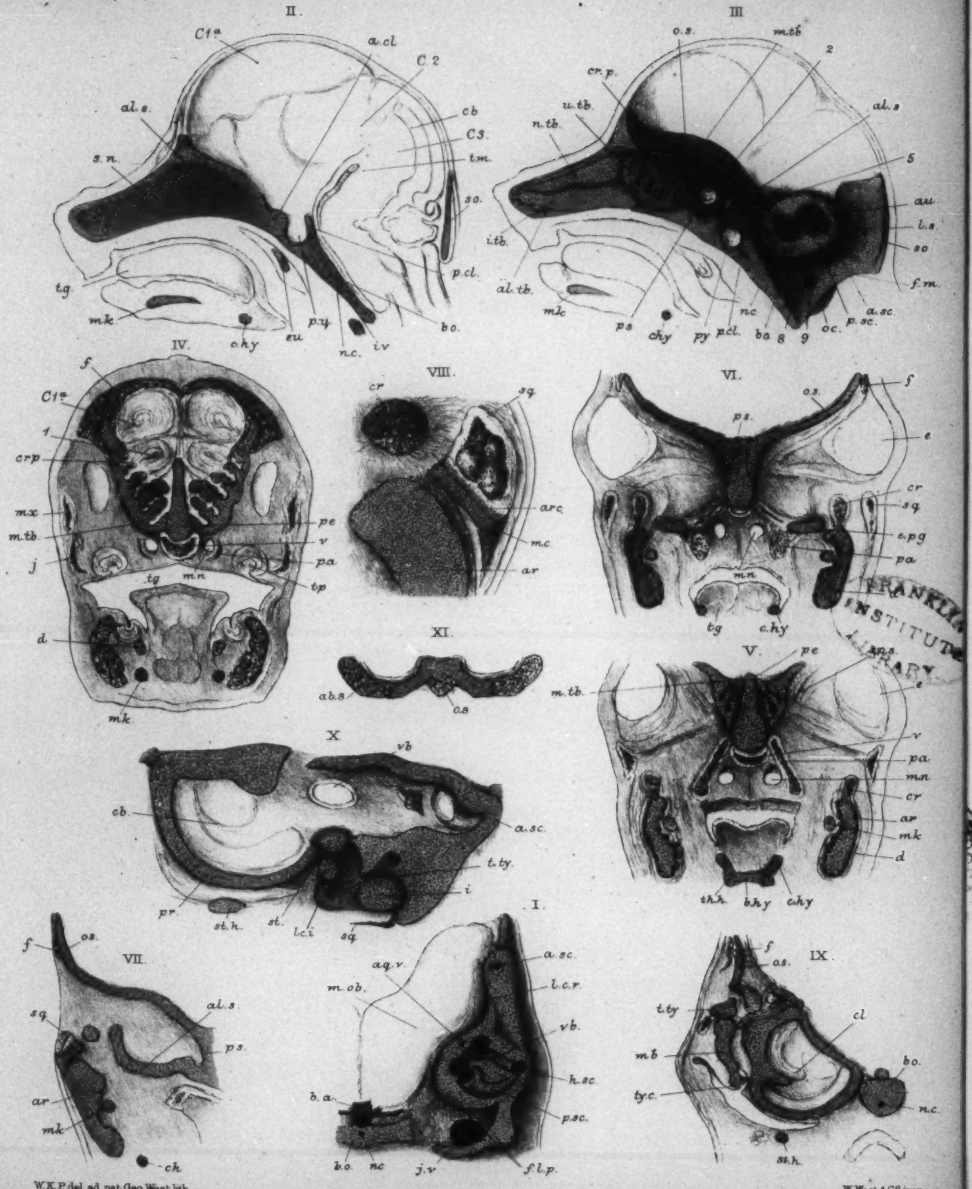
WK. P. del. ad. nat, Geo. West lith.

W. West & Co. imp.

Pig's Skull. 3rd Stage.



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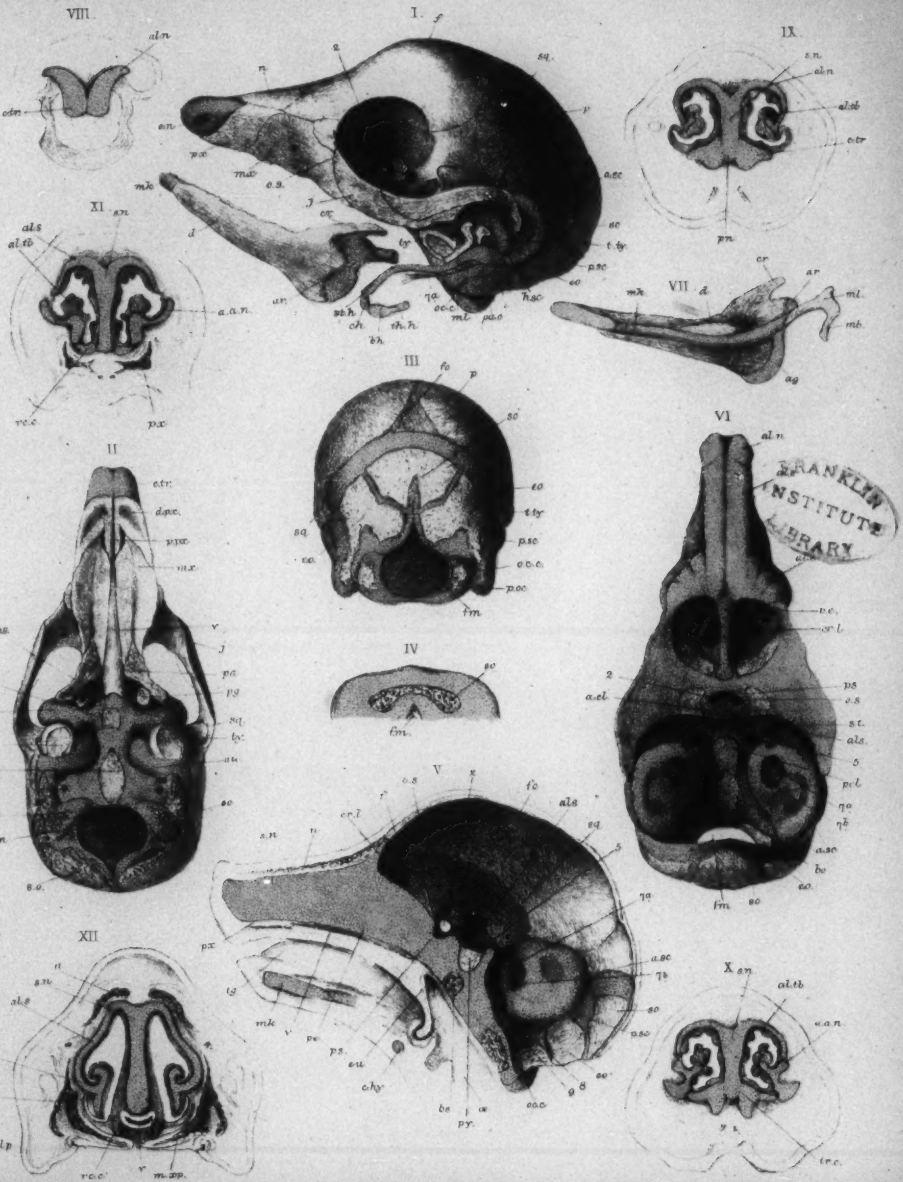
W.K.P. del. ad. nat. Geo. West lith.

W. West & Co. imp.

Pig's Skull.

Figs. 1–3, 3rd Stage. Figs. 4–10, 4th Stage. Fig. 11. 5th Stage.



Pig's Skull 4th Stage.

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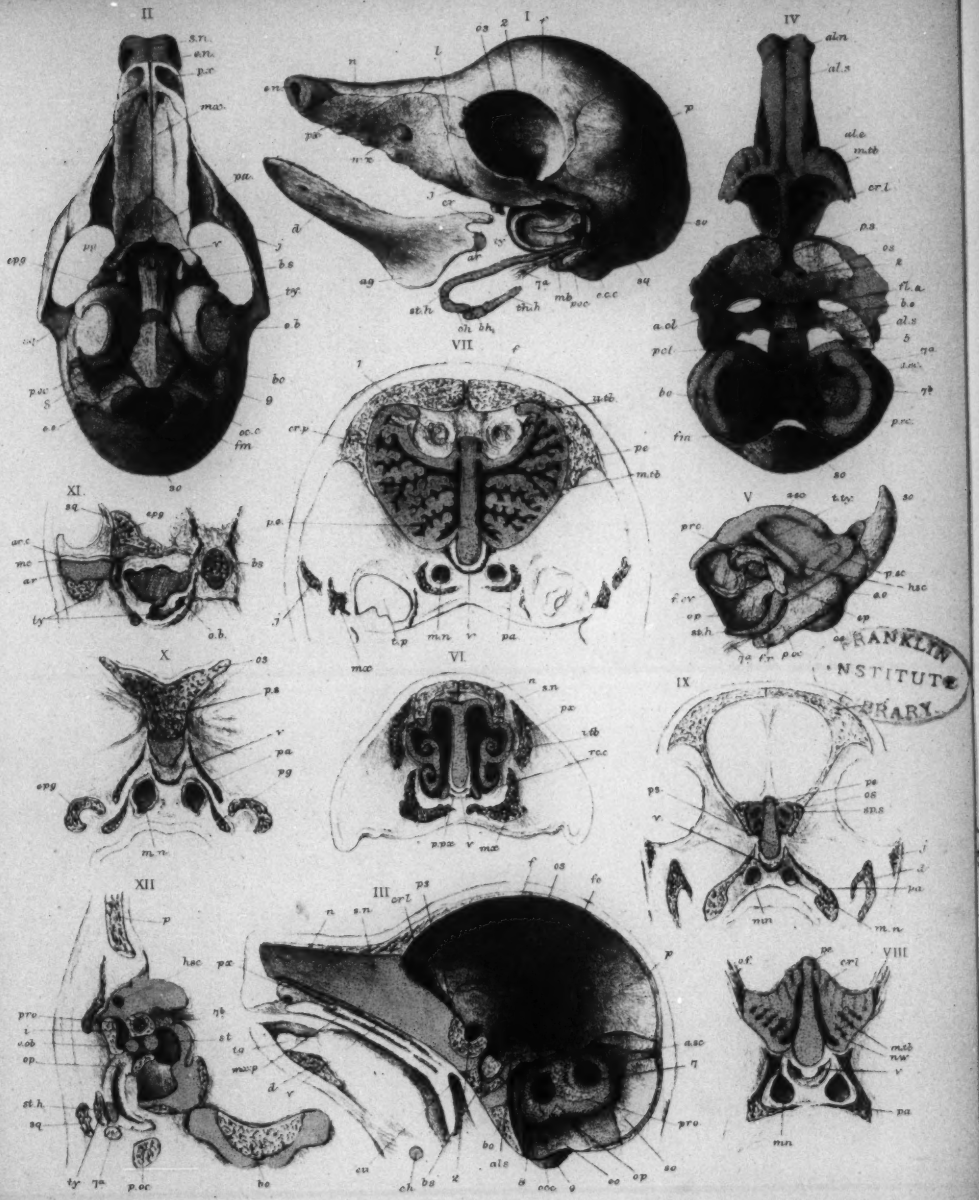
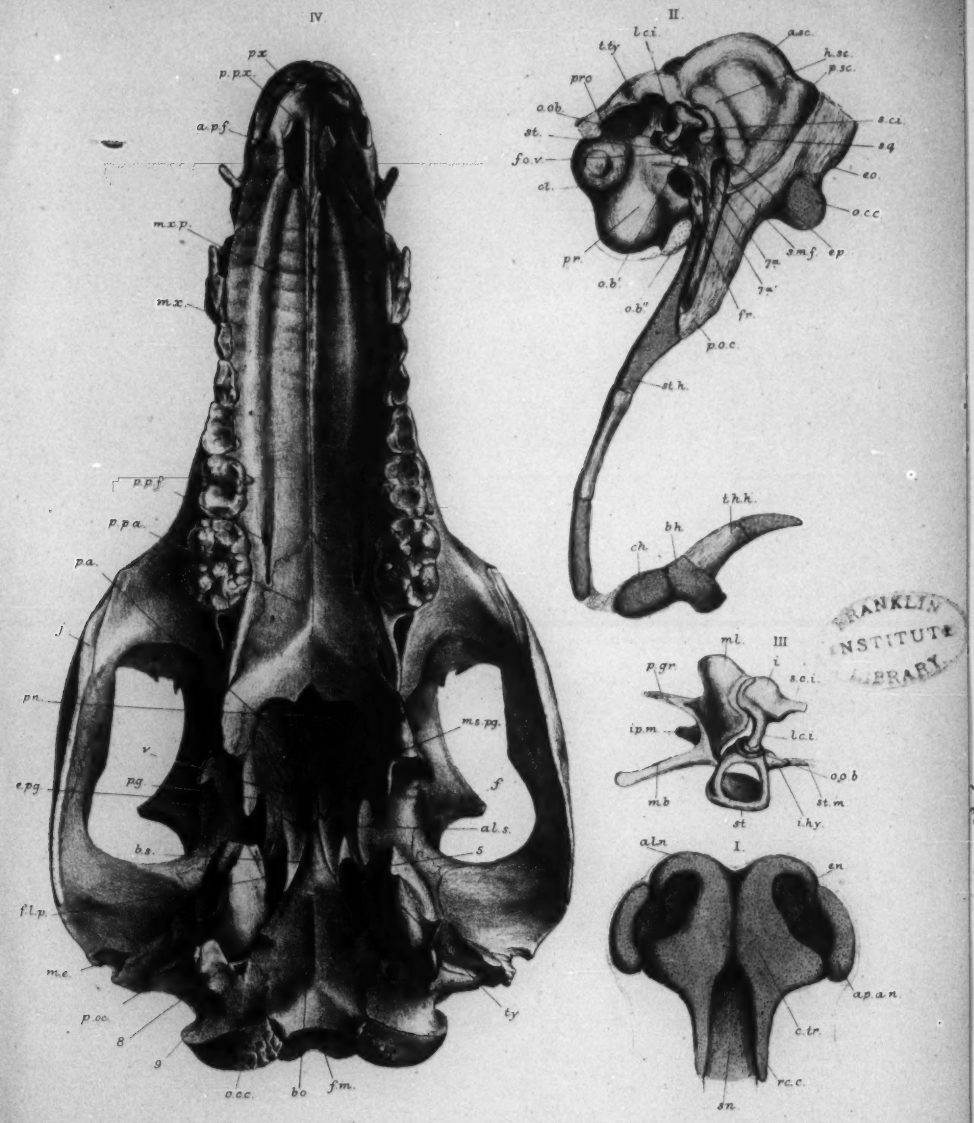
W.K.P. del ad nat. G.West lith. W.West & Co imp.



Pig's Skull. 6th Stage.

W K P. del. ad nat. Geo West lith.

W. West & Co.ª imp.



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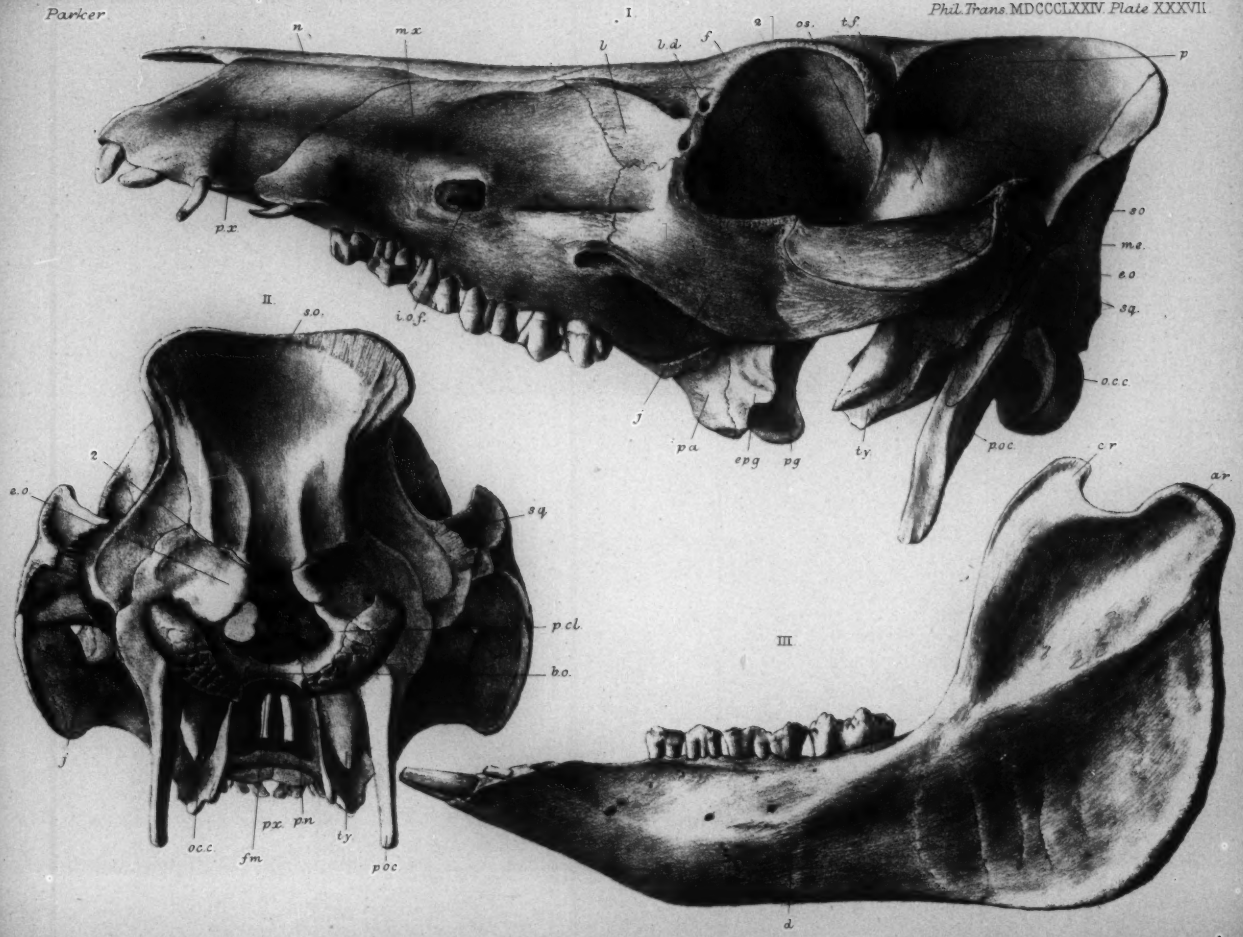
W.K.P. & G.W. del. et lat. nat. G.West lith.

W. West & Co. imp.

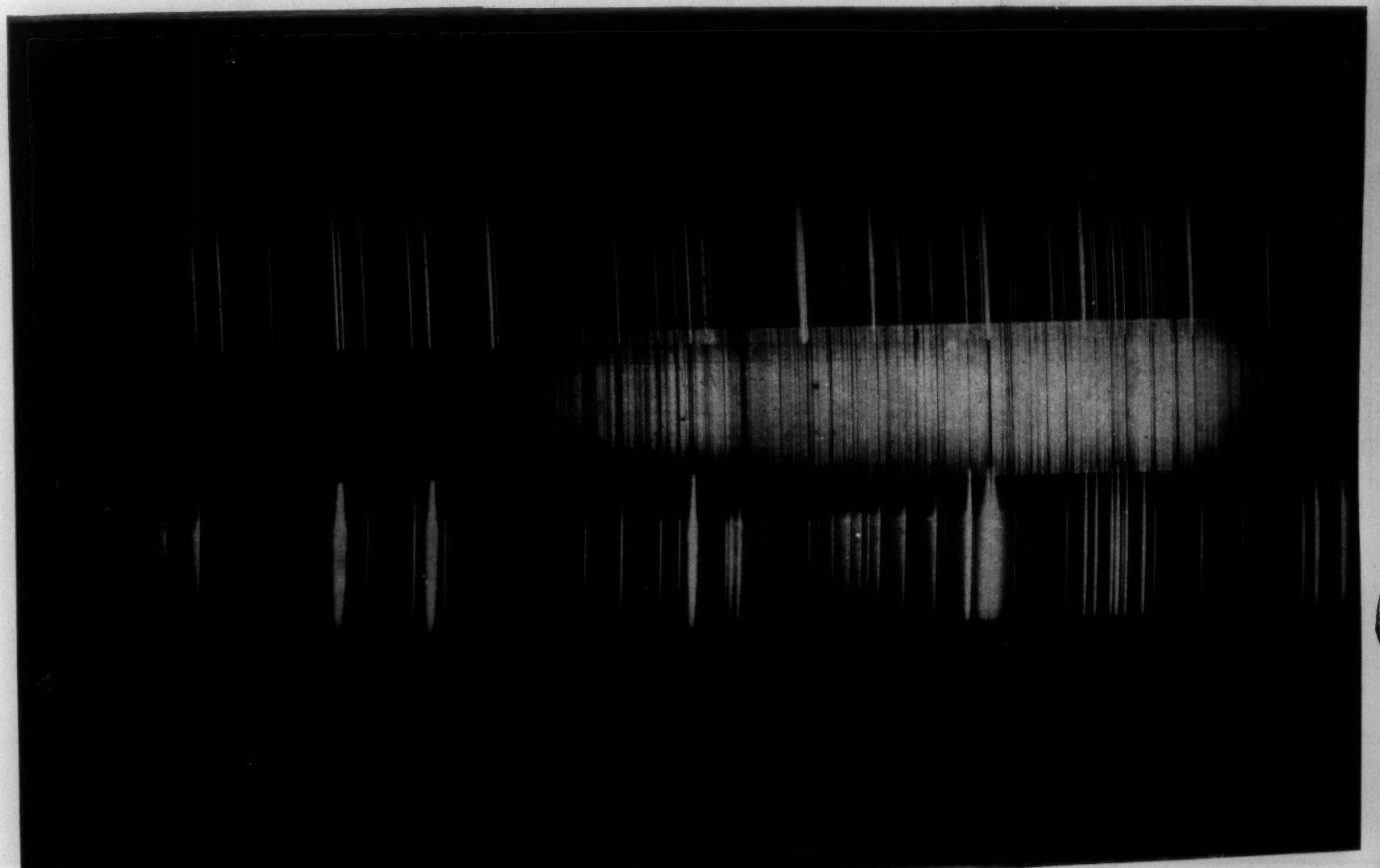
Pig's Skull, 7th & 8th Stages.

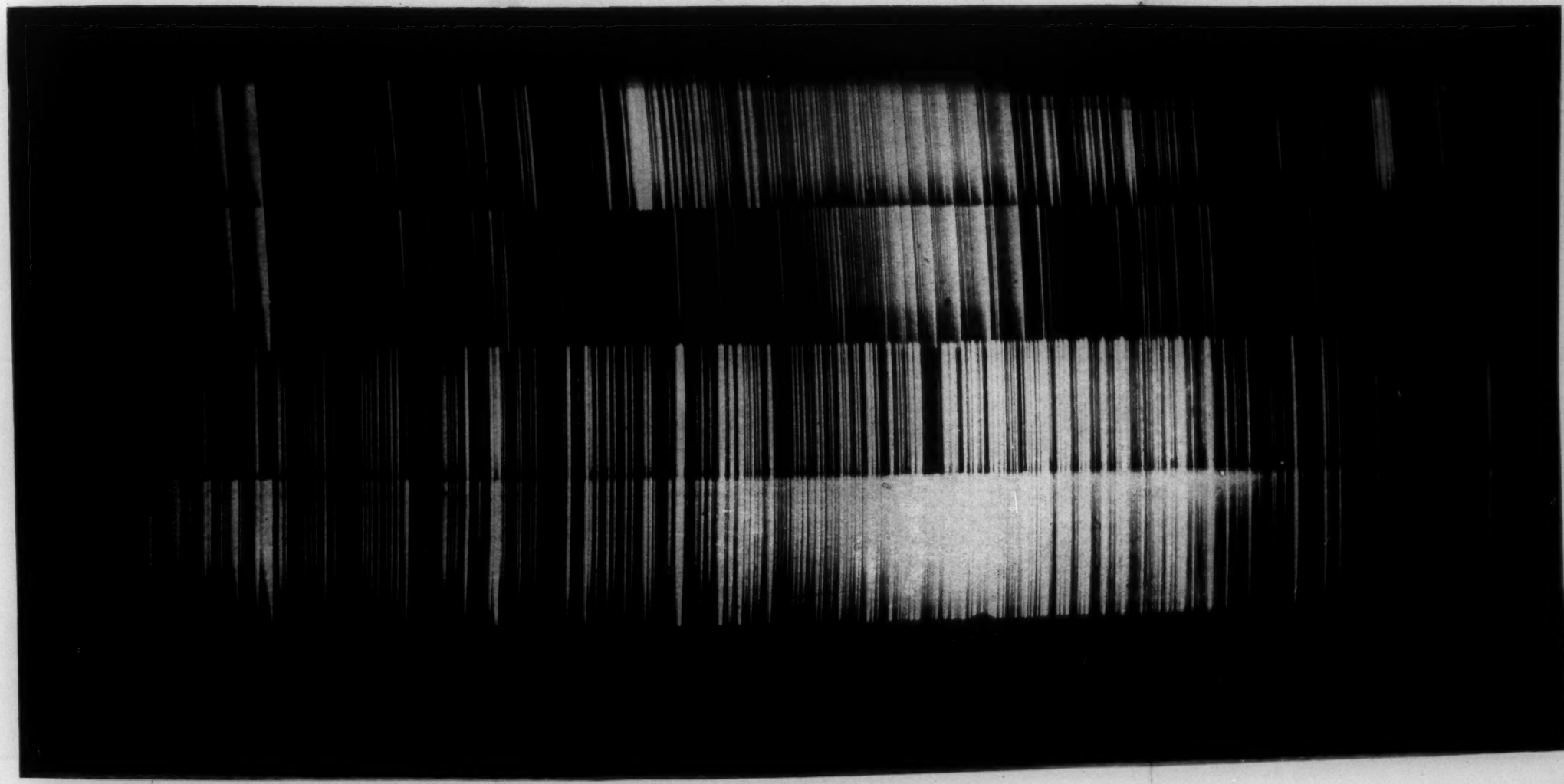


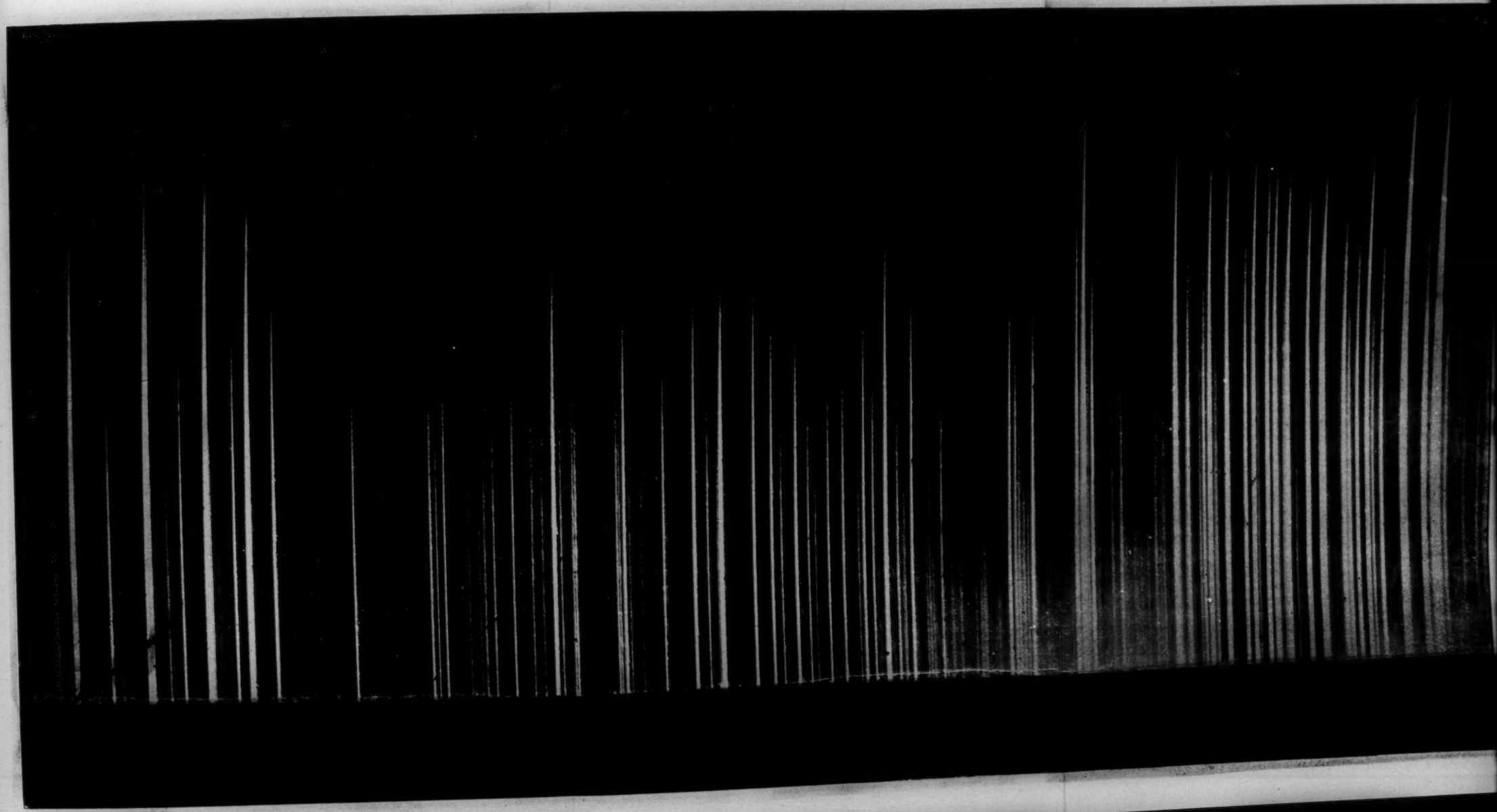
from nat: on Stone by G. West

Pig's Skull. 8th Stage

W. West & Co. Imp.







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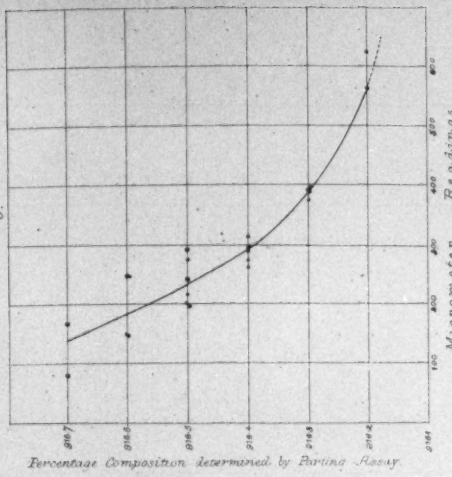

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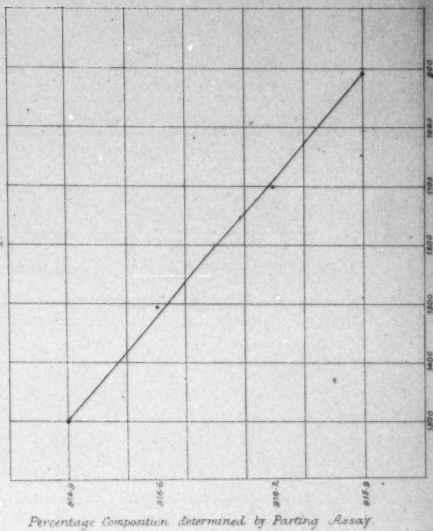

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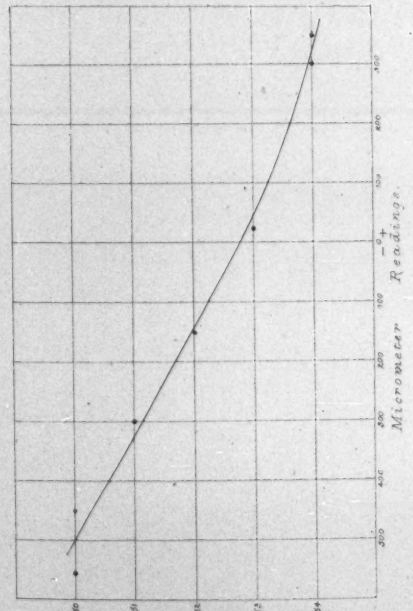

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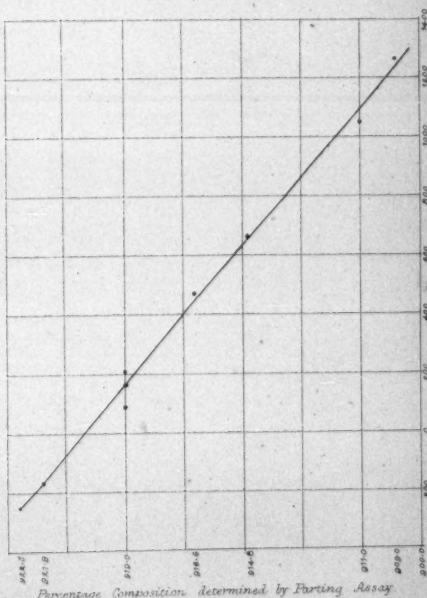
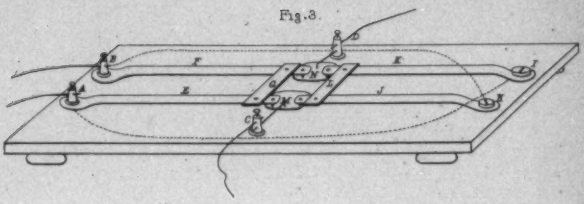


W H. Wesley lith.

W. West & Co. imp.

Fig. 3.



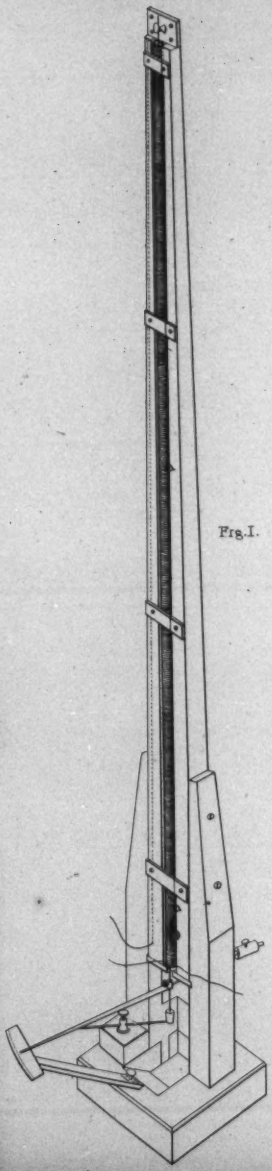
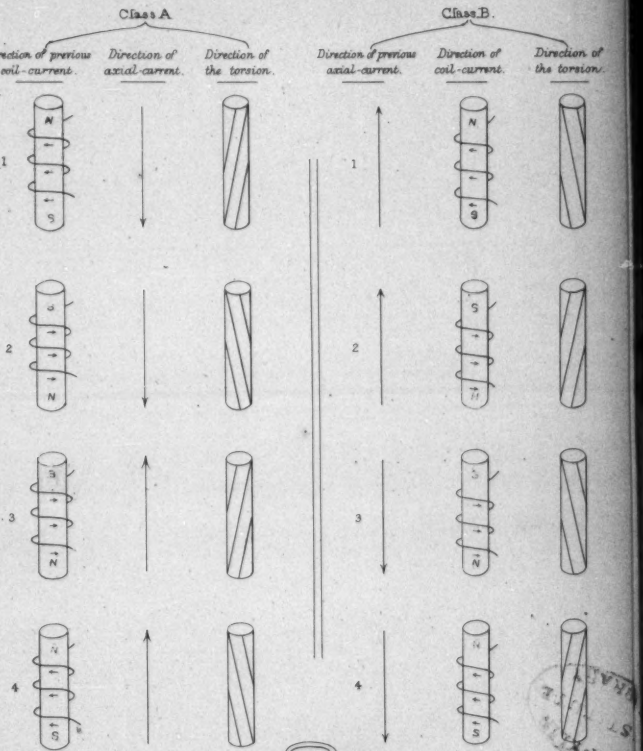


Fig. I.



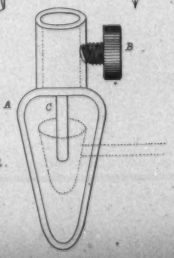


Fig. 2.

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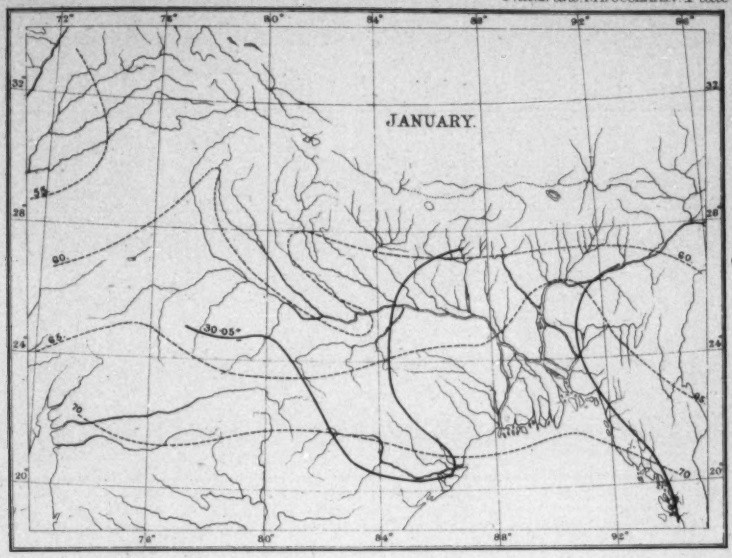

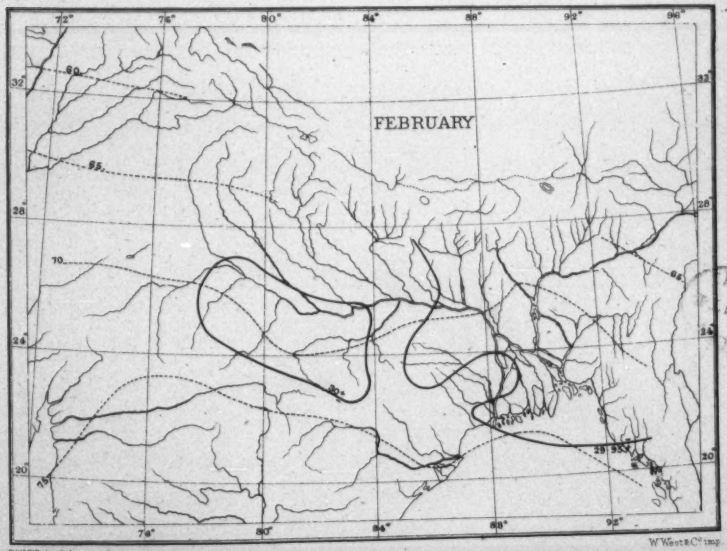

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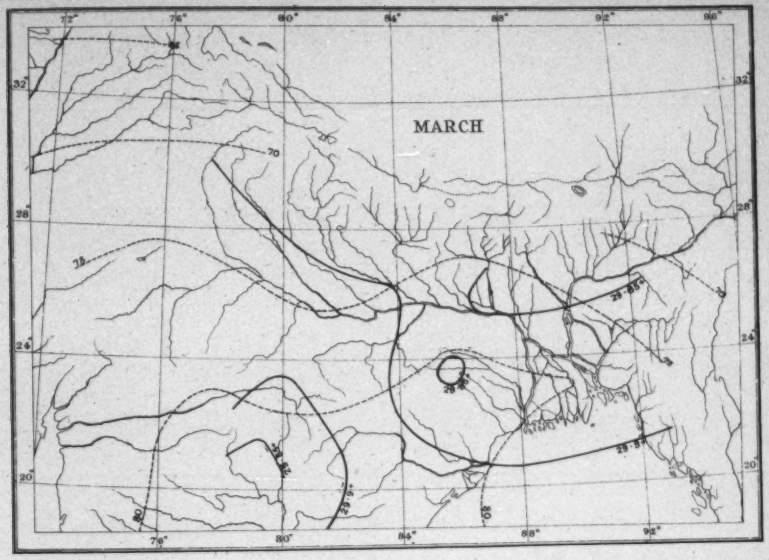
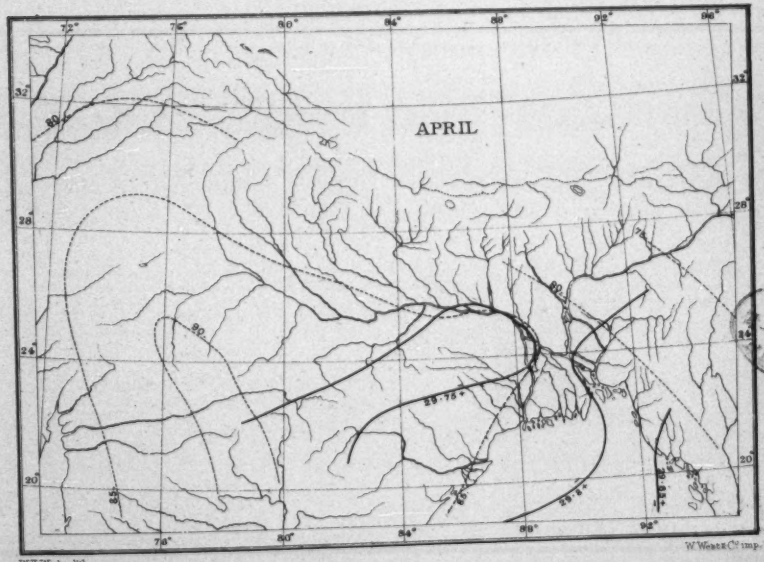
W.H. Wesley lith. W. West & Co. imp.





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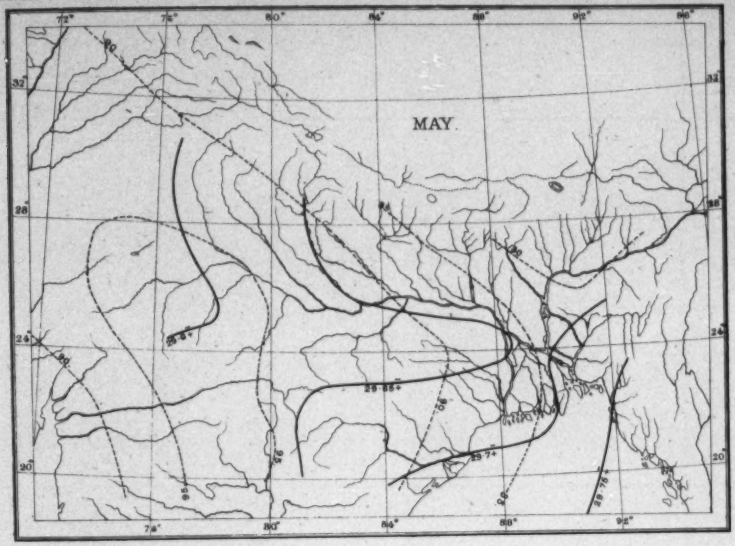

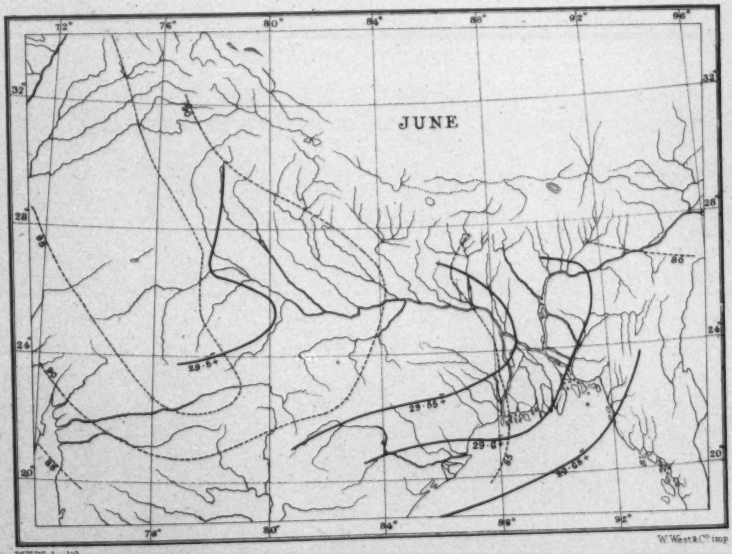

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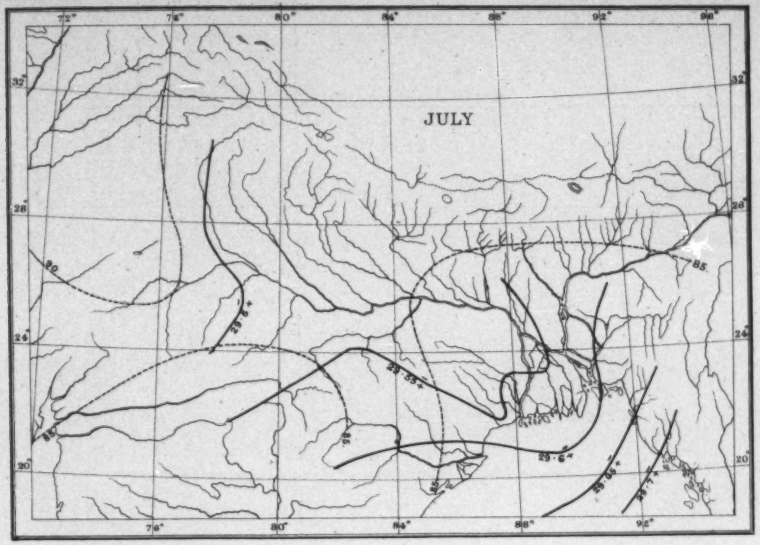

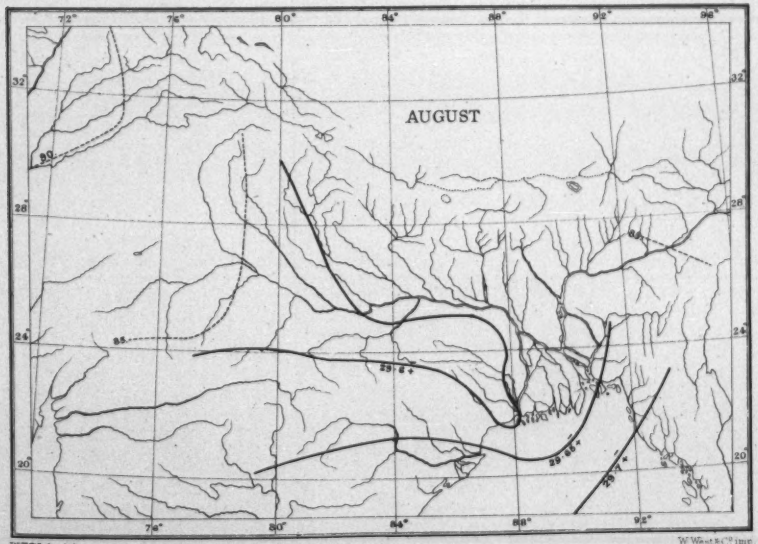

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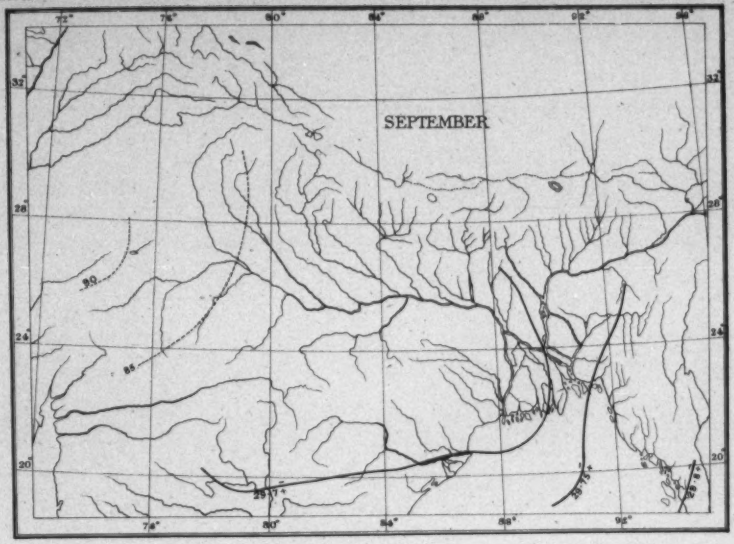

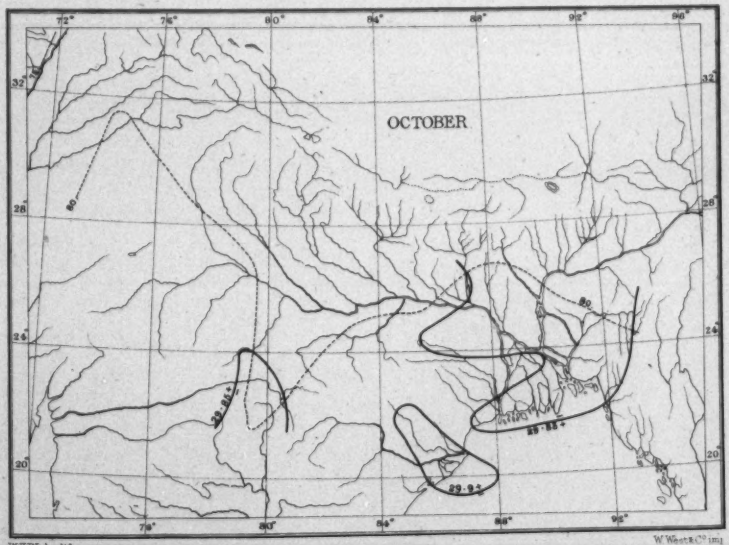

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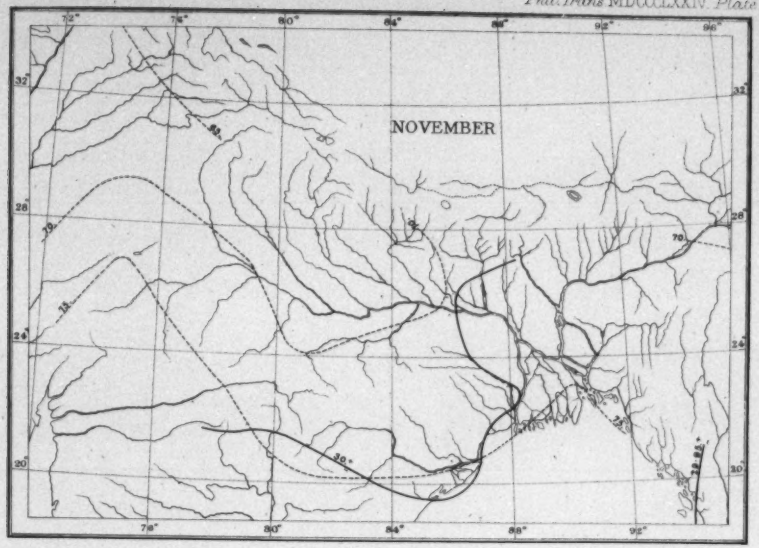

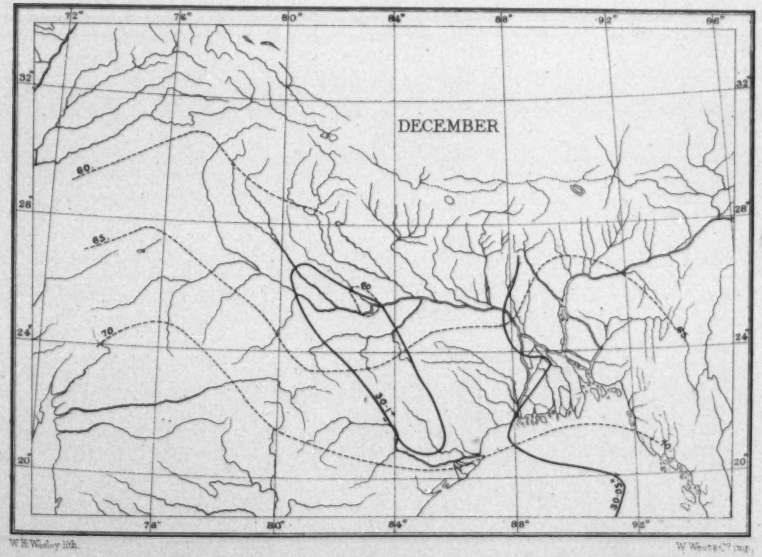

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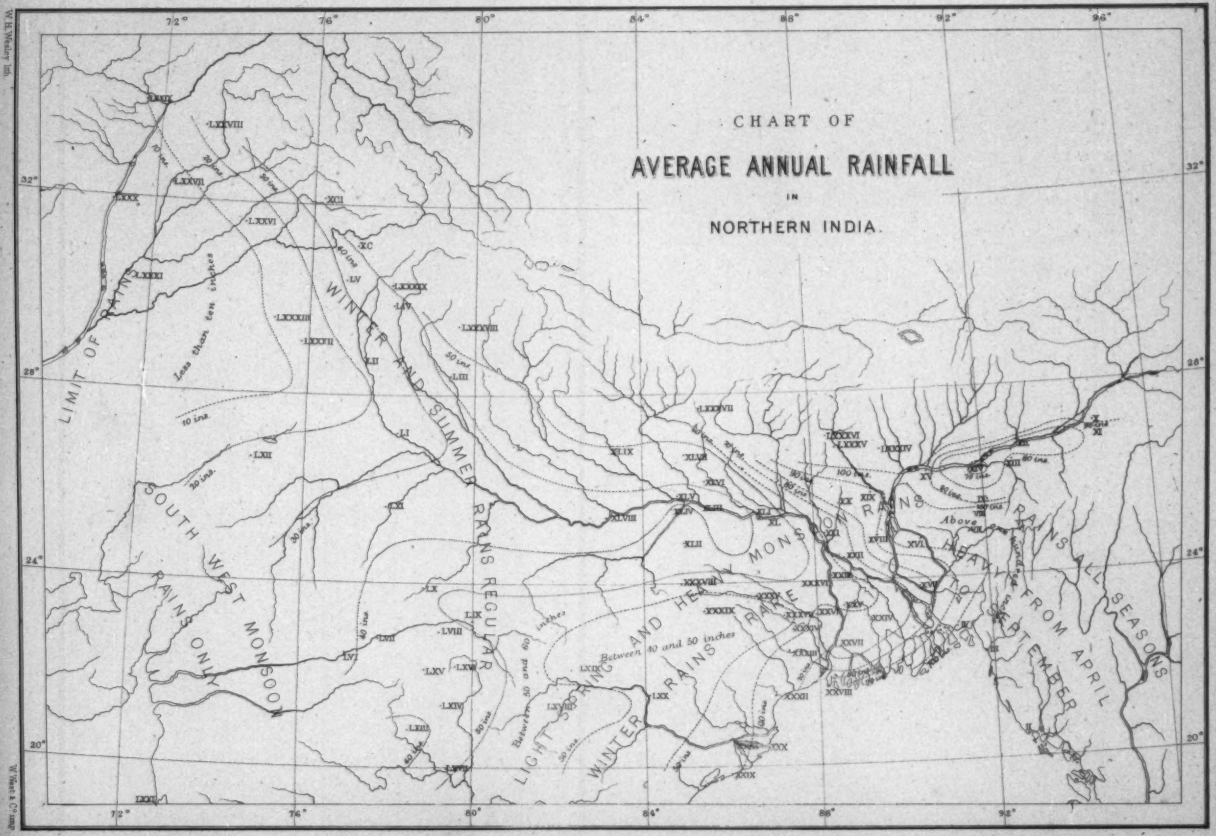
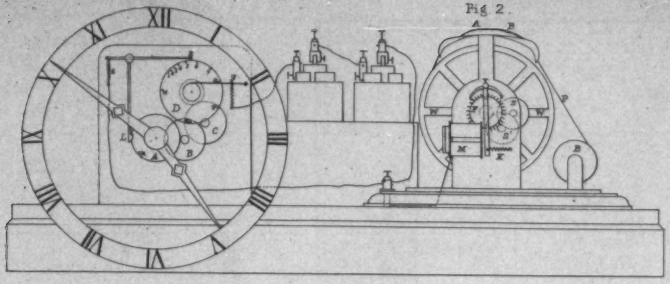


Blanford.

Phil. Trans. MDCCCLXXIV. Plate XLIX.

Fig. 1.

Fig. 2.



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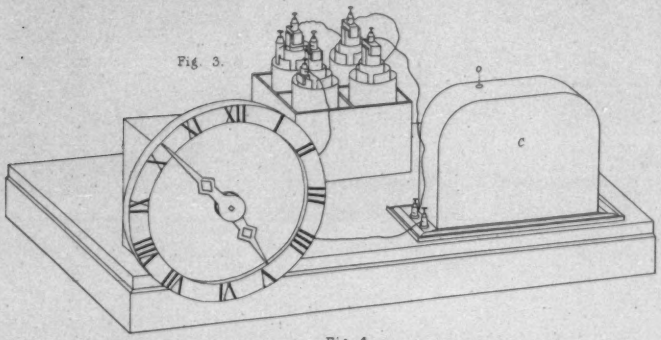

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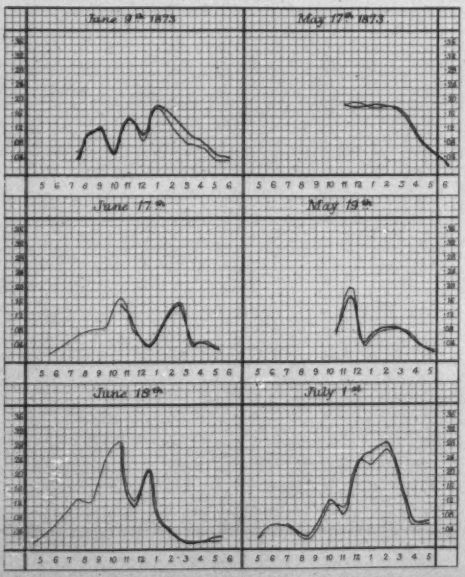


ROYAL INSTITUTION

Fig 1.

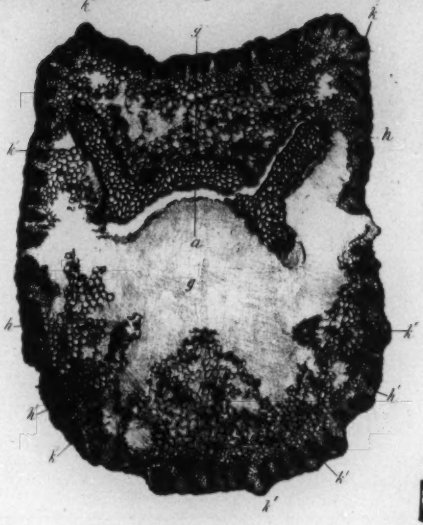


Fig 4

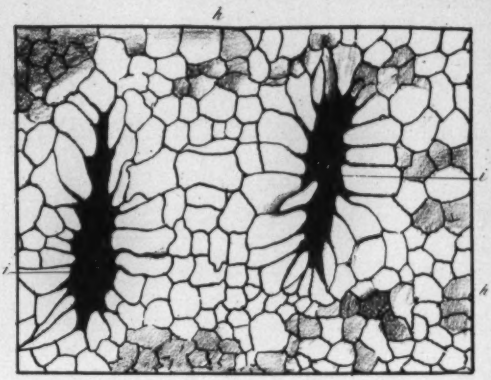


Fig 3.C.

Fig 3.B

Fig 3.A.

Fig. 2.

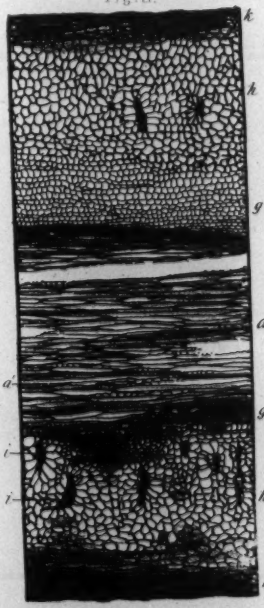


Fig. 5.

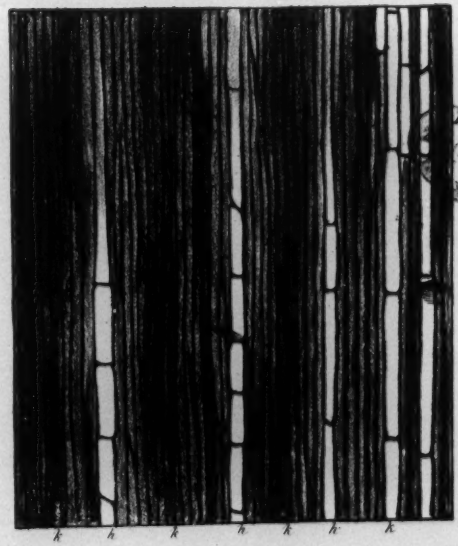
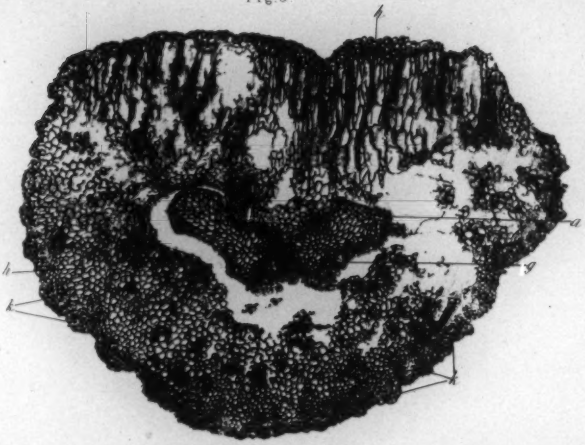


Fig. 6



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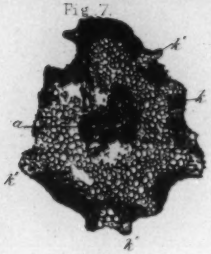


Fig 10

Fig. 8.

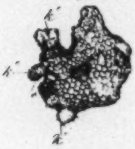


Fig. 9.

Fig 13.

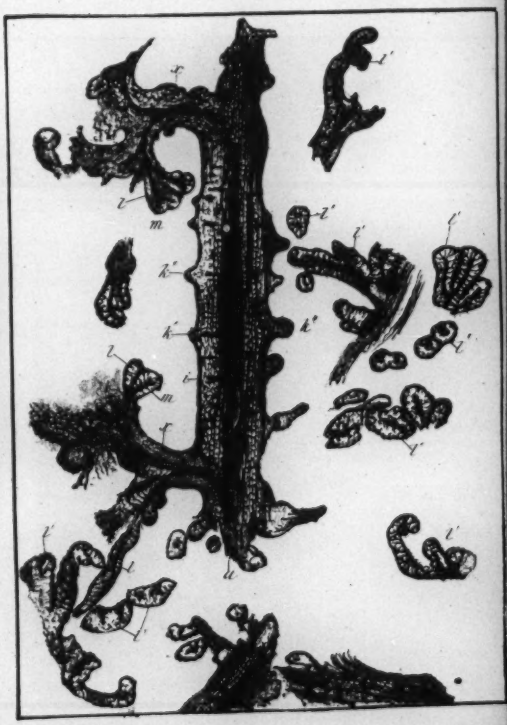


Fig. 11.

Fig. 12.

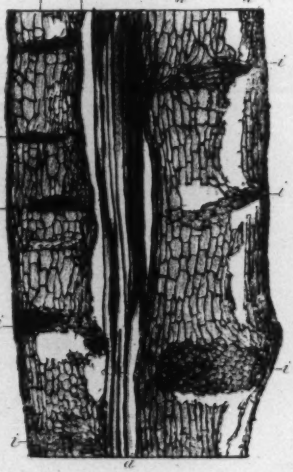


Fig. 16.

Fig. 15.

Fig 14.

Fig 17.

Fig 18.

Fig 19.

Fig 22.

Fig 25.B

Fig 21

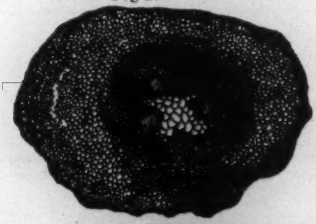


Fig 23

Fig 2[illegible]

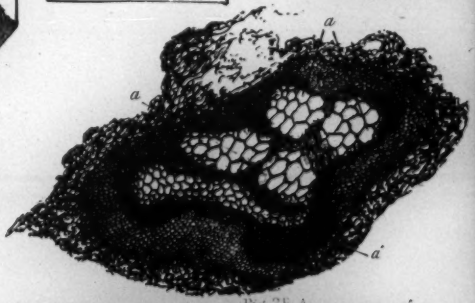
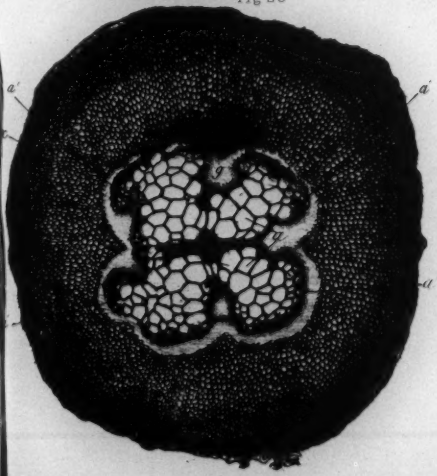


Fig 20



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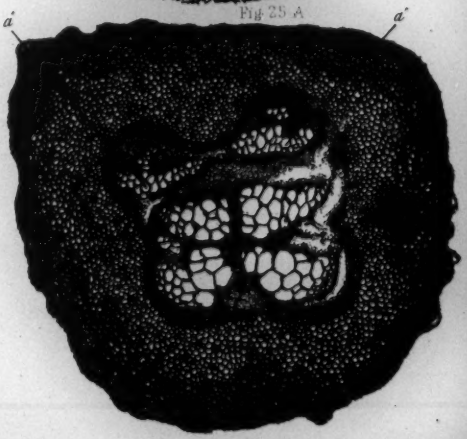
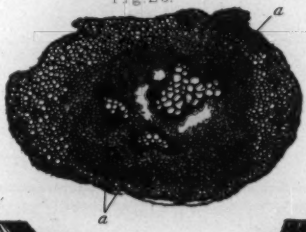


Fig. 25.

Fig. 31.

Fig. 26.



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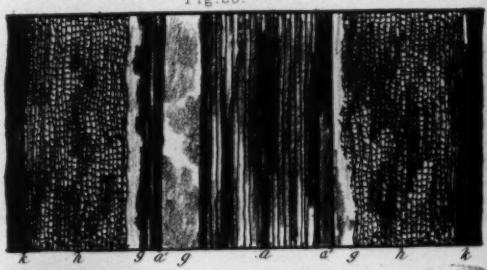

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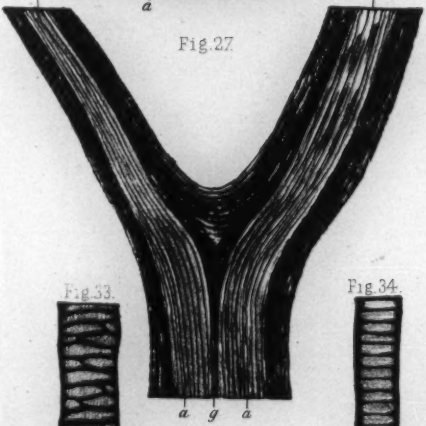

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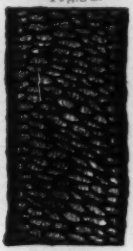

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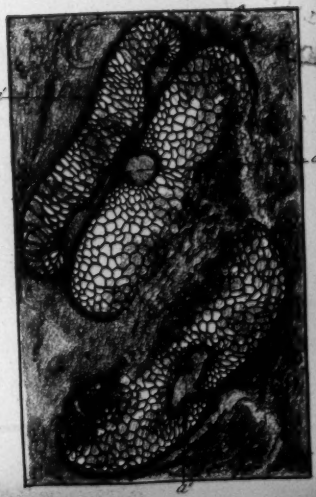

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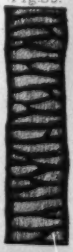


Fig. 34.

Fig. 36.

Fig.28

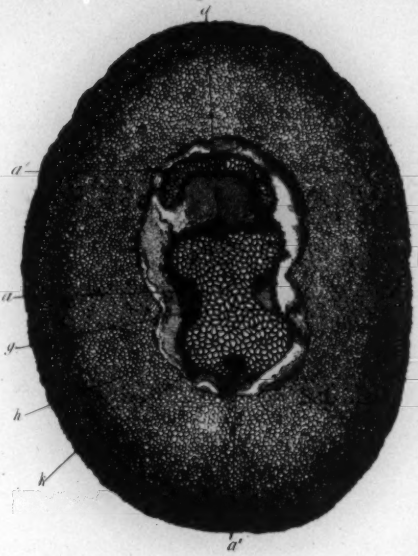


Fig.37

Fig.38

Fig.41.

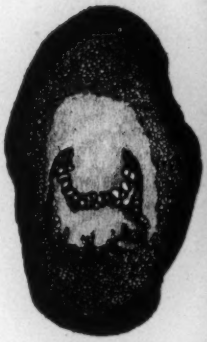


Fig.40.

Fig.35.

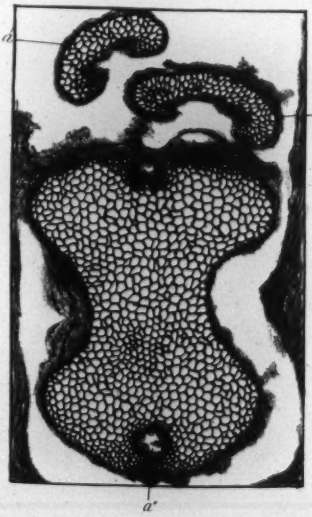


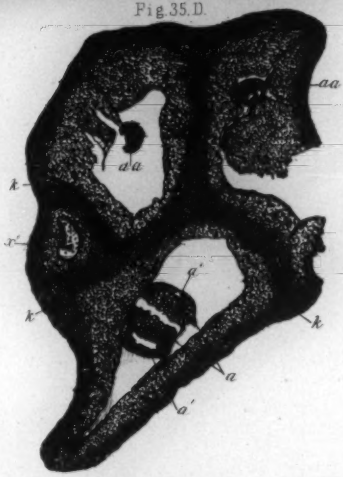
Fig.35.A.

Fig.35.B.

Fig.35.C.

Fig.35.F.

Fig. 35. D.



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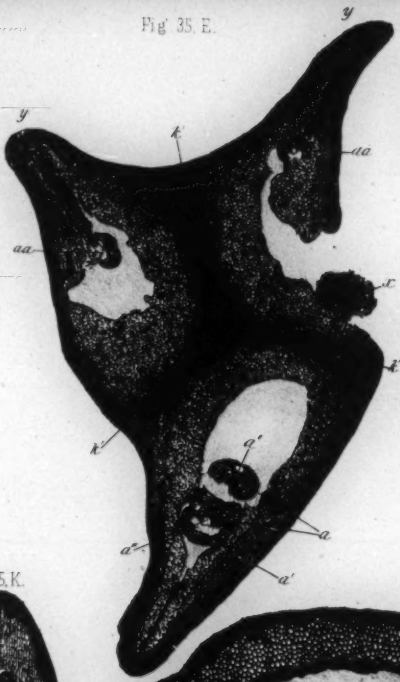


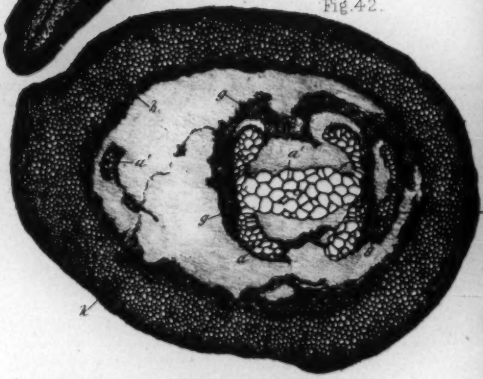
Fig. 35. H.

Fig. 35. G.

Fig. 35. I.

Fig. 35. K.

Fig. 42.



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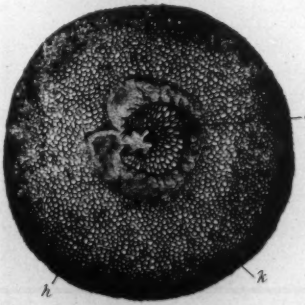

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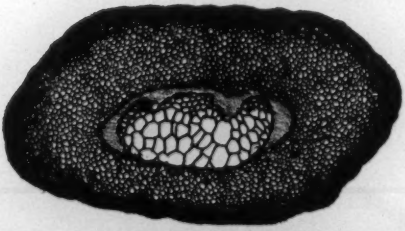
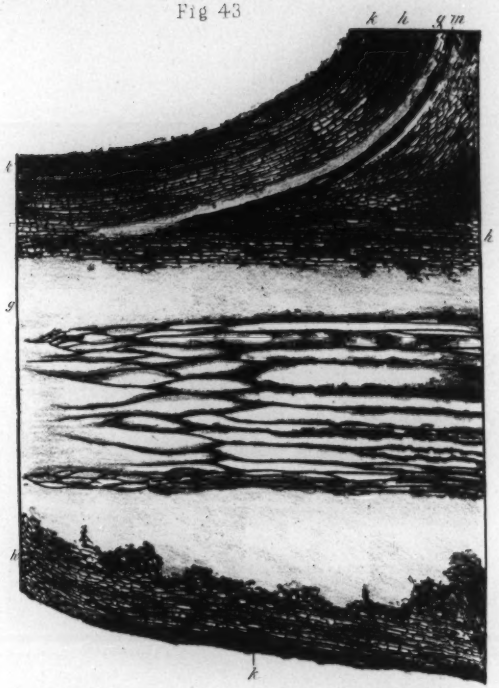


Fig 43



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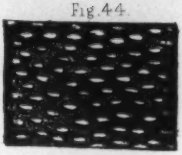
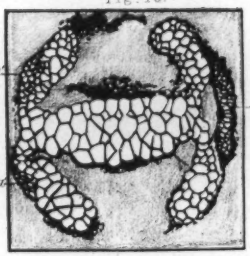


Fig 45



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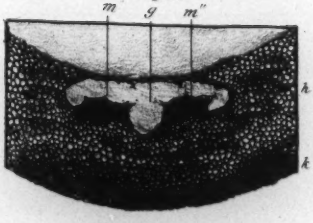

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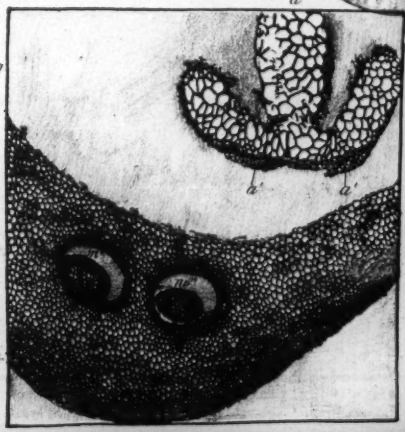
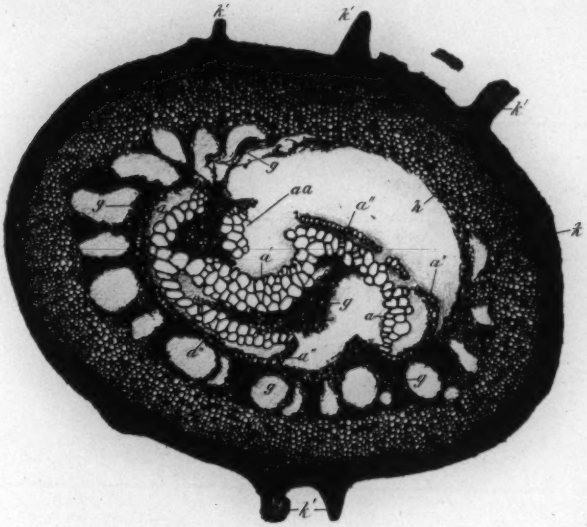


Fig. 49



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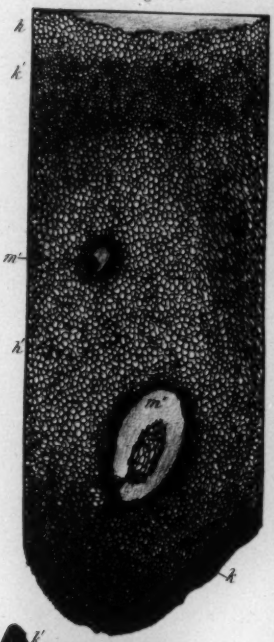

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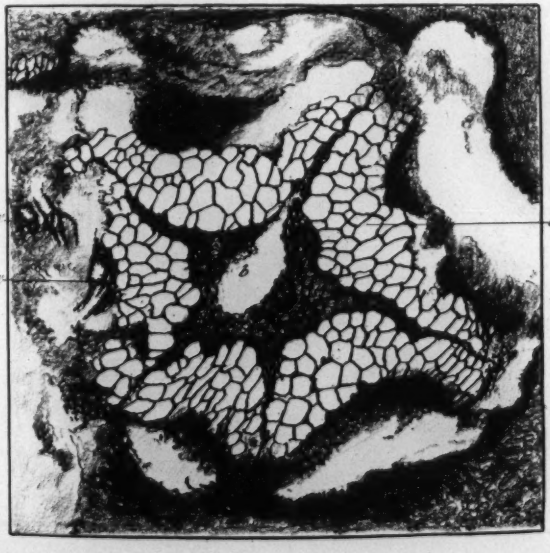

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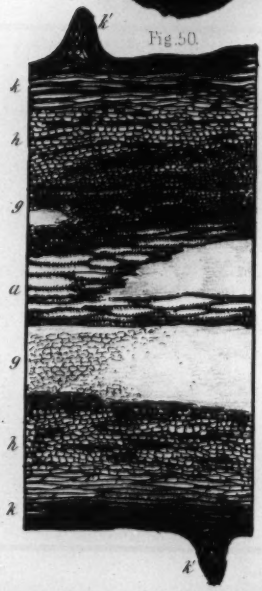
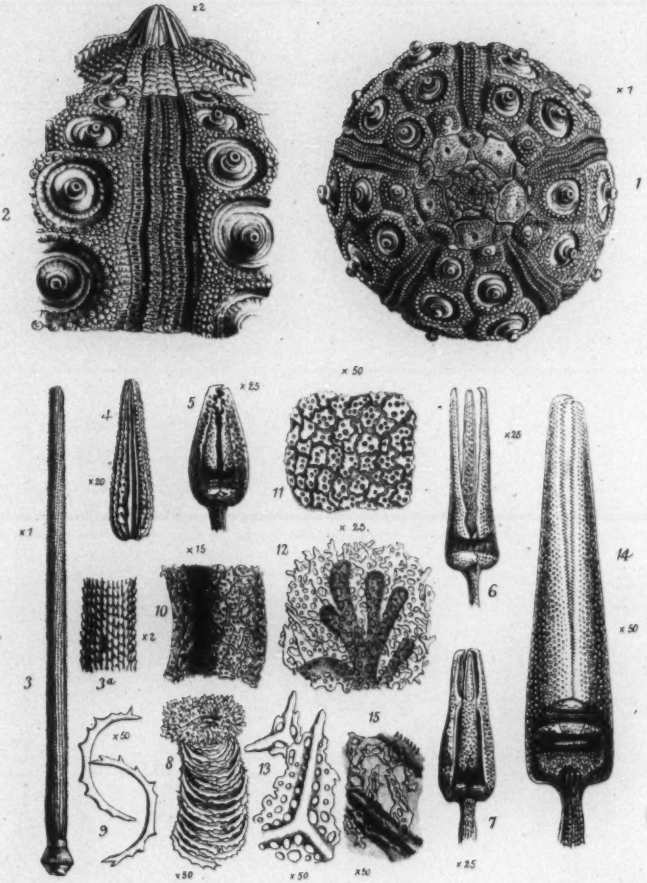
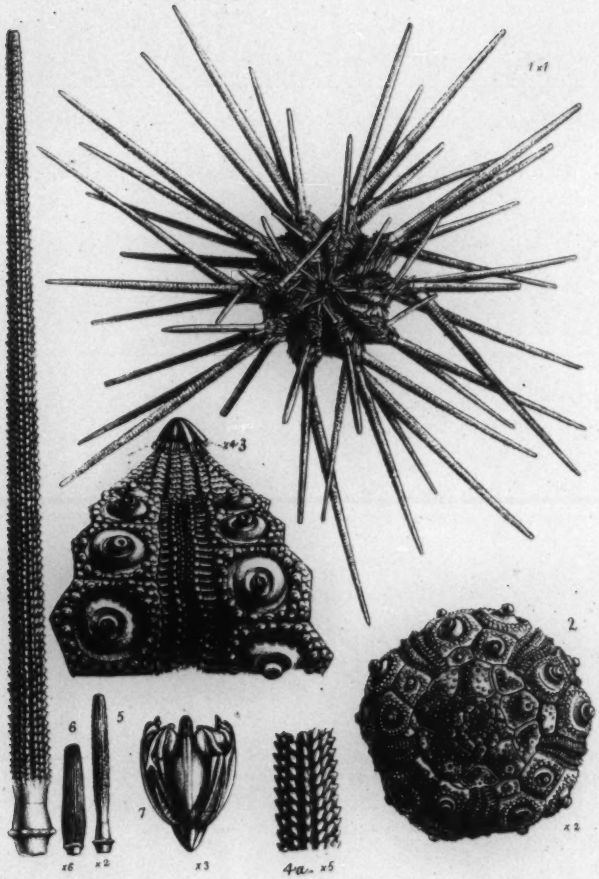




J.J.Wild lith.

M & N.Hanhart imp.

Fig 1. 13. CIDARIS PUPILLATA. *Leske.*
14.15. POROCIDARIS PURPURATA. *Wy.T.*



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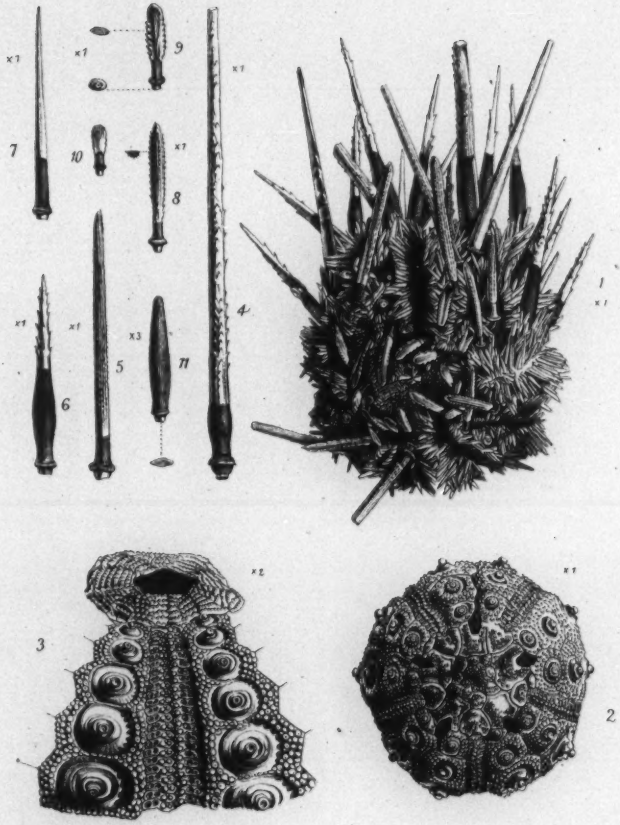
J. J. Wild lith.

M&N Hanhart imp.

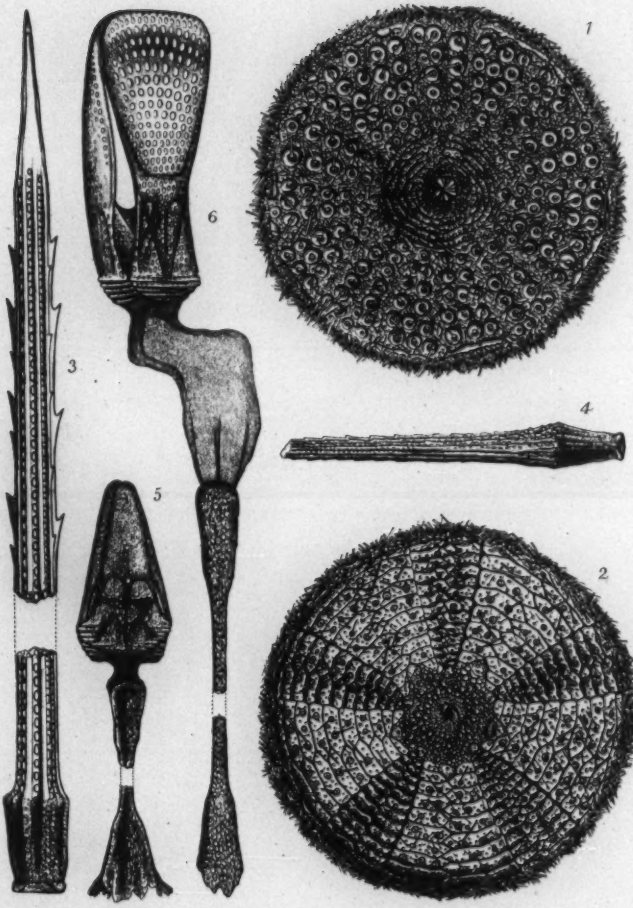
CIDARIS AFFINIS. *Philippi*



J. J. Wild, lith.

M & N. Hanhart. imp.

POROCIDARIS PURPURATA. *Wy T.*



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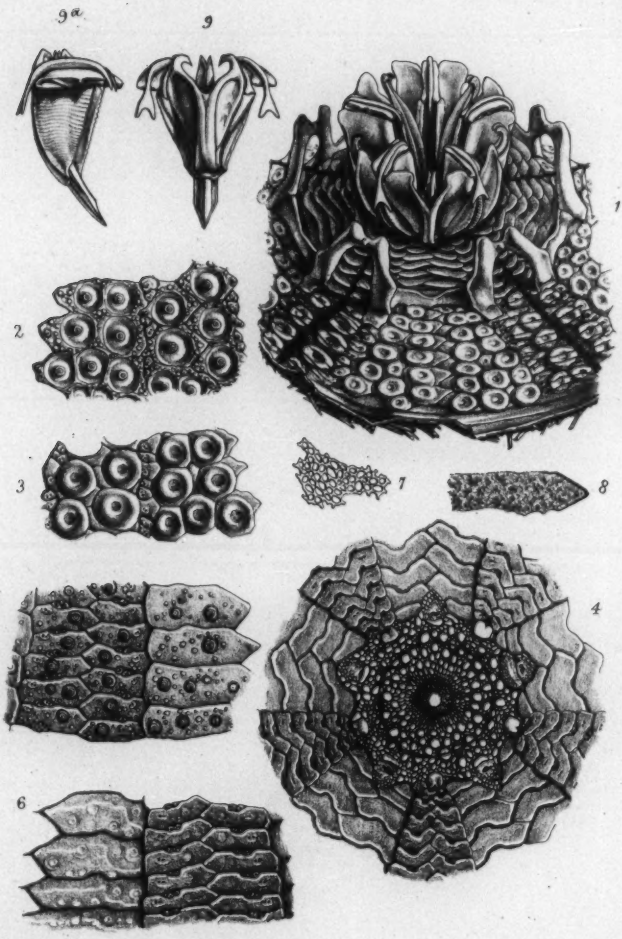
J.J.Wild, lith.

M&N Hanhart imp.

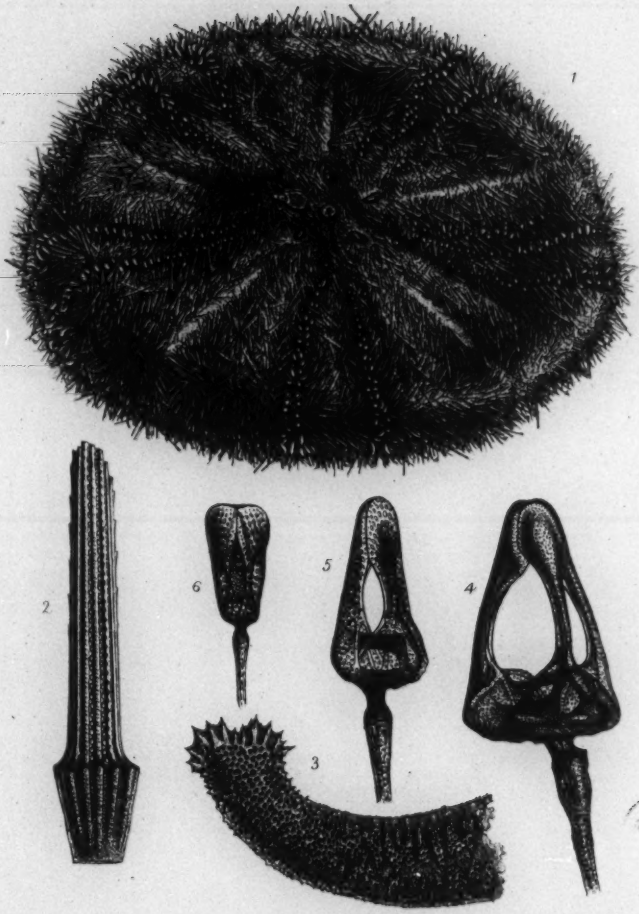
PHORMOSOMA PLACENTA *Wy. T.*



J.J. Wild lith.

M&N Hanhart imp.

PHORMOSOMA PLACENTA *Wy T.*



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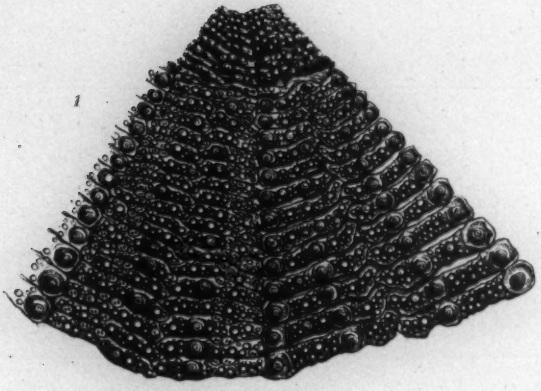
M&N Hanhart imp.

CALVERIA HYSTRIX. *Wyv. T.*

1



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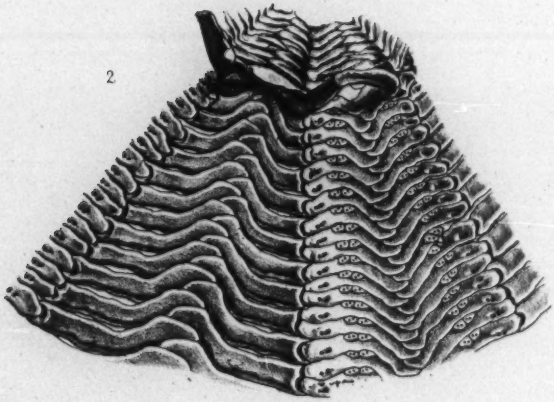

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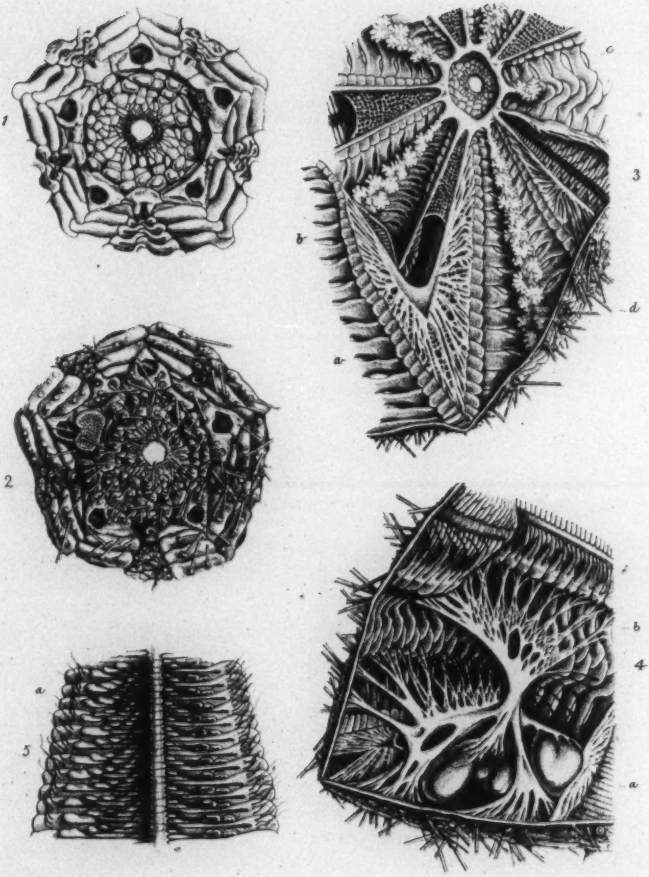
J. J. Wild lith.

M & N Hanhart imp.

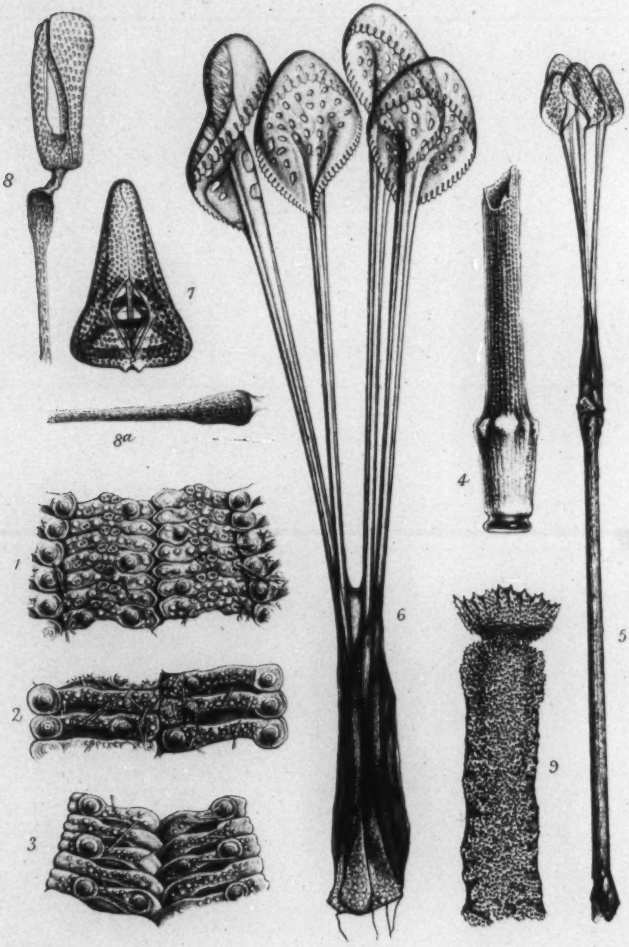
CALVERIA HYSTRIX *Wy. T.*



J.J. Wild lith.

M & N. Hanhart imp.

CALVERIA FENESTRATA. *Wy. T.*



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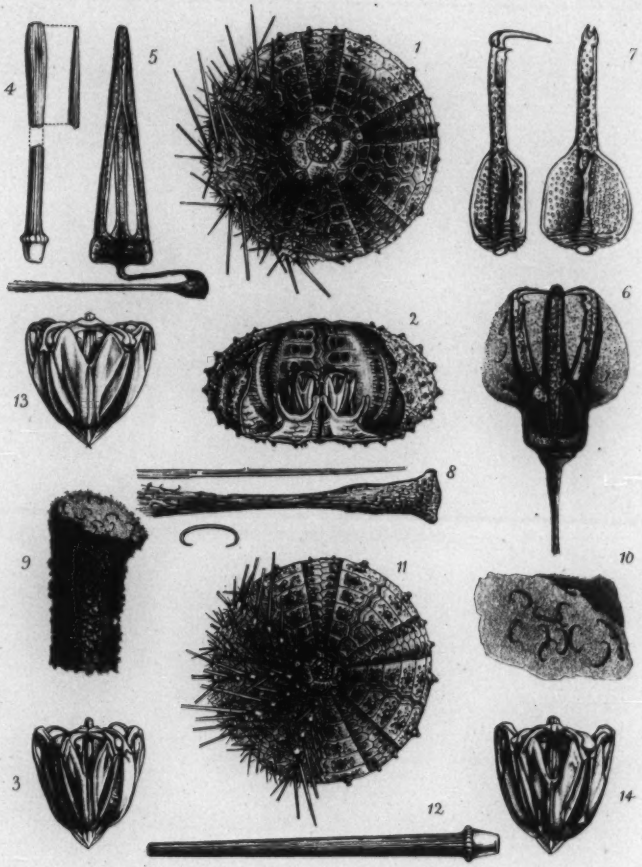
J.Wild lith.

M&N Hanhart imp.

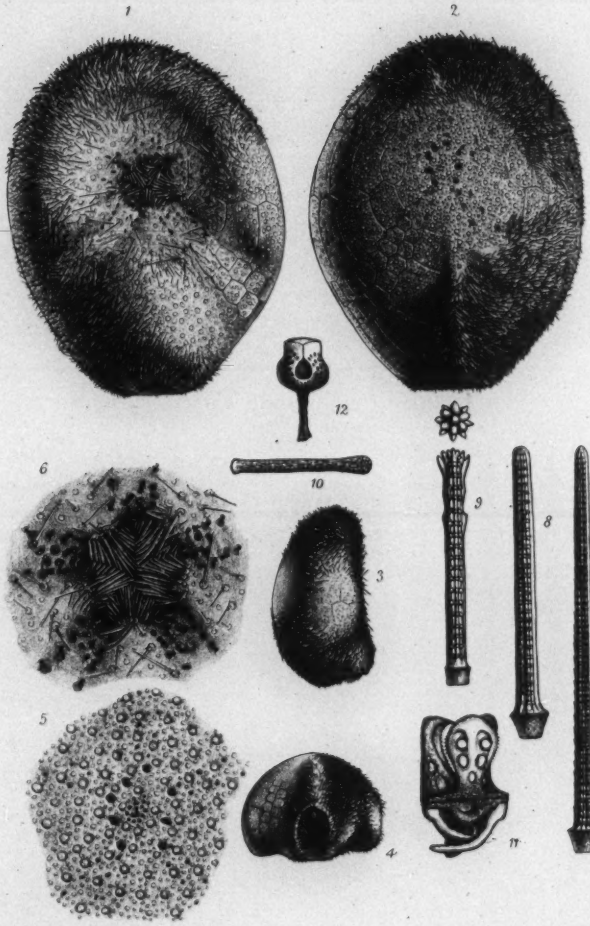
CALVERIA FENESTRATA *Wy T.*



J. J. Wild lith.

M & N. Hanhart imp.

Fig 1 10. ECHINUS MICROSTOMA. *Wy. T.*
11 13. ECHINUS ELEGANS *von Düben and Koren.*
14. ECHINUS FLEMINGII. *Ball.*



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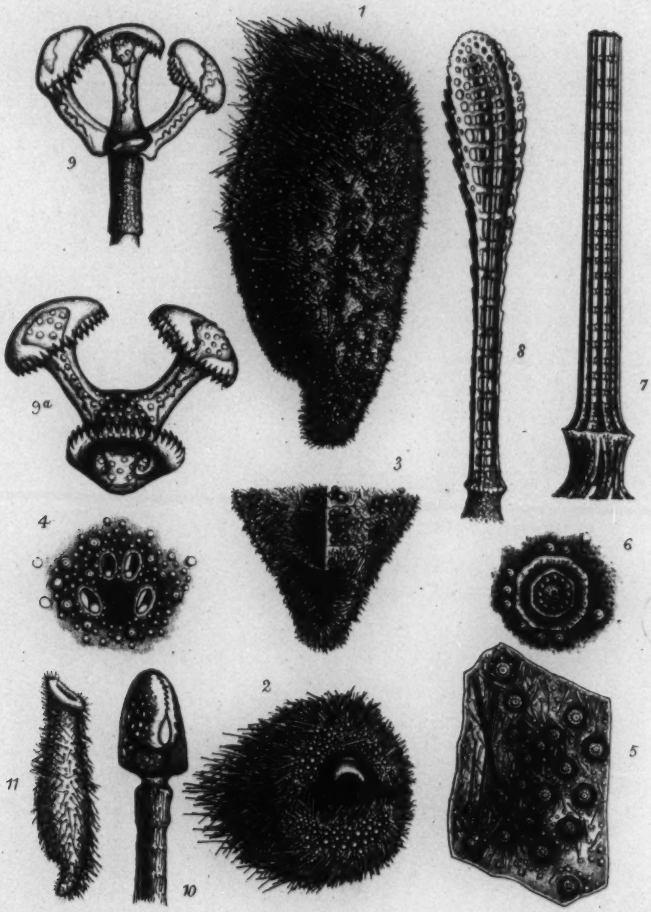
M. & N. Hanhart, imp.

NEOLAMPAS ROSTELLATA, *A. Agassiz.*



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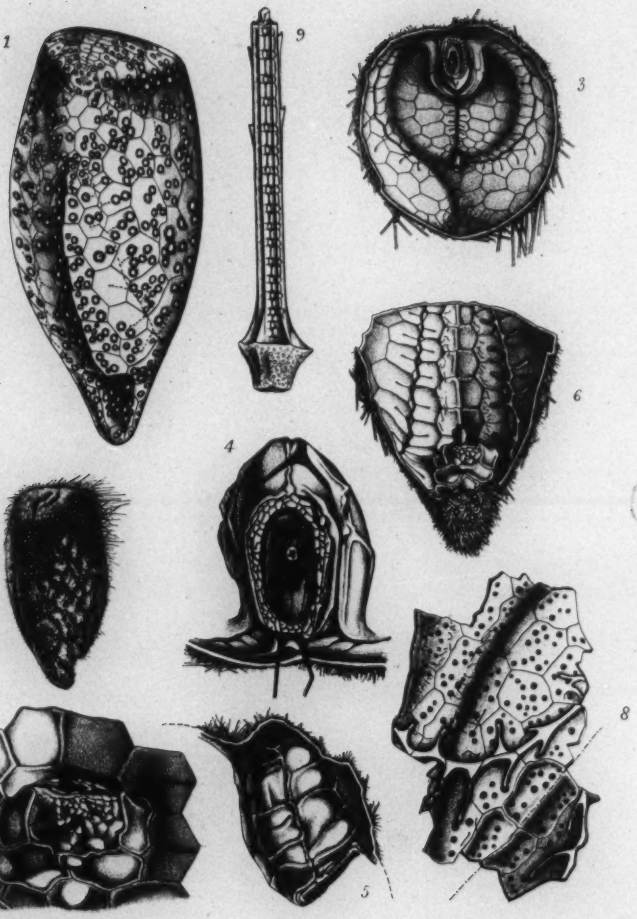
M&N.Hanhart, imp.

Fig 1 10. POURTALESIA JEFFREYSI, *Wy. T.*
11. POURTALESIA PHIALE, *Wy. T.*



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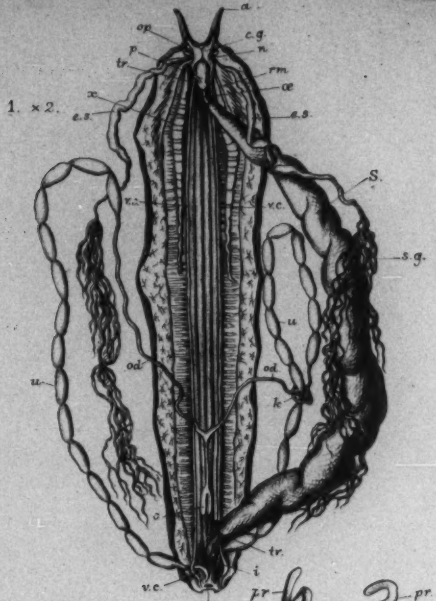
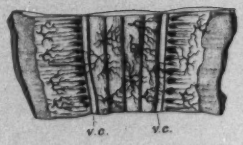
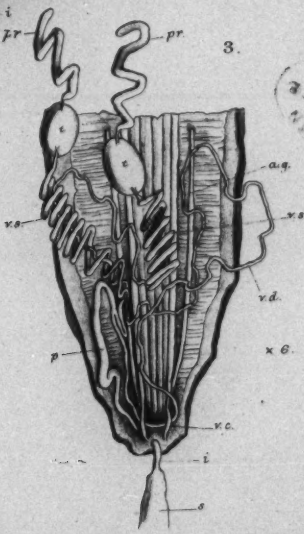
J.J.Wild, lith.

M&N.Hanhart, imp.

POURTALESIA JEFFREYSI *Wy. T.*







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J.J.Wild. ad. nat. del. W.H.Wesley lith.
W. West & Co. imp.

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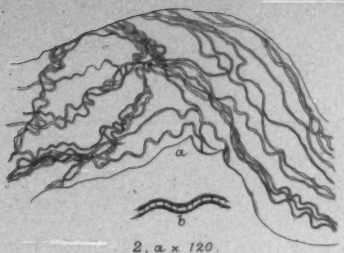

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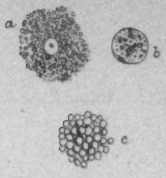

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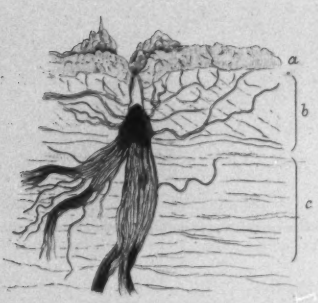

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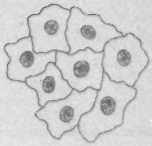

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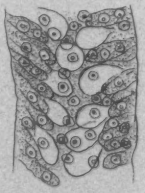

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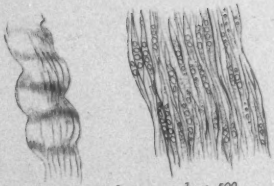
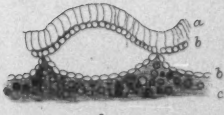


a × 200. 8. b × 500.



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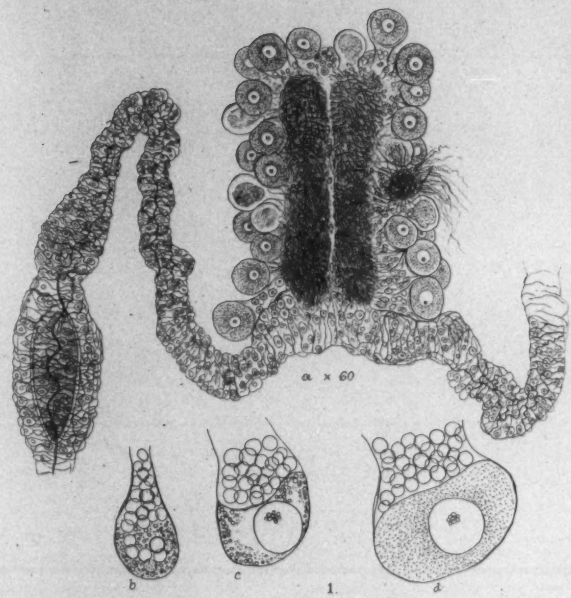
W. West & Co. imp.

W. H. Wesley lith.



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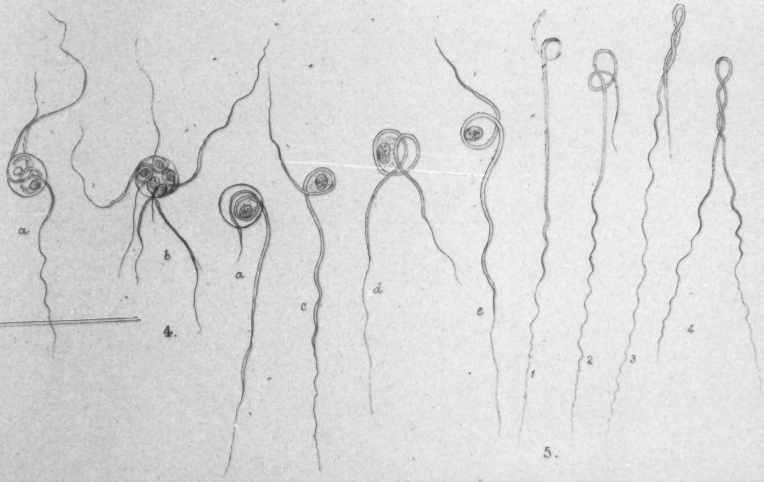

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W.H. Wesley lith.

W. West & Co. imp.

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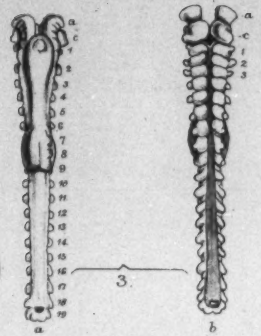

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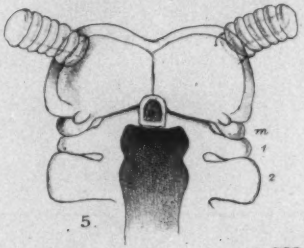

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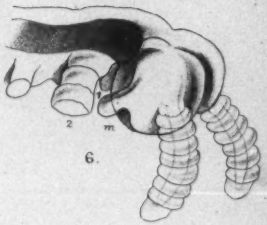

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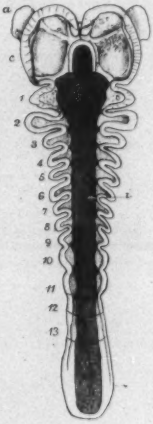

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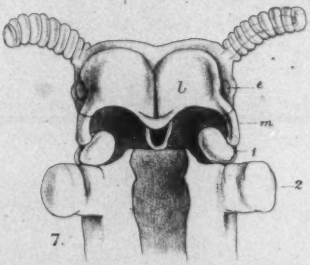

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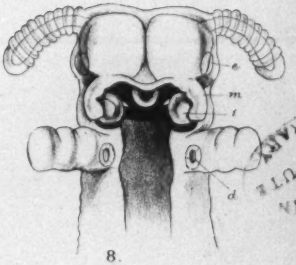

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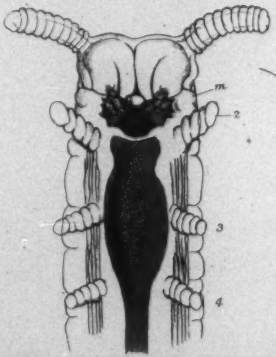

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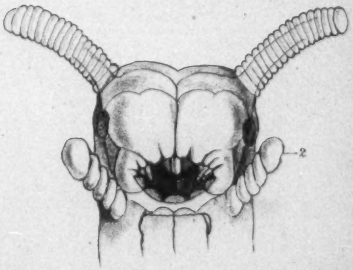

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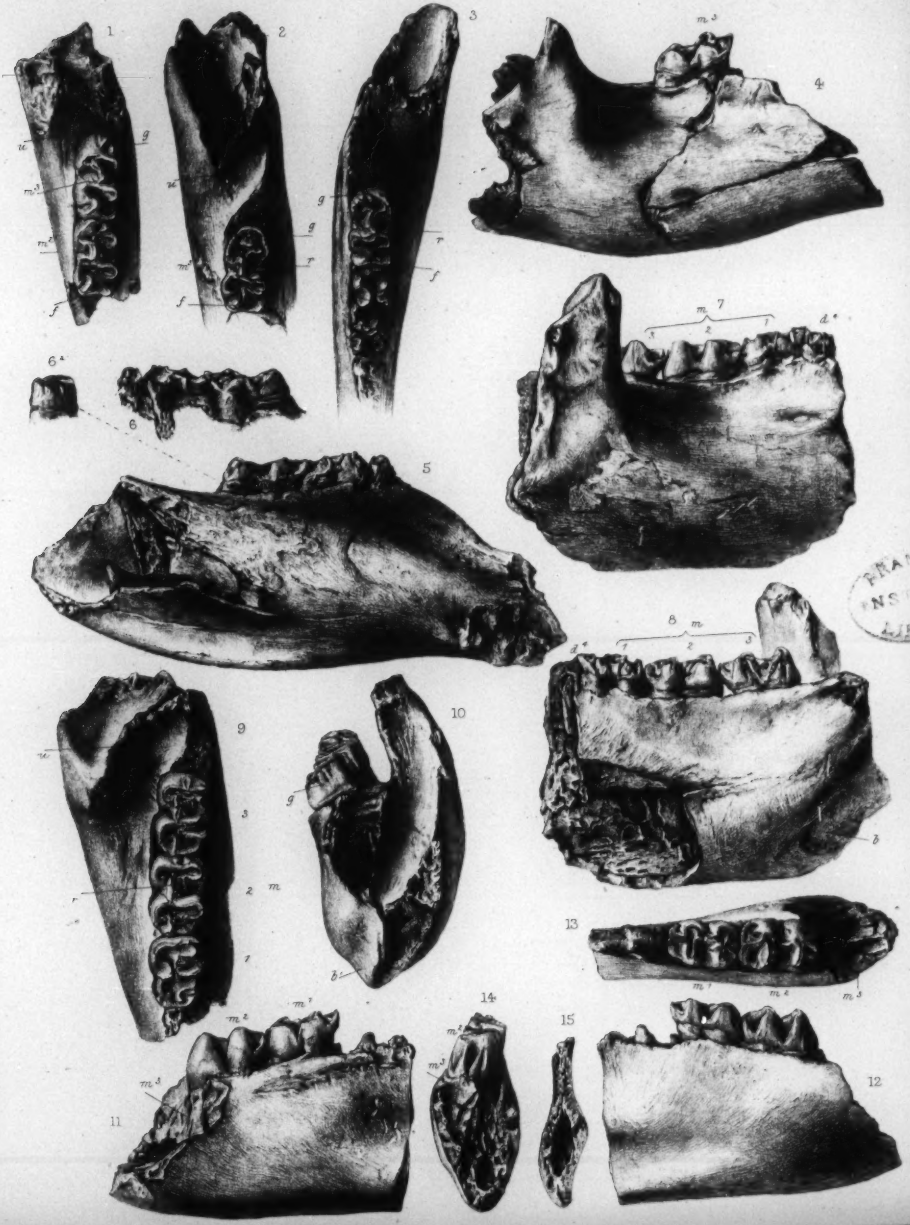
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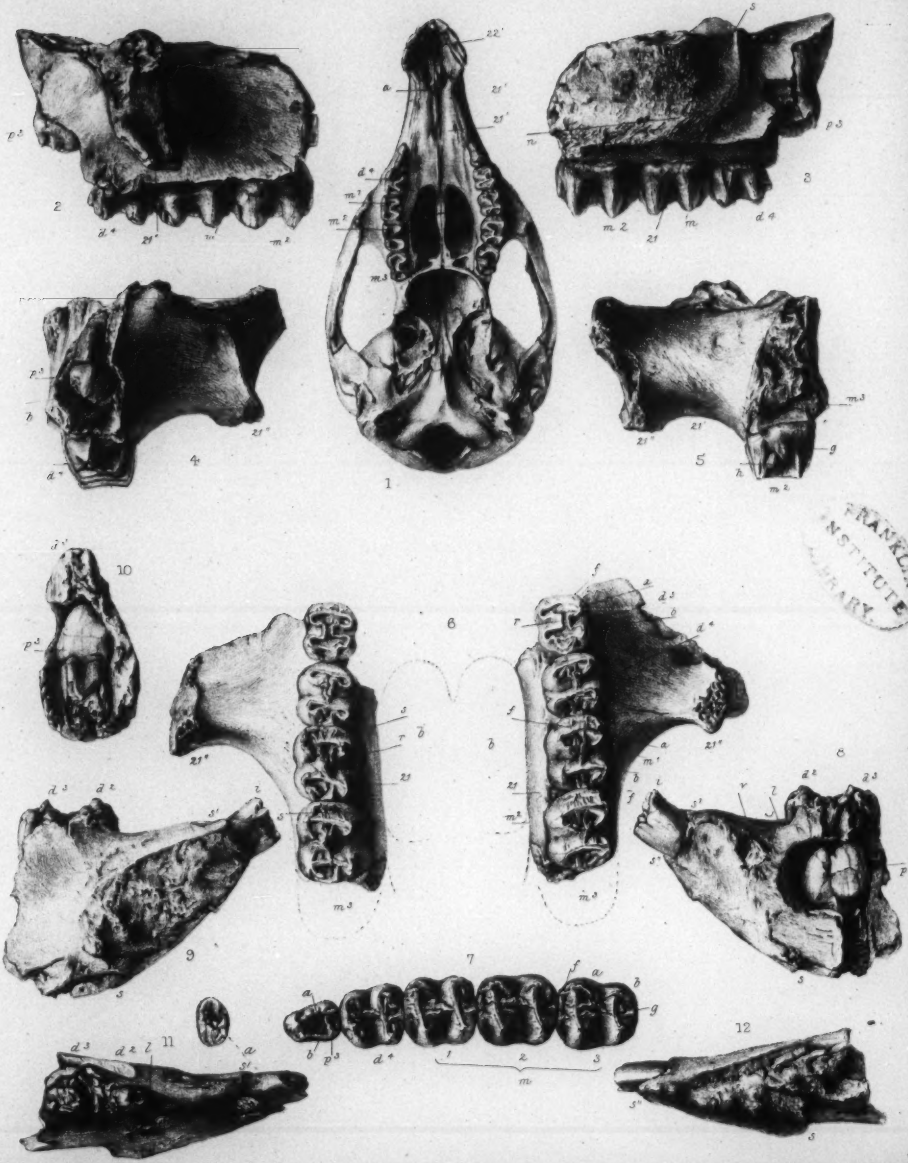

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W.H. Wesley, ad nat. Auto. Lith.

Machure & Macdonald, Lith. London.

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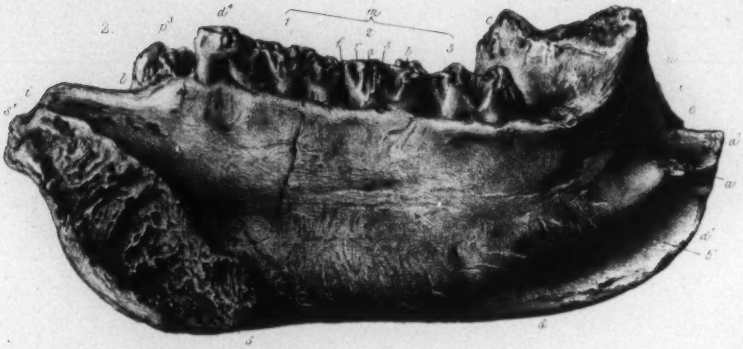

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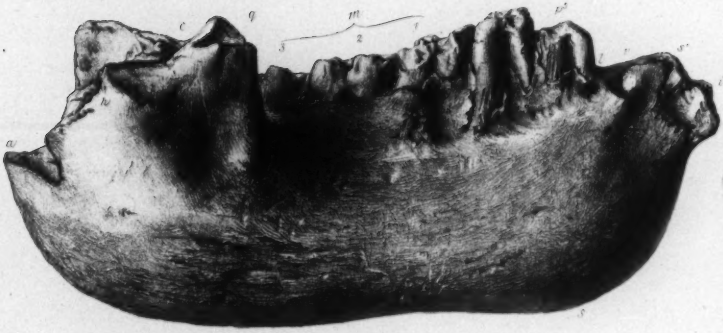

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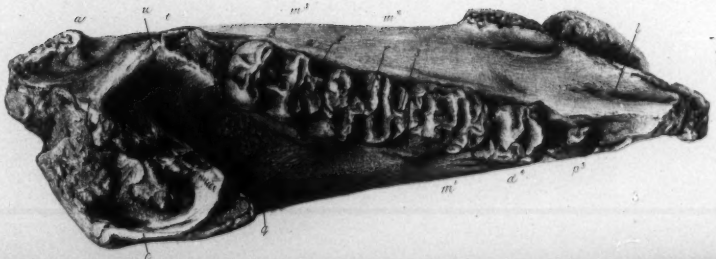

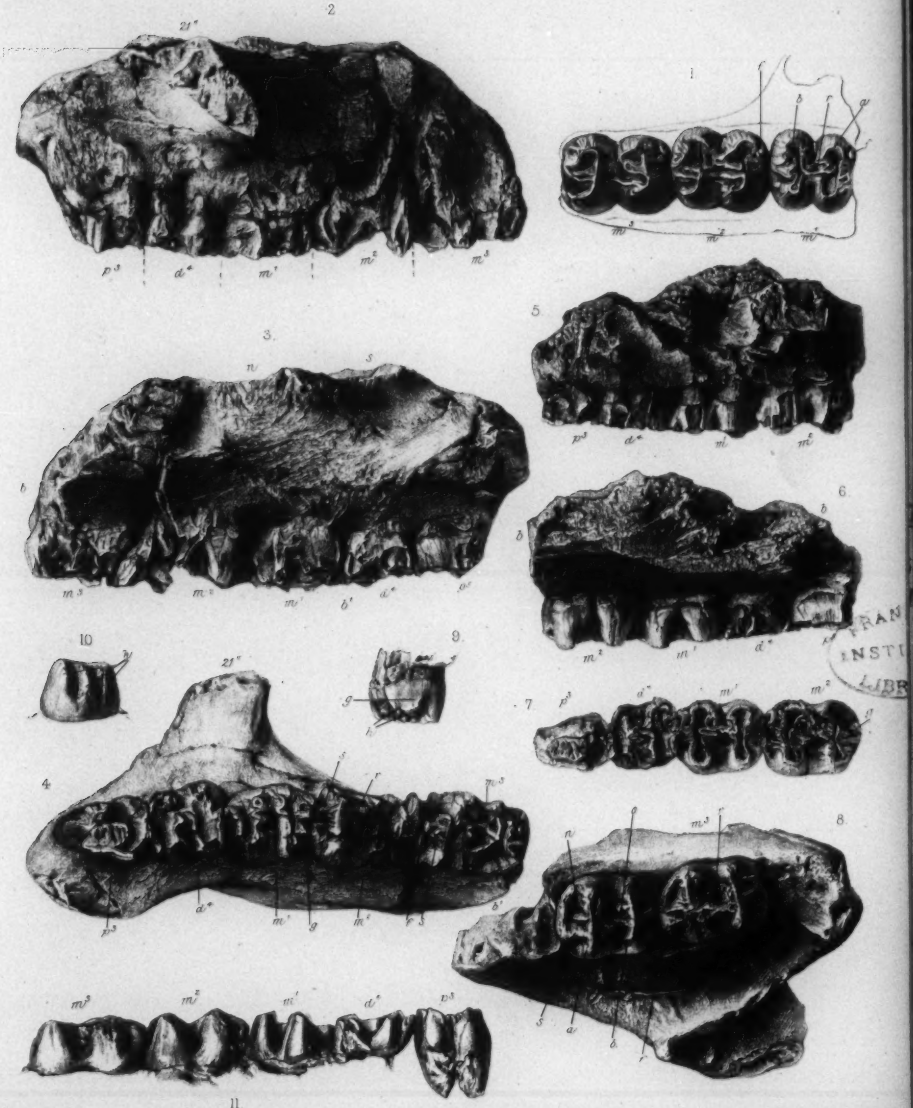
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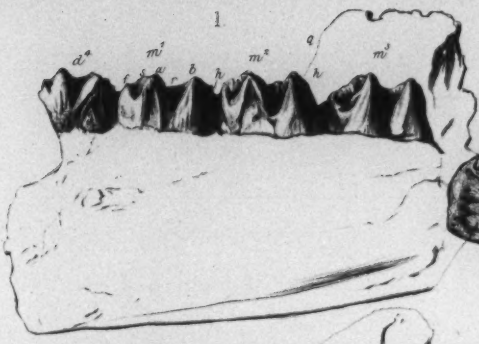

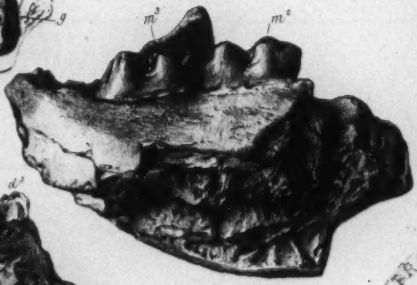

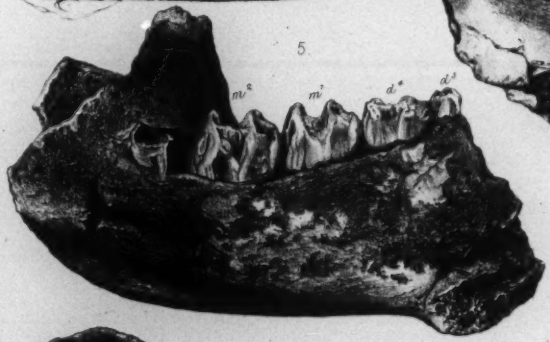

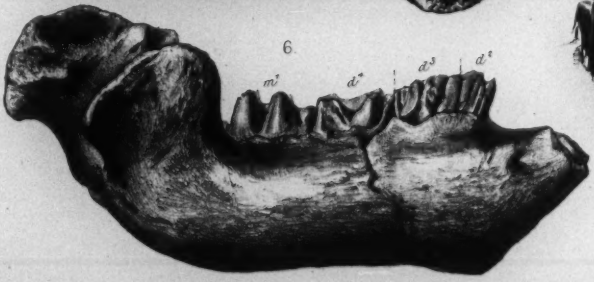

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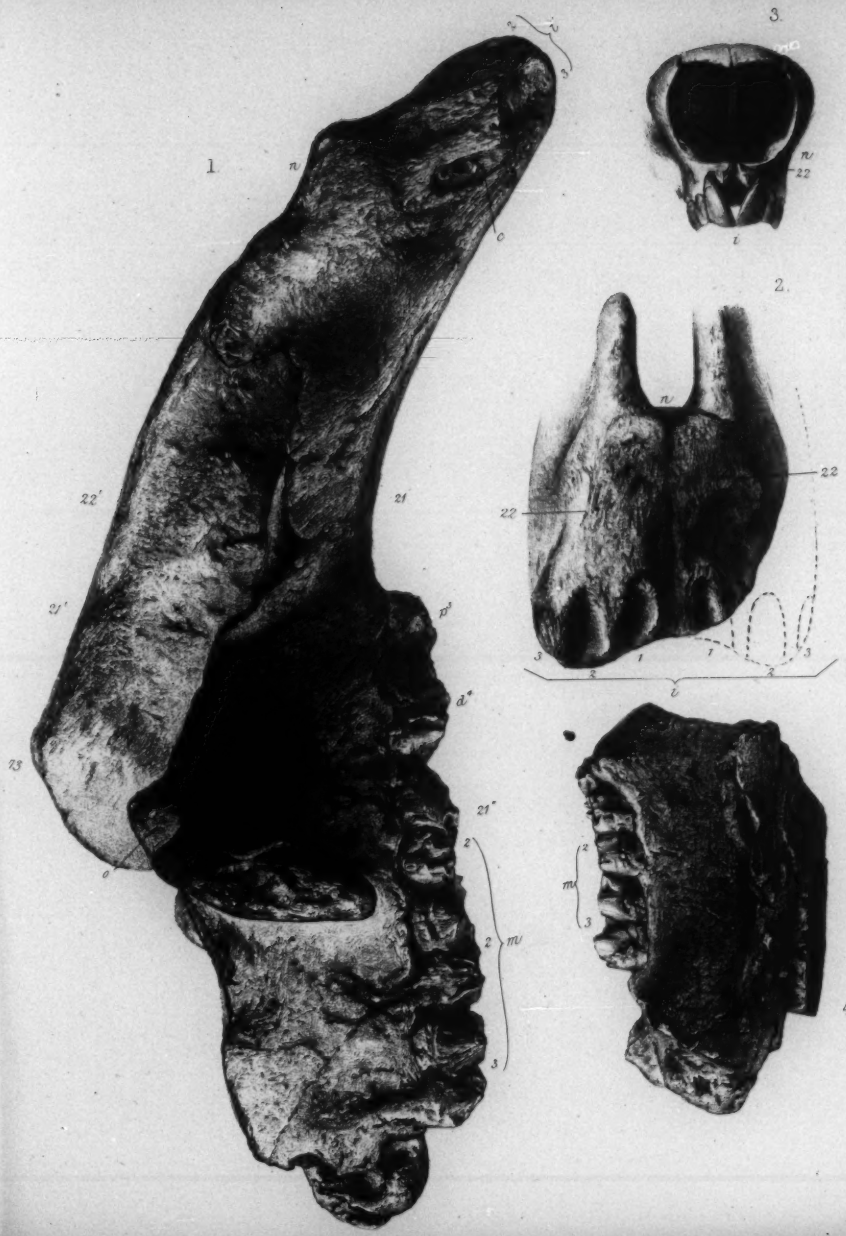

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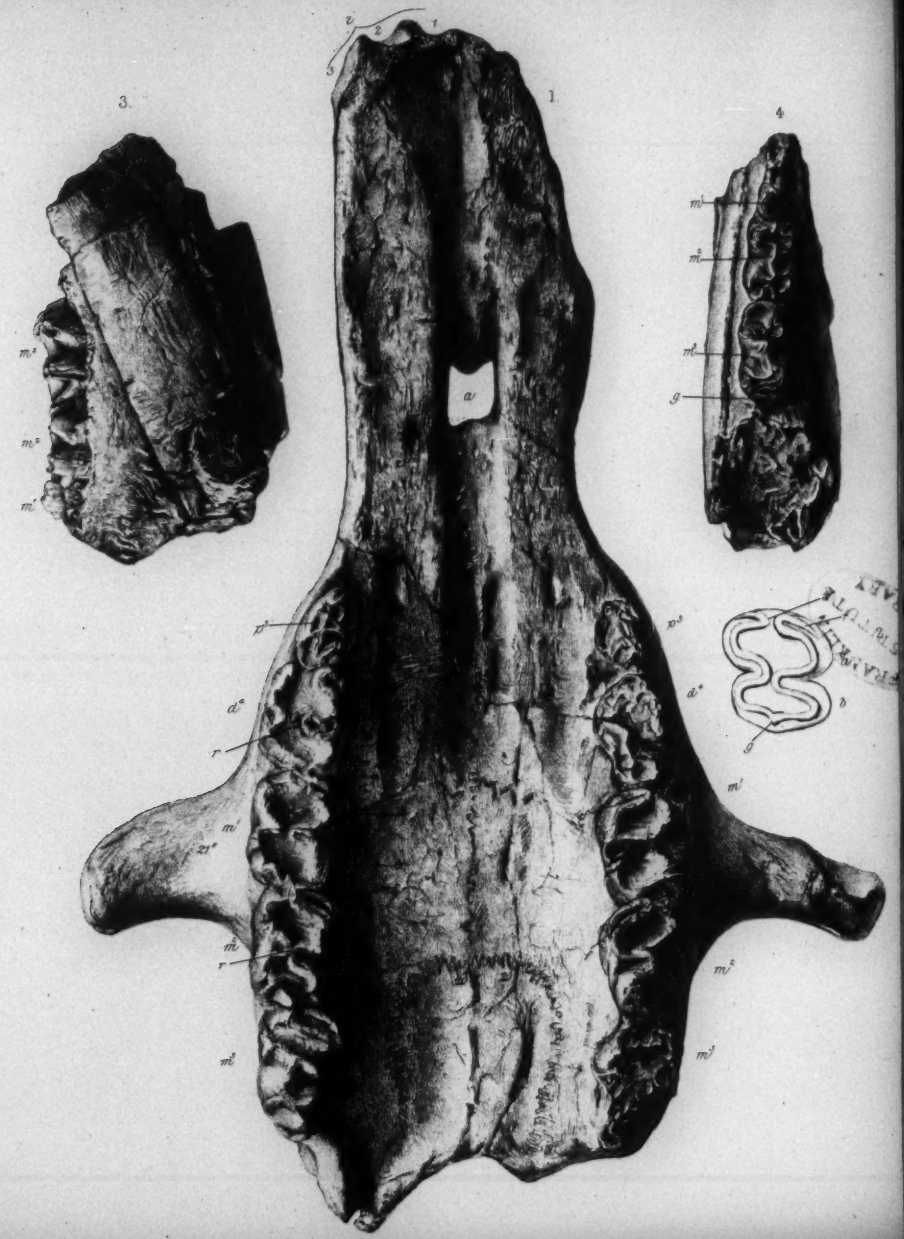
Maclure & Macdonald, Lith^rs, Lon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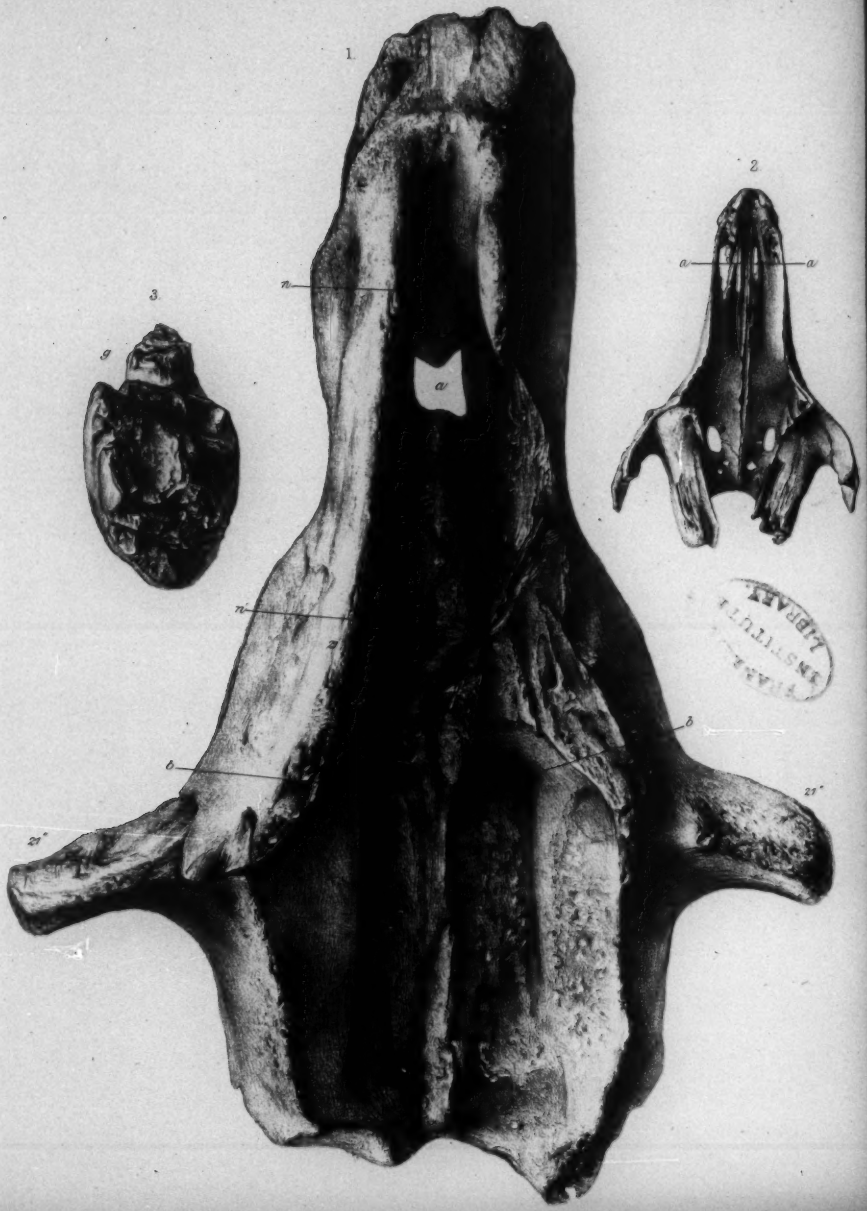
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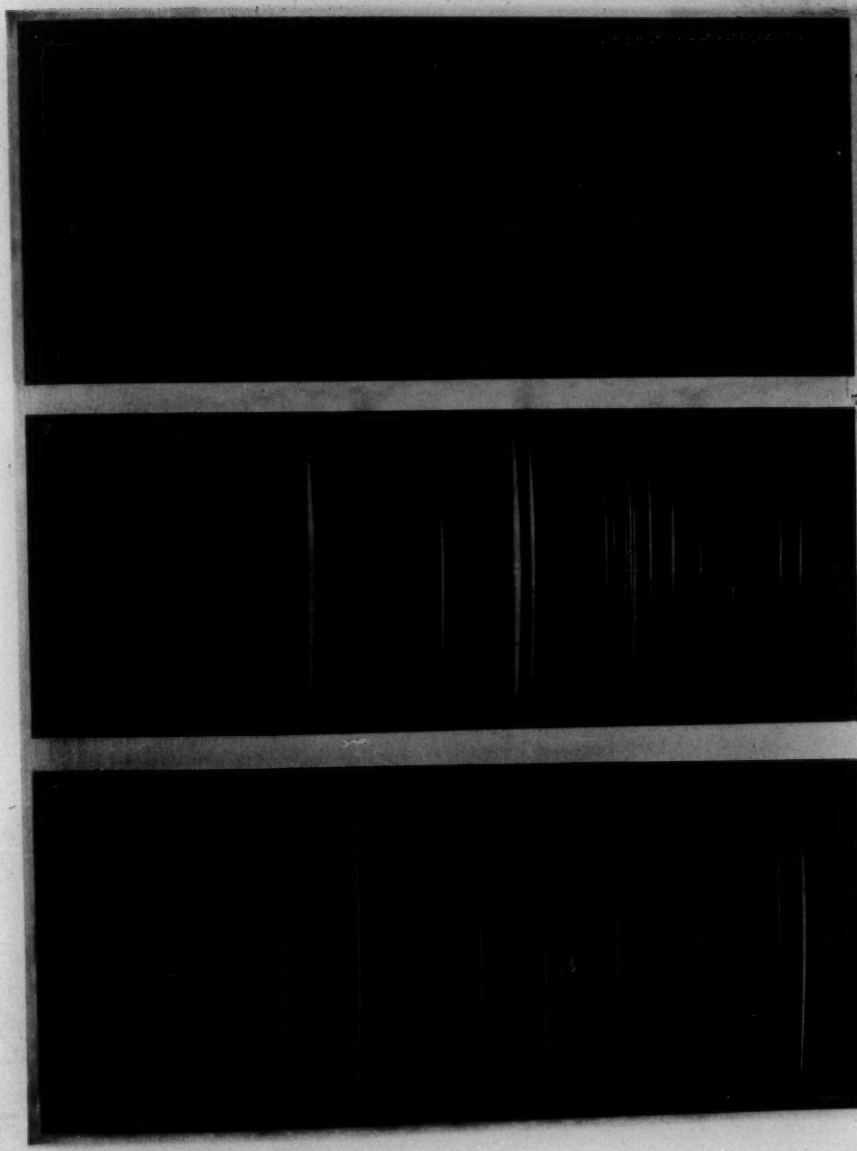

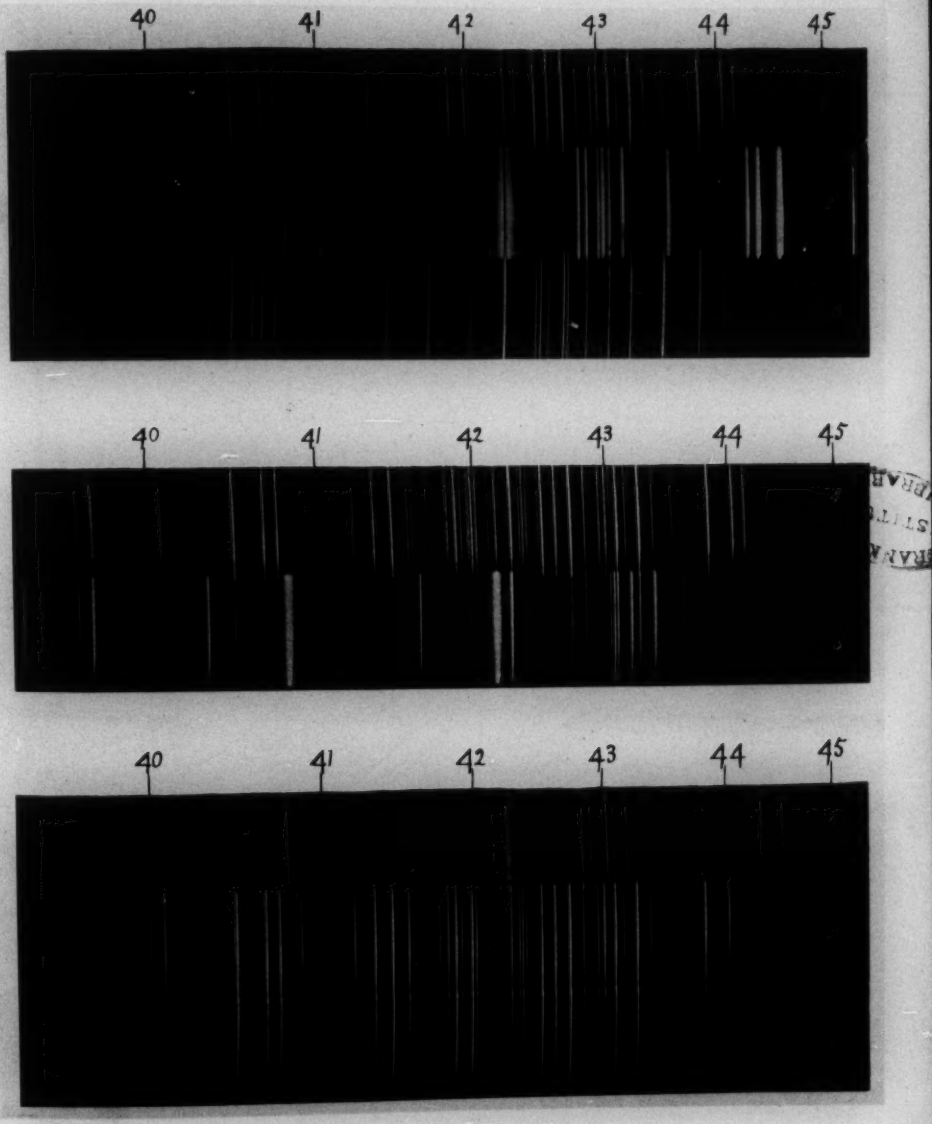
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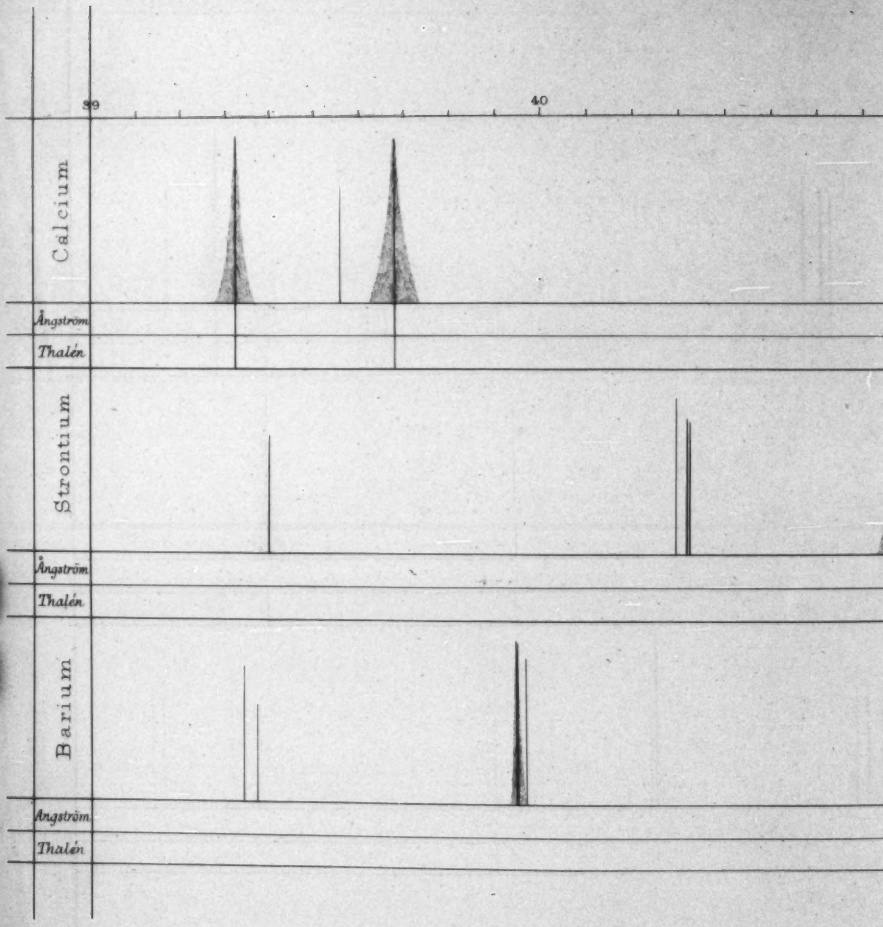

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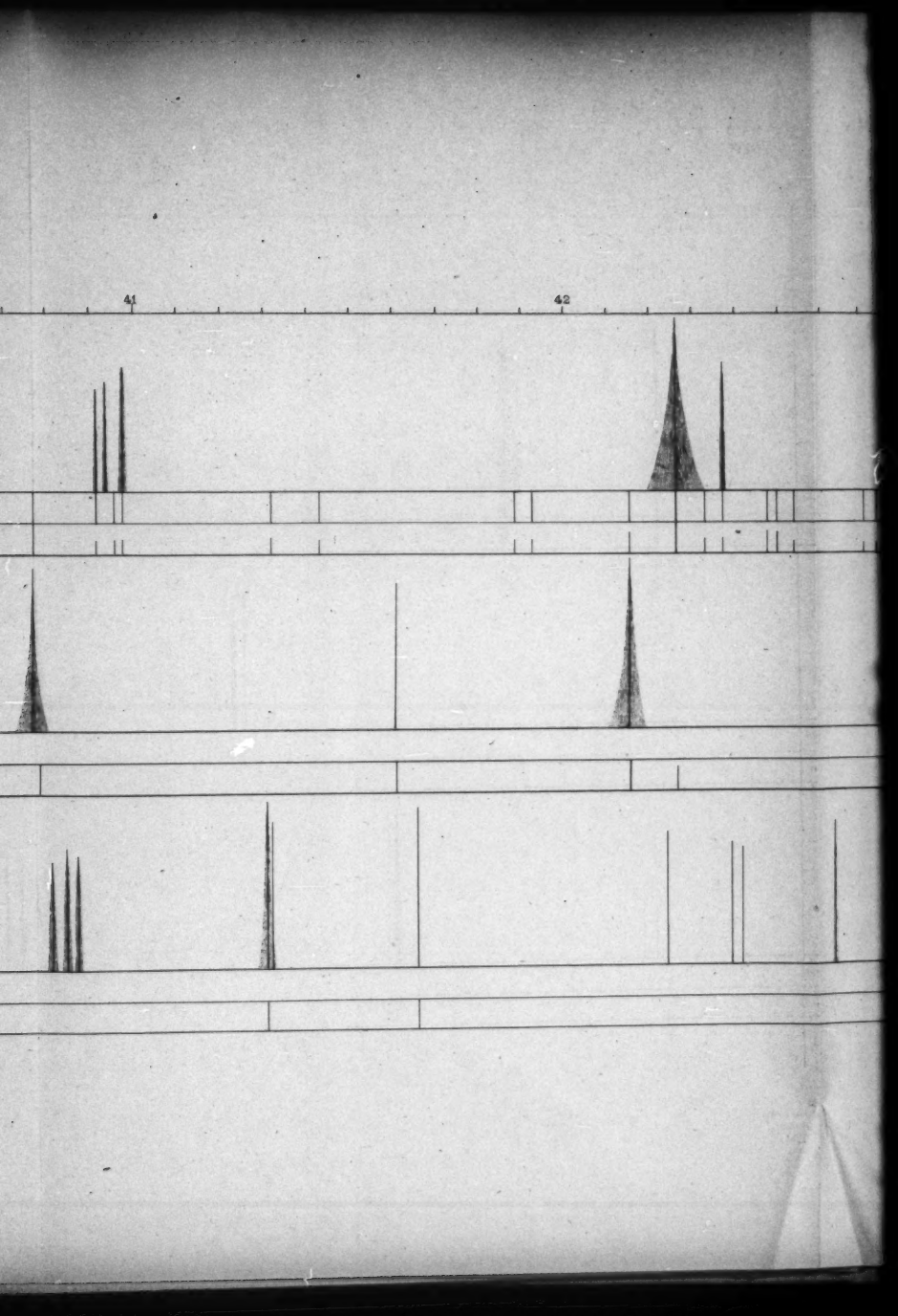
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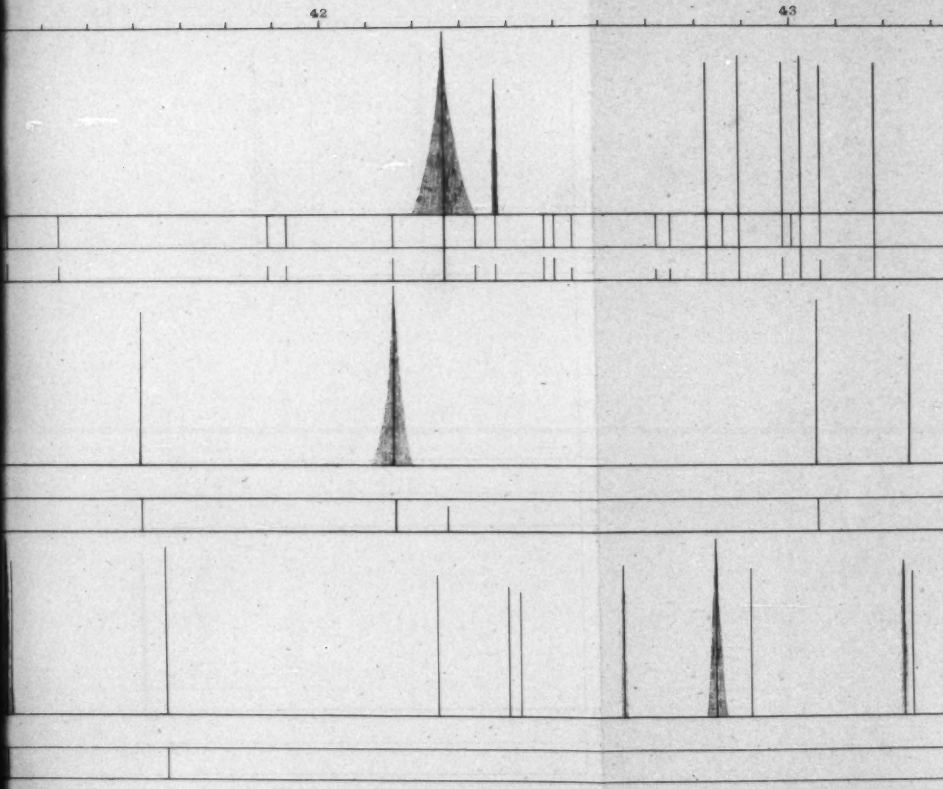
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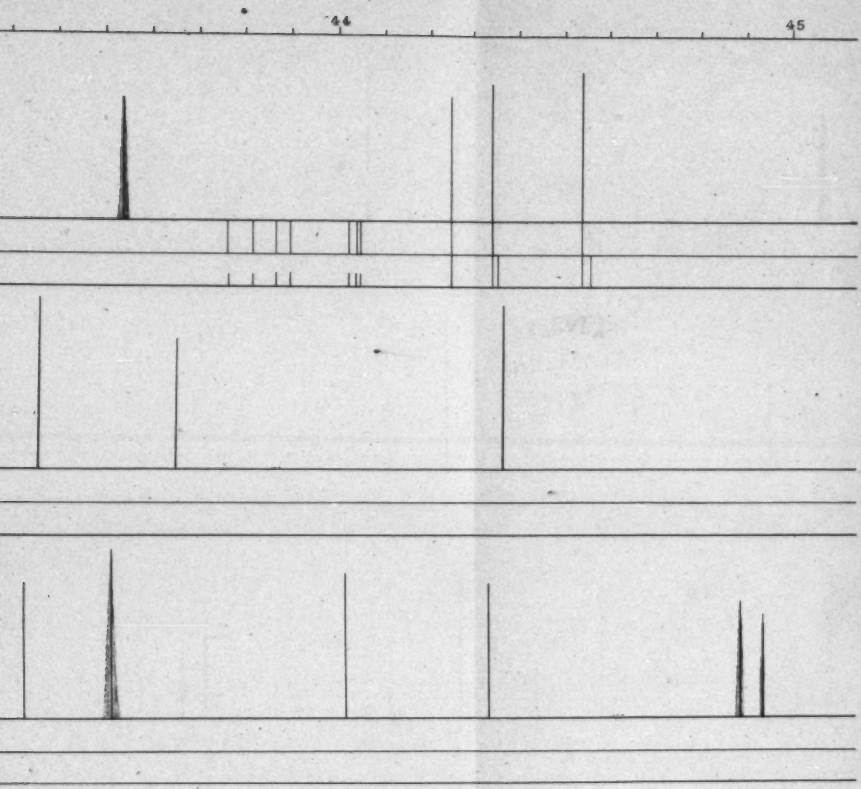

W. West & Co. imp.